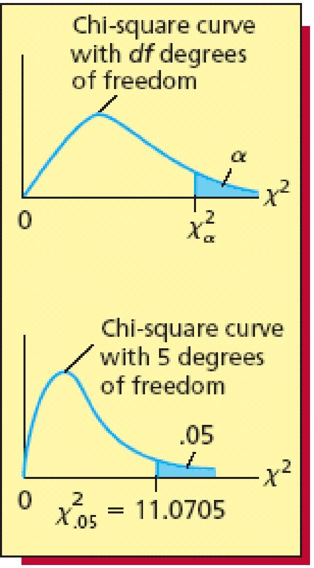
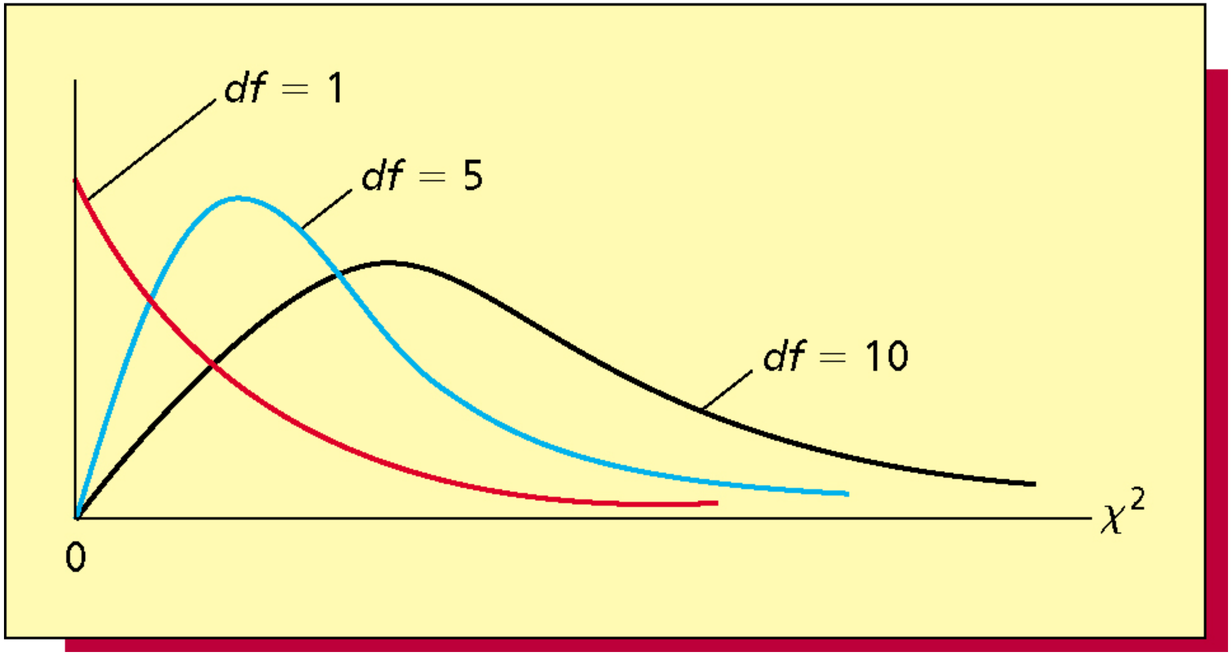
Statistical Inferences for Population Variances

The Chi-Square Distribution

- Sometimes make inferences using the chi-square distribution
  - Denoted χ^2
- Skewed to the right
- Exact shape depends on the degrees of freedom
  - Denoted *df*
- A **chi-square point** χ^2_α is the point under a chi-square distribution that gives right-hand tail area α

The Chi-Square Distribution Continued





A Portion of the Chi-Square Table

TABLE 11.1 A Portion of the Chi-Square Table

Degrees of Freedom (*df*)	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$
1	2.70554	3.84146	5.02389	6.63490	7.87944
2	4.60517	5.99147	7.37776	9.21034	10.5966
3	6.25139	7.81473	9.34840	11.3449	12.8381
4	7.77944	9.48773	11.1433	13.2767	14.8602
5	9.23635	11.0705	12.8325	15.0863	16.7496
6	10.6446	12.5916	14.4494	16.8119	18.5476

Statistical Inference for Population Variance

- If s^2 is the variance of a random sample of n measurements from a normal population with variance σ^2
- The sampling distribution of the statistic
  $(n - 1)\, s^2 / \sigma^2$ is a chi-square distribution with $(n – 1)$ degrees of freedom
- Can calculate confidence interval and perform hypothesis testing
- $100(1-\alpha)\%$ confidence interval for σ^2

Formulas

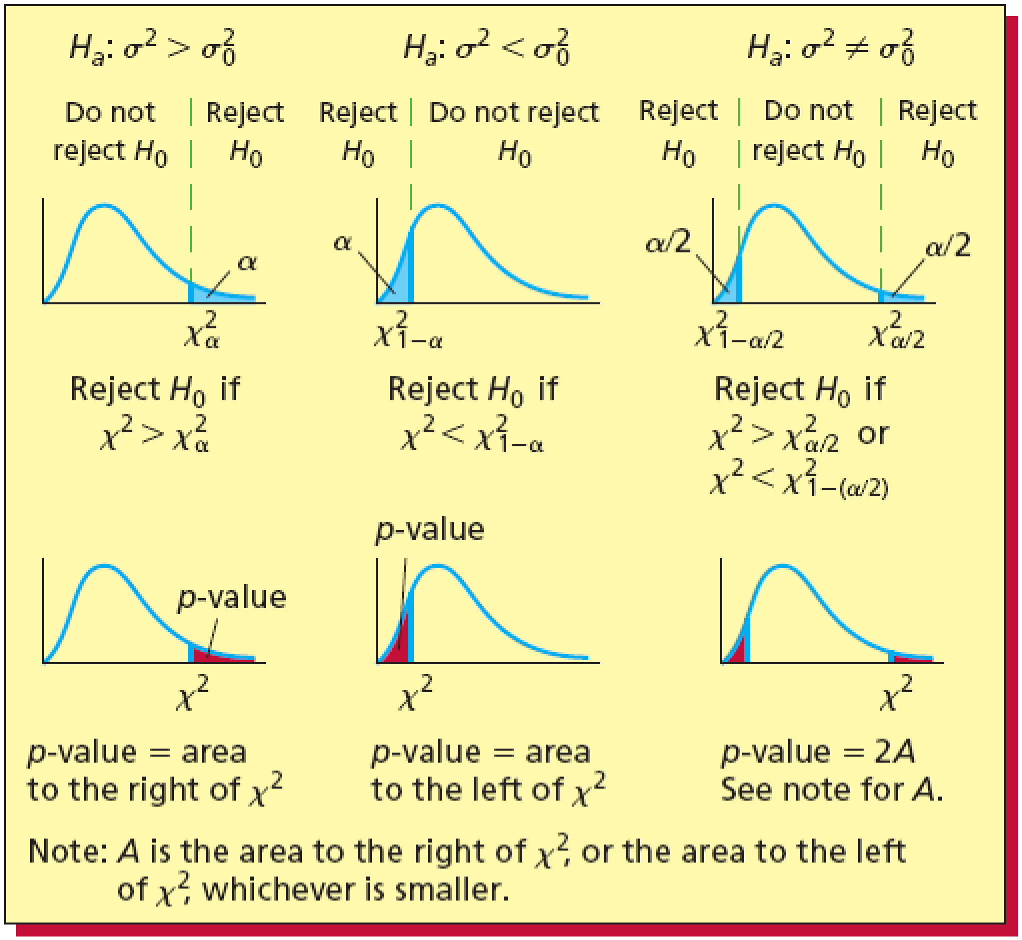
A 100(1 - α) percent confidence interval for σ^2 is

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} \right)$$

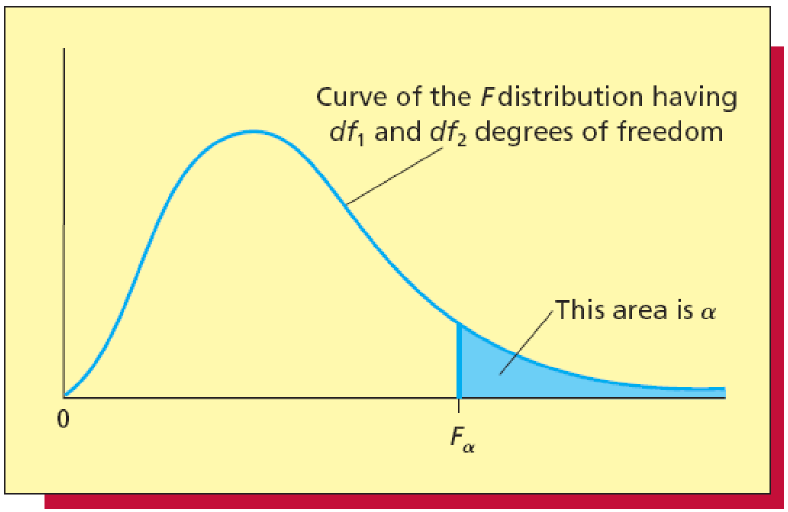
We can test $H_0 : \sigma^2 = \sigma_0^2$ at α using the test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Hypothesis Testing for Population Variance



F Distribution



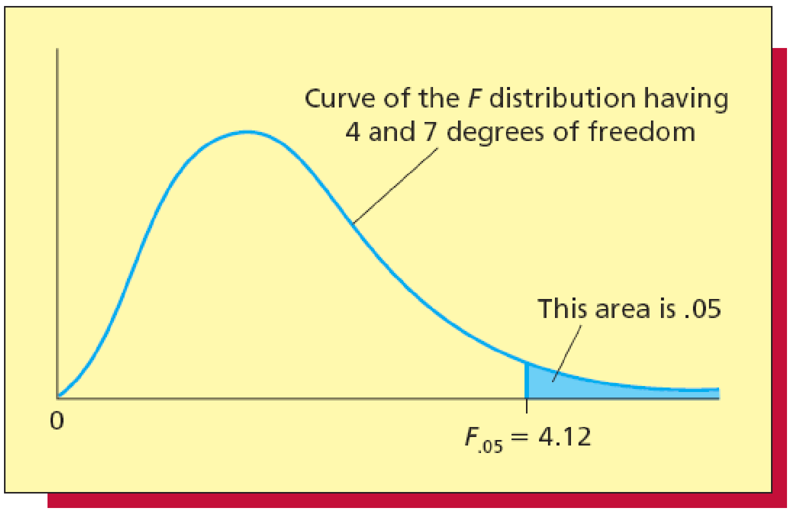
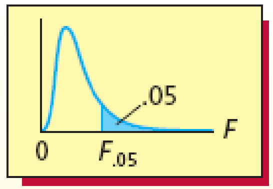


Figure 11.5

F Distribution Tables

- The *F* point F_α is the point on the horizontal axis under the curve of the *F* distribution that gives a right-hand tail area equal to α
- The value of F_α depends on a (the size of the right-hand tail area) and df_1 and df_2
- Different *F* tables for different values of α
  - Tables A.6 for $\alpha = 0.10$
  - Tables A.7 for $\alpha = 0.05$
  - Tables A.8 for $\alpha = 0.025$
  - Tables A.9 for $\alpha = 0.01$

A Portion of an *F* Table: Values of $F_{.05}$



df_2 \ df_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24
	Numerator Degrees of Freedom (df_1)													
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29

Denominator Degrees of Freedom (df_2)

Table 11.2

Comparing Two Population Variances by Using Independent Samples

- Population 1 has variance σ_1^2 and population 2 has variance σ_2^2
- The null hypothesis H_0 is that the variances are the same
  - H_0: $\sigma_1^2 = \sigma_2^2$
- The alternative is that one is smaller than the other
  - That population has less variable measurements
  - Suppose $\sigma_1^2 > \sigma_2^2$
  - More usual to normalize
- Test H_0: $\sigma_1^2/\sigma_2^2 = 1$ vs. $\sigma_1^2/\sigma_2^2 > 1$

Comparing Two Population Variances Continued

- Reject H_0 in favor of H_a if s_1^2/s_2^2 is significantly greater than 1
- s_1^2 is the variance of a random of size n_1 from a population with variance σ_1^2
- s_2^2 is the variance of a random of size n_2 from a population with variance σ_2^2
- To decide how large s_1^2/s_2^2 must be to reject H_0, describe the sampling distribution of s_1^2/s_2^2
- The sampling distribution of s_1^2/s_2^2 is the F distribution

Two Tailed Alternative

- The null and alternative hypotheses are:
  - $H_0: \sigma_1^2 = \sigma_2^2$
    $H_0: \sigma_1^2 \neq \sigma_2^2$
- Test statistic F is the ratio of the larger sample variance divided by the smaller sample variance
- df_1 = n-1 for sample having the larger variance and df_2 = n-2 for smaller variance
- Reject if $F > F_{\alpha/2}$ or p-value < α