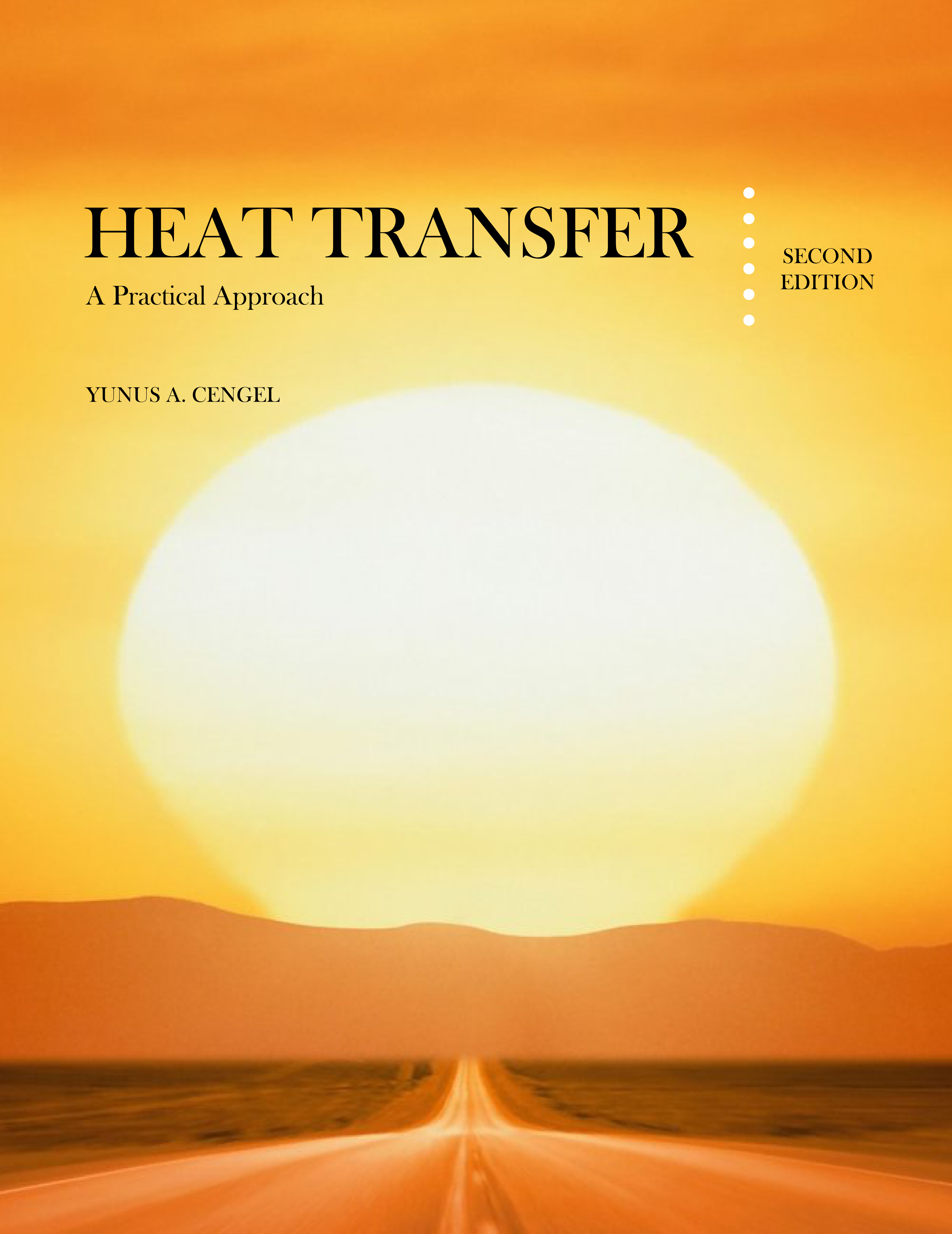
HEAT TRANSFER

A Practical Approach

SECOND EDITION

YUNUS A. CENGEL



CONTENTS

CHAPTER SEVEN
EXTERNAL FORCED CONVECTION 367

CHAPTER EIGHT
INTERNAL FORCED CONVECTION 419

CHAPTER NINE
NATURAL CONVECTION 459

TABLE OF EXAMPLES

CHAPTER ONE
BASICS OF HEAT TRANSFER 1

CHAPTER TWO
HEAT CONDUCTION EQUATION 61

CHAPTER THREE
STEADY HEAT CONDUCTION 127

CHAPTER FOUR
TRANSIENT HEAT CONDUCTION 209

CHAPTER FIVE
NUMERICAL METHODS IN HEAT CONDUCTION 265

CHAPTER SIX
FUNDAMENTALS OF CONVECTION 333

CHAPTER SEVEN
EXTERNAL FORCED CONVECTION 367

CHAPTER EIGHT
INTERNAL FORCED CONVECTION 419

CHAPTER NINE
NATURAL CONVECTION 459

CHAPTER TEN
BOILING AND CONDENSATION 515

PREFACE

OBJECTIVES

Heat transfer is a basic science that deals with the rate of transfer of thermal energy. This introductory text is intended for use in a first course in heat transfer for undergraduate engineering students, and as a reference book for practicing engineers. The objectives of this text are

- To cover the *basic principles* of heat transfer.
- To present a wealth of real-world *engineering applications* to give students a feel for engineering practice.
- To develop an *intuitive understanding* of the subject matter by emphasizing the physics and physical arguments.

Students are assumed to have completed their basic physics and calculus sequence. The completion of first courses in thermodynamics, fluid mechanics, and differential equations prior to taking heat transfer is desirable. The relevant concepts from these topics are introduced and reviewed as needed.

In engineering practice, an understanding of the mechanisms of heat transfer is becoming increasingly important since heat transfer plays a crucial role in the design of vehicles, power plants, refrigerators, electronic devices, buildings, and bridges, among other things. Even a chef needs to have an intuitive understanding of the heat transfer mechanism in order to cook the food "right" by adjusting the rate of heat transfer. We may not be aware of it, but we already use the principles of heat transfer when seeking thermal comfort. We insulate our bodies by putting on heavy coats in winter, and we minimize heat gain by radiation by staying in shady places in summer. We speed up the cooling of hot food by blowing on it and keep warm in cold weather by cuddling up and thus minimizing the exposed surface area. That is, we already use heat transfer whether we realize it or not.

GENERAL APPROACH

This text is the outcome of an attempt to have a textbook for a practically oriented heat transfer course for engineering students. The text covers the standard topics of heat transfer with an emphasis on physics and real-world applications, while de-emphasizing intimidating heavy mathematical aspects. This approach is more in line with students' intuition and makes learning the subject matter much easier.

The philosophy that contributed to the warm reception of the first edition of this book has remained unchanged. The goal throughout this project has been to offer an engineering textbook that

- Talks directly to the minds of tomorrow's engineers in a *simple yet precise* manner.
- Encourages *creative thinking* and development of a *deeper understanding* of the subject matter.
- Is *read* by students with *interest* and *enthusiasm* rather than being used as just an aid to solve problems.

Special effort has been made to appeal to readers' natural curiosity and to help students explore the various facets of the exciting subject area of heat transfer. The enthusiastic response we received from the users of the first edition all over the world indicates that our objectives have largely been achieved.

Yesterday's engineers spent a major portion of their time substituting values into the formulas and obtaining numerical results. However, now formula manipulations and number crunching are being left to computers. Tomorrow's engineer will have to have a clear understanding and a firm grasp of the *basic principles* so that he or she can understand even the most complex problems, formulate them, and interpret the results. A conscious effort is made to emphasize these basic principles while also providing students with a look at how modern tools are used in engineering practice.

NEW IN THIS EDITION

All the popular features of the previous edition are retained while new ones are added. The main body of the text remains largely unchanged except that the coverage of forced convection is expanded to three chapters and the coverage of radiation to two chapters. Of the three applications chapters, only the *Cooling of Electronic Equipment* is retained, and the other two are deleted to keep the book at a reasonable size. The most significant changes in this edition are highlighted next.

EXPANDED COVERAGE OF CONVECTION

Forced convection is now covered in three chapters instead of one. In Chapter 6, the basic concepts of convection and the theoretical aspects are introduced. Chapter 7 deals with the practical analysis of external convection while Chapter 8 deals with the practical aspects of internal convection. See the Content Changes and Reorganization section for more details.

ADDITIONAL CHAPTER ON RADIATION

Radiation is now covered in two chapters instead of one. The basic concepts associated with thermal radiation, including radiation intensity and spectral quantities, are covered in Chapter 11. View factors and radiation exchange between surfaces through participating and nonparticipating media are covered in Chapter 12. See the Content Changes and Reorganization section for more details.

TOPICS OF SPECIAL INTEREST

Most chapters now contain a new end-of-chapter optional section called "Topic of Special Interest" where interesting applications of heat transfer are discussed. Some existing sections such as *A Brief Review of Differential Equations* in Chapter 2, *Thermal Insulation* in Chapter 7, and *Controlling Numerical Error* in Chapter 5 are moved to these sections as topics of special interest. Some sections from the two deleted chapters such as the *Refrigeration and Freezing of Foods, Solar Heat Gain through Windows,* and *Heat Transfer through the Walls and Roofs* are moved to the relevant chapters as special topics. Most topics selected for these sections provide real-world applications of heat transfer, but they can be ignored if desired without a loss in continuity.

COMPREHENSIVE PROBLEMS WITH PARAMETRIC STUDIES

A distinctive feature of this edition is the incorporation of about 130 comprehensive problems that require conducting extensive parametric studies, using the enclosed EES (or other suitable) software. Students are asked to study the effects of certain variables in the problems on some quantities of interest, to plot the results, and to draw conclusions from the results obtained. These problems are designated by computer-EES and EES-CD icons for easy recognition, and can be ignored if desired. Solutions of these problems are given in the Instructor's Solutions Manual.

CONTENT CHANGES AND REORGANIZATION

With the exception of the changes already mentioned, the main body of the text remains largely unchanged. This edition involves over 500 new or revised problems. The noteworthy changes in various chapters are summarized here for those who are familiar with the previous edition.

- In Chapter 1, *surface energy balance* is added to Section 1-4. In a new section *Problem-Solving Technique,* the problem-solving technique is introduced, the engineering software packages are discussed, and overviews of EES (Engineering Equation Solver) and HTT (Heat Transfer Tools) are given. The optional Topic of Special Interest in this chapter is *Thermal Comfort.*
- In Chapter 2, the section *A Brief Review of Differential Equations* is moved to the end of chapter as the Topic of Special Interest.
- In Chapter 3, the section on *Thermal Insulation* is moved to Chapter 7, External Forced Convection, as a special topic. The optional Topic of Special Interest in this chapter is *Heat Transfer through Walls and Roofs.*
- Chapter 4 remains mostly unchanged. The Topic of Special Interest in this chapter is *Refrigeration and Freezing of Foods.*
- In Chapter 5, the section *Solutions Methods for Systems of Algebraic Equations* and the FORTRAN programs in the margin are deleted, and the section *Controlling Numerical Error* is designated as the Topic of Special Interest.
- Chapter 6, Forced Convection, is now replaced by three chapters: Chapter 6 *Fundamentals of Convection,* where the basic concepts of convection are introduced and the fundamental convection equations and relations (such as the differential momentum and energy equations and the Reynolds analogy) are developed; Chapter 7 *External Forced Convection,* where drag and heat transfer for flow over surfaces, including flow over tube banks, are discussed; and Chapter 8 *Internal Forced Convection,* where pressure drop and heat transfer for flow in tubes are

presented. *Reducing Heat Transfer through Surfaces* is added to Chapter 7 as the Topic of Special Interest.

- Chapter 7 (now Chapter 9) *Natural Convection* is completely rewritten. The Grashof number is derived from a momentum balance on a differential volume element, some Nusselt number relations (especially those for rectangular enclosures) are updated, and the section *Natural Convection from Finned Surfaces* is expanded to include heat transfer from PCBs. The optional Topic of Special Interest in this chapter is *Heat Transfer through Windows.*
- Chapter 8 (now Chapter 10) *Boiling and Condensation* remained largely unchanged. The Topic of Special Interest in this chapter is *Heat Pipes.*
- Chapter 9 is split in two chapters: Chapter 11 *Fundamentals of Thermal Radiation,* where the basic concepts associated with thermal radiation, including radiation intensity and spectral quantities, are introduced, and Chapter 12 *Radiation Heat Transfer,* where the view factors and radiation exchange between surfaces through participating and nonparticipating media are discussed. The Topic of Special Interest are *Solar Heat Gain through Windows* in Chapter 11, and *Heat Transfer from the Human Body* in Chapter 12.
- There are no significant changes in the remaining three chapters of *Heat Exchangers, Mass Transfer,* and *Cooling of Electronic Equipment.*
- In the appendices, the values of the physical constants are updated; new tables for the properties of saturated ammonia, refrigerant-134a, and propane are added; and the tables on the properties of air, gases, and liquids (including liquid metals) are replaced by those obtained using EES. Therefore, property values in tables for air, other ideal gases, ammonia, refrigerant-134a, propane, and liquids are identical to those obtained from EES.

LEARNING TOOLS

EMPHASIS ON PHYSICS

A distinctive feature of this book is its emphasis on the physical aspects of subject matter rather than mathematical representations and manipulations. The author believes that the emphasis in undergraduate education should remain on *developing a sense of underlying physical mechanism* and a *mastery of solving practical problems* an engineer is likely to face in the real world. Developing an intuitive understanding should also make the course a more motivating and worthwhile experience for the students.

EFFECTIVE USE OF ASSOCIATION

An observant mind should have no difficulty understanding engineering sciences. After all, the principles of engineering sciences are based on our *everyday experiences* and *experimental observations.* A more physical, intuitive approach is used throughout this text. Frequently *parallels are drawn* between the subject matter and students' everyday experiences so that they can relate the subject matter to what they already know. The process of cooking, for example, serves as an excellent vehicle to demonstrate the basic principles of heat transfer.

SELF-INSTRUCTING

The material in the text is introduced at a level that an average student can follow comfortably. It speaks to students, not over students. In fact, it is *self-instructive.* Noting that the principles of sciences are based on experimental observations, the derivations in this text are based on physical arguments, and thus they are easy to follow and understand.

EXTENSIVE USE OF ARTWORK

Figures are important learning tools that help the students "get the picture." The text makes effective use of graphics. It contains more figures and illustrations than any other book in this category. Figures attract attention and stimulate curiosity and interest. Some of the figures in this text are intended to serve as a means of emphasizing some key concepts that would otherwise go unnoticed; some serve as paragraph summaries.

CHAPTER OPENERS AND SUMMARIES

Each chapter begins with an overview of the material to be covered and its relation to other chapters. A *summary* is included at the end of each chapter for a quick review of basic concepts and important relations.

NUMEROUS WORKED-OUT EXAMPLES

Each chapter contains several worked-out *examples* that clarify the material and illustrate the use of the basic principles. An *intuitive* and *systematic* approach is used in the solution of the example problems, with particular attention to the proper use of units.

A WEALTH OF REAL-WORLD END-OF-CHAPTER PROBLEMS

The end-of-chapter problems are grouped under specific topics in the order they are covered to make problem selection easier for both instructors and students. The problems within each group start with concept questions, indicated by "C," to check the students' level of understanding of basic concepts. The problems under *Review Problems* are more comprehensive in nature and are not directly tied to any specific section of a chapter. The problems under the *Design and Essay Problems* title are intended to encourage students to make engineering judgments, to conduct independent exploration of topics of interest, and to communicate their findings in a professional manner. Several economics- and safety-related problems are incorporated throughout to enhance cost and safety awareness among engineering students. Answers to selected problems are listed immediately following the problem for convenience to students.

A SYSTEMATIC SOLUTION PROCEDURE

A well-structured approach is used in problem solving while maintaining an informal conversational style. The problem is first stated and the objectives are identified, and the assumptions made are stated together with their justifications. The properties needed to solve the problem are listed separately. Numerical values are used together with their units to emphasize that numbers without units are meaningless, and unit manipulations are as important as manipulating the numerical values with a calculator. The significance of the findings is discussed following the solutions. This approach is also used consistently in the solutions presented in the Instructor's Solutions Manual.

A CHOICE OF SI ALONE OR SI/ENGLISH UNITS

In recognition of the fact that English units are still widely used in some industries, both SI and English units are used in this text, with an emphasis on SI. The material in this text can be covered using combined SI/English units or SI units alone, depending on the preference of the instructor. The property tables and charts in the appendices are presented in both units, except the ones that involve dimensionless quantities. Problems, tables, and charts in English units are designated by "E" after the number for easy recognition, and they can be ignored easily by the SI users.

CONVERSION FACTORS

Frequently used conversion factors and the physical constants are listed on the inner cover pages of the text for easy reference.

SUPPLEMENTS

These supplements are available to the adopters of the book.

COSMOS SOLUTIONS MANUAL

Available to instructors only.

The detailed solutions for all text problems will be delivered in our new electronic Complete Online Solution Manual Organization System (COSMOS). COSMOS is a database management tool geared towards assembling homework assignments, tests and quizzes. No longer do instructors need to wade through thick solutions manuals and huge Word files. COSMOS helps you to quickly find solutions and also keeps a record of problems assigned to avoid duplication in subsequent semesters. Instructors can contact their McGraw-Hill sales representative at http://www.mhhe.com/catalogs/rep/ to obtain a copy of the COSMOS solutions manual.

EES SOFTWARE

Developed by Sanford Klein and William Beckman from the University of Wisconsin–Madison, this software program allows students to solve problems, especially design problems, and to ask "what if" questions. EES (pronounced "ease") is an acronym for Engineering Equation Solver. EES is very easy to master since equations can be entered in any form and in any order. The combination of equation-solving capability and engineering property data makes EES an extremely powerful tool for students.

EES can do optimization, parametric analysis, and linear and nonlinear regression and provides publication-quality plotting capability. Equations can be entered in any form and in any order. EES automatically rearranges the equations to solve them in the most efficient manner. EES is particularly useful for heat transfer problems since most of the property data needed for solving such problems are provided in the program. For example, the steam tables are implemented such that any thermodynamic property can be obtained from a built-in function call in terms of any two properties. Similar capability is provided for many organic refrigerants, ammonia, methane, carbon dioxide, and many other fluids. Air tables are built-in, as are psychrometric functions and JANAF table data for many common gases. Transport properties are also provided for all substances. EES also allows the user to enter property data or functional relationships with look-up tables, with internal functions written with EES, or with externally compiled functions written in Pascal, C, C++, or FORTRAN.

The *Student Resources CD* that accompanies the text contains the *Limited Academic Version* of the EES program and the scripted EES solutions of about 30 homework problems (indicated by the "EES-CD" logo in the text). Each EES solution provides detailed comments and on-line help, and can easily be modified to solve similar problems. These solutions should help students master the important concepts without the calculational burden that has been previously required.

HEAT TRANSFER TOOLS (HTT)

One software package specifically designed to help bridge the gap between the textbook fundamentals and commercial software packages is *Heat Transfer Tools,* which can be ordered "bundled" with this text (Robert J. Ribando, ISBN 0-07-246328-7). While it does not have the power and functionality of the professional, commercial packages, HTT uses research-grade numerical algorithms behind the scenes and modern graphical user interfaces. Each module is custom designed and applicable to a single, fundamental topic in heat transfer.

BOOK-SPECIFIC WEBSITE

The book website can be found at www.mhhe.com/cengel/. Visit this site for book and supplement information, author information, and resources for further study or reference. At this site you will also find PowerPoints of selected text figures.

ACKNOWLEDGMENTS

I would like to acknowledge with appreciation the numerous and valuable comments, suggestions, criticisms, and praise of these academic evaluators:

Sanjeev Chandra
University of Toronto, Canada

Fan-Bill Cheung
The Pennsylvania State University

Nicole DeJong
San Jose State University

David M. Doner
West Virginia University Institute of Technology

Mark J. Holowach
The Pennsylvania State University

Mehmet Kanoglu
Gaziantep University, Turkey

Francis A. Kulacki
University of Minnesota

Sai C. Lau
Texas A&M University

Joseph Majdalani
Marquette University

Jed E. Marquart
Ohio Northern University

Robert J. Ribando
University of Virginia

Jay M. Ochterbeck
Clemson University

James R. Thomas
Virginia Polytechnic Institute and State University

John D. Wellin
Rochester Institute of Technology

Their suggestions have greatly helped to improve the quality of this text. I also would like to thank my students who provided plenty of feedback from their perspectives. Finally, I would like to express my appreciation to my wife Zehra and my children for their continued patience, understanding, and support throughout the preparation of this text.

Yunus A. Çengel

CHAPTER

1

BASICS OF HEAT TRANSFER

The science of thermodynamics deals with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another, and makes no reference to *how long* the process will take. But in engineering, we are often interested in the *rate* of heat transfer, which is the topic of the science of *heat transfer.*

We start this chapter with a review of the fundamental concepts of thermodynamics that form the framework for heat transfer. We first present the relation of heat to other forms of energy and review the first law of thermodynamics. We then present the three basic mechanisms of heat transfer, which are conduction, convection, and radiation, and discuss thermal conductivity. *Conduction* is the transfer of energy from the more energetic particles of a substance to the adjacent, less energetic ones as a result of interactions between the particles. *Convection* is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. *Radiation* is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. We close this chapter with a discussion of simultaneous heat transfer.

CONTENTS

1–1 ▪ THERMODYNAMICS AND HEAT TRANSFER

We all know from experience that a cold canned drink left in a room warms up and a warm canned drink left in a refrigerator cools down. This is accomplished by the transfer of *energy* from the warm medium to the cold one. The energy transfer is always from the higher temperature medium to the lower temperature one, and the energy transfer stops when the two mediums reach the same temperature.

You will recall from thermodynamics that energy exists in various forms. In this text we are primarily interested in **heat,** which is *the form of energy that can be transferred from one system to another as a result of temperature difference.* The science that deals with the determination of the *rates* of such energy transfers is **heat transfer.**

You may be wondering why we need to undertake a detailed study on heat transfer. After all, we can determine the amount of heat transfer for any system undergoing any process using a thermodynamic analysis alone. The reason is that thermodynamics is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another, and it gives no indication about *how long* the process will take. A thermodynamic analysis simply tells us how much heat must be transferred to realize a specified change of state to satisfy the conservation of energy principle.

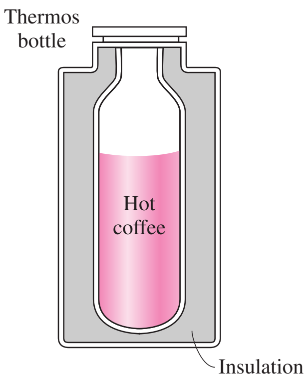


FIGURE 1–1
We are normally interested in how long it takes for the hot coffee in a thermos to cool to a certain temperature, which cannot be determined from a thermodynamic analysis alone.

In practice we are more concerned about the rate of heat transfer (heat transfer per unit time) than we are with the amount of it. For example, we can determine the amount of heat transferred from a thermos bottle as the hot coffee inside cools from 90°C to 80°C by a thermodynamic analysis alone. But a typical user or designer of a thermos is primarily interested in *how long* it will be before the hot coffee inside cools to 80°C, and a thermodynamic analysis cannot answer this question. Determining the rates of heat transfer to or from a system and thus the times of cooling or heating, as well as the variation of the temperature, is the subject of *heat transfer* (Fig. 1–1).

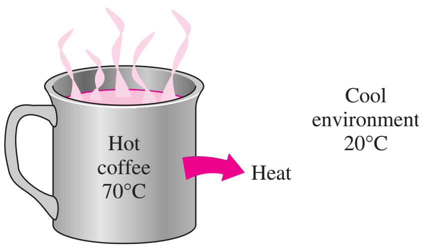


FIGURE 1–2
Heat flows in the direction of decreasing temperature.

Thermodynamics deals with equilibrium states and changes from one equilibrium state to another. Heat transfer, on the other hand, deals with systems that lack thermal equilibrium, and thus it is a *nonequilibrium* phenomenon. Therefore, the study of heat transfer cannot be based on the principles of thermodynamics alone. However, the laws of thermodynamics lay the framework for the science of heat transfer. The *first law* requires that the rate of energy transfer into a system be equal to the rate of increase of the energy of that system. The *second law* requires that heat be transferred in the direction of decreasing temperature (Fig. 1–2). This is like a car parked on an inclined road that must go downhill in the direction of decreasing elevation when its brakes are released. It is also analogous to the electric current flowing in the direction of decreasing voltage or the fluid flowing in the direction of decreasing total pressure.

The basic requirement for heat transfer is the presence of a *temperature difference.* There can be no net heat transfer between two mediums that are at the same temperature. The temperature difference is the *driving force* for heat transfer, just as the *voltage difference* is the driving force for electric current flow and *pressure difference* is the driving force for fluid flow. The rate of heat transfer in a certain direction depends on the magnitude of the *temperature gradient* (the temperature difference per unit length or the rate of change of temperature) in that direction. The larger the temperature gradient, the higher the rate of heat transfer.

Application Areas of Heat Transfer

Heat transfer is commonly encountered in engineering systems and other aspects of life, and one does not need to go very far to see some application areas of heat transfer. In fact, one does not need to go anywhere. The human body is constantly rejecting heat to its surroundings, and human comfort is closely tied to the rate of this heat rejection. We try to control this heat transfer rate by adjusting our clothing to the environmental conditions.

Many ordinary household appliances are designed, in whole or in part, by using the principles of heat transfer. Some examples include the electric or gas range, the heating and air-conditioning system, the refrigerator and freezer, the water heater, the iron, and even the computer, the TV, and the VCR. Of course, energy-efficient homes are designed on the basis of minimizing heat loss in winter and heat gain in summer. Heat transfer plays a major role in the design of many other devices, such as car radiators, solar collectors, various components of power plants, and even spacecraft. The optimal insulation thickness in the walls and roofs of the houses, on hot water or steam pipes, or on water heaters is again determined on the basis of a heat transfer analysis with economic consideration (Fig. 1–3).

Historical Background

Heat has always been perceived to be something that produces in us a sensation of warmth, and one would think that the nature of heat is one of the first things understood by mankind. But it was only in the middle of the nineteenth

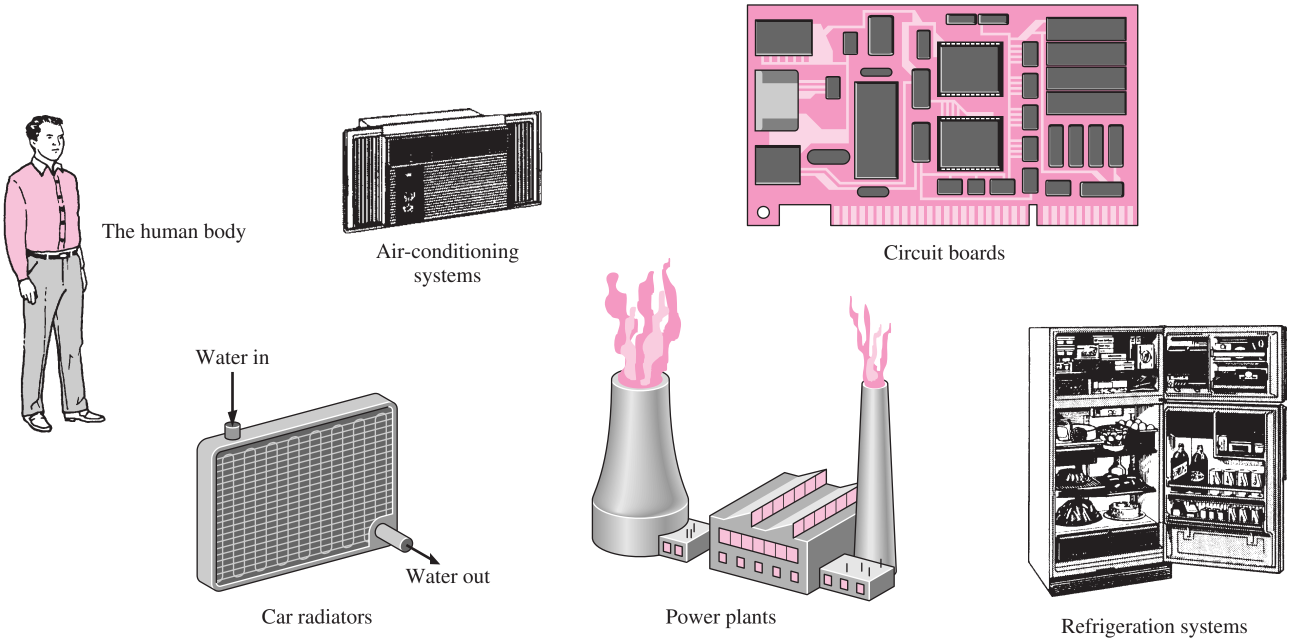


FIGURE 1–3
Some application areas of heat transfer.

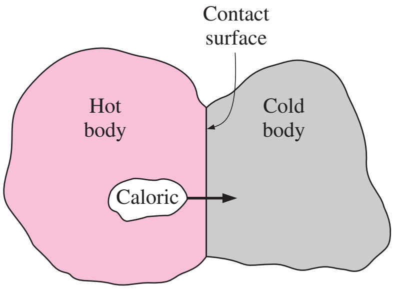


FIGURE 1–4
In the early nineteenth century, heat was thought to be an invisible fluid called the *caloric* that flowed from warmer bodies to the cooler ones.

century that we had a true physical understanding of the nature of heat, thanks to the development at that time of the **kinetic theory,** which treats molecules as tiny balls that are in motion and thus possess kinetic energy. Heat is then defined as the energy associated with the random motion of atoms and molecules. Although it was suggested in the eighteenth and early nineteenth centuries that heat is the manifestation of motion at the molecular level (called the *live force*), the prevailing view of heat until the middle of the nineteenth century was based on the **caloric theory** proposed by the French chemist Antoine Lavoisier (1743–1794) in 1789. The caloric theory asserts that heat is a fluid-like substance called the **caloric** that is a massless, colorless, odorless, and tasteless substance that can be poured from one body into another (Fig. 1–4). When caloric was added to a body, its temperature increased; and when caloric was removed from a body, its temperature decreased. When a body could not contain any more caloric, much the same way as when a glass of water could not dissolve any more salt or sugar, the body was said to be saturated with caloric. This interpretation gave rise to the terms *saturated liquid* and *saturated vapor* that are still in use today.

The caloric theory came under attack soon after its introduction. It maintained that heat is a substance that could not be created or destroyed. Yet it was known that heat can be generated indefinitely by rubbing one's hands together or rubbing two pieces of wood together. In 1798, the American Benjamin Thompson (Count Rumford) (1753–1814) showed in his papers that heat can be generated continuously through friction. The validity of the caloric theory was also challenged by several others. But it was the careful experiments of the Englishman James P. Joule (1818–1889) published in 1843 that finally convinced the skeptics that heat was not a substance after all, and thus put the caloric theory to rest. Although the caloric theory was totally abandoned in the middle of the nineteenth century, it contributed greatly to the development of thermodynamics and heat transfer.

1–2 ▪ ENGINEERING HEAT TRANSFER

Heat transfer equipment such as heat exchangers, boilers, condensers, radiators, heaters, furnaces, refrigerators, and solar collectors are designed primarily on the basis of heat transfer analysis. The heat transfer problems encountered in practice can be considered in two groups: (1) *rating* and (2) *sizing* problems. The rating problems deal with the determination of the heat transfer rate for an existing system at a specified temperature difference. The sizing problems deal with the determination of the size of a system in order to transfer heat at a specified rate for a specified temperature difference.

A heat transfer process or equipment can be studied either *experimentally* (testing and taking measurements) or *analytically* (by analysis or calculations). The experimental approach has the advantage that we deal with the actual physical system, and the desired quantity is determined by measurement, within the limits of experimental error. However, this approach is expensive, time-consuming, and often impractical. Besides, the system we are analyzing may not even exist. For example, the size of a heating system of a building must usually be determined *before* the building is actually built on the basis of the dimensions and specifications given. The analytical approach (including numerical approach) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions and idealizations made in the analysis. In heat transfer studies, often a good compromise is reached by reducing the choices to just a few by analysis, and then verifying the findings experimentally.

Modeling in Heat Transfer

The descriptions of most scientific problems involve expressions that relate the changes in some key variables to each other. Usually the smaller the increment chosen in the changing variables, the more general and accurate the description. In the limiting case of infinitesimal or differential changes in variables, we obtain *differential equations* that provide precise mathematical formulations for the physical principles and laws by representing the rates of changes as *derivatives.* Therefore, differential equations are used to investigate a wide variety of problems in sciences and engineering, including heat transfer. However, most heat transfer problems encountered in practice can be solved without resorting to differential equations and the complications associated with them.

The study of physical phenomena involves two important steps. In the first step, all the variables that affect the phenomena are identified, reasonable assumptions and approximations are made, and the interdependence of these variables is studied. The relevant physical laws and principles are invoked, and the problem is formulated mathematically. The equation itself is very instructive as it shows the degree of dependence of some variables on others, and the relative importance of various terms. In the second step, the problem is solved using an appropriate approach, and the results are interpreted.

Many processes that seem to occur in nature randomly and without any order are, in fact, being governed by some visible or not-so-visible physical laws. Whether we notice them or not, these laws are there, governing consistently and predictably what seem to be ordinary events. Most of these laws are well defined and well understood by scientists. This makes it possible to predict the course of an event before it actually occurs, or to study various aspects of an event mathematically without actually running expensive and time-consuming experiments. This is where the power of analysis lies. Very accurate results to meaningful practical problems can be obtained with relatively little effort by using a suitable and realistic mathematical model. The preparation of such models requires an adequate knowledge of the natural phenomena involved and the relevant laws, as well as a sound judgment. An unrealistic model will obviously give inaccurate and thus unacceptable results.

An analyst working on an engineering problem often finds himself or herself in a position to make a choice between a very accurate but complex model, and a simple but not-so-accurate model. The right choice depends on the situation at hand. The right choice is usually the simplest model that yields adequate results. For example, the process of baking potatoes or roasting a round chunk of beef in an oven can be studied analytically in a simple way by modeling the potato or the roast as a spherical solid ball that has the properties of water (Fig. 1–5). The model is quite simple, but the results obtained are sufficiently accurate for most practical purposes. As another example, when we analyze the heat losses from a building in order to select the right size for a heater, we determine the heat losses under anticipated worst conditions and select a furnace that will provide sufficient heat to make up for those losses.

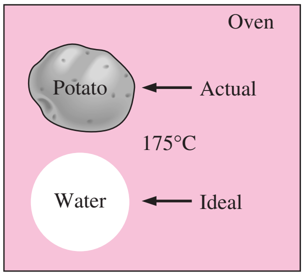


FIGURE 1–5
Modeling is a powerful engineering tool that provides great insight and simplicity at the expense of some accuracy.

Often we tend to choose a larger furnace in anticipation of some future expansion, or just to provide a factor of safety. A very simple analysis will be adequate in this case.

When selecting heat transfer equipment, it is important to consider the actual operating conditions. For example, when purchasing a heat exchanger that will handle hard water, we must consider that some calcium deposits will form on the heat transfer surfaces over time, causing fouling and thus a gradual decline in performance. The heat exchanger must be selected on the basis of operation under these adverse conditions instead of under new conditions.

Preparing very accurate but complex models is usually not so difficult. But such models are not much use to an analyst if they are very difficult and time-consuming to solve. At the minimum, the model should reflect the essential features of the physical problem it represents. There are many significant real-world problems that can be analyzed with a simple model. But it should always be kept in mind that the results obtained from an analysis are as accurate as the assumptions made in simplifying the problem. Therefore, the solution obtained should not be applied to situations for which the original assumptions do not hold.

A solution that is not quite consistent with the observed nature of the problem indicates that the mathematical model used is too crude. In that case, a more realistic model should be prepared by eliminating one or more of the questionable assumptions. This will result in a more complex problem that, of course, is more difficult to solve. Thus any solution to a problem should be interpreted within the context of its formulation.

1–3 ▪ HEAT AND OTHER FORMS OF ENERGY

Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, and nuclear, and their sum constitutes the **total energy** E (or e on a unit mass basis) of a system. The forms of energy related to the molecular structure of a system and the degree of the molecular activity are referred to as the *microscopic energy.* The sum of all microscopic forms of energy is called the **internal energy** of a system, and is denoted by U (or u on a unit mass basis).

The international unit of energy is *joule* (J) or *kilojoule* (1 kJ = 1000 J). In the English system, the unit of energy is the *British thermal unit* (Btu), which is defined as the energy needed to raise the temperature of 1 lbm of water at 60°F by 1°F. The magnitudes of kJ and Btu are almost identical (1 Btu = 1.055056 kJ). Another well-known unit of energy is the *calorie* (1 cal = 4.1868 J), which is defined as the energy needed to raise the temperature of 1 gram of water at 14.5°C by 1°C.

Internal energy may be viewed as the sum of the kinetic and potential energies of the molecules. The portion of the internal energy of a system associated with the kinetic energy of the molecules is called **sensible energy** or **sensible heat.** The average velocity and the degree of activity of the molecules are proportional to the temperature. Thus, at higher temperatures the molecules will possess higher kinetic energy, and as a result, the system will have a higher internal energy.

The internal energy is also associated with the intermolecular forces between the molecules of a system. These are the forces that bind the molecules to each other, and, as one would expect, they are strongest in solids and weakest in gases. If sufficient energy is added to the molecules of a solid or liquid, they will overcome these molecular forces and simply break away, turning the system to a gas. This is a *phase change* process and because of this added energy, a system in the gas phase is at a higher internal energy level than it is in the solid or the liquid phase. The internal energy associated with the phase of a system is called **latent energy** or **latent heat.**

The changes mentioned above can occur without a change in the chemical composition of a system. Most heat transfer problems fall into this category, and one does not need to pay any attention to the forces binding the atoms in a molecule together. The internal energy associated with the atomic bonds in a molecule is called **chemical** (or **bond**) **energy,** whereas the internal energy associated with the bonds within the nucleus of the atom itself is called **nuclear energy.** The chemical and nuclear energies are absorbed or released during chemical or nuclear reactions, respectively.

In the analysis of systems that involve fluid flow, we frequently encounter the combination of properties u and Pv. For the sake of simplicity and convenience, this combination is defined as **enthalpy** h. That is, $h = u + Pv$ where the term Pv represents the *flow energy* of the fluid (also called the *flow work*), which is the energy needed to push a fluid and to maintain flow. In the energy analysis of flowing fluids, it is convenient to treat the flow energy as part of the energy of the fluid and to represent the microscopic energy of a fluid stream by enthalpy h (Fig. 1–6).

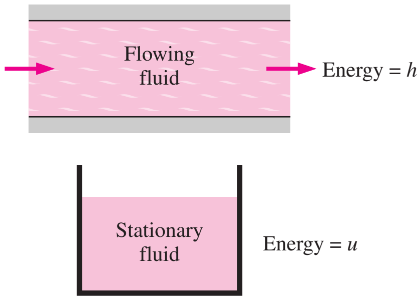


FIGURE 1–6
The *internal energy u* represents the microscopic energy of a nonflowing fluid, whereas *enthalpy h* represents the microscopic energy of a flowing fluid.

Specific Heats of Gases, Liquids, and Solids

You may recall that an **ideal gas** is defined as a gas that obeys the relation

$$Pv = RT \qquad \text{or} \qquad P = \rho RT \tag{1-1}$$

where P is the absolute pressure, v is the specific volume, T is the absolute temperature, ρ is the density, and R is the gas constant. It has been experimentally observed that the ideal gas relation given above closely approximates the P-v-T behavior of real gases at low densities. At low pressures and high temperatures, the density of a gas decreases and the gas behaves like an ideal gas. In the range of practical interest, many familiar gases such as air, nitrogen, oxygen, hydrogen, helium, argon, neon, and krypton and even heavier gases such as carbon dioxide can be treated as ideal gases with negligible error (often less than one percent). Dense gases such as water vapor in steam power plants and refrigerant vapor in refrigerators, however, should not always be treated as ideal gases since they usually exist at a state near saturation.

You may also recall that **specific heat** is defined as *the energy required to raise the temperature of a unit mass of a substance by one degree* (Fig. 1–7). In general, this energy depends on how the process is executed. In thermodynamics, we are interested in two kinds of specific heats: specific heat at constant volume C_v and specific heat at constant pressure C_p. The **specific heat at constant volume** C_v can be viewed as the energy required to raise the temperature of a unit mass of a substance by one degree as the volume is held constant. The energy required to do the same as the pressure is held constant is the **specific heat at constant pressure** C_p. The specific heat at constant

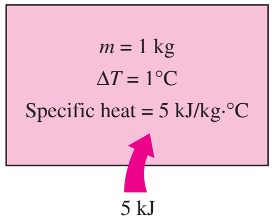


FIGURE 1–7
Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

pressure C_p is greater than C_v because at constant pressure the system is allowed to expand and the energy for this expansion work must also be supplied to the system. For ideal gases, these two specific heats are related to each other by $C_p = C_v + R$.

A common unit for specific heats is kJ/kg · °C or kJ/kg · K. Notice that these two units are *identical* since $\Delta T(°\text{C}) = \Delta T(\text{K})$, and 1°C change in temperature is equivalent to a change of 1 K. Also,

$$1 \text{ kJ/kg} \cdot °\text{C} \equiv 1 \text{ J/g} \cdot °\text{C} \equiv 1 \text{ kJ/kg} \cdot \text{K} \equiv 1 \text{ J/g} \cdot \text{K}$$

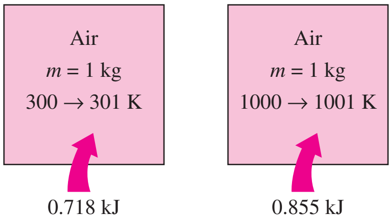


FIGURE 1–8
The specific heat of a substance changes with temperature.

The specific heats of a substance, in general, depend on two independent properties such as temperature and pressure. For an *ideal gas,* however, they depend on *temperature* only (Fig. 1–8). At low pressures all real gases approach ideal gas behavior, and therefore their specific heats depend on temperature only.

The differential changes in the internal energy u and enthalpy h of an ideal gas can be expressed in terms of the specific heats as

$$du = C_v\, dT \qquad \text{and} \qquad dh = C_p\, dT \tag{1-2}$$

The finite changes in the internal energy and enthalpy of an ideal gas during a process can be expressed approximately by using specific heat values at the average temperature as

$$\Delta u = C_{v,\,\text{ave}}\Delta T \qquad \text{and} \qquad \Delta h = C_{p,\,\text{ave}}\Delta T \qquad \text{(J/g)} \tag{1-3}$$

or

$$\Delta U = mC_{v,\,\text{ave}}\Delta T \qquad \text{and} \qquad \Delta H = mC_{p,\,\text{ave}}\Delta T \qquad \text{(J)} \tag{1-4}$$

where m is the mass of the system.

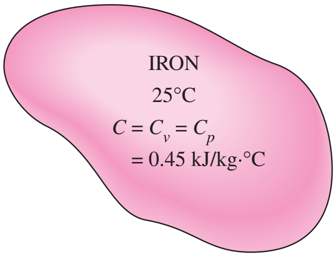


FIGURE 1–9
The C_v and C_p values of incompressible substances are identical and are denoted by C.

A substance whose specific volume (or density) does not change with temperature or pressure is called an **incompressible substance.** The specific volumes of solids and liquids essentially remain constant during a process, and thus they can be approximated as incompressible substances without sacrificing much in accuracy.

The constant-volume and constant-pressure specific heats are identical for incompressible substances (Fig. 1–9). Therefore, for solids and liquids the subscripts on C_v and C_p can be dropped and both specific heats can be represented by a single symbol, C. That is, $C_p \cong C_v \cong C$. This result could also be deduced from the physical definitions of constant-volume and constant-pressure specific heats. Specific heats of several common gases, liquids, and solids are given in the Appendix.

The specific heats of incompressible substances depend on temperature only. Therefore, the change in the internal energy of solids and liquids can be expressed as

$$\Delta U = mC_{\text{ave}}\Delta T \qquad \text{(J)} \tag{1-5}$$

where C_{ave} is the average specific heat evaluated at the average temperature. Note that the internal energy change of the systems that remain in a single phase (liquid, solid, or gas) during the process can be determined very easily using average specific heats.

Energy Transfer

Energy can be transferred to or from a given mass by two mechanisms: *heat* Q and *work* W. An energy interaction is heat transfer if its driving force is a temperature difference. Otherwise, it is work. A rising piston, a rotating shaft, and an electrical wire crossing the system boundaries are all associated with work interactions. Work done *per unit time* is called **power,** and is denoted by $\dot{W}$. The unit of power is W or hp (1 hp = 746 W). Car engines and hydraulic, steam, and gas turbines produce work; compressors, pumps, and mixers consume work. Notice that the energy of a system decreases as it does work, and increases as work is done on it.

In daily life, we frequently refer to the sensible and latent forms of internal energy as **heat,** and we talk about the heat content of bodies (Fig. 1–10). In thermodynamics, however, those forms of energy are usually referred to as **thermal energy** to prevent any confusion with *heat transfer*.

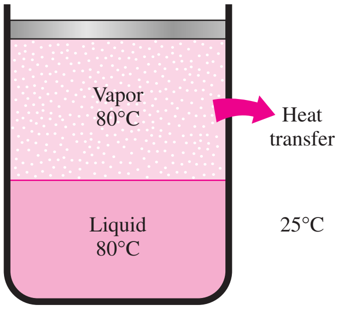


FIGURE 1–10
The sensible and latent forms of internal energy can be transferred as a result of a temperature difference, and they are referred to as *heat* or *thermal energy*.

The term *heat* and the associated phrases such as *heat flow, heat addition, heat rejection, heat absorption, heat gain, heat loss, heat storage, heat generation, electrical heating, latent heat, body heat,* and *heat source* are in common use today, and the attempt to replace *heat* in these phrases by *thermal energy* had only limited success. These phrases are deeply rooted in our vocabulary and they are used by both the ordinary people and scientists without causing any misunderstanding. For example, the phrase *body heat* is understood to mean the *thermal energy content* of a body. Likewise, *heat flow* is understood to mean the *transfer of thermal energy,* not the flow of a fluid-like substance called *heat,* although the latter incorrect interpretation, based on the caloric theory, is the origin of this phrase. Also, the transfer of heat into a system is frequently referred to as *heat addition* and the transfer of heat out of a system as *heat rejection*.

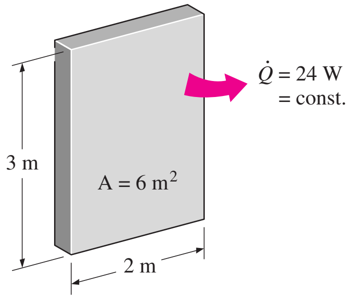
Keeping in line with current practice, we will refer to the thermal energy as *heat* and the transfer of thermal energy as *heat transfer*. The amount of heat transferred during the process is denoted by Q. The amount of heat transferred per unit time is called **heat transfer rate,** and is denoted by $\dot{Q}$. The overdot stands for the time derivative, or "per unit time." The heat transfer rate $\dot{Q}$ has the unit J/s, which is equivalent to W.

When the *rate* of heat transfer $\dot{Q}$ is available, then the total amount of heat transfer Q during a time interval Δt can be determined from

$$Q = \int_0^{\Delta t} \dot{Q}\,dt \qquad \text{(J)} \tag{1-6}$$

provided that the variation of $\dot{Q}$ with time is known. For the special case of $\dot{Q}$ = constant, the equation above reduces to

$$Q = \dot{Q}\Delta t \qquad \text{(J)} \tag{1-7}$$



$\dot{q} = \frac{\dot{Q}}{A} = \frac{24\text{ W}}{6\text{ m}^2} = 4\text{ W/m}^2$

FIGURE 1–11
Heat flux is heat transfer *per unit time* and *per unit area*, and is equal to $\dot{q} = \dot{Q}/A$ when $\dot{Q}$ is uniform over the area A.

The rate of heat transfer per unit area normal to the direction of heat transfer is called **heat flux,** and the average heat flux is expressed as (Fig. 1–11)

$$\dot{q} = \frac{\dot{Q}}{A} \qquad (\text{W/m}^2) \tag{1-8}$$

where A is the heat transfer area. The unit of heat flux in English units is Btu/h · ft^2. Note that heat flux may vary with time as well as position on a surface.

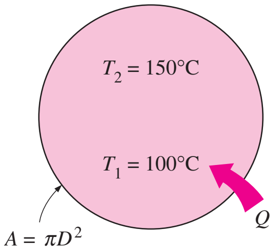


FIGURE 1–12
Schematic for Example 1–1.

EXAMPLE 1–1 Heating of a Copper Ball

A 10-cm diameter copper ball is to be heated from 100°C to an average temperature of 150°C in 30 minutes (Fig. 1–12). Taking the average density and specific heat of copper in this temperature range to be ρ = 8950 kg/m^3 and C_p = 0.395 kJ/kg · °C, respectively, determine (*a*) the total amount of heat transfer to the copper ball, (*b*) the average rate of heat transfer to the ball, and (*c*) the average heat flux.

SOLUTION The copper ball is to be heated from 100°C to 150°C. The total heat transfer, the average rate of heat transfer, and the average heat flux are to be determined.

Assumptions Constant properties can be used for copper at the average temperature.

Properties The average density and specific heat of copper are given to be ρ = 8950 kg/m^3 and C_p = 0.395 kJ/kg · °C.

Analysis (*a*) The amount of heat transferred to the copper ball is simply the change in its internal energy, and is determined from

$$\text{Energy transfer to the system} = \text{Energy increase of the system}$$
$$Q = \Delta U = mC_{\text{ave}}(T_2 - T_1)$$

where

$$m = \rho V = \frac{\pi}{6}\rho D^3 = \frac{\pi}{6}(8950\text{ kg/m}^3)(0.1\text{ m})^3 = 4.69\text{ kg}$$

Substituting,

$$Q = (4.69\text{ kg})(0.395\text{ kJ/kg}\cdot{}^\circ\text{C})(150 - 100)^\circ\text{C} = \mathbf{92.6\text{ kJ}}$$

Therefore, 92.6 kJ of heat needs to be transferred to the copper ball to heat it from 100°C to 150°C.

(*b*) The rate of heat transfer normally changes during a process with time. However, we can determine the *average* rate of heat transfer by dividing the total amount of heat transfer by the time interval. Therefore,

$$\dot{Q}_{\text{ave}} = \frac{Q}{\Delta t} = \frac{92.6\text{ kJ}}{1800\text{ s}} = 0.0514\text{ kJ/s} = \mathbf{51.4\text{ W}}$$

(*c*) Heat flux is defined as the heat transfer per unit time per unit area, or the rate of heat transfer per unit area. Therefore, the average heat flux in this case is

$$\dot{q}_{\text{ave}} = \frac{\dot{Q}_{\text{ave}}}{A} = \frac{\dot{Q}_{\text{ave}}}{\pi D^2} = \frac{51.4 \text{ W}}{\pi(0.1 \text{ m})^2} = \mathbf{1636 \ W/m^2}$$

Discussion Note that heat flux may vary with location on a surface. The value calculated above is the *average* heat flux over the entire surface of the ball.

1–4 ▪ THE FIRST LAW OF THERMODYNAMICS

The **first law of thermodynamics,** also known as the **conservation of energy principle,** states that *energy can neither be created nor destroyed; it can only change forms.* Therefore, every bit of energy must be accounted for during a process. The conservation of energy principle (or the energy balance) for *any system* undergoing *any process* may be expressed as follows: *The net change (increase or decrease) in the total energy of the system during a process is equal to the difference between the total energy entering and the total energy leaving the system during that process.* That is,

$$\left(\begin{array}{c}\text{Total energy}\\ \text{entering the}\\ \text{system}\end{array}\right) - \left(\begin{array}{c}\text{Total energy}\\ \text{leaving the}\\ \text{system}\end{array}\right) = \left(\begin{array}{c}\text{Change in the}\\ \text{total energy of}\\ \text{the system}\end{array}\right) \qquad \text{(1-9)}$$

Noting that energy can be transferred to or from a system by *heat, work,* and *mass flow,* and that the total energy of a simple compressible system consists of internal, kinetic, and potential energies, the **energy balance** for any system undergoing any process can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc., energies}}} \qquad \text{(J)} \qquad \text{(1-10)}$$

or, in the **rate form,** as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal}\\ \text{kinetic, potential, etc., energies}}} \qquad \text{(W)} \qquad \text{(1-11)}$$

Energy is a property, and the value of a property does not change unless the state of the system changes. Therefore, the energy change of a system is zero ($\Delta E_{\text{system}} = 0$) if the state of the system does not change during the process, that is, the process is steady. The energy balance in this case reduces to (Fig. 1–13)

Steady, rate form:

$$\underbrace{\dot{E}_{\text{in}}}_{\substack{\text{Rate of net energy transfer in}\\ \text{by heat, work, and mass}}} = \underbrace{\dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer out}\\ \text{by heat, work, and mass}}} \qquad \text{(1-12)}$$

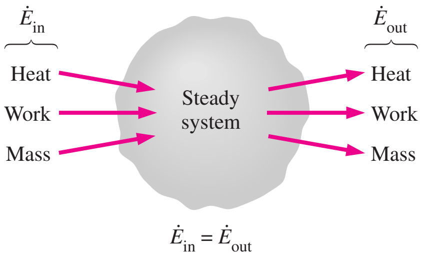


FIGURE 1–13
In steady operation, the rate of energy transfer to a system is equal to the rate of energy transfer from the system.

In the absence of significant electric, magnetic, motion, gravity, and surface tension effects (i.e., for stationary simple compressible systems), the change in the *total energy* of a system during a process is simply the change in its *internal energy*. That is, $\Delta E_{\text{system}} = \Delta U_{\text{system}}$.

In heat transfer analysis, we are usually interested only in the forms of energy that can be transferred as a result of a temperature difference, that is, heat or thermal energy. In such cases it is convenient to write a **heat balance** and to treat the conversion of nuclear, chemical, and electrical energies into thermal energy as *heat generation*. The *energy balance* in that case can be expressed as

$$\underbrace{Q_{\text{in}} - Q_{\text{out}}}_{\substack{\text{Net heat}\\ \text{transfer}}} + \underbrace{E_{\text{gen}}}_{\substack{\text{Heat}\\ \text{generation}}} = \underbrace{\Delta E_{\text{thermal, system}}}_{\substack{\text{Change in thermal}\\ \text{energy of the system}}} \qquad \text{(J)} \qquad \textbf{(1-13)}$$

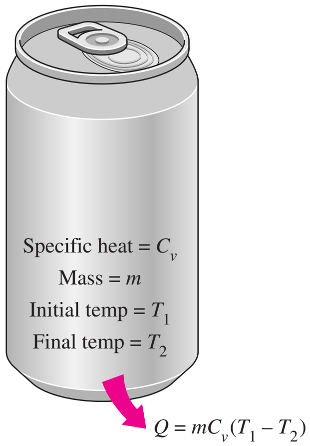


FIGURE 1–14
In the absence of any work interactions, the change in the energy content of a closed system is equal to the net heat transfer.

Energy Balance for Closed Systems *(Fixed Mass)*

A closed system consists of a *fixed mass*. The total energy E for most systems encountered in practice consists of the internal energy U. This is especially the case for stationary systems since they don't involve any changes in their velocity or elevation during a process. The energy balance relation in that case reduces to

Stationary closed system: $$E_{\text{in}} - E_{\text{out}} = \Delta U = mC_v\Delta T \qquad \text{(J)} \qquad \textbf{(1-14)}$$

where we expressed the internal energy change in terms of mass m, the specific heat at constant volume C_v, and the temperature change ΔT of the system. When the system involves heat transfer only and no work interactions across its boundary, the energy balance relation further reduces to (Fig. 1–14)

Stationary closed system, no work: $$Q = mC_v\Delta T \qquad \text{(J)} \qquad \textbf{(1-15)}$$

where Q is the net amount of heat transfer to or from the system. This is the form of the energy balance relation we will use most often when dealing with a fixed mass.

Energy Balance for Steady-Flow Systems

A large number of engineering devices such as water heaters and car radiators involve mass flow in and out of a system, and are modeled as *control volumes*. Most control volumes are analyzed under steady operating conditions. The term *steady* means *no change with time* at a specified location. The opposite of steady is *unsteady* or *transient*. Also, the term *uniform* implies *no change with position* throughout a surface or region at a specified time. These meanings are consistent with their everyday usage (steady girlfriend, uniform distribution, etc.). The total energy content of a control volume during a *steady-flow process* remains constant (E_{CV} = constant). That is, the change in the total energy of the control volume during such a process is zero ($\Delta E_{\text{CV}} = 0$). Thus the amount of energy entering a control volume in all forms (heat, work, mass transfer) for a steady-flow process must be equal to the amount of energy leaving it.

The amount of mass flowing through a cross section of a flow device per unit time is called the **mass flow rate,** and is denoted by $\dot{m}$. A fluid may flow in and out of a control volume through pipes or ducts. The mass flow rate of a fluid flowing in a pipe or duct is proportional to the cross-sectional area A_c of the pipe or duct, the density ρ, and the velocity $\mathcal{V}$ of the fluid. The mass flow rate through a differential area dA_c can be expressed as $\delta\dot{m} = \rho\mathcal{V}_n\,dA_c$ where $\mathcal{V}_n$ is the velocity component normal to dA_c. The mass flow rate through the entire cross-sectional area is obtained by integration over A_c.

The flow of a fluid through a pipe or duct can often be approximated to be *one-dimensional.* That is, the properties can be assumed to vary in one direction only (the direction of flow). As a result, all properties are assumed to be uniform at any cross section normal to the flow direction, and the properties are assumed to have *bulk average values* over the entire cross section. Under the one-dimensional flow approximation, the mass flow rate of a fluid flowing in a pipe or duct can be expressed as (Fig. 1–15)

$$\dot{m} = \rho\mathcal{V}A_c \qquad \text{(kg/s)} \tag{1-16}$$

where ρ is the fluid density, $\mathcal{V}$ is the average fluid velocity in the flow direction, and A_c is the cross-sectional area of the pipe or duct.

The volume of a fluid flowing through a pipe or duct per unit time is called the **volume flow rate** $\dot{V}$, and is expressed as

$$\dot{V} = \mathcal{V}A_c = \frac{\dot{m}}{\rho} \qquad \text{(m}^3\text{/s)} \tag{1-17}$$

Note that the mass flow rate of a fluid through a pipe or duct remains constant during steady flow. This is not the case for the volume flow rate, however, unless the density of the fluid remains constant.

For a steady-flow system with one inlet and one exit, the rate of mass flow into the control volume must be equal to the rate of mass flow out of it. That is, $\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m}$. When the changes in kinetic and potential energies are negligible, which is usually the case, and there is no work interaction, the energy balance for such a steady-flow system reduces to (Fig. 1–16)

$$\dot{Q} = \dot{m}\Delta h = \dot{m}C_p\Delta T \qquad \text{(kJ/s)} \tag{1-18}$$

where $\dot{Q}$ is the rate of net heat transfer into or out of the control volume. This is the form of the energy balance relation that we will use most often for steady-flow systems.

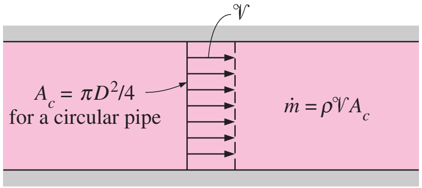


FIGURE 1–15
The mass flow rate of a fluid at a cross section is equal to the product of the fluid density, average fluid velocity, and the cross-sectional area.

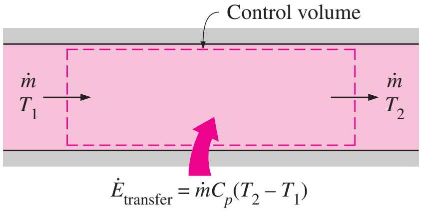


FIGURE 1–16
Under steady conditions, the net rate of energy transfer to a fluid in a control volume is equal to the rate of increase in the energy of the fluid stream flowing through the control volume.

Surface Energy Balance

As mentioned in the chapter opener, heat is transferred by the mechanisms of conduction, convection, and radiation, and heat often changes vehicles as it is transferred from one medium to another. For example, the heat conducted to the outer surface of the wall of a house in winter is convected away by the cold outdoor air while being radiated to the cold surroundings. In such cases, it may be necessary to keep track of the energy interactions at the surface, and this is done by applying the conservation of energy principle to the surface.

A surface contains no volume or mass, and thus no energy. Thereore, a surface can be viewed as a fictitious system whose energy content remains constant during a process (just like a steady-state or steady-flow system). Then the energy balance for a surface can be expressed as

Surface energy balance:
$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}} \tag{1-19}$$

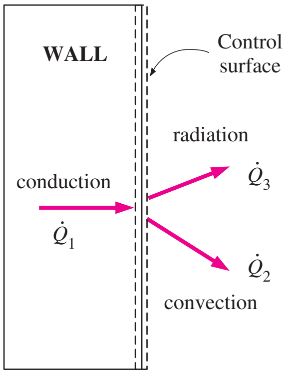


FIGURE 1–17
Energy interactions at the outer wall surface of a house.

This relation is valid for both steady and transient conditions, and the surface energy balance does not involve heat generation since a surface does not have a volume. The energy balance for the outer surface of the wall in Fig. 1–17, for example, can be expressed as

$$\dot{Q}_1 = \dot{Q}_2 + \dot{Q}_3 \tag{1-20}$$

where $\dot{Q}_1$ is conduction through the wall to the surface, $\dot{Q}_2$ is convection from the surface to the outdoor air, and $\dot{Q}_3$ is net radiation from the surface to the surroundings.

When the directions of interactions are not known, all energy interactions can be assumed to be towards the surface, and the surface energy balance can be expressed as $\Sigma \dot{E}_{\text{in}} = 0$. Note that the interactions in opposite direction will end up having negative values, and balance this equation.

EXAMPLE 1–2 Heating of Water in an Electric Teapot

1.2 kg of liquid water initially at 15°C is to be heated to 95°C in a teapot equipped with a 1200-W electric heating element inside (Fig. 1–18). The teapot is 0.5 kg and has an average specific heat of 0.7 kJ/kg · °C. Taking the specific heat of water to be 4.18 kJ/kg · °C and disregarding any heat loss from the teapot, determine how long it will take for the water to be heated.

SOLUTION Liquid water is to be heated in an electric teapot. The heating time is to be determined.

Assumptions **1** Heat loss from the teapot is negligible. **2** Constant properties can be used for both the teapot and the water.

Properties The average specific heats are given to be 0.7 kJ/kg · °C for the teapot and 4.18 kJ/kg · °C for water.

Analysis We take the teapot and the water in it as the system, which is a closed system (fixed mass). The energy balance in this case can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$
$$E_{\text{in}} = \Delta U_{\text{system}} = \Delta U_{\text{water}} + \Delta U_{\text{teapot}}$$

Then the amount of energy needed to raise the temperature of water and the teapot from 15°C to 95°C is

$$\begin{aligned} E_{\text{in}} &= (mC\Delta T)_{\text{water}} + (mC\Delta T)_{\text{teapot}} \\ &= (1.2\text{ kg})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C})(95 - 15)^\circ\text{C} + (0.5\text{ kg})(0.7\text{ kJ/kg}\cdot{}^\circ\text{C}) \\ &\quad (95 - 15)^\circ\text{C} \\ &= 429.3\text{ kJ} \end{aligned}$$

The 1200-W electric heating unit will supply energy at a rate of 1.2 kW or 1.2 kJ per second. Therefore, the time needed for this heater to supply 429.3 kJ of heat is determined from

$$\Delta t = \frac{\text{Total energy transferred}}{\text{Rate of energy transfer}} = \frac{E_{\text{in}}}{\dot{E}_{\text{transfer}}} = \frac{429.3\text{ kJ}}{1.2\text{ kJ/s}} = 358\text{ s} = \mathbf{6.0\ min}$$

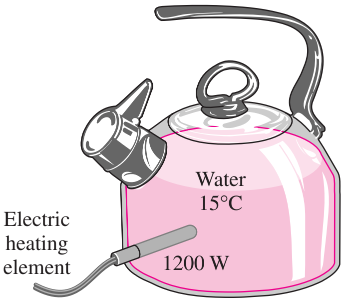


FIGURE 1–18
Schematic for Example 1–2.

Discussion In reality, it will take more than 6 minutes to accomplish this heating process since some heat loss is inevitable during heating.

EXAMPLE 1–3 Heat Loss from Heating Ducts in a Basement

A 5-m-long section of an air heating system of a house passes through an unheated space in the basement (Fig. 1–19). The cross section of the rectangular duct of the heating system is 20 cm × 25 cm. Hot air enters the duct at 100 kPa and 60°C at an average velocity of 5 m/s. The temperature of the air in the duct drops to 54°C as a result of heat loss to the cool space in the basement. Determine the rate of heat loss from the air in the duct to the basement under steady conditions. Also, determine the cost of this heat loss per hour if the house is heated by a natural gas furnace that has an efficiency of 80 percent, and the cost of the natural gas in that area is \$0.60/therm (1 therm = 100,000 Btu = 105,500 kJ).

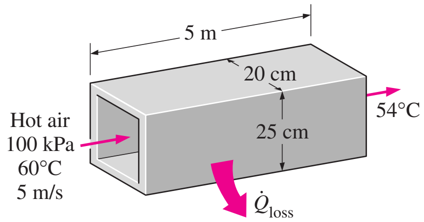


FIGURE 1–19
Schematic for Example 1–3.

SOLUTION The temperature of the air in the heating duct of a house drops as a result of heat loss to the cool space in the basement. The rate of heat loss from the hot air and its cost are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air can be treated as an ideal gas with constant properties at room temperature.

Properties The constant pressure specific heat of air at the average temperature of (54 + 60)/2 = 57°C is 1.007 kJ/kg · °C (Table A-15).

Analysis We take the basement section of the heating system as our system, which is a steady-flow system. The rate of heat loss from the air in the duct can be determined from

$$\dot{Q} = \dot{m}C_p\Delta T$$

where $\dot{m}$ is the mass flow rate and ΔT is the temperature drop. The density of air at the inlet conditions is

$$\rho = \frac{P}{RT} = \frac{100 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(60 + 273)\text{K}} = 1.046 \text{ kg/m}^3$$

The cross-sectional area of the duct is

$$A_c = (0.20 \text{ m})(0.25 \text{ m}) = 0.05 \text{ m}^2$$

Then the mass flow rate of air through the duct and the rate of heat loss become

$$\dot{m} = \rho \mathcal{V} A_c = (1.046 \text{ kg/m}^3)(5 \text{ m/s})(0.05 \text{ m}^2) = 0.2615 \text{ kg/s}$$

and

$$\begin{aligned}\dot{Q}_{\text{loss}} &= \dot{m}C_p(T_{\text{in}} - T_{\text{out}}) \\ &= (0.2615 \text{ kg/s})(1.007 \text{ kJ/kg} \cdot °\text{C})(60 - 54)°\text{C} \\ &= \mathbf{1.580 \text{ kJ/s}}\end{aligned}$$

or 5688 kJ/h. The cost of this heat loss to the home owner is

$$\begin{aligned}\text{Cost of heat loss} &= \frac{(\text{Rate of heat loss})(\text{Unit cost of energy input})}{\text{Furnace efficiency}}\\ &= \frac{(5688\text{ kJ/h})(\$0.60/\text{therm})}{0.80}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right)\\ &= \mathbf{\$0.040/h}\end{aligned}$$

Discussion The heat loss from the heating ducts in the basement is costing the home owner 4 cents per hour. Assuming the heater operates 2000 hours during a heating season, the annual cost of this heat loss adds up to \$80. Most of this money can be saved by insulating the heating ducts in the unheated areas.

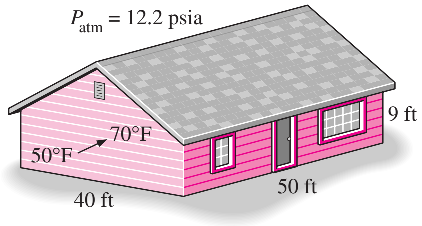


FIGURE 1–20
Schematic for Example 1–4.

EXAMPLE 1–4 Electric Heating of a House at High Elevation

Consider a house that has a floor space of 2000 ft^2 and an average height of 9 ft at 5000 ft elevation where the standard atmospheric pressure is 12.2 psia (Fig. 1–20). Initially the house is at a uniform temperature of 50°F. Now the electric heater is turned on, and the heater runs until the air temperature in the house rises to an average value of 70°F. Determine the amount of energy transferred to the air assuming (*a*) the house is air-tight and thus no air escapes during the heating process and (*b*) some air escapes through the cracks as the heated air in the house expands at constant pressure. Also determine the cost of this heat for each case if the cost of electricity in that area is \$0.075/kWh.

SOLUTION The air in the house is heated from 50°F to 70°F by an electric heater. The amount and cost of the energy transferred to the air are to be determined for constant-volume and constant-pressure cases.

Assumptions **1** Air can be treated as an ideal gas with constant properties at room temperature. **2** Heat loss from the house during heating is negligible. **3** The volume occupied by the furniture and other things is negligible.

Properties The specific heats of air at the average temperature of (50 + 70)/2 = 60°F are C_p = 0.240 Btu/lbm · °F and $C_v = C_p - R$ = 0.171 Btu/lbm · °F (Tables A-1E and A-15E).

Analysis The volume and the mass of the air in the house are

$$V = (\text{Floor area})(\text{Height}) = (2000\text{ ft}^2)(9\text{ ft}) = 18{,}000\text{ ft}^3$$

$$m = \frac{PV}{RT} = \frac{(12.2\text{ psia})(18{,}000\text{ ft}^3)}{(0.3704\text{ psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(50+460)\text{R}} = 1162\text{ lbm}$$

(*a*) The amount of energy transferred to air at constant volume is simply the change in its internal energy, and is determined from

$$\begin{aligned}E_{in} - E_{out} &= \Delta E_{system}\\ E_{in,\text{ constant volume}} &= \Delta U_{air} = mC_v\Delta T\\ &= (1162\text{ lbm})(0.171\text{ Btu/lbm}\cdot{}^\circ\text{F})(70-50)^\circ\text{F}\\ &= \mathbf{3974\ Btu}\end{aligned}$$

At a unit cost of \$0.075/kWh, the total cost of this energy is

$$\text{Cost of energy} = (\text{Amount of energy})(\text{Unit cost of energy})$$
$$= (3974 \text{ Btu})(\$0.075/\text{kWh})\left(\frac{1 \text{ kWh}}{3412 \text{ Btu}}\right)$$
$$= \mathbf{\$0.087}$$

(*b*) The amount of energy transferred to air at constant pressure is the change in its enthalpy, and is determined from

$$E_{\text{in, constant pressure}} = \Delta H_{\text{air}} = mC_p\Delta T$$
$$= (1162 \text{ lbm})(0.240 \text{ Btu/lbm} \cdot {}^\circ\text{F})(70 - 50)^\circ\text{F}$$
$$= \mathbf{5578 \text{ Btu}}$$

At a unit cost of \$0.075/kWh, the total cost of this energy is

$$\text{Cost of energy} = (\text{Amount of energy})(\text{Unit cost of energy})$$
$$= (5578 \text{ Btu})(\$0.075/\text{kWh})\left(\frac{1 \text{ kWh}}{3412 \text{ Btu}}\right)$$
$$= \mathbf{\$0.123}$$

Discussion It will cost about 12 cents to raise the temperature of the air in this house from 50°F to 70°F. The second answer is more realistic since every house has cracks, especially around the doors and windows, and the pressure in the house remains essentially constant during a heating process. Therefore, the second approach is used in practice. This conservative approach somewhat overpredicts the amount of energy used, however, since some of the air will escape through the cracks before it is heated to 70°F.

1–5 ▪ HEAT TRANSFER MECHANISMS

In Section 1–1 we defined **heat** as the form of energy that can be transferred from one system to another as a result of temperature difference. A thermodynamic analysis is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another. The science that deals with the determination of the *rates* of such energy transfers is the **heat transfer.** The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two mediums reach the same temperature.

Heat can be transferred in three different modes: *conduction, convection,* and *radiation.* All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one. Below we give a brief description of each mode. A detailed study of these modes is given in later chapters of this text.

1–6 ▪ CONDUCTION

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids, or gases. In gases and liquids, conduction is due to the *collisions* and *diffusion* of the molecules during their random motion. In solids, it is due to the combination of *vibrations* of the molecules in a lattice and the energy transport by *free electrons.* A cold canned drink in a warm room, for example, eventually warms up to the room temperature as a result of heat transfer from the room to the drink through the aluminum can by conduction.

The *rate* of heat conduction through a medium depends on the *geometry* of the medium, its *thickness,* and the *material* of the medium, as well as the *temperature difference* across the medium. We know that wrapping a hot water tank with glass wool (an insulating material) reduces the rate of heat loss from the tank. The thicker the insulation, the smaller the heat loss. We also know that a hot water tank will lose heat at a higher rate when the temperature of the room housing the tank is lowered. Further, the larger the tank, the larger the surface area and thus the rate of heat loss.

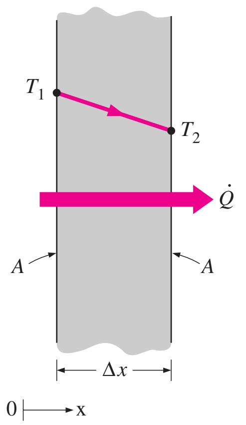


FIGURE 1–21
Heat conduction through a large plane wall of thickness Δx and area A.

Consider steady heat conduction through a large plane wall of thickness $\Delta x = L$ and area A, as shown in Fig. 1–21. The temperature difference across the wall is $\Delta T = T_2 - T_1$. Experiments have shown that the rate of heat transfer $\dot{Q}$ through the wall is *doubled* when the temperature difference ΔT across the wall or the area A normal to the direction of heat transfer is doubled, but is *halved* when the wall thickness L is doubled. Thus we conclude that *the rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer.* That is,

$$\text{Rate of heat conduction} \propto \frac{\text{(Area)(Temperature difference)}}{\text{Thickness}}$$

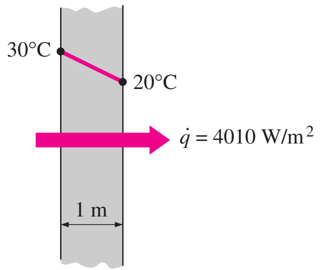
or,

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \qquad \text{(W)} \qquad \textbf{(1-21)}$$

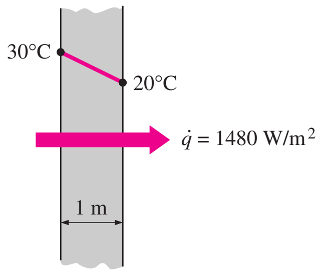
where the constant of proportionality k is the **thermal conductivity** of the material, which is a *measure of the ability of a material to conduct heat* (Fig. 1–22). In the limiting case of $\Delta x \to 0$, the equation above reduces to the differential form

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx} \qquad \text{(W)} \qquad \textbf{(1-22)}$$

which is called **Fourier's law of heat conduction** after J. Fourier, who expressed it first in his heat transfer text in 1822. Here dT/dx is the **temperature gradient,** which is the slope of the temperature curve on a T-x diagram (the rate of change of T with x), at location x. The relation above indicates that the rate of heat conduction in a direction is proportional to the temperature gradient in that direction. Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing x. The *negative sign* in Eq. 1–22 ensures that heat transfer in the positive x direction is a positive quantity.



(*a*) Copper (k = 401 W/m·°C)



(*b*) Silicon (k = 148 W/m·°C)

FIGURE 1–22
The rate of heat conduction through a solid is directly proportional to its thermal conductivity.

The heat transfer area A is always *normal* to the direction of heat transfer. For heat loss through a 5-m-long, 3-m-high, and 25-cm-thick wall, for example, the heat transfer area is $A = 15$ m^2. Note that the thickness of the wall has no effect on A (Fig. 1–23).

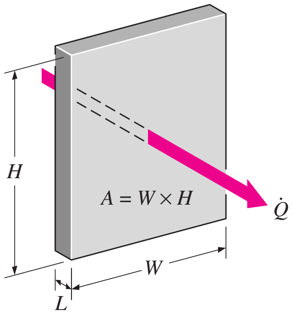


FIGURE 1–23
In heat conduction analysis, A represents the area *normal* to the direction of heat transfer.

EXAMPLE 1–5 The Cost of Heat Loss through a Roof

The roof of an electrically heated home is 6 m long, 8 m wide, and 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is $k = 0.8$ W/m · °C (Fig. 1–24). The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15°C and 4°C, respectively, for a period of 10 hours. Determine (*a*) the rate of heat loss through the roof that night and (*b*) the cost of that heat loss to the home owner if the cost of electricity is \$0.08/kWh.

SOLUTION The inner and outer surfaces of the flat concrete roof of an electrically heated home are maintained at specified temperatures during a night. The heat loss through the roof and its cost that night are to be determined.

Assumptions **1** Steady operating conditions exist during the entire night since the surface temperatures of the roof remain constant at the specified values. **2** Constant properties can be used for the roof.

Properties The thermal conductivity of the roof is given to be $k = 0.8$ W/m · °C.

Analysis (*a*) Noting that heat transfer through the roof is by conduction and the area of the roof is $A = 6 \text{ m} \times 8 \text{ m} = 48 \text{ m}^2$, the steady rate of heat transfer through the roof is determined to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8 \text{ W/m} \cdot \text{°C})(48 \text{ m}^2)\frac{(15 - 4)\text{°C}}{0.25 \text{ m}} = \mathbf{1690 \text{ W} = 1.69 \text{ kW}}$$

(*b*) The amount of heat lost through the roof during a 10-hour period and its cost are determined from

$$Q = \dot{Q}\,\Delta t = (1.69 \text{ kW})(10 \text{ h}) = 16.9 \text{ kWh}$$

$$\text{Cost} = (\text{Amount of energy})(\text{Unit cost of energy})$$

$$= (16.9 \text{ kWh})(\$0.08/\text{kWh}) = \mathbf{\$1.35}$$

Discussion The cost to the home owner of the heat loss through the roof that night was \$1.35. The total heating bill of the house will be much larger since the heat losses through the walls are not considered in these calculations.

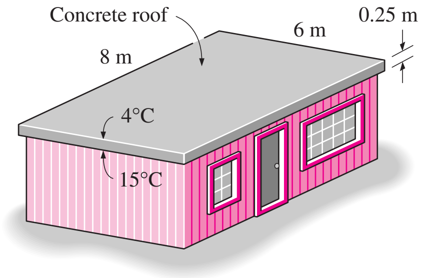


FIGURE 1–24
Schematic for Example 1–5.

Thermal Conductivity

We have seen that different materials store heat differently, and we have defined the property specific heat C_p as a measure of a material's ability to store thermal energy. For example, $C_p = 4.18$ kJ/kg · °C for water and $C_p = 0.45$ kJ/kg · °C for iron at room temperature, which indicates that water can store almost 10 times the energy that iron can per unit mass. Likewise, the thermal conductivity k is a measure of a material's ability to conduct heat. For example, $k = 0.608$ W/m · °C for water and $k = 80.2$ W/m · °C for iron at room temperature, which indicates that iron conducts heat more than 100 times faster than water can. Thus we say that water is a poor heat conductor relative to iron, although water is an excellent medium to store thermal energy.

Equation 1–22 for the rate of conduction heat transfer under steady conditions can also be viewed as the defining equation for thermal conductivity. Thus the **thermal conductivity** of a material can be defined as *the rate of*

TABLE 1–1

The thermal conductivities of some materials at room temperature

Material	k, W/m · °C*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.613
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

*Multiply by 0.5778 to convert to Btu/h · ft · °F.

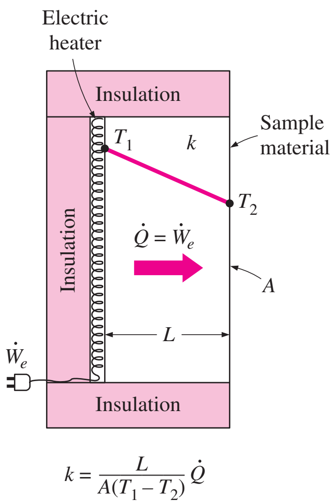


FIGURE 1–25

A simple experimental setup to determine the thermal conductivity of a material.

heat transfer through a unit thickness of the material per unit area per unit temperature difference. The thermal conductivity of a material is a measure of the ability of the material to conduct heat. A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator.* The thermal conductivities of some common materials at room temperature are given in Table 1–1. The thermal conductivity of pure copper at room temperature is $k = 401$ W/m · °C, which indicates that a 1-m-thick copper wall will conduct heat at a rate of 401 W per m^2 area per °C temperature difference across the wall. Note that materials such as copper and silver that are good electric conductors are also good heat conductors, and have high values of thermal conductivity. Materials such as rubber, wood, and styrofoam are poor conductors of heat and have low conductivity values.

A layer of material of known thickness and area can be heated from one side by an electric resistance heater of known output. If the outer surfaces of the heater are well insulated, all the heat generated by the resistance heater will be transferred through the material whose conductivity is to be determined. Then measuring the two surface temperatures of the material when steady heat transfer is reached and substituting them into Eq. 1–22 together with other known quantities give the thermal conductivity (Fig. 1–25).

The thermal conductivities of materials vary over a wide range, as shown in Fig. 1–26. The thermal conductivities of gases such as air vary by a factor of 10^4 from those of pure metals such as copper. Note that pure crystals and metals have the highest thermal conductivities, and gases and insulating materials the lowest.

Temperature is a measure of the kinetic energies of the particles such as the molecules or atoms of a substance. In a liquid or gas, the kinetic energy of the molecules is due to their random translational motion as well as their vibrational and rotational motions. When two molecules possessing different kinetic energies collide, part of the kinetic energy of the more energetic (higher-temperature) molecule is transferred to the less energetic (lower-temperature) molecule, much the same as when two elastic balls of the same mass at different velocities collide, part of the kinetic energy of the faster ball is transferred to the slower one. The higher the temperature, the faster the molecules move and the higher the number of such collisions, and the better the heat transfer.

The *kinetic theory* of gases predicts and the experiments confirm that the thermal conductivity of gases is proportional to the *square root of the absolute temperature T,* and inversely proportional to the *square root of the molar mass M.* Therefore, the thermal conductivity of a gas increases with increasing temperature and decreasing molar mass. So it is not surprising that the thermal conductivity of helium ($M = 4$) is much higher than those of air ($M = 29$) and argon ($M = 40$).

The thermal conductivities of *gases* at 1 atm pressure are listed in Table A-16. However, they can also be used at pressures other than 1 atm, since the thermal conductivity of gases is *independent of pressure* in a wide range of pressures encountered in practice.

The mechanism of heat conduction in a *liquid* is complicated by the fact that the molecules are more closely spaced, and they exert a stronger intermolecular force field. The thermal conductivities of liquids usually lie between those

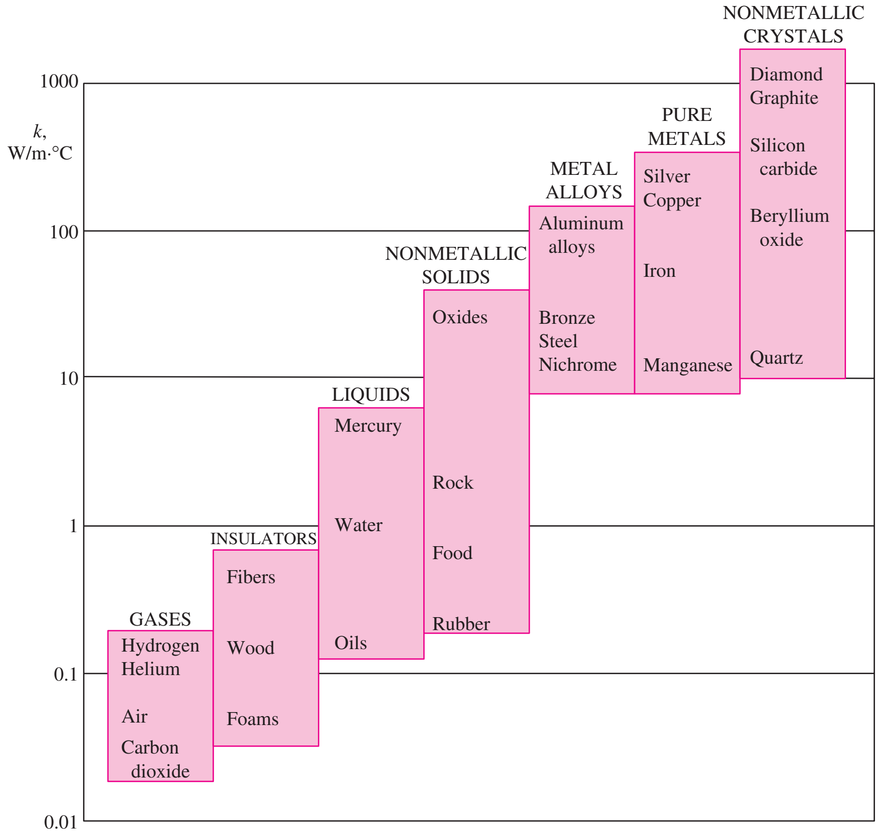


FIGURE 1–26
The range of thermal conductivity of various materials at room temperature.

of solids and gases. The thermal conductivity of a substance is normally highest in the solid phase and lowest in the gas phase. Unlike gases, the thermal conductivities of most liquids decrease with increasing temperature, with water being a notable exception. Like gases, the conductivity of liquids decreases with increasing molar mass. Liquid metals such as mercury and sodium have high thermal conductivities and are very suitable for use in applications where a high heat transfer rate to a liquid is desired, as in nuclear power plants.

In *solids,* heat conduction is due to two effects: the *lattice vibrational waves* induced by the vibrational motions of the molecules positioned at relatively fixed positions in a periodic manner called a lattice, and the energy transported via the *free flow of electrons* in the solid (Fig. 1–27). The thermal conductivity of a solid is obtained by adding the lattice and electronic components. The relatively high thermal conductivities of pure metals are primarily due to the electronic component. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. For example, diamond, which is a highly ordered crystalline solid, has the highest known thermal conductivity at room temperature.

Unlike metals, which are good electrical and heat conductors, *crystalline solids* such as diamond and semiconductors such as silicon are good heat conductors but poor electrical conductors. As a result, such materials find widespread use in the electronics industry. Despite their higher price, diamond heat sinks are used in the cooling of sensitive electronic components because of the

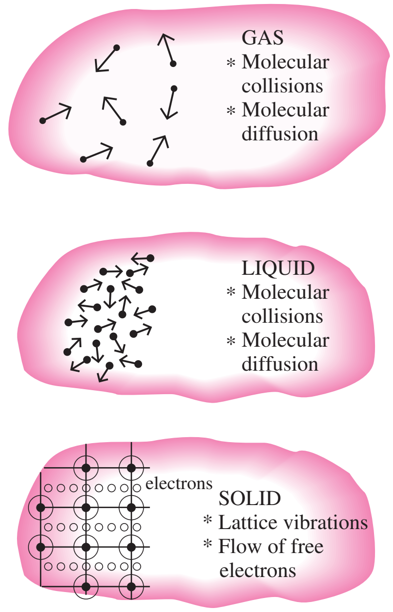


FIGURE 1–27
The mechanisms of heat conduction in different phases of a substance.

TABLE 1–2

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure metal or alloy	k, W/m · °C, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% Cu, 10% Al)	52

TABLE 1–3

Thermal conductivities of materials vary with temperature

T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

excellent thermal conductivity of diamond. Silicon oils and gaskets are commonly used in the packaging of electronic components because they provide both good thermal contact and good electrical insulation.

Pure metals have high thermal conductivities, and one would think that *metal alloys* should also have high conductivities. One would expect an alloy made of two metals of thermal conductivities k_1 and k_2 to have a conductivity k between k_1 and k_2. But this turns out not to be the case. The thermal conductivity of an alloy of two metals is usually much lower than that of either metal, as shown in Table 1–2. Even small amounts in a pure metal of "foreign" molecules that are good conductors themselves seriously disrupt the flow of heat in that metal. For example, the thermal conductivity of steel containing just 1 percent of chrome is 62 W/m · °C, while the thermal conductivities of iron and chromium are 83 and 95 W/m · °C, respectively.

The thermal conductivities of materials vary with temperature (Table 1–3). The variation of thermal conductivity over certain temperature ranges is negligible for some materials, but significant for others, as shown in Fig. 1–28. The thermal conductivities of certain solids exhibit dramatic increases at temperatures near absolute zero, when these solids become *superconductors*. For example, the conductivity of copper reaches a maximum value of about 20,000 W/m · °C at 20 K, which is about 50 times the conductivity at room temperature. The thermal conductivities and other thermal properties of various materials are given in Tables A-3 to A-16.

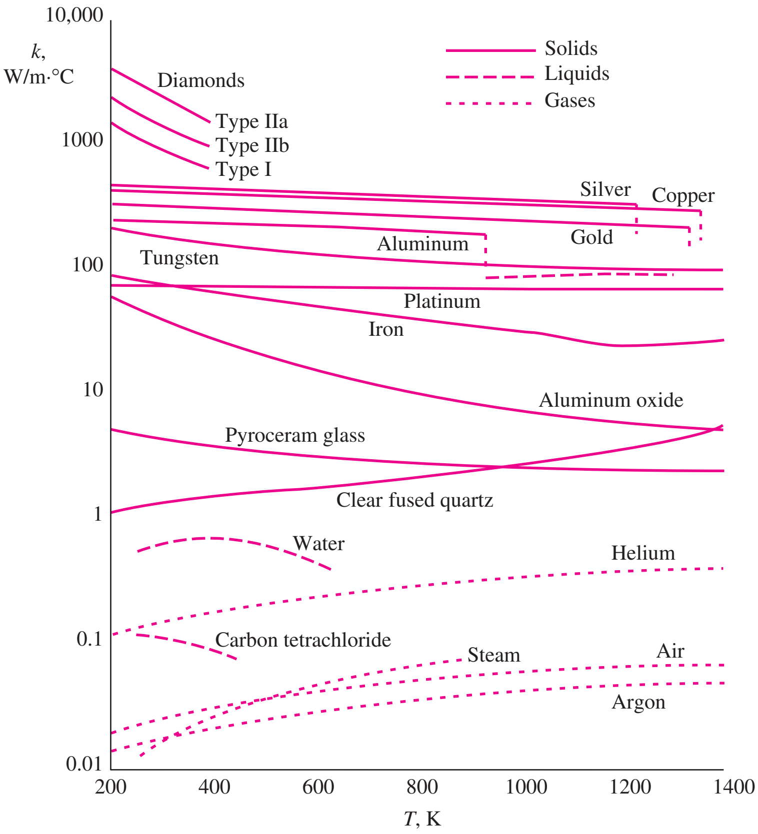


FIGURE 1–28

The variation of the thermal conductivity of various solids, liquids, and gases with temperature (from White, Ref. 10).

The temperature dependence of thermal conductivity causes considerable complexity in conduction analysis. Therefore, it is common practice to evaluate the thermal conductivity k at the *average temperature* and treat it as a *constant* in calculations.

In heat transfer analysis, a material is normally assumed to be *isotropic;* that is, to have uniform properties in all directions. This assumption is realistic for most materials, except those that exhibit different structural characteristics in different directions, such as laminated composite materials and wood. The thermal conductivity of wood across the grain, for example, is different than that parallel to the grain.

Thermal Diffusivity

The product ρC_p, which is frequently encountered in heat transfer analysis, is called the **heat capacity** of a material. Both the specific heat C_p and the heat capacity ρC_p represent the heat storage capability of a material. But C_p expresses it *per unit mass* whereas ρC_p expresses it *per unit volume,* as can be noticed from their units J/kg · °C and J/m^3 · °C, respectively.

Another material property that appears in the transient heat conduction analysis is the **thermal diffusivity,** which represents how fast heat diffuses through a material and is defined as

$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho C_p} \qquad (\text{m}^2/\text{s}) \tag{1-23}$$

Note that the thermal conductivity k represents how well a material conducts heat, and the heat capacity ρC_p represents how much energy a material stores per unit volume. Therefore, the thermal diffusivity of a material can be viewed as the ratio of the *heat conducted* through the material to the *heat stored* per unit volume. A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity. The larger the thermal diffusivity, the faster the propagation of heat into the medium. A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat will be conducted further.

The thermal diffusivities of some common materials at 20°C are given in Table 1–4. Note that the thermal diffusivity ranges from $\alpha = 0.14 \times 10^{-6}$ m^2/s for water to 174×10^{-6} m^2/s for silver, which is a difference of more than a thousand times. Also note that the thermal diffusivities of beef and water are the same. This is not surprising, since meat as well as fresh vegetables and fruits are mostly water, and thus they possess the thermal properties of water.

TABLE 1–4

The thermal diffusivities of some materials at room temperature

Material	α, m^2/s*
Silver	149×10^{-6}
Gold	127×10^{-6}
Copper	113×10^{-6}
Aluminum	97.5×10^{-6}
Iron	22.8×10^{-6}
Mercury (l)	4.7×10^{-6}
Marble	1.2×10^{-6}
Ice	1.2×10^{-6}
Concrete	0.75×10^{-6}
Brick	0.52×10^{-6}
Heavy soil (dry)	0.52×10^{-6}
Glass	0.34×10^{-6}
Glass wool	0.23×10^{-6}
Water (l)	0.14×10^{-6}
Beef	0.14×10^{-6}
Wood (oak)	0.13×10^{-6}

*Multiply by 10.76 to convert to ft^2/s.

EXAMPLE 1–6 Measuring the Thermal Conductivity of a Material

A common way of measuring the thermal conductivity of a material is to sandwich an electric thermofoil heater between two identical samples of the material, as shown in Fig. 1–29. The thickness of the resistance heater, including its cover, which is made of thin silicon rubber, is usually less than 0.5 mm. A circulating fluid such as tap water keeps the exposed ends of the samples at constant temperature. The lateral surfaces of the samples are well insulated to ensure that heat transfer through the samples is one-dimensional. Two thermocouples are embedded into each sample some distance L apart, and a

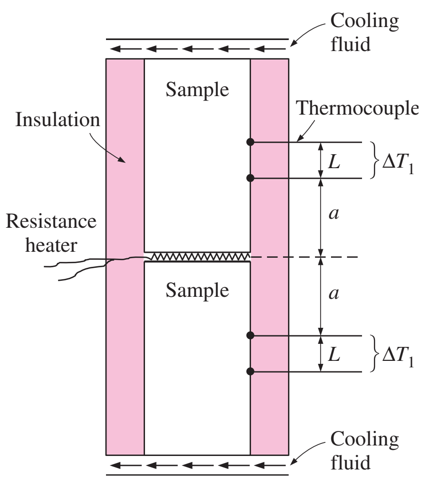


FIGURE 1–29

Apparatus to measure the thermal conductivity of a material using two identical samples and a thin resistance heater (Example 1–6).

differential thermometer reads the temperature drop ΔT across this distance along each sample. When steady operating conditions are reached, the total rate of heat transfer through both samples becomes equal to the electric power drawn by the heater, which is determined by multiplying the electric current by the voltage.

In a certain experiment, cylindrical samples of diameter 5 cm and length 10 cm are used. The two thermocouples in each sample are placed 3 cm apart. After initial transients, the electric heater is observed to draw 0.4 A at 110 V, and both differential thermometers read a temperature difference of 15°C. Determine the thermal conductivity of the sample.

SOLUTION The thermal conductivity of a material is to be determined by ensuring one-dimensional heat conduction, and by measuring temperatures when steady operating conditions are reached.

Assumptions **1** Steady operating conditions exist since the temperature readings do not change with time. **2** Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well insulated, and thus the entire heat generated by the heater is conducted through the samples. **3** The apparatus possesses thermal symmetry.

Analysis The electrical power consumed by the resistance heater and converted to heat is

$$\dot{W}_e = VI = (110\text{ V})(0.4\text{ A}) = 44\text{ W}$$

The rate of heat flow through each sample is

$$\dot{Q} = \tfrac{1}{2}\dot{W}_e = \tfrac{1}{2} \times (44\text{ W}) = 22\text{ W}$$

since only half of the heat generated will flow through each sample because of symmetry. Reading the same temperature difference across the same distance in each sample also confirms that the apparatus possesses thermal symmetry. The heat transfer area is the area normal to the direction of heat flow, which is the cross-sectional area of the cylinder in this case:

$$A = \tfrac{1}{4}\pi D^2 = \tfrac{1}{4}\pi(0.05\text{ m})^2 = 0.00196\text{ m}^2$$

Noting that the temperature drops by 15°C within 3 cm in the direction of heat flow, the thermal conductivity of the sample is determined to be

$$\dot{Q} = kA\frac{\Delta T}{L} \quad \rightarrow \quad k = \frac{\dot{Q}L}{A\,\Delta T} = \frac{(22\text{ W})(0.03\text{ m})}{(0.00196\text{ m}^2)(15^\circ\text{C})} = \mathbf{22.4\ W/m\cdot{}^\circ C}$$

Discussion Perhaps you are wondering if we really need to use two samples in the apparatus, since the measurements on the second sample do not give any additional information. It seems like we can replace the second sample by insulation. Indeed, we do not need the second sample; however, it enables us to verify the temperature measurements on the first sample and provides thermal symmetry, which reduces experimental error.

EXAMPLE 1–7 **Conversion between SI and English Units**

An engineer who is working on the heat transfer analysis of a brick building in English units needs the thermal conductivity of brick. But the only value he can find from his handbooks is 0.72 W/m · °C, which is in SI units. To make matters worse, the engineer does not have a direct conversion factor between the two unit systems for thermal conductivity. Can you help him out?

SOLUTION The situation this engineer is facing is not unique, and most engineers often find themselves in a similar position. A person must be very careful during unit conversion not to fall into some common pitfalls and to avoid some costly mistakes. Although unit conversion is a simple process, it requires utmost care and careful reasoning.

The conversion factors for W and m are straightforward and are given in conversion tables to be

$$1 \text{ W} = 3.41214 \text{ Btu/h}$$
$$1 \text{ m} = 3.2808 \text{ ft}$$

But the conversion of °C into °F is not so simple, and it can be a source of error if one is not careful. Perhaps the first thought that comes to mind is to replace °C by (°F − 32)/1.8 since $T(°\text{C}) = [T(°\text{F}) - 32]/1.8$. But this will be wrong since the °C in the unit W/m · °C represents *per °C change in temperature.* Noting that 1°C change in temperature corresponds to 1.8°F, the proper conversion factor to be used is

$$1°\text{C} = 1.8°\text{F}$$

Substituting, we get

$$1 \text{ W/m} \cdot °\text{C} = \frac{3.41214 \text{ Btu/h}}{(3.2808 \text{ ft})(1.8°\text{F})} = 0.5778 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F}$$

which is the desired conversion factor. Therefore, the thermal conductivity of the brick in English units is

$$\begin{aligned} k_{\text{brick}} &= 0.72 \text{ W/m} \cdot °\text{C} \\ &= 0.72 \times (0.5778 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F}) \\ &= \mathbf{0.42 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F}} \end{aligned}$$

Discussion Note that the thermal conductivity value of a material in English units is about half that in SI units (Fig. 1–30). Also note that we rounded the result to two significant digits (the same number in the original value) since expressing the result in more significant digits (such as 0.4160 instead of 0.42) would falsely imply a more accurate value than the original one.

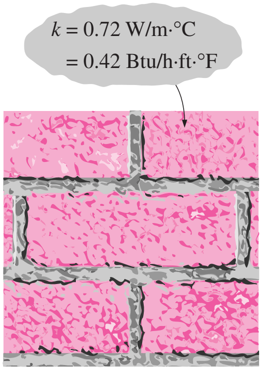


FIGURE 1–30
The thermal conductivity value in English units is obtained by multiplying the value in SI units by 0.5778.

1–7 ▪ CONVECTION

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of *conduction* and *fluid motion.* The faster the fluid motion, the greater the convection heat transfer. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction. The presence of bulk motion of the fluid enhances the heat transfer between the solid surface and the fluid, but it also complicates the determination of heat transfer rates.

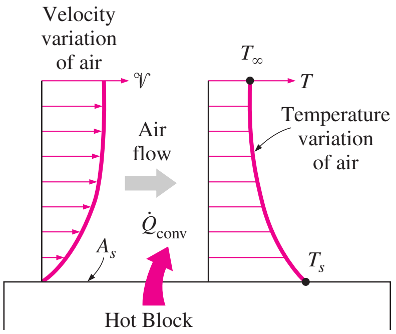


FIGURE 1–31
Heat transfer from a hot surface to air by convection.

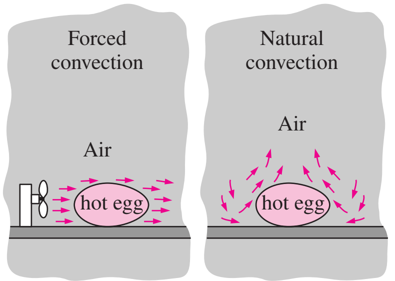


FIGURE 1–32
The cooling of a boiled egg by forced and natural convection.

TABLE 1–5

Typical values of convection heat transfer coefficient

Type of convection	h, W/m$^2 \cdot$ °C*
Free convection of gases	2–25
Free convection of liquids	10–1000
Forced convection of gases	25–250
Forced convection of liquids	50–20,000
Boiling and condensation	2500–100,000

*Multiply by 0.176 to convert to Btu/h · ft^2 · °F.

Consider the cooling of a hot block by blowing cool air over its top surface (Fig. 1–31). Energy is first transferred to the air layer adjacent to the block by conduction. This energy is then carried away from the surface by convection, that is, by the combined effects of conduction within the air that is due to random motion of air molecules and the bulk or macroscopic motion of the air that removes the heated air near the surface and replaces it by the cooler air.

Convection is called **forced convection** if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind. In contrast, convection is called **natural** (or **free**) **convection** if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid (Fig. 1–32). For example, in the absence of a fan, heat transfer from the surface of the hot block in Fig. 1–31 will be by natural convection since any motion in the air in this case will be due to the rise of the warmer (and thus lighter) air near the surface and the fall of the cooler (and thus heavier) air to fill its place. Heat transfer between the block and the surrounding air will be by conduction if the temperature difference between the air and the block is not large enough to overcome the resistance of air to movement and thus to initiate natural convection currents.

Heat transfer processes that involve *change of phase* of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

Despite the complexity of convection, the rate of *convection heat transfer* is observed to be proportional to the temperature difference, and is conveniently expressed by **Newton's law of cooling** as

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad \text{(W)} \tag{1-24}$$

where h is the *convection heat transfer coefficient* in W/m$^2 \cdot$ °C or Btu/h · ft$^2 \cdot$ °F, A_s is the surface area through which convection heat transfer takes place, T_s is the surface temperature, and T_∞ is the temperature of the fluid sufficiently far from the surface. Note that at the surface, the fluid temperature equals the surface temperature of the solid.

The convection heat transfer coefficient h is not a property of the fluid. It is an experimentally determined parameter whose value depends on all the variables influencing convection such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity. Typical values of h are given in Table 1–5.

Some people do not consider convection to be a fundamental mechanism of heat transfer since it is essentially heat conduction in the presence of fluid motion. But we still need to give this combined phenomenon a name, unless we are willing to keep referring to it as "conduction with fluid motion." Thus, it is practical to recognize convection as a separate heat transfer mechanism despite the valid arguments to the contrary.

EXAMPLE 1–8 Measuring Convection Heat Transfer Coefficient

A 2-m-long, 0.3-cm-diameter electrical wire extends across a room at 15°C, as shown in Fig. 1–33. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152°C in steady

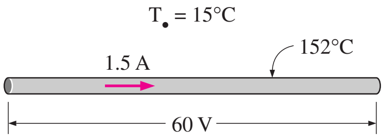


FIGURE 1–33
Schematic for Example 1–8.

operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

SOLUTION The convection heat transfer coefficient for heat transfer from an electrically heated wire to air is to be determined by measuring temperatures when steady operating conditions are reached and the electric power consumed.

Assumptions **1** Steady operating conditions exist since the temperature readings do not change with time. **2** Radiation heat transfer is negligible.

Analysis When steady operating conditions are reached, the rate of heat loss from the wire will equal the rate of heat generation in the wire as a result of resistance heating. That is,

$$\dot{Q} = \dot{E}_{\text{generated}} = VI = (60\text{ V})(1.5\text{ A}) = 90\text{ W}$$

The surface area of the wire is

$$A_s = \pi DL = \pi(0.003\text{ m})(2\text{ m}) = 0.01885\text{ m}^2$$

Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q}_{\text{conv}} = hA_s\,(T_s - T_\infty)$$

Disregarding any heat transfer by radiation and thus assuming all the heat loss from the wire to occur by convection, the convection heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}_{\text{conv}}}{A_s(T_s - T_\infty)} = \frac{90\text{ W}}{(0.01885\text{ m}^2)(152 - 15)°\text{C}} = \mathbf{34.9\ W/m^2\cdot°C}$$

Discussion Note that the simple setup described above can be used to determine the average heat transfer coefficients from a variety of surfaces in air. Also, heat transfer by radiation can be eliminated by keeping the surrounding surfaces at the temperature of the wire.

1–8 ▪ RADIATION

Radiation is the energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules. Unlike conduction and convection, the transfer of energy by radiation does not require the presence of an *intervening medium.* In fact, energy transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.

In heat transfer studies we are interested in *thermal radiation,* which is the form of radiation emitted by bodies because of their temperature. It differs from other forms of electromagnetic radiation such as x-rays, gamma rays, microwaves, radio waves, and television waves that are not related to temperature. All bodies at a temperature above absolute zero emit thermal radiation.

Radiation is a *volumetric phenomenon,* and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees. However, radiation is

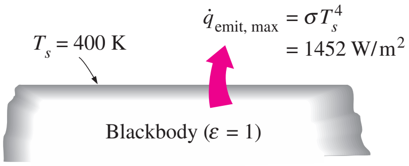


FIGURE 1–34
Blackbody radiation represents the *maximum amount of radiation that can be emitted from a surface at a specified temperature.*

TABLE 1–6

Emissivities of some materials at 300 K

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92–0.97
Asphalt pavement	0.85–0.93
Red brick	0.93–0.96
Human skin	0.95
Wood	0.82–0.92
Soil	0.93–0.96
Water	0.96
Vegetation	0.92–0.96

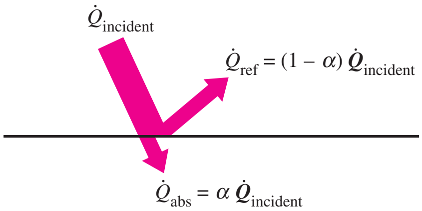


FIGURE 1–35
The absorption of radiation incident on an opaque surface of absorptivity α.

usually considered to be a *surface phenomenon* for solids that are opaque to thermal radiation such as metals, wood, and rocks since the radiation emitted by the interior regions of such material can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few microns from the surface.

The maximum rate of radiation that can be emitted from a surface at an absolute temperature T_s (in K or R) is given by the **Stefan–Boltzmann law** as

$$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4 \qquad \text{(W)} \tag{1-25}$$

where $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$ K^4 or 0.1714×10^{-8} Btu/h $\cdot$ ft$^2 \cdot$ R^4 is the *Stefan–Boltzmann constant.* The idealized surface that emits radiation at this maximum rate is called a **blackbody,** and the radiation emitted by a blackbody is called **blackbody radiation** (Fig. 1–34). The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$\dot{Q}_{\text{emit}} = \varepsilon\sigma A_s T_s^4 \qquad \text{(W)} \tag{1-26}$$

where ε is the **emissivity** of the surface. The property emissivity, whose value is in the range $0 \le \varepsilon \le 1$, is a measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$. The emissivities of some surfaces are given in Table 1–6.

Another important radiation property of a surface is its **absorptivity** α, which is the fraction of the radiation energy incident on a surface that is absorbed by the surface. Like emissivity, its value is in the range $0 \le \alpha \le 1$. A blackbody absorbs the entire radiation incident on it. That is, a blackbody is a perfect absorber ($\alpha = 1$) as it is a perfect emitter.

In general, both ε and α of a surface depend on the temperature and the wavelength of the radiation. **Kirchhoff's law** of radiation states that the emissivity and the absorptivity of a surface at a given temperature and wavelength are equal. In many practical applications, the surface temperature and the temperature of the source of incident radiation are of the same order of magnitude, and the average absorptivity of a surface is taken to be equal to its average emissivity. The rate at which a surface absorbs radiation is determined from (Fig. 1–35)

$$\dot{Q}_{\text{absorbed}} = \alpha\dot{Q}_{\text{incident}} \qquad \text{(W)} \tag{1-27}$$

where $\dot{Q}_{\text{incident}}$ is the rate at which radiation is incident on the surface and α is the absorptivity of the surface. For opaque (nontransparent) surfaces, the portion of incident radiation not absorbed by the surface is reflected back.

The difference between the rates of radiation emitted by the surface and the radiation absorbed is the *net* radiation heat transfer. If the rate of radiation absorption is greater than the rate of radiation emission, the surface is said to be *gaining* energy by radiation. Otherwise, the surface is said to be *losing* energy by radiation. In general, the determination of the net rate of heat transfer by radiation between two surfaces is a complicated matter since it depends on the properties of the surfaces, their orientation relative to each other, and the interaction of the medium between the surfaces with radiation.

When a surface of emissivity ε and surface area A_s at an *absolute temperature* T_s is *completely enclosed* by a much larger (or black) surface at absolute temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by (Fig. 1–36)

$$\dot{Q}_{rad} = \varepsilon\sigma A_s (T_s^4 - T_{surr}^4) \qquad \text{(W)} \tag{1-28}$$

In this special case, the emissivity and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

Radiation heat transfer to or from a surface surrounded by a gas such as air occurs *parallel* to conduction (or convection, if there is bulk gas motion) between the surface and the gas. Thus the total heat transfer is determined by *adding* the contributions of both heat transfer mechanisms. For simplicity and convenience, this is often done by defining a **combined heat transfer coefficient** $h_{combined}$ that includes the effects of both convection and radiation. Then the *total* heat transfer rate to or from a surface by convection and radiation is expressed as

$$\dot{Q}_{total} = h_{combined} A_s (T_s - T_\infty) \qquad \text{(W)} \tag{1-29}$$

Note that the combined heat transfer coefficient is essentially a convection heat transfer coefficient modified to include the effects of radiation.

Radiation is usually significant relative to conduction or natural convection, but negligible relative to forced convection. Thus radiation in forced convection applications is usually disregarded, especially when the surfaces involved have low emissivities and low to moderate temperatures.

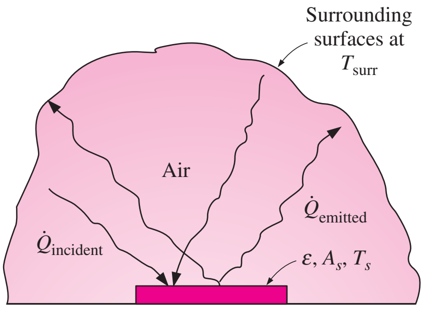


FIGURE 1–36
Radiation heat transfer between a surface and the surfaces surrounding it.

EXAMPLE 1–9 Radiation Effect on Thermal Comfort

It is a common experience to feel "chilly" in winter and "warm" in summer in our homes even when the thermostat setting is kept the same. This is due to the so called "radiation effect" resulting from radiation heat exchange between our bodies and the surrounding surfaces of the walls and the ceiling.

Consider a person standing in a room maintained at 22°C at all times. The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 10°C in winter and 25°C in summer. Determine the rate of radiation heat transfer between this person and the surrounding surfaces if the exposed surface area and the average outer surface temperature of the person are 1.4 m^2 and 30°C, respectively (Fig. 1–37).

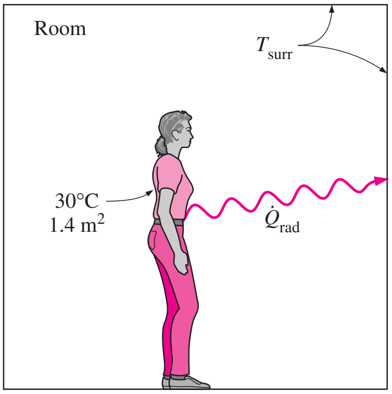


FIGURE 1–37
Schematic for Example 1–9.

SOLUTION The rates of radiation heat transfer between a person and the surrounding surfaces at specified temperatures are to be determined in summer and winter.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is not considered. **3** The person is completely surrounded by the interior surfaces of the room. **4** The surrounding surfaces are at a uniform temperature.

Properties The emissivity of a person is $\varepsilon = 0.95$ (Table 1–6).

Analysis The net rates of radiation heat transfer from the body to the surrounding walls, ceiling, and floor in winter and summer are

$$\dot{Q}_{\text{rad, winter}} = \varepsilon\sigma A_s (T_s^4 - T_{\text{surr, winter}}^4)$$
$$= (0.95)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(1.4\ \text{m}^2)$$
$$\times [(30 + 273)^4 - (10 + 273)^4]\ \text{K}^4$$
$$= \mathbf{152\ W}$$

and

$$\dot{Q}_{\text{rad, summer}} = \varepsilon\sigma A_s (T_s^4 - T_{\text{surr, summer}}^4)$$
$$= (0.95)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(1.4\ \text{m}^2)$$
$$\times [(30 + 273)^4 - (25 + 273)^4]\ \text{K}^4$$
$$= \mathbf{40.9\ W}$$

Discussion Note that we must use *absolute temperatures* in radiation calculations. Also note that the rate of heat loss from the person by radiation is almost four times as large in winter than it is in summer, which explains the "chill" we feel in winter even if the thermostat setting is kept the same.

1–9 ▪ SIMULTANEOUS HEAT TRANSFER MECHANISMS

We mentioned that there are three mechanisms of heat transfer, but not all three can exist simultaneously in a medium. For example, heat transfer is only by conduction in *opaque solids,* but by conduction and radiation in *semitransparent solids.* Thus, a solid may involve conduction and radiation but not convection. However, a solid may involve heat transfer by convection and/or radiation on its surfaces exposed to a fluid or other surfaces. For example, the outer surfaces of a cold piece of rock will warm up in a warmer environment as a result of heat gain by convection (from the air) and radiation (from the sun or the warmer surrounding surfaces). But the inner parts of the rock will warm up as this heat is transferred to the inner region of the rock by conduction.

Heat transfer is by conduction and possibly by radiation in a *still fluid* (no bulk fluid motion) and by convection and radiation in a *flowing fluid.* In the absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion. Convection can be viewed as combined conduction and fluid motion, and conduction in a fluid can be viewed as a special case of convection in the absence of any fluid motion (Fig. 1–38).

Thus, when we deal with heat transfer through a *fluid,* we have either *conduction* or *convection,* but not both. Also, gases are practically transparent to radiation, except that some gases are known to absorb radiation strongly at certain wavelengths. Ozone, for example, strongly absorbs ultraviolet radiation. But in most cases, a gas between two solid surfaces does not interfere with radiation and acts effectively as a vacuum. Liquids, on the other hand, are usually strong absorbers of radiation.

Finally, heat transfer through a *vacuum* is by radiation only since conduction or convection requires the presence of a material medium.

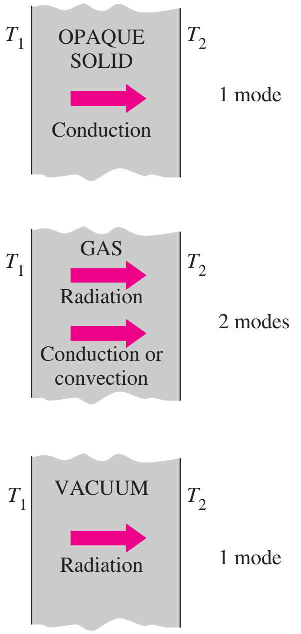


FIGURE 1–38
Although there are three mechanisms of heat transfer, a medium may involve only two of them simultaneously.

EXAMPLE 1–10 Heat Loss from a Person

Consider a person standing in a breezy room at 20°C. Determine the total rate of heat transfer from this person if the exposed surface area and the average outer surface temperature of the person are 1.6 m^2 and 29°C, respectively, and the convection heat transfer coefficient is 6 W/m^2 · °C (Fig. 1–39).

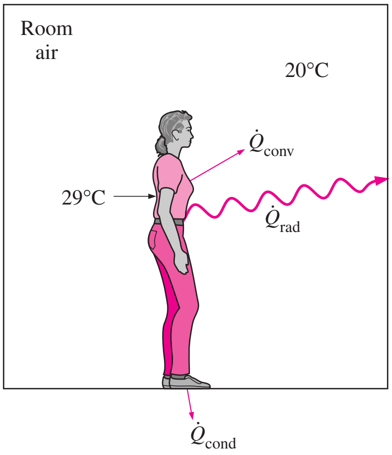


FIGURE 1–39
Heat transfer from the person described in Example 1–10.

SOLUTION The total rate of heat transfer from a person by both convection and radiation to the surrounding air and surfaces at specified temperatures is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The person is completely surrounded by the interior surfaces of the room. **3** The surrounding surfaces are at the same temperature as the air in the room. **4** Heat conduction to the floor through the feet is negligible.

Properties The emissivity of a person is $\varepsilon = 0.95$ (Table 1–6).

Analysis The heat transfer between the person and the air in the room will be by convection (instead of conduction) since it is conceivable that the air in the vicinity of the skin or clothing will warm up and rise as a result of heat transfer from the body, initiating natural convection currents. It appears that the experimentally determined value for the rate of convection heat transfer in this case is 6 W per unit surface area (m^2) per unit temperature difference (in K or °C) between the person and the air away from the person. Thus, the rate of convection heat transfer from the person to the air in the room is

$$\begin{aligned}\dot{Q}_{conv} &= hA_s\,(T_s - T_\infty)\\ &= (6\ \text{W/m}^2\cdot{}^\circ\text{C})(1.6\ \text{m}^2)(29 - 20)^\circ\text{C}\\ &= 86.4\ \text{W}\end{aligned}$$

The person will also lose heat by radiation to the surrounding wall surfaces. We take the temperature of the surfaces of the walls, ceiling, and floor to be equal to the air temperature in this case for simplicity, but we recognize that this does not need to be the case. These surfaces may be at a higher or lower temperature than the average temperature of the room air, depending on the outdoor conditions and the structure of the walls. Considering that air does not intervene with radiation and the person is completely enclosed by the surrounding surfaces, the net rate of radiation heat transfer from the person to the surrounding walls, ceiling, and floor is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon\sigma A_s\,(T_s^4 - T_{surr}^4)\\ &= (0.95)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.6\ \text{m}^2)\\ &\quad\times[(29+273)^4 - (20+273)^4]\ \text{K}^4\\ &= 81.7\ \text{W}\end{aligned}$$

Note that we must use *absolute* temperatures in radiation calculations. Also note that we used the emissivity value for the skin and clothing at room temperature since the emissivity is not expected to change significantly at a slightly higher temperature.

Then the rate of total heat transfer from the body is determined by adding these two quantities:

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = (86.4 + 81.7)\ \text{W} = \mathbf{168.1\ W}$$

Discussion The heat transfer would be much higher if the person were not dressed since the exposed surface temperature would be higher. Thus, an important function of the clothes is to serve as a barrier against heat transfer.

In these calculations, heat transfer through the feet to the floor by conduction, which is usually very small, is neglected. Heat transfer from the skin by perspiration, which is the dominant mode of heat transfer in hot environments, is not considered here.

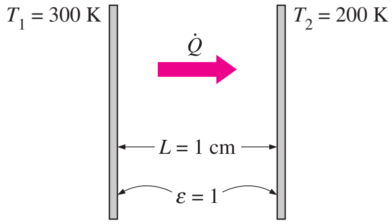


FIGURE 1–40
Schematic for Example 1–11.

EXAMPLE 1–11 Heat Transfer between Two Isothermal Plates

Consider steady heat transfer between two large parallel plates at constant temperatures of $T_1 = 300$ K and $T_2 = 200$ K that are $L = 1$ cm apart, as shown in Fig. 1–40. Assuming the surfaces to be black (emissivity $\varepsilon = 1$), determine the rate of heat transfer between the plates per unit surface area assuming the gap between the plates is (*a*) filled with atmospheric air, (*b*) evacuated, (*c*) filled with urethane insulation, and (*d*) filled with superinsulation that has an apparent thermal conductivity of 0.00002 W/m · °C.

SOLUTION The total rate of heat transfer between two large parallel plates at specified temperatures is to be determined for four different cases.

Assumptions **1** Steady operating conditions exist. **2** There are no natural convection currents in the air between the plates. **3** The surfaces are black and thus $\varepsilon = 1$.

Properties The thermal conductivity at the average temperature of 250 K is $k = 0.0219$ W/m · °C for air (Table A-11), 0.026 W/m · °C for urethane insulation (Table A-6), and 0.00002 W/m · °C for the superinsulation.

Analysis (*a*) The rates of conduction and radiation heat transfer between the plates through the air layer are

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.0219 \text{ W/m} \cdot {}^\circ\text{C})(1 \text{ m}^2)\frac{(300 - 200){}^\circ\text{C}}{0.01 \text{ m}} = 219 \text{ W}$$

and

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A(T_1^4 - T_2^4) \\ &= (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1 \text{ m}^2)[(300 \text{ K})^4 - (200 \text{ K})^4] = 368 \text{ W}\end{aligned}$$

Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} + \dot{Q}_{\text{rad}} = 219 + 368 = \mathbf{587\ W}$$

The heat transfer rate in reality will be higher because of the natural convection currents that are likely to occur in the air space between the plates.

(*b*) When the air space between the plates is evacuated, there will be no conduction or convection, and the only heat transfer between the plates will be by radiation. Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{rad}} = \mathbf{368\ W}$$

(*c*) An opaque solid material placed between two plates blocks direct radiation heat transfer between the plates. Also, the thermal conductivity of an insulating material accounts for the radiation heat transfer that may be occurring through

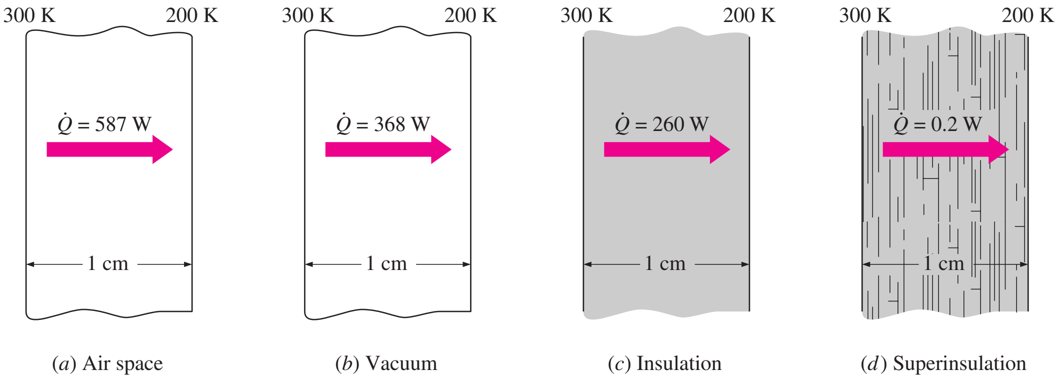


FIGURE 1–41
Different ways of reducing heat transfer between two isothermal plates, and their effectiveness.

the voids in the insulating material. The rate of heat transfer through the urethane insulation is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.026 \text{ W/m}\cdot{}^\circ\text{C})(1 \text{ m}^2)\frac{(300 - 200)^\circ\text{C}}{0.01 \text{ m}} = \mathbf{260\ W}$$

Note that heat transfer through the urethane material is less than the heat transfer through the air determined in (*a*), although the thermal conductivity of the insulation is higher than that of air. This is because the insulation blocks the radiation whereas air transmits it.

(*d*) The layers of the superinsulation prevent any direct radiation heat transfer between the plates. However, radiation heat transfer between the sheets of superinsulation does occur, and the apparent thermal conductivity of the superinsulation accounts for this effect. Therefore,

$$\dot{Q}_{\text{total}} = kA\frac{T_1 - T_2}{L} = (0.00002 \text{ W/m}\cdot{}^\circ\text{C})(1 \text{ m}^2)\frac{(300 - 200)^\circ\text{C}}{0.01 \text{ m}} = \mathbf{0.2\ W}$$

which is $\frac{1}{1840}$ of the heat transfer through the vacuum. The results of this example are summarized in Fig. 1–41 to put them into perspective.

Discussion This example demonstrates the effectiveness of superinsulations, which are discussed in the next chapter, and explains why they are the insulation of choice in critical applications despite their high cost.

EXAMPLE 1–12 Heat Transfer in Conventional and Microwave Ovens

The fast and efficient cooking of microwave ovens made them one of the essential appliances in modern kitchens (Fig. 1–42). Discuss the heat transfer mechanisms associated with the cooking of a chicken in microwave and conventional ovens, and explain why cooking in a microwave oven is more efficient.

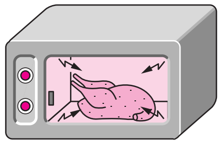


FIGURE 1–42
A chicken being cooked in a microwave oven (Example 1–12).

SOLUTION Food is cooked in a microwave oven by absorbing the electromagnetic radiation energy generated by the microwave tube, called the magnetron.

The radiation emitted by the magnetron is not thermal radiation, since its emission is not due to the temperature of the magnetron; rather, it is due to the conversion of electrical energy into electromagnetic radiation at a specified wavelength. The wavelength of the microwave radiation is such that it is *reflected* by metal surfaces; *transmitted* by the cookware made of glass, ceramic, or plastic; and *absorbed* and converted to internal energy by food (especially the water, sugar, and fat) molecules.

In a microwave oven, the *radiation* that strikes the chicken is absorbed by the skin of the chicken and the outer parts. As a result, the temperature of the chicken at and near the skin rises. Heat is then *conducted* toward the inner parts of the chicken from its outer parts. Of course, some of the heat absorbed by the outer surface of the chicken is lost to the air in the oven by *convection*.

In a conventional oven, the air in the oven is first heated to the desired temperature by the electric or gas heating element. This preheating may take several minutes. The heat is then transferred from the air to the skin of the chicken by *natural convection* in most ovens or by *forced convection* in the newer convection ovens that utilize a fan. The air motion in convection ovens increases the convection heat transfer coefficient and thus decreases the cooking time. Heat is then *conducted* toward the inner parts of the chicken from its outer parts as in microwave ovens.

Microwave ovens replace the slow convection heat transfer process in conventional ovens by the instantaneous radiation heat transfer. As a result, microwave ovens transfer energy to the food at full capacity the moment they are turned on, and thus they cook faster while consuming less energy.

EXAMPLE 1–13 Heating of a Plate by Solar Energy

A thin metal plate is insulated on the back and exposed to solar radiation at the front surface (Fig. 1–43). The exposed surface of the plate has an absorptivity of 0.6 for solar radiation. If solar radiation is incident on the plate at a rate of 700 W/m^2 and the surrounding air temperature is 25°C, determine the surface temperature of the plate when the heat loss by convection and radiation equals the solar energy absorbed by the plate. Assume the combined convection and radiation heat transfer coefficient to be 50 W/m$^2 \cdot$ °C.

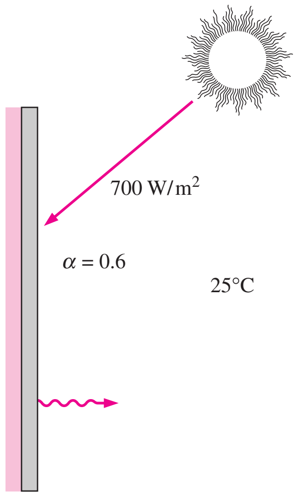


FIGURE 1–43
Schematic for Example 1–13.

SOLUTION The back side of the thin metal plate is insulated and the front side is exposed to solar radiation. The surface temperature of the plate is to be determined when it stabilizes.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulated side of the plate is negligible. **3** The heat transfer coefficient remains constant.

Properties The solar absorptivity of the plate is given to be $\alpha = 0.6$.

Analysis The absorptivity of the plate is 0.6, and thus 60 percent of the solar radiation incident on the plate will be absorbed continuously. As a result, the temperature of the plate will rise, and the temperature difference between the plate and the surroundings will increase. This increasing temperature difference will cause the rate of heat loss from the plate to the surroundings to increase. At some point, the rate of heat loss from the plate will equal the rate of solar energy absorbed, and the temperature of the plate will no longer change. The temperature of the plate when steady operation is established is determined from

$$\dot{E}_{\text{gained}} = \dot{E}_{\text{lost}} \qquad \text{or} \qquad \alpha A_s\, \dot{q}_{\text{incident, solar}} = h_{\text{combined}} A_s\, (T_s - T_\infty)$$

Solving for T_s and substituting, the plate surface temperature is determined to be

$$T_s = T_\infty + \alpha \frac{\dot{q}_{\text{incident, solar}}}{h_{\text{combined}}} = 25°\text{C} + \frac{0.6 \times (700 \text{ W/m}^2)}{50 \text{ W/m}^2 \cdot °\text{C}} = \mathbf{33.4°C}$$

Discussion Note that the heat losses will prevent the plate temperature from rising above 33.4°C. Also, the combined heat transfer coefficient accounts for the effects of both convection and radiation, and thus it is very convenient to use in heat transfer calculations when its value is known with reasonable accuracy.

1–10 ▪ PROBLEM-SOLVING TECHNIQUE

The first step in learning any science is to grasp the fundamentals, and to gain a sound knowledge of it. The next step is to master the fundamentals by putting this knowledge to test. This is done by solving significant real-world problems. Solving such problems, especially complicated ones, requires a systematic approach. By using a step-by-step approach, an engineer can reduce the solution of a complicated problem into the solution of a series of simple problems (Fig. 1–44). When solving a problem, we recommend that you use the following steps zealously as applicable. This will help you avoid some of the common pitfalls associated with problem solving.

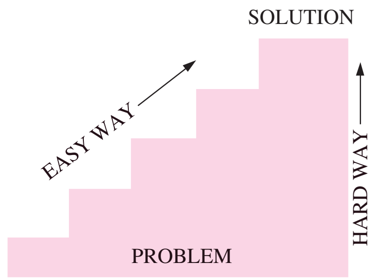


FIGURE 1–44
A step-by-step approach can greatly simplify problem solving.

Step 1: Problem Statement

In your own words, briefly state the problem, the key information given, and the quantities to be found. This is to make sure that you understand the problem and the objectives before you attempt to solve the problem.

Step 2: Schematic

Draw a realistic sketch of the physical system involved, and list the relevant information on the figure. The sketch does not have to be something elaborate, but it should resemble the actual system and show the key features. Indicate any energy and mass interactions with the surroundings. Listing the given information on the sketch helps one to see the entire problem at once. Also, check for properties that remain constant during a process (such as temperature during an isothermal process), and indicate them on the sketch.

Step 3: Assumptions

State any appropriate assumptions made to simplify the problem to make it possible to obtain a solution. Justify the questionable assumptions. Assume reasonable values for missing quantities that are necessary. For example, in the absence of specific data for atmospheric pressure, it can be taken to be

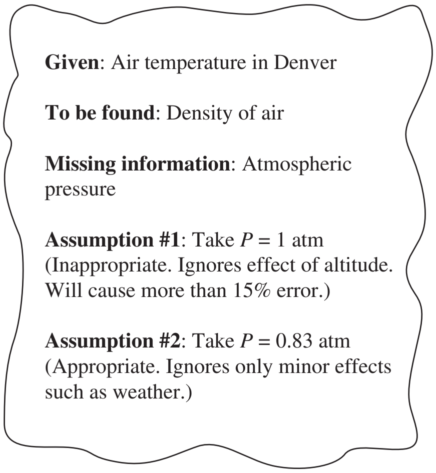


FIGURE 1–45
The assumptions made while solving an engineering problem must be reasonable and justifiable.

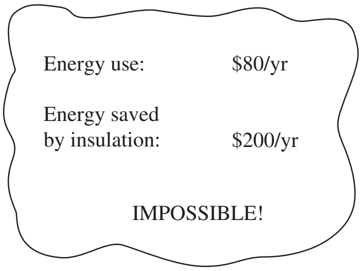


FIGURE 1–46
The results obtained from an engineering analysis must be checked for reasonableness.

1 atm. However, it should be noted in the analysis that the atmospheric pressure decreases with increasing elevation. For example, it drops to 0.83 atm in Denver (elevation 1610 m) (Fig. 1–45).

Step 4: Physical Laws

Apply all the relevant basic physical laws and principles (such as the conservation of energy), and reduce them to their simplest form by utilizing the assumptions made. However, the region to which a physical law is applied must be clearly identified first. For example, the heating or cooling of a canned drink is usually analyzed by applying the conservation of energy principle to the entire can.

Step 5: Properties

Determine the unknown properties at known states necessary to solve the problem from property relations or tables. List the properties separately, and indicate their source, if applicable.

Step 6: Calculations

Substitute the known quantities into the simplified relations and perform the calculations to determine the unknowns. Pay particular attention to the units and unit cancellations, and remember that a dimensional quantity without a unit is meaningless. Also, don't give a false implication of high accuracy by copying all the digits from the screen of the calculator—round the results to an appropriate number of significant digits.

Step 7: Reasoning, Verification, and Discussion

Check to make sure that the results obtained are reasonable and intuitive, and verify the validity of the questionable assumptions. Repeat the calculations that resulted in unreasonable values. For example, insulating a water heater that uses $80 worth of natural gas a year cannot result in savings of $200 a year (Fig. 1–46).

Also, point out the significance of the results, and discuss their implications. State the conclusions that can be drawn from the results, and any recommendations that can be made from them. Emphasize the limitations under which the results are applicable, and caution against any possible misunderstandings and using the results in situations where the underlying assumptions do not apply. For example, if you determined that wrapping a water heater with a $20 insulation jacket will reduce the energy cost by $30 a year, indicate that the insulation will pay for itself from the energy it saves in less than a year. However, also indicate that the analysis does not consider labor costs, and that this will be the case if you install the insulation yourself.

Keep in mind that you present the solutions to your instructors, and any engineering analysis presented to others is a form of communication. Therefore neatness, organization, completeness, and visual appearance are of utmost importance for maximum effectiveness. Besides, neatness also serves as a great checking tool since it is very easy to spot errors and inconsistencies in a neat work. Carelessness and skipping steps to save time often ends up costing more time and unnecessary anxiety.

The approach just described is used in the solved example problems without explicitly stating each step, as well as in the Solutions Manual of this text. For some problems, some of the steps may not be applicable or necessary. However, we cannot overemphasize the importance of a logical and orderly approach to problem solving. Most difficulties encountered while solving a problem are not due to a lack of knowledge; rather, they are due to a lack of coordination. You are strongly encouraged to follow these steps in problem solving until you develop your own approach that works best for you.

A Remark on Significant Digits

In engineering calculations, the information given is not known to more than a certain number of significant digits, usually three digits. Consequently, the results obtained cannot possibly be accurate to more significant digits. Reporting results in more significant digits implies greater accuracy than exists, and it should be avoided.

For example, consider a 3.75-L container filled with gasoline whose density is 0.845 kg/L, and try to determine its mass. Probably the first thought that comes to your mind is to multiply the volume and density to obtain 3.16875 kg for the mass, which falsely implies that the mass determined is accurate to six significant digits. In reality, however, the mass cannot be more accurate than three significant digits since both the volume and the density are accurate to three significant digits only. Therefore, the result should be rounded to three significant digits, and the mass should be reported to be 3.17 kg instead of what appears in the screen of the calculator. The result 3.16875 kg would be correct only if the volume and density were given to be 3.75000 L and 0.845000 kg/L, respectively. The value 3.75 L implies that we are fairly confident that the volume is accurate within $\pm$0.01 L, and it cannot be 3.74 or 3.76 L. However, the volume can be 3.746, 3.750, 3.753, etc., since they all round to 3.75 L (Fig. 1–47). It is more appropriate to retain all the digits during intermediate calculations, and to do the rounding in the final step since this is what a computer will normally do.

When solving problems, we will assume the given information to be accurate to at least three significant digits. Therefore, if the length of a pipe is given to be 40 m, we will assume it to be 40.0 m in order to justify using three significant digits in the final results. You should also keep in mind that all experimentally determined values are subject to measurement errors, and such errors will reflect in the results obtained. For example, if the density of a substance has an uncertainty of 2 percent, then the mass determined using this density value will also have an uncertainty of 2 percent.

You should also be aware that we sometimes knowingly introduce small errors in order to avoid the trouble of searching for more accurate data. For example, when dealing with liquid water, we just use the value of 1000 kg/m^3 for density, which is the density value of pure water at 0°C. Using this value at 75°C will result in an error of 2.5 percent since the density at this temperature is 975 kg/m^3. The minerals and impurities in the water will introduce additional error. This being the case, you should have no reservation in rounding the final results to a reasonable number of significant digits. Besides, having a few percent uncertainty in the results of engineering analysis is usually the norm, not the exception.

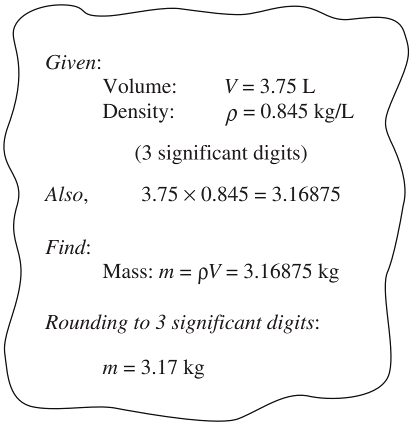


FIGURE 1–47
A result with more significant digits than that of given data falsely implies more accuracy.

Engineering Software Packages

Perhaps you are wondering why we are about to undertake a painstaking study of the fundamentals of heat transfer. After all, almost all such problems we are likely to encounter in practice can be solved using one of several sophisticated software packages readily available in the market today. These software packages not only give the desired numerical results, but also supply the outputs in colorful graphical form for impressive presentations. It is unthinkable to practice engineering today without using some of these packages. This tremendous computing power available to us at the touch of a button is both a blessing and a curse. It certainly enables engineers to solve problems easily and quickly, but it also opens the door for abuses and misinformation. In the hands of poorly educated people, these software packages are as dangerous as sophisticated powerful weapons in the hands of poorly trained soldiers.

Thinking that a person who can use the engineering software packages without proper training on fundamentals can practice engineering is like thinking that a person who can use a wrench can work as a car mechanic. If it were true that the engineering students do not need all these fundamental courses they are taking because practically everything can be done by computers quickly and easily, then it would also be true that the employers would no longer need high-salaried engineers since any person who knows how to use a word-processing program can also learn how to use those software packages. However, the statistics show that the need for engineers is on the rise, not on the decline, despite the availability of these powerful packages.

FIGURE 1–48
An excellent word-processing program does not make a person a good writer; it simply makes a good writer a better and more efficient writer.

We should always remember that all the computing power and the engineering software packages available today are just *tools*, and tools have meaning only in the hands of masters. Having the best word-processing program does not make a person a good writer, but it certainly makes the job of a good writer much easier and makes the writer more productive (Fig. 1–48). Hand calculators did not eliminate the need to teach our children how to add or subtract, and the sophisticated medical software packages did not take the place of medical school training. Neither will engineering software packages replace the traditional engineering education. They will simply cause a shift in emphasis in the courses from mathematics to physics. That is, more time will be spent in the classroom discussing the physical aspects of the problems in greater detail, and less time on the mechanics of solution procedures.

All these marvelous and powerful tools available today put an extra burden on today's engineers. They must still have a thorough understanding of the fundamentals, develop a "feel" of the physical phenomena, be able to put the data into proper perspective, and make sound engineering judgments, just like their predecessors. However, they must do it much better, and much faster, using more realistic models because of the powerful tools available today. The engineers in the past had to rely on hand calculations, slide rules, and later hand calculators and computers. Today they rely on software packages. The easy access to such power and the possibility of a simple misunderstanding or misinterpretation causing great damage make it more important today than ever to have a solid training in the fundamentals of engineering. In this text we make an extra effort to put the emphasis on developing an intuitive and physical understanding of natural phenomena instead of on the mathematical details of solution procedures.

Engineering Equation Solver (EES)

EES is a program that solves systems of linear or nonlinear algebraic or differential equations numerically. It has a large library of built-in thermodynamic property functions as well as mathematical functions, and allows the user to supply additional property data. Unlike some software packages, EES does not solve thermodynamic problems; it only solves the equations supplied by the user. Therefore, the user must understand the problem and formulate it by applying any relevant physical laws and relations. EES saves the user considerable time and effort by simply solving the resulting mathematical equations. This makes it possible to attempt significant engineering problems not suitable for hand calculations, and to conduct parametric studies quickly and conveniently. EES is a very powerful yet intuitive program that is very easy to use, as shown in the examples below. The use and capabilities of EES are explained in Appendix 3.

Heat Transfer Tools (HTT)

One software package specifically designed to help bridge the gap between the textbook fundamentals and these powerful software packages is *Heat Transfer Tools,* which may be ordered "bundled" with this text. The software included in that package was developed for instructional use only and thus is applicable only to fundamental problems in heat transfer. While it does not have the power and functionality of the professional, commercial packages, HTT uses research-grade numerical algorithms behind the scenes and modern graphical user interfaces. Each module is custom designed and applicable to a single, fundamental topic in heat transfer to ensure that almost all time at the computer is spent learning heat transfer. Nomenclature and all inputs and outputs are consistent with those used in this and most other textbooks in the field. In addition, with the capability of testing parameters so readily available, one can quickly gain a physical feel for the effects of all the nondimensional numbers that arise in heat transfer.

EXAMPLE 1–14 Solving a System of Equations with EES

The difference of two numbers is 4, and the sum of the squares of these two numbers is equal to the sum of the numbers plus 20. Determine these two numbers.

SOLUTION Relations are given for the difference and the sum of the squares of two numbers. They are to be determined.

Analysis We start the EES program by double-clicking on its icon, open a new file, and type the following on the blank screen that appears:

```
x-y=4
x^2+y^2=x+y+20
```

which is an exact mathematical expression of the problem statement with x and y denoting the unknown numbers. The solution to this system of two nonlinear equations with two unknowns is obtained by a single click on the "calculator" symbol on the taskbar. It gives

$$x=5 \quad \text{and} \quad y=1$$

Discussion Note that all we did is formulate the problem as we would on paper; EES took care of all the mathematical details of solution. Also note that equations can be linear or nonlinear, and they can be entered in any order with unknowns on either side. Friendly equation solvers such as EES allow the user to concentrate on the physics of the problem without worrying about the mathematical complexities associated with the solution of the resulting system of equations.

Throughout the text, problems that are unsuitable for hand calculations and are intended to be solved using EES are indicated by a computer icon.

TOPIC OF SPECIAL INTEREST*

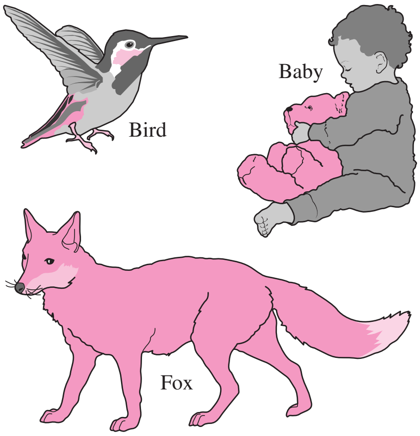


FIGURE 1–49
Most animals come into this world with built-in insulation, but human beings come with a delicate skin.

Thermal Comfort

Unlike animals such as a fox or a bear that are born with built-in furs, human beings come into this world with little protection against the harsh environmental conditions (Fig. 1–49). Therefore, we can claim that the search for thermal comfort dates back to the beginning of human history. It is believed that early human beings lived in caves that provided shelter as well as protection from extreme thermal conditions. Probably the first form of heating system used was *open fire,* followed by fire in dwellings through the use of a *chimney* to vent out the combustion gases. The concept of *central heating* dates back to the times of the Romans, who heated homes by utilizing double-floor construction techniques and passing the fire's fumes through the opening between the two floor layers. The Romans were also the first to use *transparent windows* made of mica or glass to keep the wind and rain out while letting the light in. Wood and coal were the primary energy sources for heating, and oil and candles were used for lighting. The ruins of south-facing houses indicate that the value of *solar heating* was recognized early in the history.

The term **air-conditioning** is usually used in a restricted sense to imply cooling, but in its broad sense it means *to condition* the air to the desired level by heating, cooling, humidifying, dehumidifying, cleaning, and deodorizing. The purpose of the air-conditioning system of a building is to provide *complete thermal comfort* for its occupants. Therefore, we need to understand the thermal aspects of the *human body* in order to design an effective air-conditioning system.

The building blocks of living organisms are *cells,* which resemble miniature factories performing various functions necessary for the survival of organisms. The human body contains about 100 trillion cells with an average diameter of 0.01 mm. In a typical cell, thousands of chemical reactions

*This section can be skipped without a loss in continuity.

occur every second during which some molecules are broken down and energy is released and some new molecules are formed. The high level of chemical activity in the cells that maintain the human body temperature at a temperature of 37.0°C (98.6°F) while performing the necessary bodily functions is called the **metabolism.** In simple terms, metabolism refers to the burning of foods such as carbohydrates, fat, and protein. The metabolizable energy content of foods is usually expressed by nutritionists in terms of the capitalized Calorie. One Calorie is equivalent to 1 Cal = 1 kcal = 4.1868 kJ.

The rate of metabolism at the resting state is called the *basal metabolic rate,* which is the rate of metabolism required to keep a body performing the necessary bodily functions such as breathing and blood circulation at zero external activity level. The metabolic rate can also be interpreted as the energy consumption rate for a body. For an *average man* (30 years old, 70 kg, 1.73 m high, 1.8 m^2 surface area), the basal metabolic rate is 84 W. That is, the body is converting chemical energy of the food (or of the body fat if the person had not eaten) into heat at a rate of 84 J/s, which is then dissipated to the surroundings. The metabolic rate increases with the *level of activity,* and it may exceed 10 times the basal metabolic rate when someone is doing strenuous exercise. That is, two people doing heavy exercising in a room may be supplying more energy to the room than a 1-kW resistance heater (Fig. 1–50). An average man generates heat at a rate of 108 W while reading, writing, typing, or listening to a lecture in a classroom in a seated position. The maximum metabolic rate of an average man is 1250 W at age 20 and 730 at age 70. The corresponding rates for women are about 30 percent lower. Maximum metabolic rates of trained athletes can exceed 2000 W.

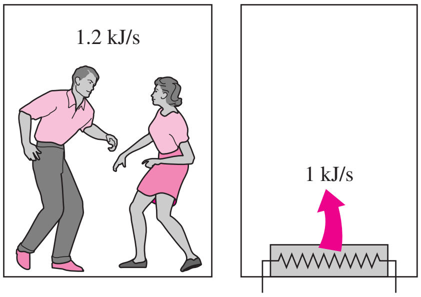


FIGURE 1–50
Two fast-dancing people supply more heat to a room than a 1-kW resistance heater.

Metabolic rates during various activities are given in Table 1–7 per unit body surface area. The **surface area** of a nude body was given by D. DuBois in 1916 as

$$A_s = 0.202 m^{0.425} h^{0.725} \qquad (\text{m}^2) \tag{1-30}$$

where m is the mass of the body in kg and h is the height in m. *Clothing* increases the exposed surface area of a person by up to about 50 percent. The metabolic rates given in the table are sufficiently accurate for most purposes, but there is considerable uncertainty at high activity levels. More accurate values can be determined by measuring the rate of respiratory *oxygen consumption,* which ranges from about 0.25 L/min for an average resting man to more than 2 L/min during extremely heavy work. The entire energy released during metabolism can be assumed to be released as *heat* (in sensible or latent forms) since the external mechanical work done by the muscles is very small. Besides, the work done during most activities such as walking or riding an exercise bicycle is eventually converted to heat through friction.

The comfort of the human body depends primarily on three environmental factors: the temperature, relative humidity, and air motion. The temperature of the environment is the single most important index of comfort. Extensive research is done on human subjects to determine the "**thermal comfort zone**" and to identify the conditions under which the body feels

TABLE 1–7

Metabolic rates during various activities (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 8, Table 4).

Activity	Metabolic rate* W/m^2
Resting:	
Sleeping	40
Reclining	45
Seated, quiet	60
Standing, relaxed	70
Walking (on the level):	
2 mph (0.89 m/s)	115
3 mph (1.34 m/s)	150
4 mph (1.79 m/s)	220
Office Activities:	
Reading, seated	55
Writing	60
Typing	65
Filing, seated	70
Filing, standing	80
Walking about	100
Lifting/packing	120
Driving/Flying:	
Car	60–115
Aircraft, routine	70
Heavy vehicle	185
Miscellaneous Occupational Activities:	
Cooking	95–115
Cleaning house	115–140
Machine work:	
Light	115–140
Heavy	235
Handling 50-kg bags	235
Pick and shovel work	235–280
Miscellaneous Leisure Activities:	
Dancing, social	140–255
Calisthenics/exercise	175–235
Tennis, singles	210–270
Basketball	290–440
Wrestling, competitive	410–505

*Multiply by 1.8 m^2 to obtain metabolic rates for an average man. Multiply by 0.3171 to convert to Btu/h · ft^2.

comfortable in an environment. It has been observed that most normally clothed people resting or doing light work feel comfortable in the *operative temperature* (roughly, the average temperature of air and surrounding surfaces) range of 23°C to 27°C or 73°C to 80°F (Fig. 1–51). For unclothed people, this range is 29°C to 31°C. Relative humidity also has a considerable effect on comfort since it is a measure of air's ability to absorb moisture and thus it affects the amount of heat a body can dissipate by evaporation. High relative humidity slows down heat rejection by evaporation, especially at high temperatures, and low relative humidity speeds it up. The desirable level of *relative humidity* is the broad range of 30 to 70 percent, with 50 percent being the most desirable level. Most people at these conditions feel neither hot nor cold, and the body does not need to activate any of the defense mechanisms to maintain the normal body temperature (Fig. 1–52).

Another factor that has a major effect on thermal comfort is **excessive air motion** or **draft,** which causes undesired local cooling of the human body. Draft is identified by many as a most annoying factor in work places, automobiles, and airplanes. Experiencing discomfort by draft is most common among people wearing indoor clothing and doing light sedentary work, and least common among people with high activity levels. The air velocity should be kept below 9 m/min (30 ft/min) in winter and 15 m/min (50 ft/min) in summer to minimize discomfort by draft, especially when the air is cool. A low level of air motion is desirable as it removes the warm, moist air that builds around the body and replaces it with fresh air. Therefore, air motion should be strong enough to remove heat and moisture from the vicinity of the body, but gentle enough to be unnoticed. High speed air motion causes discomfort outdoors as well. For example, an environment at 10°C (50°F) with 48 km/h winds feels as cold as an environment at −7°C (20°F) with 3 km/h winds because of the chilling effect of the air motion (the wind-chill factor).

A comfort system should provide *uniform conditions* throughout the living space to avoid discomfort caused by nonuniformities such as *drafts, asymmetric thermal radiation, hot or cold floors,* and *vertical temperature stratification.* **Asymmetric thermal radiation** is caused by the *cold surfaces* of large windows, uninsulated walls, or cold products and the *warm surfaces* of gas or electric radiant heating panels on the walls or ceiling, solar-heated masonry walls or ceilings, and warm machinery. Asymmetric radiation causes discomfort by exposing different sides of the body to surfaces at different temperatures and thus to different heat loss or gain by radiation. A person whose left side is exposed to a cold window, for example, will feel like heat is being drained from that side of his or her body (Fig. 1–53). For thermal comfort, the radiant temperature asymmetry should not exceed 5°C in the vertical direction and 10°C in the horizontal direction. The unpleasant effect of radiation asymmetry can be minimized by properly sizing and installing heating panels, using double-pane windows, and providing generous insulation at the walls and the roof.

Direct contact with **cold** or **hot floor surfaces** also causes localized discomfort in the feet. The temperature of the floor depends on the way it is *constructed* (being directly on the ground or on top of a heated room, being made of wood or concrete, the use of insulation, etc.) as well as the *floor* *covering used* such as pads, carpets, rugs, and linoleum. A floor temperature of 23 to 25°C is found to be comfortable to most people. The floor asymmetry loses its significance for people with footwear. An effective and economical way of raising the floor temperature is to use radiant heating panels instead of turning the thermostat up. Another nonuniform condition that causes discomfort is **temperature stratification** in a room that exposes the head and the feet to different temperatures. For thermal comfort, the temperature difference between the head and foot levels should not exceed 3°C. This effect can be minimized by using destratification fans.

It should be noted that no thermal environment will please everyone. No matter what we do, some people will express some discomfort. The thermal comfort zone is based on a 90 percent acceptance rate. That is, an environment is deemed comfortable if only 10 percent of the people are dissatisfied with it. Metabolism decreases somewhat with *age,* but it has no effect on the comfort zone. Research indicates that there is no appreciable difference between the environments preferred by old and young people. Experiments also show that *men* and *women* prefer almost the same environment. The metabolism rate of women is somewhat lower, but this is compensated by their slightly lower skin temperature and evaporative loss. Also, there is no significant variation in the comfort zone from one part of the world to another and from winter to summer. Therefore, the same thermal comfort conditions can be used *throughout the world* in any season. Also, people cannot *acclimatize* themselves to prefer different comfort conditions.

In a **cold environment,** the rate of heat loss from the body may exceed the rate of metabolic heat generation. Average specific heat of the human body is 3.49 kJ/kg · °C, and thus each 1°C drop in body temperature corresponds to a deficit of 244 kJ in body heat content for an average 70-kg man. A drop of 0.5°C in mean body temperature causes noticeable but acceptable discomfort. A drop of 2.6°C causes extreme discomfort. A sleeping person will wake up when his or her mean body temperature drops by 1.3°C (which normally shows up as a 0.5°C drop in the deep body and 3°C in the skin area). The drop of deep body temperature below 35°C may damage the body temperature regulation mechanism, while a drop below 28°C may be fatal. Sedentary people reported to feel *comfortable* at a *mean skin temperature* of 33.3°C, *uncomfortably cold* at 31°C, *shivering cold* at 30°C, and *extremely cold* at 29°C. People doing heavy work reported to feel comfortable at much lower temperatures, which shows that the activity level affects human performance and comfort. The extremities of the body such as hands and feet are most easily affected by cold weather, and their temperature is a better indication of comfort and performance. A hand-skin temperature of 20°C is perceived to be uncomfortably cold, 15°C to be extremely cold, and 5°C to be painfully cold. Useful work can be performed by hands without difficulty as long as the skin temperature of fingers remains above 16°C (ASHRAE *Handbook of Fundamentals,* Ref. 1, Chapter 8).

The first line of defense of the body against excessive heat loss in a cold environment is *to reduce the skin temperature* and thus the rate of heat loss from the skin by constricting the veins and decreasing the blood flow to the skin. This measure decreases the temperature of the tissues subjacent to the skin, but maintains the inner body temperature. The next preventive

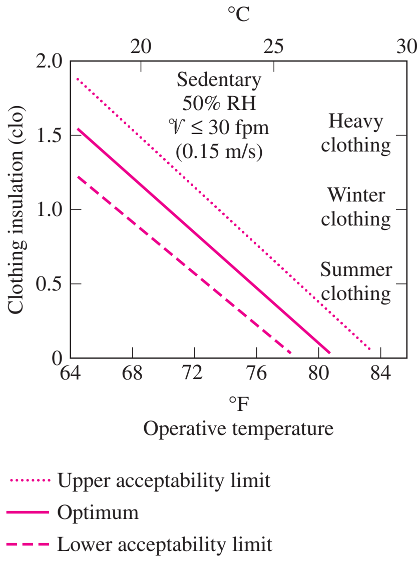


FIGURE 1–51
The effect of clothing on the environment temperature that feels comfortable (1 clo = 0.155 m² · °C/W = 0.880 ft² · °F · h/Btu) (from ASHRAE Standard 55-1981).

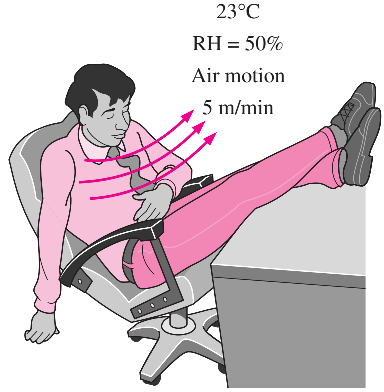


FIGURE 1–52
A thermally comfortable environment.

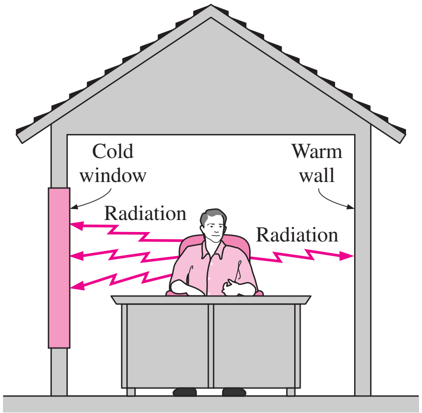


FIGURE 1–53
Cold surfaces cause excessive heat loss from the body by radiation, and thus discomfort on that side of the body.

FIGURE 1–54
The rate of metabolic heat generation may go up by six times the resting level during total body shivering in cold weather.

measure is increasing the rate of *metabolic heat generation* in the body by *shivering,* unless the person does it voluntarily by increasing his or her level of activity or puts on additional clothing. Shivering begins slowly in small muscle groups and may double the rate of metabolic heat production of the body at its initial stages. In the extreme case of total body shivering, the rate of heat production may reach six times the resting levels (Fig. 1–54). If this measure also proves inadequate, the deep body temperature starts *falling.* Body parts furthest away from the core such as the hands and feet are at greatest danger for tissue damage.

In **hot environments,** the rate of heat loss from the body may drop below the metabolic heat generation rate. This time the body activates the opposite mechanisms. First the body increases the *blood flow* and thus heat transport to the skin, causing the temperature of the skin and the subjacent tissues to rise and approach the deep body temperature. Under extreme heat conditions, the *heart rate* may reach 180 beats per minute in order to maintain adequate blood supply to the brain and the skin. At higher heart rates, the *volumetric efficiency* of the heart drops because of the very short time between the beats to fill the heart with blood, and the blood supply to the skin and more importantly to the brain drops. This causes the person to faint as a result of *heat exhaustion.* Dehydration makes the problem worse. A similar thing happens when a person working very hard for a long time stops suddenly. The blood that has flooded the skin has difficulty returning to the heart in this case since the relaxed muscles no longer force the blood back to the heart, and thus there is less blood available for pumping to the brain.

The next line of defense is releasing water from sweat glands and resorting to *evaporative cooling,* unless the person removes some clothing and reduces the activity level (Fig. 1–55). The body can maintain its core temperature at 37°C in this evaporative cooling mode indefinitely, even in environments at higher temperatures (as high as 200°C during military endurance tests), if the person drinks plenty of liquids to replenish his or her water reserves and the ambient air is sufficiently dry to allow the sweat to evaporate instead of rolling down the skin. If this measure proves inadequate, the body will have to start absorbing the metabolic heat and the deep body temperature will rise. A person can tolerate a temperature rise of 1.4°C without major discomfort but may *collapse* when the temperature rise reaches 2.8°C. People feel sluggish and their efficiency drops considerably when the core body temperature rises above 39°C. A core temperature above 41°C may damage hypothalamic proteins, resulting in cessation of sweating, increased heat production by shivering, and a *heat stroke* with irreversible and life-threatening damage. Death can occur above 43°C.

A surface temperature of 46°C causes pain on the skin. Therefore, direct contact with a metal block at this temperature or above is painful. However, a person can stay in a room at 100°C for up to 30 min without any damage or pain on the skin because of the convective resistance at the skin surface and evaporative cooling. We can even put our hands into an oven at 200°C for a short time without getting burned.

Another factor that affects thermal comfort, health, and productivity is **ventilation.** Fresh outdoor air can be provided to a building *naturally* by doing nothing, or *forcefully* by a mechanical ventilation system. In the first case, which is the norm in residential buildings, the necessary ventilation is provided by *infiltration through cracks and leaks* in the living space and by the opening of the windows and doors. The additional ventilation needed in the bathrooms and kitchens is provided by *air vents with dampers* or *exhaust fans.* With this kind of uncontrolled ventilation, however, the fresh air supply will be either too high, wasting energy, or too low, causing poor indoor air quality. But the current practice is not likely to change for residential buildings since there is not a public outcry for energy waste or air quality, and thus it is difficult to justify the cost and complexity of mechanical ventilation systems.

Mechanical ventilation systems are part of any heating and air conditioning system in *commercial buildings,* providing the necessary amount of fresh outdoor air and distributing it uniformly throughout the building. This is not surprising since many rooms in large commercial buildings have no windows and thus rely on mechanical ventilation. Even the rooms with windows are in the same situation since the windows are tightly sealed and cannot be opened in most buildings. It is not a good idea to oversize the ventilation system just to be on the "safe side" since exhausting the heated or cooled indoor air wastes energy. On the other hand, reducing the ventilation rates below the required minimum to conserve energy should also be avoided so that the indoor air quality can be maintained at the required levels. The minimum fresh air ventilation requirements are listed in Table 1–8. The values are based on controlling the CO_2 and other contaminants with an adequate margin of safety, which requires each person be supplied with at least 7.5 L/s (15 ft^3/min) of fresh air.

Another function of the mechanical ventilation system is to **clean** the air by filtering it as it enters the building. Various types of filters are available for this purpose, depending on the cleanliness requirements and the allowable pressure drop.

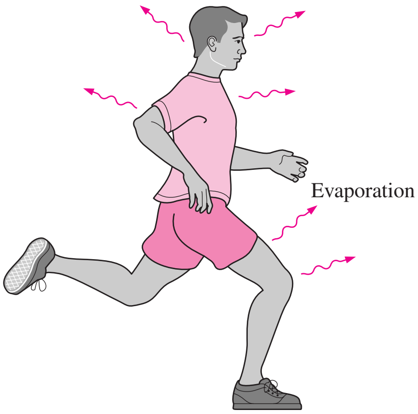


FIGURE 1–55
In hot environments, a body can dissipate a large amount of metabolic heat by sweating since the sweat absorbs the body heat and evaporates.

TABLE 1–8
Minimum fresh air requirements in buildings (from ASHRAE Standard 62-1989)

	Requirement (per person)	
Application	L/s	ft^3/min
Classrooms, libraries, supermarkets	8	15
Dining rooms, conference rooms, offices	10	20
Hospital rooms	13	25
Hotel rooms	15 (per room)	30 (per room)
Smoking lounges	30	60
Retail stores	1.0–1.5 (per m^2)	0.2–0.3 (per ft^2)
Residential buildings	0.35 air change per hour, but not less than 7.5 L/s (or 15 ft^3/min) per person	

SUMMARY

In this chapter, the basics of heat transfer are introduced and discussed. The science of *thermodynamics* deals with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, whereas the science of *heat transfer* deals with the rate of heat transfer, which is the main quantity of interest in the design and evaluation of heat transfer equipment. The sum of all forms of energy of a system is called *total energy,* and it includes the internal, kinetic, and potential energies. The *internal energy* represents the molecular energy of a system, and it consists of sensible, latent, chemical, and nuclear forms. The sensible and latent forms of internal energy can be transferred from one medium to another as a result of a temperature difference, and are referred to as *heat* or *thermal energy.* Thus, *heat transfer* is the exchange of the sensible and latent forms of internal energy between two mediums as a result of a temperature difference. The amount of heat transferred per unit time is called *heat transfer rate* and is denoted by $\dot{Q}$. The rate of heat transfer per unit area is called *heat flux,* $\dot{q}$.

A system of fixed mass is called a *closed system* and a system that involves mass transfer across its boundaries is called an *open system* or *control volume.* The *first law of thermodynamics* or the *energy balance* for any system undergoing any process can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

When a stationary closed system involves heat transfer only and no work interactions across its boundary, the energy balance relation reduces to

$$Q = mC_v\Delta T$$

where Q is the amount of net heat transfer to or from the system. When heat is transferred at a constant rate of $\dot{Q}$, the amount of heat transfer during a time interval Δt can be determined from $Q = \dot{Q}\Delta t$.

Under steady conditions and in the absence of any work interactions, the conservation of energy relation for a control volume with one inlet and one exit with negligible changes in kinetic and potential energies can be expressed as

$$\dot{Q} = \dot{m}C_p\Delta T$$

where $\dot{m} = \rho\mathcal{V}A_c$ is the mass flow rate and $\dot{Q}$ is the rate of net heat transfer into or out of the control volume.

Heat can be transferred in three different modes: conduction, convection, and radiation. *Conduction* is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles, and is expressed by *Fourier's law of heat conduction* as

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx}$$

where k is the *thermal conductivity* of the material, A is the *area* normal to the direction of heat transfer, and dT/dx is the *temperature gradient.* The magnitude of the rate of heat conduction across a plane layer of thickness L is given by

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L}$$

where ΔT is the temperature difference across the layer.

Convection is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion, and involves the combined effects of conduction and fluid motion. The rate of convection heat transfer is expressed by *Newton's law of cooling* as

$$\dot{Q}_{\text{convection}} = hA_s\,(T_s - T_\infty)$$

where h is the *convection heat transfer coefficient* in $\text{W/m}^2 \cdot {}^\circ\text{C}$ or $\text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$, A_s is the *surface area* through which convection heat transfer takes place, T_s is the *surface temperature,* and T_∞ is the *temperature of the fluid* sufficiently far from the surface.

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. The maximum rate of radiation that can be emitted from a surface at an absolute temperature T_s is given by the *Stefan–Boltzmann law* as $\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4$, where $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$ or $0.1714 \times 10^{-8}\ \text{Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4$ is the *Stefan–Boltzmann constant.*

When a surface of emissivity ε and surface area A_s at an absolute temperature T_s is completely enclosed by a much larger (or black) surface at absolute temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s\,(T_s^4 - T_{\text{surr}}^4)$$

In this case, the emissivity and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

The rate at which a surface absorbs radiation is determined from $\dot{Q}_{\text{absorbed}} = \alpha\dot{Q}_{\text{incident}}$ where $\dot{Q}_{\text{incident}}$ is the rate at which radiation is incident on the surface and α is the absorptivity of the surface.

REFERENCES AND SUGGESTED READING

1. American Society of Heating, Refrigeration, and Air-Conditioning Engineers, *Handbook of Fundamentals.* Atlanta: ASHRAE, 1993.
2. Y. A. Çengel and R. H. Turner. *Fundamentals of Thermal-Fluid Sciences.* New York: McGraw-Hill, 2001.
3. Y. A. Çengel and M. A. Boles. *Thermodynamics—An Engineering Approach.* 4th ed. New York: McGraw-Hill, 2002.
4. J. P. Holman. *Heat Transfer.* 9th ed. New York: McGraw-Hill, 2002.
5. F. P. Incropera and D. P. DeWitt. *Introduction to Heat Transfer.* 4th ed. New York: John Wiley & Sons, 2002.
6. F. Kreith and M. S. Bohn. *Principles of Heat Transfer.* 6th ed. Pacific Grove, CA: Brooks/Cole, 2001.
7. A. F. Mills. *Basic Heat and Mass Transfer.* 2nd ed. Upper Saddle River, NJ: Prentice-Hall, 1999.
8. M. N. Ozisik. *Heat Transfer—A Basic Approach.* New York: McGraw-Hill, 1985.
9. Robert J. Ribando. *Heat Transfer Tools.* New York: McGraw-Hill, 2002.
10. F. M. White. *Heat and Mass Transfer.* Reading, MA: Addison-Wesley, 1988.

PROBLEMS*

Thermodynamics and Heat Transfer

1–1C How does the science of heat transfer differ from the science of thermodynamics?

1–2C What is the driving force for (*a*) heat transfer, (*b*) electric current flow, and (*c*) fluid flow?

1–3C What is the caloric theory? When and why was it abandoned?

1–4C How do rating problems in heat transfer differ from the sizing problems?

1–5C What is the difference between the analytical and experimental approach to heat transfer? Discuss the advantages and disadvantages of each approach.

1–6C What is the importance of modeling in engineering? How are the mathematical models for engineering processes prepared?

1–7C When modeling an engineering process, how is the right choice made between a simple but crude and a complex but accurate model? Is the complex model necessarily a better choice since it is more accurate?

Heat and Other Forms of Energy

1–8C What is heat flux? How is it related to the heat transfer rate?

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with a CD-EES icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

1–9C What are the mechanisms of energy transfer to a closed system? How is heat transfer distinguished from the other forms of energy transfer?

1–10C How are heat, internal energy, and thermal energy related to each other?

1–11C An ideal gas is heated from 50°C to 80°C (*a*) at constant volume and (*b*) at constant pressure. For which case do you think the energy required will be greater? Why?

1–12 A cylindrical resistor element on a circuit board dissipates 0.6 W of power. The resistor is 1.5 cm long, and has a diameter of 0.4 cm. Assuming heat to be transferred uniformly from all surfaces, determine (*a*) the amount of heat this resistor dissipates during a 24-hour period, (*b*) the heat flux, and (*c*) the fraction of heat dissipated from the top and bottom surfaces.

1–13E A logic chip used in a computer dissipates 3 W of power in an environment at 120°F, and has a heat transfer surface area of 0.08 in^2. Assuming the heat transfer from the surface to be uniform, determine (*a*) the amount of heat this chip dissipates during an eight-hour work day, in kWh, and (*b*) the heat flux on the surface of the chip, in W/in^2.

1–14 Consider a 150-W incandescent lamp. The filament of the lamp is 5 cm long and has a diameter of 0.5 mm. The diameter of the glass bulb of the lamp is 8 cm. Determine the heat flux, in W/m^2, (*a*) on the surface of the filament and (*b*) on the surface of the glass bulb, and (*c*) calculate how much it will cost per year to keep that lamp on for eight hours a day every day if the unit cost of electricity is \$0.08/kWh.

Answers: (*a*) 1.91×10^6 W/m^2, (*b*) 7500 W/m^2, (*c*) \$35.04/yr

1–15 A 1200-W iron is left on the ironing board with its base exposed to the air. About 90 percent of the heat generated in the iron is dissipated through its base whose surface area is 150 cm^2, and the remaining 10 percent through other surfaces. Assuming the heat transfer from the surface to be uniform,

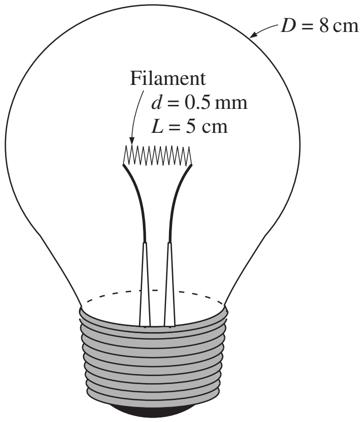


FIGURE P1–14

determine (*a*) the amount of heat the iron dissipates during a 2-hour period, in kWh, (*b*) the heat flux on the surface of the iron base, in W/m^2, and (*c*) the total cost of the electrical energy consumed during this 2-hour period. Take the unit cost of electricity to be $0.07/kWh.

1–16 A 15-cm × 20-cm circuit board houses on its surface 120 closely spaced logic chips, each dissipating 0.12 W. If the heat transfer from the back surface of the board is negligible, determine (*a*) the amount of heat this circuit board dissipates during a 10-hour period, in kWh, and (*b*) the heat flux on the surface of the circuit board, in W/m^2.

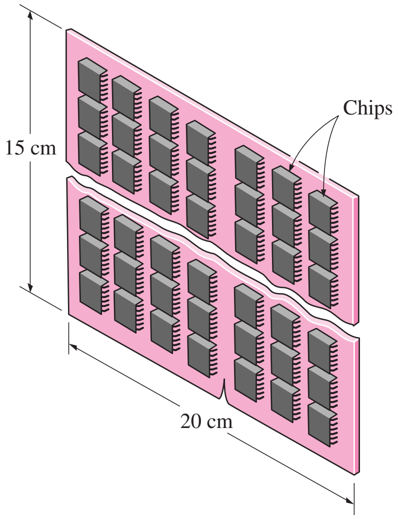


FIGURE P1–16

1–17 A 15-cm-diameter aluminum ball is to be heated from 80°C to an average temperature of 200°C. Taking the average density and specific heat of aluminum in this temperature range to be $\rho = 2700$ kg/m^3 and $C_p = 0.90$ kJ/kg · °C, respectively, determine the amount of energy that needs to be transferred to the aluminum ball. *Answer:* 515 kJ

1–18 The average specific heat of the human body is 3.6 kJ/kg · °C. If the body temperature of a 70-kg man rises from 37°C to 39°C during strenuous exercise, determine the increase in the thermal energy content of the body as a result of this rise in body temperature.

1–19 Infiltration of cold air into a warm house during winter through the cracks around doors, windows, and other openings is a major source of energy loss since the cold air that enters needs to be heated to the room temperature. The infiltration is often expressed in terms of ACH (air changes per hour). An ACH of 2 indicates that the entire air in the house is replaced twice every hour by the cold air outside.

Consider an electrically heated house that has a floor space of 200 m^2 and an average height of 3 m at 1000 m elevation, where the standard atmospheric pressure is 89.6 kPa. The house is maintained at a temperature of 22°C, and the infiltration losses are estimated to amount to 0.7 ACH. Assuming the pressure and the temperature in the house remain constant, determine the amount of energy loss from the house due to infiltration for a day during which the average outdoor temperature is 5°C. Also, determine the cost of this energy loss for that day if the unit cost of electricity in that area is $0.082/kWh.

Answers: 53.8 kWh/day, $4.41/day

1–20 Consider a house with a floor space of 200 m^2 and an average height of 3 m at sea level, where the standard atmospheric pressure is 101.3 kPa. Initially the house is at a uniform temperature of 10°C. Now the electric heater is turned on, and the heater runs until the air temperature in the house rises to an average value of 22°C. Determine how much heat is absorbed by the air assuming some air escapes through the cracks as the heated air in the house expands at constant pressure. Also, determine the cost of this heat if the unit cost of electricity in that area is $0.075/kWh.

1–21E Consider a 60-gallon water heater that is initially filled with water at 45°F. Determine how much energy needs to be transferred to the water to raise its temperature to 140°F. Take the density and specific heat of water to be 62 lbm/ft^3 and 1.0 Btu/lbm · °F, respectively.

The First Law of Thermodynamics

1–22C On a hot summer day, a student turns his fan on when he leaves his room in the morning. When he returns in the evening, will his room be warmer or cooler than the neighboring rooms? Why? Assume all the doors and windows are kept closed.

1–23C Consider two identical rooms, one with a refrigerator in it and the other without one. If all the doors and windows are closed, will the room that contains the refrigerator be cooler or warmer than the other room? Why?

1–24C Define mass and volume flow rates. How are they related to each other?

1–25 Two 800-kg cars moving at a velocity of 90 km/h have a head-on collision on a road. Both cars come to a complete rest after the crash. Assuming all the kinetic energy of cars is converted to thermal energy, determine the average temperature rise of the remains of the cars immediately after the crash. Take the average specific heat of the cars to be 0.45 kJ/kg · °C.

1–26 A classroom that normally contains 40 people is to be air-conditioned using window air-conditioning units of 5-kW cooling capacity. A person at rest may be assumed to dissipate heat at a rate of 360 kJ/h. There are 10 lightbulbs in the room, each with a rating of 100 W. The rate of heat transfer to the classroom through the walls and the windows is estimated to be 15,000 kJ/h. If the room air is to be maintained at a constant temperature of 21°C, determine the number of window air-conditioning units required. *Answer:* two units

1–27E A rigid tank contains 20 lbm of air at 50 psia and 80°F. The air is now heated until its pressure is doubled. Determine (*a*) the volume of the tank and (*b*) the amount of heat transfer. *Answers:* (*a*) 80 ft^3, (*b*) 2035 Btu

1–28 A 1-m^3 rigid tank contains hydrogen at 250 kPa and 420 K. The gas is now cooled until its temperature drops to 300 K. Determine (*a*) the final pressure in the tank and (*b*) the amount of heat transfer from the tank.

1–29 A 4-m × 5-m × 6-m room is to be heated by a baseboard resistance heater. It is desired that the resistance heater be able to raise the air temperature in the room from 7°C to 25°C within 15 minutes. Assuming no heat losses from the room and an atmospheric pressure of 100 kPa, determine the required power rating of the resistance heater. Assume constant specific heats at room temperature. *Answer:* 3.01 kW

1–30 A 4-m × 5-m × 7-m room is heated by the radiator of a steam heating system. The steam radiator transfers heat at a rate of 10,000 kJ/h and a 100-W fan is used to distribute the warm air in the room. The heat losses from the room are estimated to be at a rate of about 5000 kJ/h. If the initial temperature of the room air is 10°C, determine how long it will take for the air temperature to rise to 20°C. Assume constant specific heats at room temperature.

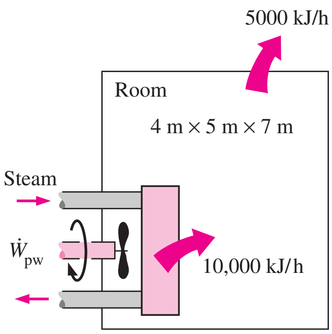


FIGURE P1–30

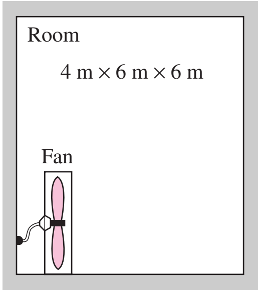


FIGURE P1–31

1–31 A student living in a 4-m × 6-m × 6-m dormitory room turns his 150-W fan on before she leaves her room on a summer day hoping that the room will be cooler when she comes back in the evening. Assuming all the doors and windows are tightly closed and disregarding any heat transfer through the walls and the windows, determine the temperature in the room when she comes back 10 hours later. Use specific heat values at room temperature and assume the room to be at 100 kPa and 15°C in the morning when she leaves. *Answer:* 58.1°C

1–32E A 10-ft^3 tank contains oxygen initially at 14.7 psia and 80°F. A paddle wheel within the tank is rotated until the pressure inside rises to 20 psia. During the process 20 Btu of heat is lost to the surroundings. Neglecting the energy stored in the paddle wheel, determine the work done by the paddle wheel.

1–33 A room is heated by a baseboard resistance heater. When the heat losses from the room on a winter day amount to 7000 kJ/h, it is observed that the air temperature in the room remains constant even though the heater operates continuously. Determine the power rating of the heater, in kW.

1–34 A 50-kg mass of copper at 70°C is dropped into an insulated tank containing 80 kg of water at 25°C. Determine the final equilibrium temperature in the tank.

1–35 A 20-kg mass of iron at 100°C is brought into contact with 20 kg of aluminum at 200°C in an insulated enclosure. Determine the final equilibrium temperature of the combined system. *Answer:* 168°C

1–36 An unknown mass of iron at 90°C is dropped into an insulated tank that contains 80 L of water at 20°C. At the same

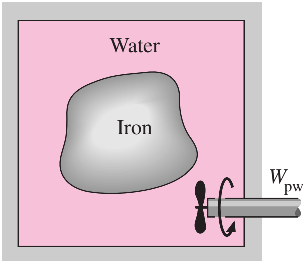


FIGURE P1–36

time, a paddle wheel driven by a 200-W motor is activated to stir the water. Thermal equilibrium is established after 25 minutes with a final temperature of 27°C. Determine the mass of the iron. Neglect the energy stored in the paddle wheel, and take the density of water to be 1000 kg/m^3. *Answer:* 72.1 kg

1–37E A 90-lbm mass of copper at 160°F and a 50-lbm mass of iron at 200°F are dropped into a tank containing 180 lbm of water at 70°F. If 600 Btu of heat is lost to the surroundings during the process, determine the final equilibrium temperature.

1–38 A 5-m × 6-m × 8-m room is to be heated by an electrical resistance heater placed in a short duct in the room. Initially, the room is at 15°C, and the local atmospheric pressure is 98 kPa. The room is losing heat steadily to the outside at a rate of 200 kJ/min. A 200-W fan circulates the air steadily through the duct and the electric heater at an average mass flow rate of 50 kg/min. The duct can be assumed to be adiabatic, and there is no air leaking in or out of the room. If it takes 15 minutes for the room air to reach an average temperature of 25°C, find (*a*) the power rating of the electric heater and (*b*) the temperature rise that the air experiences each time it passes through the heater.

1–39 A house has an electric heating system that consists of a 300-W fan and an electric resistance heating element placed in a duct. Air flows steadily through the duct at a rate of 0.6 kg/s and experiences a temperature rise of 5°C. The rate of heat loss from the air in the duct is estimated to be 250 W. Determine the power rating of the electric resistance heating element.

1–40 A hair dryer is basically a duct in which a few layers of electric resistors are placed. A small fan pulls the air in and forces it to flow over the resistors where it is heated. Air enters a 1200-W hair dryer at 100 kPa and 22°C, and leaves at 47°C. The cross-sectional area of the hair dryer at the exit is 60 cm^2. Neglecting the power consumed by the fan and the heat losses through the walls of the hair dryer, determine (*a*) the volume flow rate of air at the inlet and (*b*) the velocity of the air at the exit. *Answers:* (*a*) 0.0404 m^3/s, (*b*) 7.30 m/s

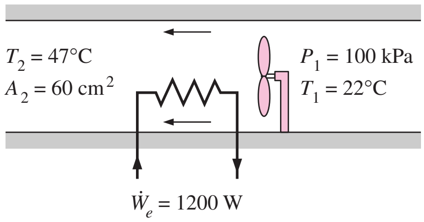


FIGURE P1–40

1–41 The ducts of an air heating system pass through an unheated area. As a result of heat losses, the temperature of the air in the duct drops by 3°C. If the mass flow rate of air is 120 kg/min, determine the rate of heat loss from the air to the cold environment.

1–42E Air enters the duct of an air-conditioning system at 15 psia and 50°F at a volume flow rate of 450 ft^3/min. The diameter of the duct is 10 inches and heat is transferred to the air in the duct from the surroundings at a rate of 2 Btu/s. Determine (*a*) the velocity of the air at the duct inlet and (*b*) the temperature of the air at the exit. *Answers:* (*a*) 825 ft/min, (*b*) 64°F

1–43 Water is heated in an insulated, constant diameter tube by a 7-kW electric resistance heater. If the water enters the heater steadily at 15°C and leaves at 70°C, determine the mass flow rate of water.

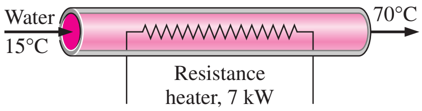


FIGURE P1–43

Heat Transfer Mechanisms

1–44C Define thermal conductivity and explain its significance in heat transfer.

1–45C What are the mechanisms of heat transfer? How are they distinguished from each other?

1–46C What is the physical mechanism of heat conduction in a solid, a liquid, and a gas?

1–47C Consider heat transfer through a windowless wall of a house in a winter day. Discuss the parameters that affect the rate of heat conduction through the wall.

1–48C Write down the expressions for the physical laws that govern each mode of heat transfer, and identify the variables involved in each relation.

1–49C How does heat conduction differ from convection?

1–50C Does any of the energy of the sun reach the earth by conduction or convection?

1–51C How does forced convection differ from natural convection?

1–52C Define emissivity and absorptivity. What is Kirchhoff's law of radiation?

1–53C What is a blackbody? How do real bodies differ from blackbodies?

1–54C Judging from its unit W/m · °C, can we define thermal conductivity of a material as the rate of heat transfer through the material per unit thickness per unit temperature difference? Explain.

1–55C Consider heat loss through the two walls of a house on a winter night. The walls are identical, except that one of them has a tightly fit glass window. Through which wall will the house lose more heat? Explain.

1–56C Which is a better heat conductor, diamond or silver?

1–57C Consider two walls of a house that are identical except that one is made of 10-cm-thick wood, while the other is made of 25-cm-thick brick. Through which wall will the house lose more heat in winter?

1–58C How do the thermal conductivity of gases and liquids vary with temperature?

1–59C Why is the thermal conductivity of superinsulation orders of magnitude lower than the thermal conductivity of ordinary insulation?

1–60C Why do we characterize the heat conduction ability of insulators in terms of their apparent thermal conductivity instead of the ordinary thermal conductivity?

1–61C Consider an alloy of two metals whose thermal conductivities are k_1 and k_2. Will the thermal conductivity of the alloy be less than k_1, greater than k_2, or between k_1 and k_2?

1–62 The inner and outer surfaces of a 5-m × 6-m brick wall of thickness 30 cm and thermal conductivity 0.69 W/m · °C are maintained at temperatures of 20°C and 5°C, respectively. Determine the rate of heat transfer through the wall, in W.
Answer: 1035 W

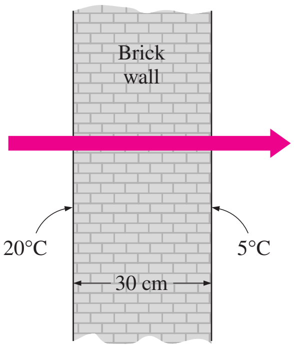


FIGURE P1–62

1–63 The inner and outer surfaces of a 0.5-cm-thick 2-m × 2-m window glass in winter are 10°C and 3°C, respectively. If the thermal conductivity of the glass is 0.78 W/m · °C, determine the amount of heat loss, in kJ, through the glass over a period of 5 hours. What would your answer be if the glass were 1 cm thick? *Answers:* 78,624 kJ, 39,312 kJ

1–64 Reconsider Problem 1–63. Using EES (or other) software, plot the amount of heat loss through the glass as a function of the window glass thickness in the range of 0.1 cm to 1.0 cm. Discuss the results.

1–65 An aluminum pan whose thermal conductivity is 237 W/m · °C has a flat bottom with diameter 20 cm and thickness 0.4 cm. Heat is transferred steadily to boiling water in the pan through its bottom at a rate of 800 W. If the inner surface of the bottom of the pan is at 105°C, determine the temperature of the outer surface of the bottom of the pan.

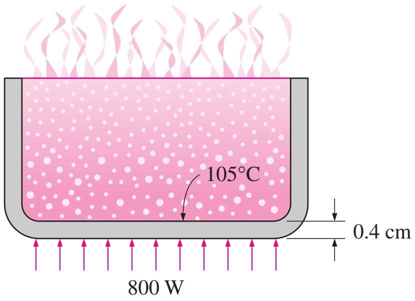


FIGURE P1–65

1–66E The north wall of an electrically heated home is 20 ft long, 10 ft high, and 1 ft thick, and is made of brick whose thermal conductivity is $k = 0.42$ Btu/h · ft · °F. On a certain winter night, the temperatures of the inner and the outer surfaces of the wall are measured to be at about 62°F and 25°F, respectively, for a period of 8 hours. Determine (*a*) the rate of heat loss through the wall that night and (*b*) the cost of that heat loss to the home owner if the cost of electricity is $0.07/kWh.

1–67 In a certain experiment, cylindrical samples of diameter 4 cm and length 7 cm are used (see Fig. 1–29). The two thermocouples in each sample are placed 3 cm apart. After initial transients, the electric heater is observed to draw 0.6 A at 110 V, and both differential thermometers read a temperature difference of 10°C. Determine the thermal conductivity of the sample. *Answer:* 78.8 W/m · °C

1–68 One way of measuring the thermal conductivity of a material is to sandwich an electric thermofoil heater between two identical rectangular samples of the material and to heavily insulate the four outer edges, as shown in the figure. Thermocouples attached to the inner and outer surfaces of the samples record the temperatures.

During an experiment, two 0.5-cm-thick samples 10 cm × 10 cm in size are used. When steady operation is reached, the heater is observed to draw 35 W of electric power, and the temperature of each sample is observed to drop from 82°C at the inner surface to 74°C at the outer surface. Determine the thermal conductivity of the material at the average temperature.

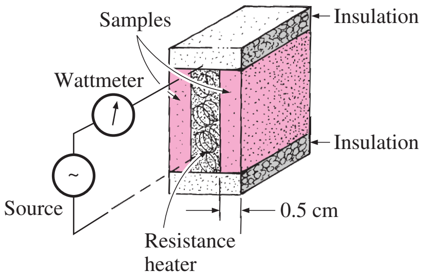


FIGURE P1–68

1–69 Repeat Problem 1–68 for an electric power consumption of 28 W.

1–70 A heat flux meter attached to the inner surface of a 3-cm-thick refrigerator door indicates a heat flux of 25 W/m^2 through the door. Also, the temperatures of the inner and the outer surfaces of the door are measured to be 7°C and 15°C, respectively. Determine the average thermal conductivity of the refrigerator door. *Answer:* 0.0938 W/m · °C

1–71 Consider a person standing in a room maintained at 20°C at all times. The inner surfaces of the walls, floors, and ceiling of the house are observed to be at an average temperature of 12°C in winter and 23°C in summer. Determine the rates of radiation heat transfer between this person and the surrounding surfaces in both summer and winter if the exposed surface area, emissivity, and the average outer surface temperature of the person are 1.6 m^2, 0.95, and 32°C, respectively.

1–72 EES Reconsider Problem 1–71. Using EES (or other) software, plot the rate of radiation heat transfer in winter as a function of the temperature of the inner surface of the room in the range of 8°C to 18°C. Discuss the results.

1–73 For heat transfer purposes, a standing man can be modeled as a 30-cm-diameter, 170-cm-long vertical cylinder with both the top and bottom surfaces insulated and with the side surface at an average temperature of 34°C. For a convection heat transfer coefficient of 15 W/m^2 · °C, determine the rate of heat loss from this man by convection in an environment at 20°C. *Answer:* 336 W

1–74 Hot air at 80°C is blown over a 2-m × 4-m flat surface at 30°C. If the average convection heat transfer coefficient is 55 W/m^2 · °C, determine the rate of heat transfer from the air to the plate, in kW. *Answer:* 22 kW

1–75 EES Reconsider Problem 1–74. Using EES (or other) software, plot the rate of heat transfer as a function of the heat transfer coefficient in the range of 20 W/m^2 · °C to 100 W/m^2 · °C. Discuss the results.

1–76 The heat generated in the circuitry on the surface of a silicon chip (k = 130 W/m · °C) is conducted to the ceramic substrate to which it is attached. The chip is 6 mm × 6 mm in size and 0.5 mm thick and dissipates 3 W of power. Disregarding any heat transfer through the 0.5-mm-high side surfaces, determine the temperature difference between the front and back surfaces of the chip in steady operation.

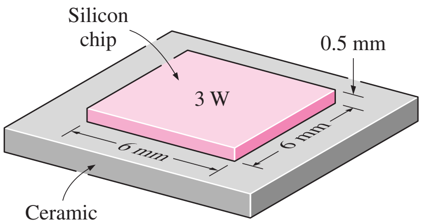


FIGURE P1–76

1–77 A 50-cm-long, 800-W electric resistance heating element with diameter 0.5 cm and surface temperature 120°C is immersed in 60 kg of water initially at 20°C. Determine how long it will take for this heater to raise the water temperature to 80°C. Also, determine the convection heat transfer coefficients at the beginning and at the end of the heating process.

1–78 A 5-cm-external-diameter, 10-m-long hot water pipe at 80°C is losing heat to the surrounding air at 5°C by natural convection with a heat transfer coefficient of 25 W/m^2 · °C. Determine the rate of heat loss from the pipe by natural convection, in W. *Answer:* 2945 W

1–79 A hollow spherical iron container with outer diameter 20 cm and thickness 0.4 cm is filled with iced water at 0°C. If the outer surface temperature is 5°C, determine the approximate rate of heat loss from the sphere, in kW, and the rate at which ice melts in the container. The heat from fusion of water is 333.7 kJ/kg.

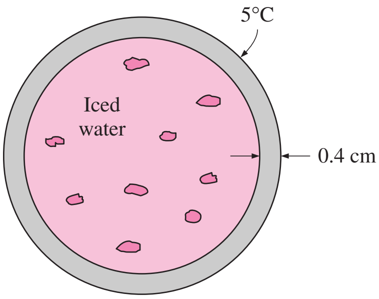


FIGURE P1–79

1–80 EES Reconsider Problem 1–79. Using EES (or other) software, plot the rate at which ice melts as a function of the container thickness in the range of 0.2 cm to 2.0 cm. Discuss the results.

1–81E The inner and outer glasses of a 6-ft × 6-ft double-pane window are at 60°F and 42°F, respectively. If the 0.25-in. space between the two glasses is filled with still air, determine the rate of heat transfer through the window.
Answer: 439 Btu/h

1–82 Two surfaces of a 2-cm-thick plate are maintained at 0°C and 80°C, respectively. If it is determined that heat is transferred through the plate at a rate of 500 W/m^2, determine its thermal conductivity.

1–83 Four power transistors, each dissipating 15 W, are mounted on a thin vertical aluminum plate 22 cm × 22 cm in size. The heat generated by the transistors is to be dissipated by both surfaces of the plate to the surrounding air at 25°C, which is blown over the plate by a fan. The entire plate can be assumed to be nearly isothermal, and the exposed surface area of the transistor can be taken to be equal to its base area. If the average convection heat transfer coefficient is 25 W/m^2 · °C, determine the temperature of the aluminum plate. Disregard any radiation effects.

1–84 An ice chest whose outer dimensions are 30 cm × 40 cm × 40 cm is made of 3-cm-thick Styrofoam (k = 0.033 W/m · °C). Initially, the chest is filled with 40 kg of ice at 0°C, and the inner surface temperature of the ice chest can be taken to be 0°C at all times. The heat of fusion of ice at 0°C is 333.7 kJ/kg, and the surrounding ambient air is at 30°C. Disregarding any heat transfer from the 40-cm × 40-cm base of the ice chest, determine how long it will take for the ice in the chest to melt completely if the outer surfaces of the ice chest are at 8°C. *Answer:* 32.7 days

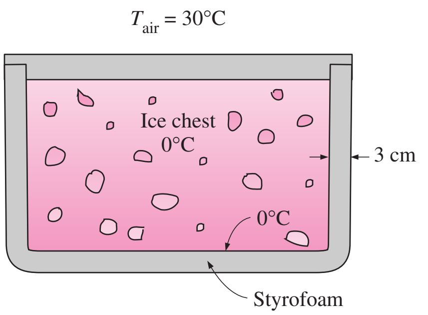


FIGURE P1–84

1–85 A transistor with a height of 0.4 cm and a diameter of 0.6 cm is mounted on a circuit board. The transistor is cooled by air flowing over it with an average heat transfer coefficient of 30 W/m² · °C. If the air temperature is 55°C and the transistor case temperature is not to exceed 70°C, determine the amount of power this transistor can dissipate safely. Disregard any heat transfer from the transistor base.

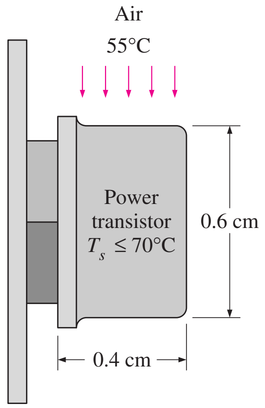


FIGURE P1–85

1–86 Reconsider Problem 1–85. Using EES (or other) software, plot the amount of power the transistor can dissipate safely as a function of the maximum case temperature in the range of 60°C to 90°C. Discuss the results.

1–87E A 200-ft-long section of a steam pipe whose outer diameter is 4 inches passes through an open space at 50°F. The average temperature of the outer surface of the pipe is measured to be 280°F, and the average heat transfer coefficient on that surface is determined to be 6 Btu/h · ft² · °F. Determine (*a*) the rate of heat loss from the steam pipe and (*b*) the annual cost of this energy loss if steam is generated in a natural gas furnace having an efficiency of 86 percent, and the price of natural gas is \$0.58/therm (1 therm = 100,000 Btu).
Answers: (*a*) 289,000 Btu/h, (*b*) \$17,074/yr

1–88 The boiling temperature of nitrogen at atmospheric pressure at sea level (1 atm) is −196°C. Therefore, nitrogen is commonly used in low temperature scientific studies since the temperature of liquid nitrogen in a tank open to the atmosphere will remain constant at −196°C until the liquid nitrogen in the tank is depleted. Any heat transfer to the tank will result in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg and a density of 810 kg/m³ at 1 atm.

Consider a 4-m-diameter spherical tank initially filled with liquid nitrogen at 1 atm and −196°C. The tank is exposed to 20°C ambient air with a heat transfer coefficient of 25 W/m² · °C. The temperature of the thin-shelled spherical tank is observed to be almost the same as the temperature of the nitrogen inside. Disregarding any radiation heat exchange, determine the rate of evaporation of the liquid nitrogen in the tank as a result of the heat transfer from the ambient air.

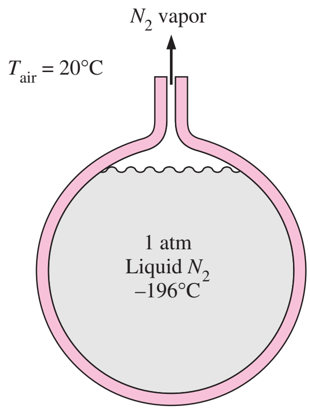


FIGURE P1–88

1–89 Repeat Problem 1–88 for liquid oxygen, which has a boiling temperature of −183°C, a heat of vaporization of 213 kJ/kg, and a density of 1140 kg/m³ at 1 atm pressure.

1–90 Reconsider Problem 1–88. Using EES (or other) software, plot the rate of evaporation of liquid nitrogen as a function of the ambient air temperature in the range of 0°C to 35°C. Discuss the results.

1–91 Consider a person whose exposed surface area is 1.7 m², emissivity is 0.7, and surface temperature is 32°C.

Determine the rate of heat loss from that person by radiation in a large room having walls at a temperature of (*a*) 300 K and (*b*) 280 K. *Answers:* (*a*) 37.4 W, (*b*) 169.2 W

1–92 A 0.3-cm-thick, 12-cm-high, and 18-cm-long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.06 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 16 W/m · °C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to the ambient air. Determine the temperature difference between the two sides of the circuit board. *Answer:* 0.042°C

1–93 Consider a sealed 20-cm-high electronic box whose base dimensions are 40 cm × 40 cm placed in a vacuum chamber. The emissivity of the outer surface of the box is 0.95. If the electronic components in the box dissipate a total of 100 W of power and the outer surface temperature of the box is not to exceed 55°C, determine the temperature at which the surrounding surfaces must be kept if this box is to be cooled by radiation alone. Assume the heat transfer from the bottom surface of the box to the stand to be negligible.

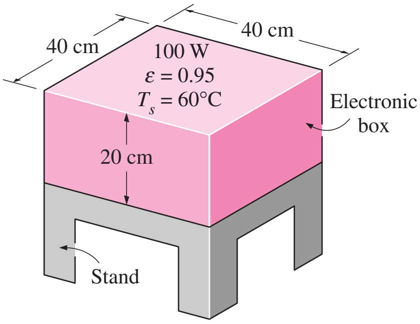


FIGURE P1–93

1–94 Using the conversion factors between W and Btu/h, m and ft, and K and R, express the Stefan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/m^2 · K^4 in the English unit Btu/h · ft^2 · R^4.

1–95 An engineer who is working on the heat transfer analysis of a house in English units needs the convection heat transfer coefficient on the outer surface of the house. But the only value he can find from his handbooks is 20 W/m^2 · °C, which is in SI units. The engineer does not have a direct conversion factor between the two unit systems for the convection heat transfer coefficient. Using the conversion factors between W and Btu/h, m and ft, and °C and °F, express the given convection heat transfer coefficient in Btu/h · ft^2 · °F.
Answer: 3.52 Btu/h · ft^2 · °F

Simultaneous Heat Transfer Mechanisms

1–96C Can all three modes of heat transfer occur simultaneously (in parallel) in a medium?

1–97C Can a medium involve (*a*) conduction and convection, (*b*) conduction and radiation, or (*c*) convection and radiation simultaneously? Give examples for the "yes" answers.

1–98C The deep human body temperature of a healthy person remains constant at 37°C while the temperature and the humidity of the environment change with time. Discuss the heat transfer mechanisms between the human body and the environment both in summer and winter, and explain how a person can keep cooler in summer and warmer in winter.

1–99C We often turn the fan on in summer to help us cool. Explain how a fan makes us feel cooler in the summer. Also explain why some people use ceiling fans also in winter.

1–100 Consider a person standing in a room at 23°C. Determine the total rate of heat transfer from this person if the exposed surface area and the skin temperature of the person are 1.7 m^2 and 32°C, respectively, and the convection heat transfer coefficient is 5 W/m^2 · °C. Take the emissivity of the skin and the clothes to be 0.9, and assume the temperature of the inner surfaces of the room to be the same as the air temperature.
Answer: 161 W

1–101 Consider steady heat transfer between two large parallel plates at constant temperatures of $T_1 = 290$ K and $T_2 = 150$ K that are $L = 2$ cm apart. Assuming the surfaces to be black (emissivity $\varepsilon = 1$), determine the rate of heat transfer between the plates per unit surface area assuming the gap between the plates is (*a*) filled with atmospheric air, (*b*) evacuated, (*c*) filled with fiberglass insulation, and (*d*) filled with superinsulation having an apparent thermal conductivity of 0.00015 W/m · °C.

1–102 A 1.4-m-long, 0.2-cm-diameter electrical wire extends across a room that is maintained at 20°C. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 240°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 110 V and 3 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.
Answer: 170.5 W/m^2 · °C

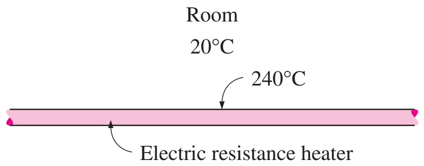


FIGURE P1–102

1–103  Reconsider Problem 1–102. Using EES (or other) software, plot the convection heat transfer coefficient as a function of the wire surface temperature in the range of 100°C to 300°C. Discuss the results.

1–104E A 2-in-diameter spherical ball whose surface is maintained at a temperature of 170°F is suspended in the middle of a room at 70°F. If the convection heat transfer coefficient is 12 Btu/h · ft^2 · °F and the emissivity of the surface is 0.8, determine the total rate of heat transfer from the ball.

1–105 A 1000-W iron is left on the iron board with its base exposed to the air at 20°C. The convection heat transfer coefficient between the base surface and the surrounding air is 35 W/m^2 · °C. If the base has an emissivity of 0.6 and a surface area of 0.02 m^2, determine the temperature of the base of the iron. *Answer:* 674°C

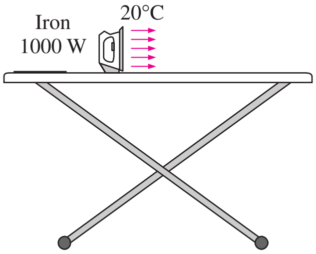


FIGURE P1–105

1–106 The outer surface of a spacecraft in space has an emissivity of 0.8 and a solar absorptivity of 0.3. If solar radiation is incident on the spacecraft at a rate of 950 W/m^2, determine the surface temperature of the spacecraft when the radiation emitted equals the solar energy absorbed.

1–107 A 3-m-internal-diameter spherical tank made of 1-cm-thick stainless steel is used to store iced water at 0°C. The tank is located outdoors at 25°C. Assuming the entire steel tank to be at 0°C and thus the thermal resistance of the tank to be negligible, determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-hour period. The heat of fusion of water at atmospheric pressure is h_{if} = 333.7 kJ/kg. The emissivity of the outer surface of the tank is 0.6, and the convection heat transfer coefficient on the outer surface can be taken to be 30 W/m^2 · °C. Assume the average surrounding surface temperature for radiation exchange to be 15°C. *Answer:* 5898 kg

1–108 The roof of a house consists of a 15-cm-thick concrete slab (k = 2 W/m · °C) that is 15 m wide and 20 m long. The emissivity of the outer surface of the roof is 0.9, and the convection heat transfer coefficient on that surface is estimated to be 15 W/m^2 · °C. The inner surface of the roof is maintained at 15°C. On a clear winter night, the ambient air is reported to be at 10°C while the night sky temperature for radiation heat transfer is 255 K. Considering both radiation and convection heat transfer, determine the outer surface temperature and the rate of heat transfer through the roof.

If the house is heated by a furnace burning natural gas with an efficiency of 85 percent, and the unit cost of natural gas is \$0.60/therm (1 therm = 105,500 kJ of energy content), determine the money lost through the roof that night during a 14-hour period.

1–109E Consider a flat plate solar collector placed horizontally on the flat roof of a house. The collector is 5 ft wide and 15 ft long, and the average temperature of the exposed surface of the collector is 100°F. The emissivity of the exposed surface of the collector is 0.9. Determine the rate of heat loss from the collector by convection and radiation during a calm day when the ambient air temperature is 70°F and the effective sky temperature for radiation exchange is 50°F. Take the convection heat transfer coefficient on the exposed surface to be 2.5 Btu/h · ft^2 · °F.

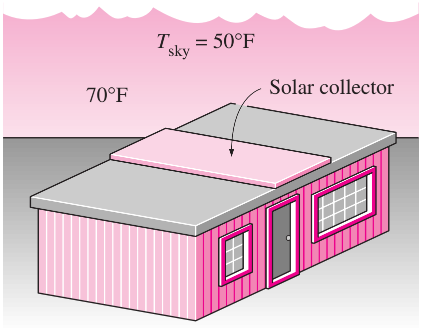


FIGURE P1–109E

Problem Solving Technique and EES

1–110C What is the value of the engineering software packages in (*a*) engineering education and (*b*) engineering practice?

1–111 Determine a positive real root of the following equation using EES:

$$2x^3 - 10x^{0.5} - 3x = -3$$

1–112 Solve the following system of two equations with two unknowns using EES:

$$x^3 - y^2 = 7.75$$
$$3xy + y = 3.5$$

1–113 Solve the following system of three equations with three unknowns using EES:

$$2x - y + z = 5$$
$$3x^2 + 2y = z + 2$$
$$xy + 2z = 8$$

1–114 Solve the following system of three equations with three unknowns using EES:

$$x^2y - z = 1$$
$$x - 3y^{0.5} + xz = -2$$
$$x + y - z = 2$$

Special Topic: Thermal Comfort

1–115C What is metabolism? What is the range of metabolic rate for an average man? Why are we interested in metabolic rate of the occupants of a building when we deal with heating and air conditioning?

1–116C Why is the metabolic rate of women, in general, lower than that of men? What is the effect of clothing on the environmental temperature that feels comfortable?

1–117C What is asymmetric thermal radiation? How does it cause thermal discomfort in the occupants of a room?

1–118C How do (*a*) draft and (*b*) cold floor surfaces cause discomfort for a room's occupants?

1–119C What is stratification? Is it likely to occur at places with low or high ceilings? How does it cause thermal discomfort for a room's occupants? How can stratification be prevented?

1–120C Why is it necessary to ventilate buildings? What is the effect of ventilation on energy consumption for heating in winter and for cooling in summer? Is it a good idea to keep the bathroom fans on all the time? Explain.

Review Problems

1–121 2.5 kg of liquid water initially at 18°C is to be heated to 96°C in a teapot equipped with a 1200-W electric heating element inside. The teapot is 0.8 kg and has an average specific heat of 0.6 kJ/kg · °C. Taking the specific heat of water to be 4.18 kJ/kg · °C and disregarding any heat loss from the teapot, determine how long it will take for the water to be heated.

1–122 A 4-m-long section of an air heating system of a house passes through an unheated space in the attic. The inner diameter of the circular duct of the heating system is 20 cm. Hot air enters the duct at 100 kPa and 65°C at an average velocity of 3 m/s. The temperature of the air in the duct drops to 60°C as a result of heat loss to the cool space in the attic. Determine the rate of heat loss from the air in the duct to the attic under steady conditions. Also, determine the cost of this heat loss per hour if the house is heated by a natural gas furnace having an efficiency of 82 percent, and the cost of the natural gas in that area is \$0.58/therm (1 therm = 105,500 kJ).
Answers: 0.488 kJ/s, \$0.012/h

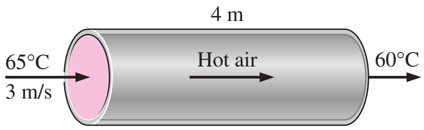


FIGURE P1–122

1–123 Reconsider Problem 1–122. Using EES (or other) software, plot the cost of the heat loss per hour as a function of the average air velocity in the range of 1 m/s to 10 m/s. Discuss the results.

1–124 Water flows through a shower head steadily at a rate of 10 L/min. An electric resistance heater placed in the water pipe heats the water from 16°C to 43°C. Taking the density of water to be 1 kg/L, determine the electric power input to the heater, in kW.

In an effort to conserve energy, it is proposed to pass the drained warm water at a temperature of 39°C through a heat exchanger to preheat the incoming cold water. If the heat exchanger has an effectiveness of 0.50 (that is, it recovers only half of the energy that can possibly be transferred from the drained water to incoming cold water), determine the electric power input required in this case. If the price of the electric energy is 8.5 ¢/kWh, determine how much money is saved during a 10-minute shower as a result of installing this heat exchanger.
Answers: 18.8 kW, 10.8 kW, \$0.0113

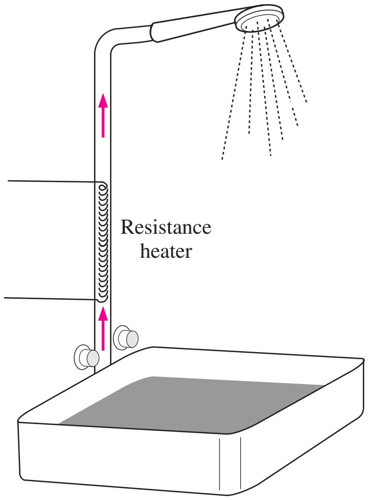


FIGURE P1–124

1–125 It is proposed to have a water heater that consists of an insulated pipe of 5 cm diameter and an electrical resistor inside. Cold water at 15°C enters the heating section steadily at a rate of 18 L/min. If water is to be heated to 50°C, determine (*a*) the power rating of the resistance heater and (*b*) the average velocity of the water in the pipe.

1–126 A passive solar house that is losing heat to the outdoors at an average rate of 50,000 kJ/h is maintained at 22°C at all times during a winter night for 10 hours. The house is to be heated by 50 glass containers each containing 20 L of water heated to 80°C during the day by absorbing solar energy. A thermostat-controlled 15-kW back-up electric resistance heater turns on whenever necessary to keep the house at 22°C. (*a*) How long did the electric heating system run that night? (*b*) How long would the electric heater have run that night if the house incorporated no solar heating?
Answers: (a) 4.77 h, (b) 9.26 h

1–127 It is well known that wind makes the cold air feel much colder as a result of the *windchill* effect that is due to the increase in the convection heat transfer coefficient with increasing air velocity. The windchill effect is usually expressed in terms of the *windchill factor,* which is the difference between the actual air temperature and the equivalent calm-air

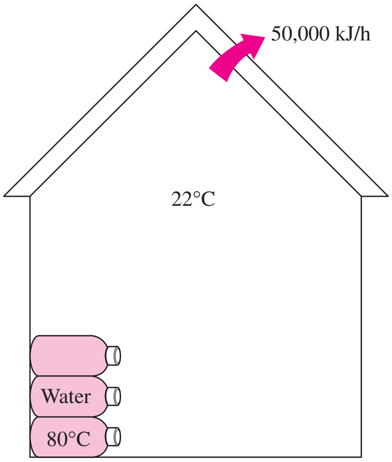


FIGURE P1–126

temperature. For example, a windchill factor of 20°C for an actual air temperature of 5°C means that the windy air at 5°C feels as cold as the still air at −15°C. In other words, a person will lose as much heat to air at 5°C with a windchill factor of 20°C as he or she would in calm air at −15°C.

For heat transfer purposes, a standing man can be modeled as a 30-cm-diameter, 170-cm-long vertical cylinder with both the top and bottom surfaces insulated and with the side surface at an average temperature of 34°C. For a convection heat transfer coefficient of 15 W/m^2 · °C, determine the rate of heat loss from this man by convection in still air at 20°C. What would your answer be if the convection heat transfer coefficient is increased to 50 W/m^2 · °C as a result of winds? What is the windchill factor in this case? *Answers:* 336 W, 1120 W, 32.7°C

1–128 A thin metal plate is insulated on the back and exposed to solar radiation on the front surface. The exposed surface of the plate has an absorptivity of 0.7 for solar radiation. If solar radiation is incident on the plate at a rate of 700 W/m^2 and the surrounding air temperature is 10°C, determine the surface temperature of the plate when the heat loss by convection equals the solar energy absorbed by the plate. Take the convection heat transfer coefficient to be 30 W/m^2 · °C, and disregard any heat loss by radiation.

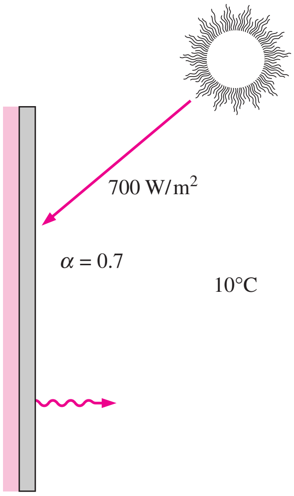


FIGURE P1–128

1–129 A 4-m × 5-m × 6-m room is to be heated by one ton (1000 kg) of liquid water contained in a tank placed in the room. The room is losing heat to the outside at an average rate of 10,000 kJ/h. The room is initially at 20°C and 100 kPa, and is maintained at an average temperature of 20°C at all times. If the hot water is to meet the heating requirements of this room for a 24-hour period, determine the minimum temperature of the water when it is first brought into the room. Assume constant specific heats for both air and water at room temperature. *Answer:* 77.4°C

1–130 Consider a 3-m × 3-m × 3-m cubical furnace whose top and side surfaces closely approximate black surfaces at a temperature of 1200 K. The base surface has an emissivity of $\varepsilon = 0.7$, and is maintained at 800 K. Determine the net rate of radiation heat transfer to the base surface from the top and side surfaces. *Answer:* 594,400 W

1–131 Consider a refrigerator whose dimensions are 1.8 m × 1.2 m × 0.8 m and whose walls are 3 cm thick. The refrigerator consumes 600 W of power when operating and has a COP of 2.5. It is observed that the motor of the refrigerator remains on for 5 minutes and then is off for 15 minutes periodically. If the average temperatures at the inner and outer surfaces of the refrigerator are 6°C and 17°C, respectively, determine the average thermal conductivity of the refrigerator walls. Also, determine the annual cost of operating this refrigerator if the unit cost of electricity is $0.08/kWh.

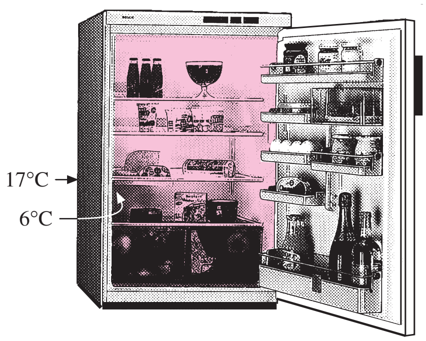


FIGURE P1–131

1–132 A 0.2-L glass of water at 20°C is to be cooled with ice to 5°C. Determine how much ice needs to be added to the water, in grams, if the ice is at 0°C. Also, determine how much water would be needed if the cooling is to be done with cold water at 0°C. The melting temperature and the heat of fusion of ice at atmospheric pressure are 0°C and 333.7 kJ/kg, respectively, and the density of water is 1 kg/L.

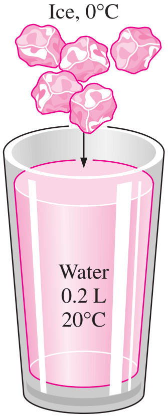


FIGURE P1–132

1–133  Reconsider Problem 1–132. Using EES (or other) software, plot the amount of ice that needs to be added to the water as a function of the ice temperature in the range of −24°C to 0°C. Discuss the results.

1–134E In order to cool 1 short ton (2000 lbm) of water at 70°F in a tank, a person pours 160 lbm of ice at 25°F into the water. Determine the final equilibrium temperature in the tank. The melting temperature and the heat of fusion of ice at atmospheric pressure are 32°F and 143.5 Btu/lbm, respectively.
Answer: 56.3°F

1–135 Engine valves (C_p = 440 J/kg · °C and ρ = 7840 kg/m^3) are to be heated from 40°C to 800°C in 5 minutes in the heat treatment section of a valve manufacturing facility. The valves have a cylindrical stem with a diameter of 8 mm and a length of 10 cm. The valve head and the stem may be assumed to be of equal surface area, with a total mass of 0.0788 kg. For a single valve, determine (*a*) the amount of heat transfer, (*b*) the average rate of heat transfer, and (*c*) the average heat flux, (*d*) the number of valves that can be heat treated per day if the heating section can hold 25 valves, and it is used 10 hours per day.

1–136 The hot water needs of a household are met by an electric 60-L hot water tank equipped with a 1.6-kW heating element. The tank is initially filled with hot water at 80°C, and the cold water temperature is 20°C. Someone takes a shower by mixing constant flow rates of hot and cold waters. After a showering period of 8 minutes, the average water temperature in the tank is measured to be 60°C. The heater is kept on during the shower and hot water is replaced by cold water. If the cold water is mixed with the hot water stream at a rate of 0.06 kg/s, determine the flow rate of hot water and the average temperature of mixed water used during the shower.

1–137 Consider a flat plate solar collector placed at the roof of a house. The temperatures at the inner and outer surfaces of glass cover are measured to be 28°C and 25°C, respectively. The glass cover has a surface area of 2.2. m^2 and a thickness of 0.6 cm and a thermal conductivity of 0.7 W/m · C. Heat is lost from the outer surface of the cover by convection and radiation with a convection heat transfer coefficient of 10 W/m^2 · °C and an ambient temperature of 15°C. Determine the fraction of heat lost from the glass cover by radiation.

1–138 The rate of heat loss through a unit surface area of a window per unit temperature difference between the indoors and the outdoors is called the *U*-factor. The value of the *U*-factor ranges from about 1.25 W/m^2 · °C (or 0.22 Btu/h · ft^2 · °F) for low-*e* coated, argon-filled, quadruple-pane windows to 6.25 W/m^2 · °C (or 1.1 Btu/h · ft^2 · °F) for a single-pane window with aluminum frames. Determine the range for the rate of heat loss through a 1.2-m × 1.8-m window of a house that is maintained at 20°C when the outdoor air temperature is −8°C.

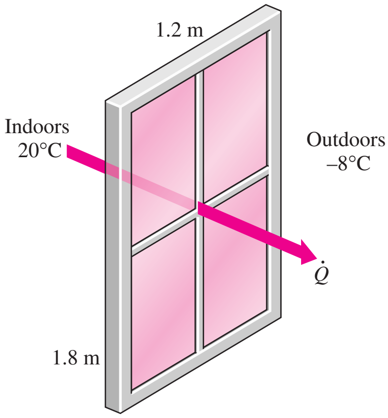


FIGURE P1–138

1–139  Reconsider Problem 1–138. Using EES (or other) software, plot the rate of heat loss through the window as a function of the *U*-factor. Discuss the results.

Design and Essay Problems

1–140 Write an essay on how microwave ovens work, and explain how they cook much faster than conventional ovens. Discuss whether conventional electric or microwave ovens consume more electricity for the same task.

1–141 Using information from the utility bills for the coldest month last year, estimate the average rate of heat loss from your house for that month. In your analysis, consider the contribution of the internal heat sources such as people, lights, and appliances. Identify the primary sources of heat loss from your house and propose ways of improving the energy efficiency of your house.

1–142 Design a 1200-W electric hair dryer such that the air temperature and velocity in the dryer will not exceed 50°C and 3/ms, respectively.

1–143 Design an electric hot water heater for a family of four in your area. The maximum water temperature in the tank and the power consumption are not to exceed 60°C and 4 kW, respectively. There are two showers in the house, and the flow rate of water through each of the shower heads is about 10 L/min. Each family member takes a 5-minute shower every morning. Explain why a hot water tank is necessary, and determine the proper size of the tank for this family.

1–144 Conduct this experiment to determine the heat transfer coefficient between an incandescent lightbulb and the surrounding air using a 60-W lightbulb. You will need an indoor–outdoor thermometer, which can be purchased for about $10 in a hardware store, and a metal glue. You will also need a piece of string and a ruler to calculate the surface area of the lightbulb. First, measure the air temperature in the room, and then glue the tip of the thermocouple wire of the thermometer to the glass of the lightbulb. Turn the light on and wait until the temperature reading stabilizes. The temperature reading will give the surface temperature of the lightbulb. Assuming 10 percent of the rated power of the bulb is converted to light, calculate the heat transfer coefficient from Newton's law of cooling.

CHAPTER

2

HEAT CONDUCTION EQUATION

Heat transfer has *direction* as well as *magnitude.* The rate of heat conduction in a specified direction is proportional to the *temperature gradient,* which is the change in temperature per unit length in that direction. Heat conduction in a medium, in general, is three-dimensional and time dependent. That is, $T = T(x, y, z, t)$ and the temperature in a medium varies with position as well as time. Heat conduction in a medium is said to be *steady* when the temperature does not vary with time, and *unsteady* or *transient* when it does. Heat conduction in a medium is said to be *one-dimensional* when conduction is significant in one dimension only and negligible in the other two dimensions, *two-dimensional* when conduction in the third dimension is negligible, and *three-dimensional* when conduction in all dimensions is significant.

We start this chapter with a description of steady, unsteady, and multidimensional heat conduction. Then we derive the differential equation that governs heat conduction in a large plane wall, a long cylinder, and a sphere, and generalize the results to three-dimensional cases in rectangular, cylindrical, and spherical coordinates. Following a discussion of the boundary conditions, we present the formulation of heat conduction problems and their solutions. Finally, we consider heat conduction problems with variable thermal conductivity.

This chapter deals with the theoretical and mathematical aspects of heat conduction, and it can be covered selectively, if desired, without causing a significant loss in continuity. The more practical aspects of heat conduction are covered in the following two chapters.

CONTENTS

2–1 ▪ INTRODUCTION

In Chapter 1 heat conduction was defined as the transfer of thermal energy from the more energetic particles of a medium to the adjacent less energetic ones. It was stated that conduction can take place in liquids and gases as well as solids provided that there is no bulk motion involved.

Although heat transfer and temperature are closely related, they are of a different nature. Unlike temperature, heat transfer has direction as well as magnitude, and thus it is a *vector* quantity (Fig. 2–1). Therefore, we must specify both direction and magnitude in order to describe heat transfer completely at a point. For example, saying that the temperature on the inner surface of a wall is 18°C describes the temperature at that location fully. But saying that the heat flux on that surface is 50 W/m^2 immediately prompts the question "in what direction?" We can answer this question by saying that heat conduction is toward the inside (indicating heat gain) or toward the outside (indicating heat loss).

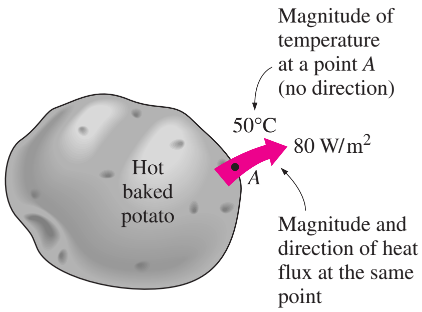


FIGURE 2–1
Heat transfer has direction as well as magnitude, and thus it is a *vector* quantity.

To avoid such questions, we can work with a coordinate system and indicate direction with plus or minus signs. The generally accepted convention is that heat transfer in the positive direction of a coordinate axis is positive and in the opposite direction it is negative. Therefore, a positive quantity indicates heat transfer in the positive direction and a negative quantity indicates heat transfer in the negative direction (Fig. 2–2).

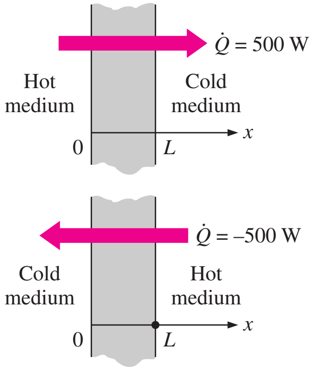
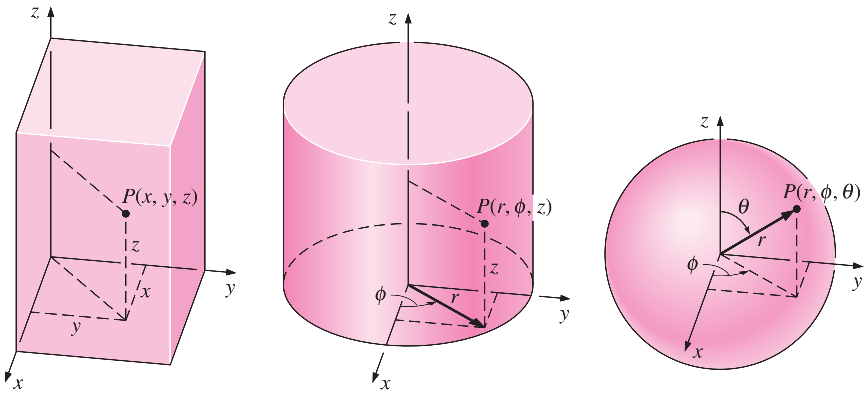


FIGURE 2–2
Indicating direction for heat transfer (positive in the positive direction; negative in the negative direction).

The driving force for any form of heat transfer is the *temperature difference,* and the larger the temperature difference, the larger the rate of heat transfer. Some heat transfer problems in engineering require the determination of the *temperature distribution* (the variation of temperature) throughout the medium in order to calculate some quantities of interest such as the local heat transfer rate, thermal expansion, and thermal stress at some critical locations at specified times. The specification of the *temperature* at a point in a medium first requires the specification of the *location* of that point. This can be done by choosing a suitable coordinate system such as the *rectangular, cylindrical,* or *spherical* coordinates, depending on the geometry involved, and a convenient reference point (the origin).

The *location* of a point is specified as (x, y, z) in rectangular coordinates, as (r, ϕ, z) in cylindrical coordinates, and as (r, ϕ, θ) in spherical coordinates, where the distances x, y, z, and r and the angles ϕ and θ are as shown in Figure 2–3. Then the temperature at a point (x, y, z) at time t in rectangular coordinates is expressed as $T(x, y, z, t)$. The best coordinate system for a given geometry is the one that describes the surfaces of the geometry best. For example, a parallelepiped is best described in rectangular coordinates since each surface can be described by a constant value of the x-, y-, or z-coordinates. A cylinder is best suited for cylindrical coordinates since its lateral surface can be described by a constant value of the radius. Similarly, the entire outer surface of a spherical body can best be described by a constant value of the radius in spherical coordinates. For an arbitrarily shaped body, we normally use rectangular coordinates since it is easier to deal with distances than with angles.

The notation just described is also used to identify the variables involved in a heat transfer problem. For example, the notation $T(x, y, z, t)$ implies that the temperature varies with the space variables x, y, and z as well as time. The



(*a*) Rectangular coordinates (*b*) Cylindrical coordinates (*c*) Spherical coordinates

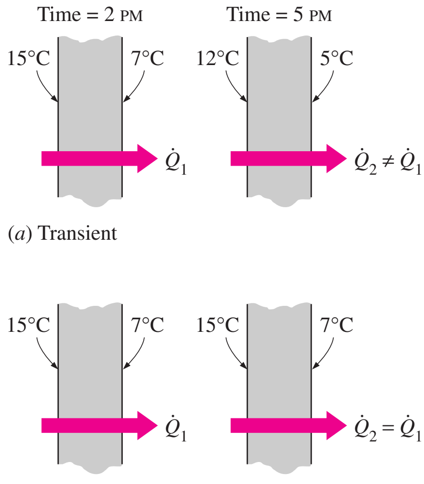
FIGURE 2–3
The various distances and angles involved when describing the location of a point in different coordinate systems.

notation $T(x)$, on the other hand, indicates that the temperature varies in the x-direction only and there is no variation with the other two space coordinates or time.

Steady versus Transient Heat Transfer

Heat transfer problems are often classified as being **steady** (also called *steady-state*) or **transient** (also called *unsteady*). The term *steady* implies *no change* with time at any point within the medium, while *transient* implies *variation with time* or *time dependence*. Therefore, the temperature or heat flux remains unchanged with time during steady heat transfer through a medium at any location, although both quantities may vary from one location to another (Fig. 2–4). For example, heat transfer through the walls of a house will be steady when the conditions inside the house and the outdoors remain constant for several hours. But even in this case, the temperatures on the inner and outer surfaces of the wall will be different unless the temperatures inside and outside the house are the same. The cooling of an apple in a refrigerator, on the other hand, is a transient heat transfer process since the temperature at any fixed point within the apple will change with time during cooling. During transient heat transfer, the temperature normally varies with time as well as position. In the special case of variation with time but not with position, the temperature of the medium changes *uniformly* with time. Such heat transfer systems are called **lumped systems.** A small metal object such as a thermocouple junction or a thin copper wire, for example, can be analyzed as a lumped system during a heating or cooling process.

Most heat transfer problems encountered in practice are *transient* in nature, but they are usually analyzed under some presumed *steady* conditions since steady processes are easier to analyze, and they provide the answers to our questions. For example, heat transfer through the walls and ceiling of a typical house is never steady since the outdoor conditions such as the temperature, the speed and direction of the wind, the location of the sun, and so on, change constantly. The conditions in a typical house are not so steady either. Therefore, it is almost impossible to perform a heat transfer analysis of a house accurately. But then, do we really need an in-depth heat transfer analysis? If the



(*a*) Transient

(*b*) Steady-state

FIGURE 2–4
Steady and transient heat conduction in a plane wall.

purpose of a heat transfer analysis of a house is to determine the proper size of a heater, which is usually the case, we need to know the *maximum* rate of heat loss from the house, which is determined by considering the heat loss from the house under *worst* conditions for an extended period of time, that is, during *steady* operation under worst conditions. Therefore, we can get the answer to our question by doing a heat transfer analysis under steady conditions. If the heater is large enough to keep the house warm under the presumed worst conditions, it is large enough for all conditions. The approach described above is a common practice in engineering.

Multidimensional Heat Transfer

Heat transfer problems are also classified as being *one-dimensional, two-dimensional,* or *three-dimensional,* depending on the relative magnitudes of heat transfer rates in different directions and the level of accuracy desired. In the most general case, heat transfer through a medium is **three-dimensional.** That is, the temperature varies along all three primary directions within the medium during the heat transfer process. The temperature distribution throughout the medium at a specified time as well as the heat transfer rate at any location in this general case can be described by a set of three coordinates such as the x, y, and z in the rectangular (or Cartesian) coordinate system; the r, ϕ, and z in the cylindrical coordinate system; and the r, ϕ, and θ in the spherical (or polar) coordinate system. The temperature distribution in this case is expressed as $T(x, y, z, t)$, $T(r, \phi, z, t)$, and $T(r, \phi, \theta, t)$ in the respective coordinate systems.

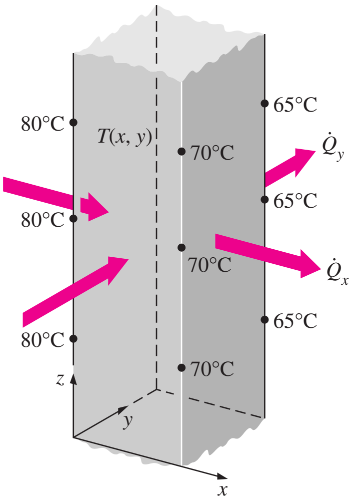


FIGURE 2–5
Two-dimensional heat transfer in a long rectangular bar.

The temperature in a medium, in some cases, varies mainly in two primary directions, and the variation of temperature in the third direction (and thus heat transfer in that direction) is negligible. A heat transfer problem in that case is said to be **two-dimensional.** For example, the steady temperature distribution in a long bar of rectangular cross section can be expressed as $T(x, y)$ if the temperature variation in the z-direction (along the bar) is negligible and there is no change with time (Fig. 2–5).

A heat transfer problem is said to be **one-dimensional** if the temperature in the medium varies in one direction only and thus heat is transferred in one direction, and the variation of temperature and thus heat transfer in other directions are negligible or zero. For example, heat transfer through the glass of a window can be considered to be one-dimensional since heat transfer through the glass will occur predominantly in one direction (the direction normal to the surface of the glass) and heat transfer in other directions (from one side edge to the other and from the top edge to the bottom) is negligible (Fig. 2–6). Likewise, heat transfer through a hot water pipe can be considered to be one-dimensional since heat transfer through the pipe occurs predominantly in the radial direction from the hot water to the ambient, and heat transfer along the pipe and along the circumference of a cross section (z- and ϕ-directions) is typically negligible. Heat transfer to an egg dropped into boiling water is also nearly one-dimensional because of symmetry. Heat will be transferred to the egg in this case in the radial direction, that is, along straight lines passing through the midpoint of the egg.

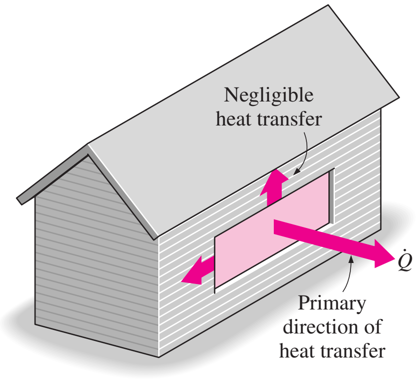


FIGURE 2–6
Heat transfer through the window of a house can be taken to be one-dimensional.

We also mentioned in Chapter 1 that the rate of heat conduction through a medium in a specified direction (say, in the x-direction) is proportional to the temperature difference across the medium and the area normal to the direction of heat transfer, but is inversely proportional to the distance in that direction. This was expressed in the differential form by **Fourier's law of heat conduction** for one-dimensional heat conduction as

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx} \qquad \text{(W)} \tag{2-1}$$

where k is the *thermal conductivity* of the material, which is a measure of the ability of a material to conduct heat, and dT/dx is the *temperature gradient,* which is the slope of the temperature curve on a T-x diagram (Fig. 2–7). The thermal conductivity of a material, in general, varies with temperature. But sufficiently accurate results can be obtained by using a constant value for thermal conductivity at the *average* temperature.

Heat is conducted in the direction of decreasing temperature, and thus the temperature gradient is negative when heat is conducted in the positive x-direction. The *negative sign* in Eq. 2–1 ensures that heat transfer in the positive x-direction is a positive quantity.

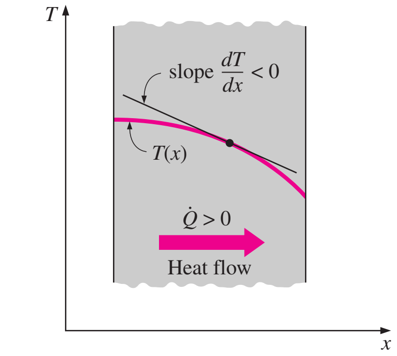


FIGURE 2–7
The temperature gradient dT/dx is simply the slope of the temperature curve on a T-x diagram.

To obtain a general relation for Fourier's law of heat conduction, consider a medium in which the temperature distribution is three-dimensional. Figure 2–8 shows an isothermal surface in that medium. The heat flux vector at a point P on this surface must be perpendicular to the surface, and it must point in the direction of decreasing temperature. If n is the normal of the isothermal surface at point P, the rate of heat conduction at that point can be expressed by Fourier's law as

$$\dot{Q}_n = -kA\frac{\partial T}{\partial n} \qquad \text{(W)} \tag{2-2}$$

In rectangular coordinates, the heat conduction vector can be expressed in terms of its components as

$$\vec{\dot{Q}}_n = \dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k} \tag{2-3}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the unit vectors, and $\dot{Q}_x$, $\dot{Q}_y$, and $\dot{Q}_z$ are the magnitudes of the heat transfer rates in the x-, y-, and z-directions, which again can be determined from Fourier's law as

$$\dot{Q}_x = -kA_x\frac{\partial T}{\partial x}, \qquad \dot{Q}_y = -kA_y\frac{\partial T}{\partial y}, \qquad \text{and} \qquad \dot{Q}_z = -kA_z\frac{\partial T}{\partial z} \tag{2-4}$$

Here A_x, A_y and A_z are heat conduction areas normal to the x-, y-, and z-directions, respectively (Fig. 2–8).

Most engineering materials are *isotropic* in nature, and thus they have the same properties in all directions. For such materials we do not need to be concerned about the variation of properties with direction. But in *anisotropic* materials such as the fibrous or composite materials, the properties may change with direction. For example, some of the properties of wood along the grain are different than those in the direction normal to the grain. In such cases the thermal conductivity may need to be expressed as a tensor quantity to account for the variation with direction. The treatment of such advanced topics is beyond the scope of this text, and we will assume the thermal conductivity of a material to be independent of direction.

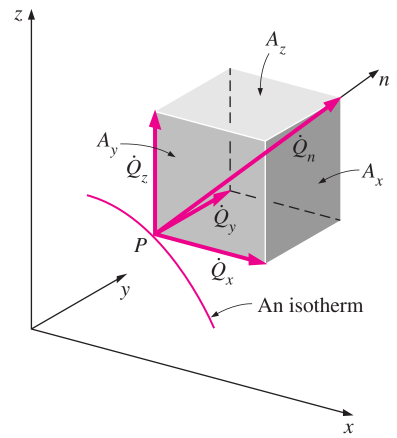


FIGURE 2–8
The heat transfer vector is always normal to an isothermal surface and can be resolved into its components like any other vector.

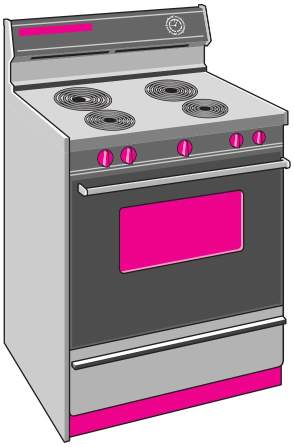


FIGURE 2–9
Heat is generated in the heating coils of an electric range as a result of the conversion of electrical energy to heat.

Heat Generation

A medium through which heat is conducted may involve the conversion of electrical, nuclear, or chemical energy into heat (or thermal) energy. In heat conduction analysis, such conversion processes are characterized as **heat generation.**

For example, the temperature of a resistance wire rises rapidly when electric current passes through it as a result of the electrical energy being converted to heat at a rate of I^2R, where I is the current and R is the electrical resistance of the wire (Fig. 2–9). The safe and effective removal of this heat away from the sites of heat generation (the electronic circuits) is the subject of *electronics cooling,* which is one of the modern application areas of heat transfer.

Likewise, a large amount of heat is generated in the fuel elements of nuclear reactors as a result of nuclear fission that serves as the *heat source* for the nuclear power plants. The natural disintegration of radioactive elements in nuclear waste or other radioactive material also results in the generation of heat throughout the body. The heat generated in the sun as a result of the fusion of hydrogen into helium makes the sun a large nuclear reactor that supplies heat to the earth.

Another source of heat generation in a medium is exothermic chemical reactions that may occur throughout the medium. The chemical reaction in this case serves as a *heat source* for the medium. In the case of endothermic reactions, however, heat is absorbed instead of being released during reaction, and thus the chemical reaction serves as a *heat sink.* The heat generation term becomes a negative quantity in this case.

Often it is also convenient to model the absorption of radiation such as solar energy or gamma rays as heat generation when these rays penetrate deep into the body while being absorbed gradually. For example, the absorption of solar energy in large bodies of water can be treated as heat generation throughout the water at a rate equal to the rate of absorption, which varies with depth (Fig. 2–10). But the absorption of solar energy by an opaque body occurs within a few microns of the surface, and the solar energy that penetrates into the medium in this case can be treated as specified heat flux on the surface.

Note that heat generation is a *volumetric phenomenon.* That is, it occurs throughout the body of a medium. Therefore, the rate of heat generation in a medium is usually specified *per unit volume* and is denoted by $\dot{g}$, whose unit is W/m^3 or $Btu/h \cdot ft^3$.

The rate of heat generation in a medium may vary with time as well as position within the medium. When the variation of heat generation with position is known, the *total* rate of heat generation in a medium of volume V can be determined from

$$\dot{G} = \int_V \dot{g}\,dV \qquad \text{(W)} \tag{2-5}$$

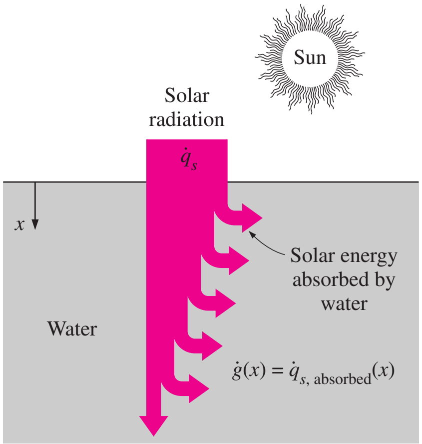


FIGURE 2–10
The absorption of solar radiation by water can be treated as heat generation.

In the special case of *uniform* heat generation, as in the case of electric resistance heating throughout a homogeneous material, the relation in Eq. 2–5 reduces to $\dot{G} = \dot{g}V$, where $\dot{g}$ is the constant rate of heat generation per unit volume.

EXAMPLE 2–1 Heat Gain by a Refrigerator

In order to size the compressor of a new refrigerator, it is desired to determine the rate of heat transfer from the kitchen air into the refrigerated space through the walls, door, and the top and bottom section of the refrigerator (Fig. 2–11). In your analysis, would you treat this as a transient or steady-state heat transfer problem? Also, would you consider the heat transfer to be one-dimensional or multidimensional? Explain.

SOLUTION The heat transfer process from the kitchen air to the refrigerated space is transient in nature since the thermal conditions in the kitchen and the refrigerator, in general, change with time. However, we would analyze this problem as a steady heat transfer problem under the worst anticipated conditions such as the lowest thermostat setting for the refrigerated space, and the anticipated highest temperature in the kitchen (the so-called design conditions). If the compressor is large enough to keep the refrigerated space at the desired temperature setting under the presumed worst conditions, then it is large enough to do so under all conditions by cycling on and off.

Heat transfer into the refrigerated space is three-dimensional in nature since heat will be entering through all six sides of the refrigerator. However, heat transfer through any wall or floor takes place in the direction normal to the surface, and thus it can be analyzed as being one-dimensional. Therefore, this problem can be simplified greatly by considering the heat transfer to be one-dimensional at each of the four sides as well as the top and bottom sections, and then by adding the calculated values of heat transfer at each surface.

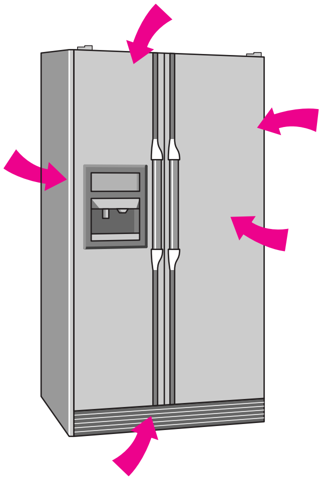


FIGURE 2–11
Schematic for Example 2–1.

EXAMPLE 2–2 Heat Generation in a Hair Dryer

The resistance wire of a 1200-W hair dryer is 80 cm long and has a diameter of $D = 0.3$ cm (Fig. 2–12). Determine the rate of heat generation in the wire per unit volume, in W/cm^3, and the heat flux on the outer surface of the wire as a result of this heat generation.

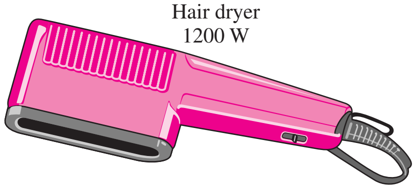


FIGURE 2–12
Schematic for Example 2–2.

SOLUTION The power consumed by the resistance wire of a hair dryer is given. The heat generation and the heat flux are to be determined.

Assumptions Heat is generated uniformly in the resistance wire.

Analysis A 1200-W hair dryer will convert electrical energy into heat in the wire at a rate of 1200 W. Therefore, the rate of heat generation in a resistance wire is equal to the power consumption of a resistance heater. Then the rate of heat generation in the wire per unit volume is determined by dividing the total rate of heat generation by the volume of the wire,

$$\dot{g} = \frac{\dot{G}}{V_{\text{wire}}} = \frac{\dot{G}}{(\pi D^2/4)L} = \frac{1200 \text{ W}}{[\pi(0.3 \text{ cm})^2/4](80 \text{ cm})} = \mathbf{212 \text{ W/cm}^3}$$

Similarly, heat flux on the outer surface of the wire as a result of this heat generation is determined by dividing the total rate of heat generation by the surface area of the wire,

$$\dot{q} = \frac{\dot{G}}{A_{\text{wire}}} = \frac{\dot{G}}{\pi D L} = \frac{1200 \text{ W}}{\pi(0.3 \text{ cm})(80 \text{ cm})} = \mathbf{15.9 \text{ W/cm}^2}$$

Discussion Note that heat generation is expressed per unit volume in W/cm^3 or $Btu/h \cdot ft^3$, whereas heat flux is expressed per unit surface area in W/cm^2 or $Btu/h \cdot ft^2$.

2–2 ▪ ONE-DIMENSIONAL HEAT CONDUCTION EQUATION

Consider heat conduction through a large plane wall such as the wall of a house, the glass of a single pane window, the metal plate at the bottom of a pressing iron, a cast iron steam pipe, a cylindrical nuclear fuel element, an electrical resistance wire, the wall of a spherical container, or a spherical metal ball that is being quenched or tempered. Heat conduction in these and many other geometries can be approximated as being *one-dimensional* since heat conduction through these geometries will be dominant in one direction and negligible in other directions. Below we will develop the one-dimensional heat conduction equation in rectangular, cylindrical, and spherical coordinates.

Heat Conduction Equation in a Large Plane Wall

Consider a thin element of thickness Δx in a large plane wall, as shown in Figure 2–13. Assume the density of the wall is ρ, the specific heat is C, and the area of the wall normal to the direction of heat transfer is A. An *energy balance* on this thin element during a small time interval Δt can be expressed as

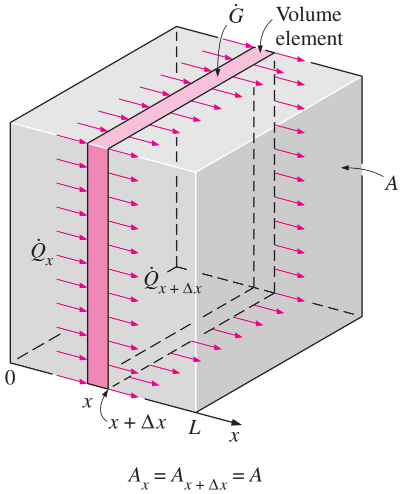


FIGURE 2–13
One-dimensional heat conduction through a volume element in a large plane wall.

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

or

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \tag{2-6}$$

But the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho C A \Delta x (T_{t+\Delta t} - T_t) \tag{2-7}$$

$$\dot{G}_{\text{element}} = \dot{g} V_{\text{element}} = \dot{g} A \Delta x \tag{2-8}$$

Substituting into Equation 2–6, we get

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{g} A \Delta x = \rho C A \Delta x \frac{T_{t+\Delta t} - T_t}{\Delta t} \tag{2-9}$$

Dividing by $A\Delta x$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{g} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t} \tag{2-10}$$

Taking the limit as $\Delta x \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \tag{2-11}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x \to 0}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{\partial \dot{Q}}{\partial x} = \frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right) \tag{2-12}$$

Noting that the area A is constant for a plane wall, the one-dimensional transient heat conduction equation in a plane wall becomes

Variable conductivity:
$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \tag{2-13}$$

The thermal conductivity k of a material, in general, depends on the temperature T (and therefore x), and thus it cannot be taken out of the derivative. However, the *thermal conductivity* in most practical applications can be assumed to remain *constant* at some average value. The equation above in that case reduces to

Constant conductivity:
$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2-14}$$

where the property $\alpha = k/\rho C$ is the **thermal diffusivity** of the material and represents how fast heat propagates through a material. It reduces to the following forms under specified conditions (Fig. 2–14):

(1) *Steady-state:*
$(\partial/\partial t = 0)$
$$\frac{d^2 T}{dx^2} + \frac{\dot{g}}{k} = 0 \tag{2-15}$$

(2) *Transient, no heat generation:*
$(\dot{g} = 0)$
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2-16}$$

(3) *Steady-state, no heat generation:*
$(\partial/\partial t = 0$ and $\dot{g} = 0)$
$$\frac{d^2 T}{dx^2} = 0 \tag{2-17}$$

Note that we replaced the partial derivatives by ordinary derivatives in the one-dimensional steady heat conduction case since the partial and ordinary derivatives of a function are identical when the function depends on a single variable only [$T = T(x)$ in this case].

General, one dimensional:

No generation — Steady-state

$$\frac{\partial^2 T}{\partial x^2} + \cancelto{0}{\frac{\dot{g}}{k}} = \frac{1}{\alpha}\cancelto{0}{\frac{\partial T}{\partial t}}$$

Steady, one-dimensional:

$$\frac{d^2 T}{dx^2} = 0$$

FIGURE 2–14
The simplification of the one-dimensional heat conduction equation in a plane wall for the case of constant conductivity for steady conduction with no heat generation.

Heat Conduction Equation in a Long Cylinder

Now consider a thin cylindrical shell element of thickness Δr in a long cylinder, as shown in Figure 2–15. Assume the density of the cylinder is ρ, the specific heat is C, and the length is L. The area of the cylinder normal to the direction of heat transfer at any location is $A = 2\pi rL$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. An *energy balance* on this thin cylindrical shell element during a small time interval Δt can be expressed as

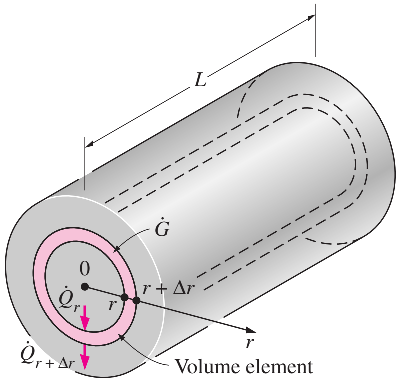


FIGURE 2–15
One-dimensional heat conduction through a volume element in a long cylinder.

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r + \Delta r\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

or

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \tag{2-18}$$

The change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho CA\Delta r(T_{t+\Delta t} - T_t) \tag{2-19}$$

$$\dot{G}_{\text{element}} = \dot{g}V_{\text{element}} = \dot{g}A\Delta r \tag{2-20}$$

Substituting into Eq. 2–18, we get

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{g}A\Delta r = \rho CA\Delta r\frac{T_{t+\Delta t} - T_t}{\Delta t} \tag{2-21}$$

where $A = 2\pi rL$. You may be tempted to express the area at the *middle* of the element using the *average* radius as $A = 2\pi(r + \Delta r/2)L$. But there is nothing we can gain from this complication since later in the analysis we will take the limit as $\Delta r \to 0$ and thus the term $\Delta r/2$ will drop out. Now dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{g} = \rho C\frac{T_{t+\Delta t} - T_t}{\Delta t} \tag{2-22}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \tag{2-23}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right) \tag{2-24}$$

Noting that the heat transfer area in this case is $A = 2\pi rL$, the one-dimensional transient heat conduction equation in a cylinder becomes

Variable conductivity:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \tag{2-25}$$

For the case of constant thermal conductivity, the equation above reduces to

Constant conductivity:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2-26}$$

where again the property $\alpha = k/\rho C$ is the thermal diffusivity of the material. Equation 2–26 reduces to the following forms under specified conditions (Fig. 2–16):

(1) *Steady-state:* ($\partial/\partial t = 0$)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0 \quad \textbf{(2-27)}$$

(2) *Transient, no heat generation:* ($\dot{g} = 0$)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \textbf{(2-28)}$$

(3) *Steady-state, no heat generation:* ($\partial/\partial t = 0$ and $\dot{g} = 0$)

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \quad \textbf{(2-29)}$$

(*a*) The form that is ready to integrate

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

(*b*) The equivalent alternative form

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$$

FIGURE 2–16
Two equivalent forms of the differential equation for the one-dimensional steady heat conduction in a cylinder with no heat generation.

Note that we again replaced the partial derivatives by ordinary derivatives in the one-dimensional steady heat conduction case since the partial and ordinary derivatives of a function are identical when the function depends on a single variable only [$T = T(r)$ in this case].

Heat Conduction Equation in a Sphere

Now consider a sphere with density ρ, specific heat C, and outer radius R. The area of the sphere normal to the direction of heat transfer at any location is $A = 4\pi r^2$, where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case also, and thus it varies with location. By considering a thin spherical shell element of thickness Δr and repeating the approach described above for the cylinder by using $A = 4\pi r^2$ instead of $A = 2\pi rL$, the one-dimensional transient heat conduction equation for a sphere is determined to be (Fig. 2–17)

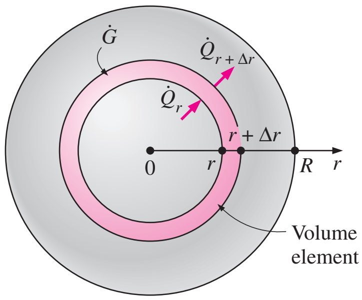


FIGURE 2–17
One-dimensional heat conduction through a volume element in a sphere.

Variable conductivity:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k\frac{\partial T}{\partial r}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \quad \textbf{(2-30)}$$

which, in the case of constant thermal conductivity, reduces to

Constant conductivity:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \textbf{(2-31)}$$

where again the property $\alpha = k/\rho C$ is the thermal diffusivity of the material. It reduces to the following forms under specified conditions:

(1) *Steady-state:* ($\partial/\partial t = 0$)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0 \quad \textbf{(2-32)}$$

(2) *Transient, no heat generation:* ($\dot{g} = 0$)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \textbf{(2-33)}$$

(3) *Steady-state, no heat generation:* ($\partial/\partial t = 0$ and $\dot{g} = 0$)

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \quad \text{or} \quad r\frac{d^2T}{dr^2} + 2\frac{dT}{dr} = 0 \quad \textbf{(2-34)}$$

where again we replaced the partial derivatives by ordinary derivatives in the one-dimensional steady heat conduction case.

Combined One-Dimensional Heat Conduction Equation

An examination of the one-dimensional transient heat conduction equations for the plane wall, cylinder, and sphere reveals that all three equations can be expressed in a compact form as

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n k \frac{\partial T}{\partial r}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t} \tag{2-35}$$

where $n = 0$ for a plane wall, $n = 1$ for a cylinder, and $n = 2$ for a sphere. In the case of a plane wall, it is customary to replace the variable r by x. This equation can be simplified for steady-state or no heat generation cases as described before.

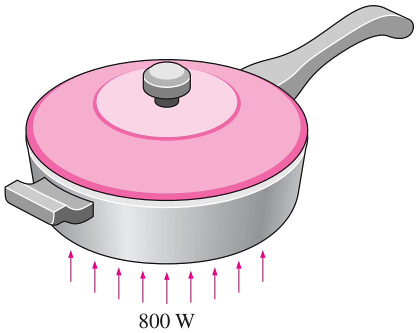


FIGURE 2–18
Schematic for Example 2–3.

EXAMPLE 2–3 Heat Conduction through the Bottom of a Pan

Consider a steel pan placed on top of an electric range to cook spaghetti (Fig. 2–18). The bottom section of the pan is $L = 0.4$ cm thick and has a diameter of $D = 18$ cm. The electric heating unit on the range top consumes 800 W of power during cooking, and 80 percent of the heat generated in the heating element is transferred uniformly to the pan. Assuming constant thermal conductivity, obtain the differential equation that describes the variation of the temperature in the bottom section of the pan during steady operation.

SOLUTION The bottom section of the pan has a large surface area relative to its thickness and can be approximated as a large plane wall. Heat flux is applied to the bottom surface of the pan uniformly, and the conditions on the inner surface are also uniform. Therefore, we expect the heat transfer through the bottom section of the pan to be from the bottom surface toward the top, and heat transfer in this case can reasonably be approximated as being one-dimensional. Taking the direction normal to the bottom surface of the pan to be the x-axis, we will have $T = T(x)$ during steady operation since the temperature in this case will depend on x only.

The thermal conductivity is given to be constant, and there is no heat generation in the medium (within the bottom section of the pan). Therefore, the differential equation governing the variation of temperature in the bottom section of the pan in this case is simply Eq. 2–17,

$$\frac{d^2T}{dx^2} = 0$$

which is the steady one-dimensional heat conduction equation in rectangular coordinates under the conditions of constant thermal conductivity and no heat generation. Note that the conditions at the surface of the medium have no effect on the differential equation.

EXAMPLE 2–4 Heat Conduction in a Resistance Heater

A 2-kW resistance heater wire with thermal conductivity $k = 15$ W/m · °C, diameter $D = 0.4$ cm, and length $L = 50$ cm is used to boil water by immersing it in water (Fig. 2–19). Assuming the variation of the thermal conductivity of the wire with temperature to be negligible, obtain the differential equation that describes the variation of the temperature in the wire during steady operation.

SOLUTION The resistance wire can be considered to be a very long cylinder since its length is more than 100 times its diameter. Also, heat is generated uniformly in the wire and the conditions on the outer surface of the wire are uniform. Therefore, it is reasonable to expect the temperature in the wire to vary in the radial r direction only and thus the heat transfer to be one-dimensional. Then we will have $T = T(r)$ during steady operation since the temperature in this case will depend on r only.

The rate of heat generation in the wire per unit volume can be determined from

$$\dot{g} = \frac{\dot{G}}{V_{\text{wire}}} = \frac{\dot{G}}{(\pi D^2/4)L} = \frac{2000 \text{ W}}{[\pi(0.004 \text{ m})^2/4](0.5 \text{ cm})} = 0.318 \times 10^9 \text{ W/m}^3$$

Noting that the thermal conductivity is given to be constant, the differential equation that governs the variation of temperature in the wire is simply Eq. 2–27,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

which is the steady one-dimensional heat conduction equation in cylindrical coordinates for the case of constant thermal conductivity. Note again that the conditions at the surface of the wire have no effect on the differential equation.

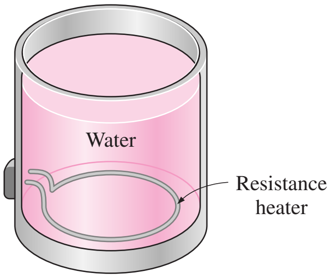


FIGURE 2–19
Schematic for Example 2–4.

EXAMPLE 2–5 Cooling of a Hot Metal Ball in Air

A spherical metal ball of radius R is heated in an oven to a temperature of 600°F throughout and is then taken out of the oven and allowed to cool in ambient air at T_∞ = 75°F by convection and radiation (Fig. 2–20). The thermal conductivity of the ball material is known to vary linearly with temperature. Assuming the ball is cooled uniformly from the entire outer surface, obtain the differential equation that describes the variation of the temperature in the ball during cooling.

SOLUTION The ball is initially at a uniform temperature and is cooled uniformly from the entire outer surface. Also, the temperature at any point in the ball will change with time during cooling. Therefore, this is a one-dimensional transient heat conduction problem since the temperature within the ball will change with the radial distance r and the time t. That is, $T = T(r, t)$.

The thermal conductivity is given to be variable, and there is no heat generation in the ball. Therefore, the differential equation that governs the variation of temperature in the ball in this case is obtained from Eq. 2–30 by setting the heat generation term equal to zero. We obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k\frac{\partial T}{\partial r}\right) = \rho C\frac{\partial T}{\partial t}$$

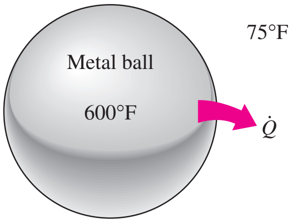


FIGURE 2–20
Schematic for Example 2–5.

which is the one-dimensional transient heat conduction equation in spherical coordinates under the conditions of variable thermal conductivity and no heat generation. Note again that the conditions at the outer surface of the ball have no effect on the differential equation.

2–3 ▪ GENERAL HEAT CONDUCTION EQUATION

In the last section we considered one-dimensional heat conduction and assumed heat conduction in other directions to be negligible. Most heat transfer problems encountered in practice can be approximated as being one-dimensional, and we will mostly deal with such problems in this text. However, this is not always the case, and sometimes we need to consider heat transfer in other directions as well. In such cases heat conduction is said to be *multidimensional,* and in this section we will develop the governing differential equation in such systems in rectangular, cylindrical, and spherical coordinate systems.

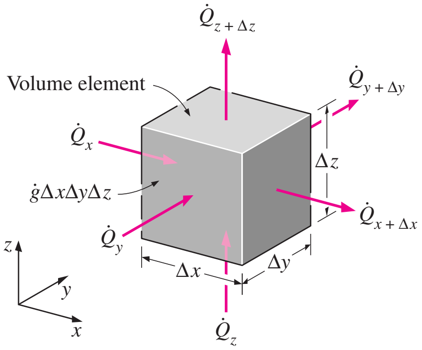


FIGURE 2–21
Three-dimensional heat conduction through a rectangular volume element.

Rectangular Coordinates

Consider a small rectangular element of length Δx, width Δy, and height Δz, as shown in Figure 2–21. Assume the density of the body is ρ and the specific heat is C. An *energy balance* on this element during a small time interval Δt can be expressed as

$$\left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction at}\\ x,\, y, \text{ and } z\end{array}\right) - \left(\begin{array}{c}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x+\Delta x,\\ y+\Delta y, \text{ and } z+\Delta z\end{array}\right) + \left(\begin{array}{c}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{content of}\\ \text{the element}\end{array}\right)$$

or

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \qquad \textbf{(2-36)}$$

Noting that the volume of the element is $V_{\text{element}} = \Delta x \Delta y \Delta z$, the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho C \Delta x \Delta y \Delta z (T_{t+\Delta t} - T_t)$$
$$\dot{G}_{\text{element}} = \dot{g} V_{\text{element}} = \dot{g} \Delta x \Delta y \Delta z$$

Substituting into Eq. 2–36, we get

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{g}\Delta x \Delta y \Delta z = \rho C \Delta x \Delta y \Delta z \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $\Delta x \Delta y \Delta z$ gives

$$-\frac{1}{\Delta y \Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x \Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} - \frac{1}{\Delta x \Delta y}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{g} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t} \qquad \textbf{(2-37)}$$

Noting that the heat transfer areas of the element for heat conduction in the x, y, and z directions are $A_x = \Delta y \Delta z$, $A_y = \Delta x \Delta z$, and $A_z = \Delta x \Delta y$, respectively, and taking the limit as Δx, Δy, Δz and $\Delta t \to 0$ yields

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t} \qquad \textbf{(2-38)}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x \to 0}\frac{1}{\Delta y \Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{1}{\Delta y \Delta z}\frac{\partial Q_x}{\partial x} = \frac{1}{\Delta y \Delta z}\frac{\partial}{\partial x}\left(-k\Delta y \Delta z\frac{\partial T}{\partial x}\right) = -\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)$$

$$\lim_{\Delta y \to 0}\frac{1}{\Delta x \Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \frac{1}{\Delta x \Delta z}\frac{\partial Q_y}{\partial y} = \frac{1}{\Delta x \Delta z}\frac{\partial}{\partial y}\left(-k\Delta x \Delta z\frac{\partial T}{\partial y}\right) = -\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)$$

$$\lim_{\Delta z \to 0}\frac{1}{\Delta x \Delta y}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \frac{1}{\Delta x \Delta y}\frac{\partial Q_z}{\partial z} = \frac{1}{\Delta x \Delta y}\frac{\partial}{\partial z}\left(-k\Delta x \Delta y\frac{\partial T}{\partial z}\right) = -\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$

Equation 2–38 is the general heat conduction equation in rectangular coordinates. In the case of constant thermal conductivity, it reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \textbf{(2-39)}$$

where the property $\alpha = k/\rho C$ is again the *thermal diffusivity* of the material. Equation 2–39 is known as the **Fourier-Biot equation,** and it reduces to these forms under specified conditions:

(1) *Steady-state:*
(called the **Poisson equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = 0 \qquad \textbf{(2-40)}$$

(2) *Transient, no heat generation:*
(called the **diffusion equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \textbf{(2-41)}$$

(3) *Steady-state, no heat generation:*
(called the **Laplace equation**)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad \textbf{(2-42)}$$

Note that in the special case of one-dimensional heat transfer in the x-direction, the derivatives with respect to y and z drop out and the equations above reduce to the ones developed in the previous section for a plane wall (Fig. 2–22).

$$\frac{\partial^2 T}{\partial x^2} \cancel{+ \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}} + \frac{\dot{g}}{k} = 0$$

$$\frac{\partial^2 T}{\partial x^2} \cancel{+ \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} \cancel{+ \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}} = 0$$

FIGURE 2–22
The three-dimensional heat conduction equations reduce to the one-dimensional ones when the temperature varies in one dimension only.

Cylindrical Coordinates

The general heat conduction equation in cylindrical coordinates can be obtained from an energy balance on a volume element in cylindrical coordinates, shown in Figure 2–23, by following the steps just outlined. It can also be obtained directly from Eq. 2–38 by coordinate transformation using the following relations between the coordinates of a point in rectangular and cylindrical coordinate systems:

$$x = r\cos\phi, \qquad y = r\sin\phi, \qquad \text{and} \qquad z = z$$

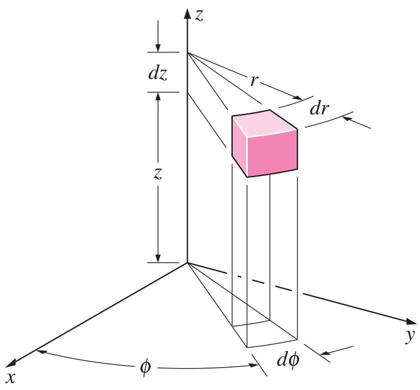


FIGURE 2–23
A differential volume element in cylindrical coordinates.

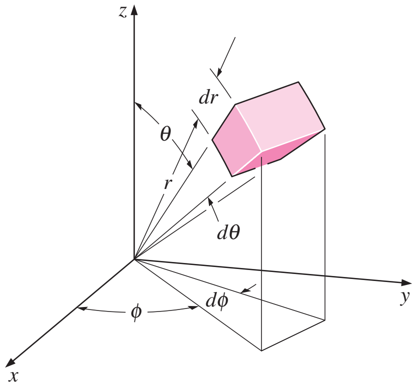


FIGURE 2–24
A differential volume element in spherical coordinates.

After lengthy manipulations, we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(kr\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{g}=\rho C\frac{\partial T}{\partial t} \tag{2-43}$$

Spherical Coordinates

The general heat conduction equations in spherical coordinates can be obtained from an energy balance on a volume element in spherical coordinates, shown in Figure 2–24, by following the steps outlined above. It can also be obtained directly from Eq. 2–38 by coordinate transformation using the following relations between the coordinates of a point in rectangular and spherical coordinate systems:

$$x = r\cos\phi\sin\theta, \qquad y = r\sin\phi\sin\theta, \qquad \text{and} \qquad z = \cos\theta$$

Again after lengthy manipulations, we obtain

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{g}=\rho C\frac{\partial T}{\partial t} \tag{2-44}$$

Obtaining analytical solutions to these differential equations requires a knowledge of the solution techniques of partial differential equations, which is beyond the scope of this introductory text. Here we limit our consideration to one-dimensional steady-state cases or lumped systems, since they result in ordinary differential equations.

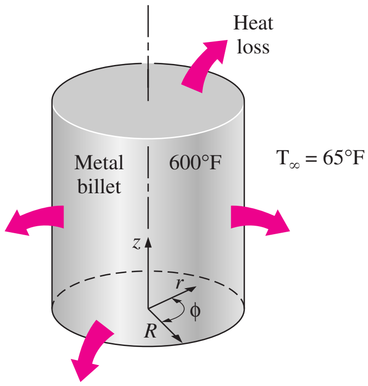


FIGURE 2–25
Schematic for Example 2–6.

EXAMPLE 2–6 **Heat Conduction in a Short Cylinder**

A short cylindrical metal billet of radius R and height h is heated in an oven to a temperature of 600°F throughout and is then taken out of the oven and allowed to cool in ambient air at $T_\infty = 65$°F by convection and radiation. Assuming the billet is cooled uniformly from all outer surfaces and the variation of the thermal conductivity of the material with temperature is negligible, obtain the differential equation that describes the variation of the temperature in the billet during this cooling process.

SOLUTION The billet shown in Figure 2–25 is initially at a uniform temperature and is cooled uniformly from the top and bottom surfaces in the z-direction as well as the lateral surface in the radial r-direction. Also, the temperature at any point in the ball will change with time during cooling. Therefore, this is a two-dimensional transient heat conduction problem since the temperature within the billet will change with the radial and axial distances r and z and with time t. That is, $T = T(r, z, t)$.

The thermal conductivity is given to be constant, and there is no heat generation in the billet. Therefore, the differential equation that governs the variation of temperature in the billet in this case is obtained from Eq. 2–43 by setting the heat generation term and the derivatives with respect to ϕ equal to zero. We obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)=\rho C\frac{\partial T}{\partial t}$$

In the case of constant thermal conductivity, it reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

which is the desired equation.

2–4 ▪ BOUNDARY AND INITIAL CONDITIONS

The heat conduction equations above were developed using an energy balance on a differential element inside the medium, and they remain the same regardless of the *thermal conditions* on the *surfaces* of the medium. That is, the differential equations do not incorporate any information related to the conditions on the surfaces such as the surface temperature or a specified heat flux. Yet we know that the heat flux and the temperature distribution in a medium depend on the conditions at the surfaces, and the description of a heat transfer problem in a medium is not complete without a full description of the thermal conditions at the bounding surfaces of the medium. The *mathematical expressions* of the thermal conditions at the boundaries are called the **boundary conditions.**

From a mathematical point of view, solving a differential equation is essentially a process of *removing derivatives,* or an *integration* process, and thus the solution of a differential equation typically involves arbitrary constants (Fig. 2–26). It follows that to obtain a unique solution to a problem, we need to specify more than just the governing differential equation. We need to specify some conditions (such as the value of the function or its derivatives at some value of the independent variable) so that forcing the solution to satisfy these conditions at specified points will result in unique values for the arbitrary constants and thus a *unique solution.* But since the differential equation has no place for the additional information or conditions, we need to supply them separately in the form of boundary or initial conditions.

Consider the variation of temperature along the wall of a brick house in winter. The temperature at any point in the wall depends on, among other things, the conditions at the two surfaces of the wall such as the air temperature of the house, the velocity and direction of the winds, and the solar energy incident on the outer surface. That is, the temperature distribution in a medium depends on the conditions at the boundaries of the medium as well as the heat transfer mechanism inside the medium. To describe a heat transfer problem completely, *two boundary conditions* must be given for *each direction* of the coordinate system along which heat transfer is significant (Fig. 2–27). Therefore, we need to specify *two boundary conditions* for one-dimensional problems, *four boundary conditions* for two-dimensional problems, and *six boundary conditions* for three-dimensional problems. In the case of the wall of a house, for example, we need to specify the conditions at two locations (the inner and the outer surfaces) of the wall since heat transfer in this case is one-dimensional. But in the case of a parallelepiped, we need to specify six boundary conditions (one at each face) when heat transfer in all three dimensions is significant.

The differential equation:

$$\frac{d^2T}{dx^2} = 0$$

General solution:

$$T(x) = C_1x + C_2$$

Arbitrary constants

Some specific solutions:

$$T(x) = 2x + 5$$
$$T(x) = -x + 12$$
$$T(x) = -3$$
$$T(x) = 6.2x$$
$$\vdots$$

FIGURE 2–26
The general solution of a typical differential equation involves arbitrary constants, and thus an infinite number of solutions.

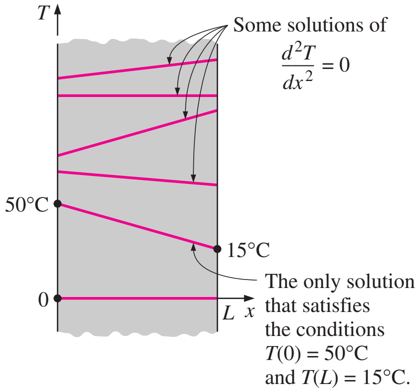


FIGURE 2–27
To describe a heat transfer problem completely, two boundary conditions must be given for each direction along which heat transfer is significant.

The physical argument presented above is consistent with the mathematical nature of the problem since the heat conduction equation is second order (i.e., involves second derivatives with respect to the space variables) in all directions along which heat conduction is significant, and the general solution of a second-order linear differential equation involves two arbitrary constants for each direction. That is, the number of boundary conditions that needs to be specified in a direction is equal to the order of the differential equation in that direction.

Reconsider the brick wall already discussed. The temperature at any point on the wall at a specified time also depends on the condition of the wall at the beginning of the heat conduction process. Such a condition, which is usually specified at time $t = 0$, is called the **initial condition,** which is a mathematical expression for the temperature distribution of the medium initially. Note that we need only one initial condition for a heat conduction problem regardless of the dimension since the conduction equation is first order in time (it involves the first derivative of temperature with respect to time).

In rectangular coordinates, the initial condition can be specified in the general form as

$$T(x, y, z, 0) = f(x, y, z) \tag{2-45}$$

where the function $f(x, y, z)$ represents the temperature distribution throughout the medium at time $t = 0$. When the medium is initially at a uniform temperature of T_i, the initial condition of Eq. 2–45 can be expressed as $T(x, y, z, 0) = T_i$. Note that under *steady* conditions, the heat conduction equation does not involve any time derivatives, and thus we do not need to specify an initial condition.

The heat conduction equation is first order in time, and thus the initial condition cannot involve any derivatives (it is limited to a specified temperature). However, the heat conduction equation is second order in space coordinates, and thus a boundary condition may involve first derivatives at the boundaries as well as specified values of temperature. Boundary conditions most commonly encountered in practice are the *specified temperature, specified heat flux, convection,* and *radiation* boundary conditions.

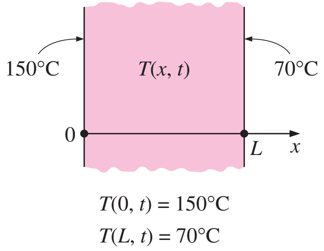


FIGURE 2–28
Specified temperature boundary conditions on both surfaces of a plane wall.

1 Specified Temperature Boundary Condition

The *temperature* of an exposed surface can usually be measured directly and easily. Therefore, one of the easiest ways to specify the thermal conditions on a surface is to specify the temperature. For one-dimensional heat transfer through a plane wall of thickness L, for example, the specified temperature boundary conditions can be expressed as (Fig. 2–28)

$$\begin{aligned} T(0, t) &= T_1 \\ T(L, t) &= T_2 \end{aligned} \tag{2-46}$$

where T_1 and T_2 are the specified temperatures at surfaces at $x = 0$ and $x = L$, respectively. The specified temperatures can be constant, which is the case for steady heat conduction, or may vary with time.

2 Specified Heat Flux Boundary Condition

When there is sufficient information about energy interactions at a surface, it may be possible to determine the rate of heat transfer and thus the *heat flux* $\dot{q}$ (heat transfer rate per unit surface area, W/m^2) on that surface, and this information can be used as one of the boundary conditions. The heat flux in the positive x-direction anywhere in the medium, including the boundaries, can be expressed by *Fourier's law* of heat conduction as

$$\dot{q} = -k\frac{\partial T}{\partial x} = \left(\begin{array}{c}\text{Heat flux in the}\\ \text{positive } x\text{-direction}\end{array}\right) \qquad (\text{W/m}^2) \tag{2-47}$$

Then the boundary condition at a boundary is obtained by setting the specified heat flux equal to $-k(\partial T/\partial x)$ at that boundary. The sign of the specified heat flux is determined by inspection: *positive* if the heat flux is in the positive direction of the coordinate axis, and *negative* if it is in the opposite direction. Note that it is extremely important to have the *correct sign* for the specified heat flux since the wrong sign will invert the direction of heat transfer and cause the heat gain to be interpreted as heat loss (Fig. 2–29).

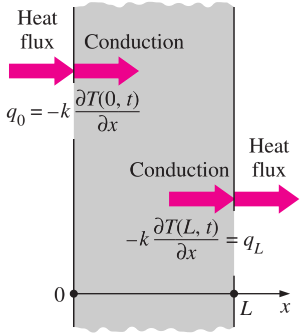


FIGURE 2–29
Specified heat flux boundary conditions on both surfaces of a plane wall.

For a plate of thickness L subjected to heat flux of 50 W/m^2 into the medium from both sides, for example, the specified heat flux boundary conditions can be expressed as

$$-k\frac{\partial T(0, t)}{\partial x} = 50 \qquad \text{and} \qquad -k\frac{\partial T(L, t)}{\partial x} = -50 \tag{2-48}$$

Note that the heat flux at the surface at $x = L$ is in the *negative* x-direction, and thus it is -50 W/m^2.

Special Case: Insulated Boundary

Some surfaces are commonly insulated in practice in order to minimize heat loss (or heat gain) through them. Insulation reduces heat transfer but does not totally eliminate it unless its thickness is infinity. However, heat transfer through a properly insulated surface can be taken to be zero since adequate insulation reduces heat transfer through a surface to negligible levels. Therefore, a well-insulated surface can be modeled as a surface with a specified heat flux of zero. Then the boundary condition on a perfectly insulated surface (at $x = 0$, for example) can be expressed as (Fig. 2–30)

$$k\frac{\partial T(0, t)}{\partial x} = 0 \qquad \text{or} \qquad \frac{\partial T(0, t)}{\partial x} = 0 \tag{2-49}$$

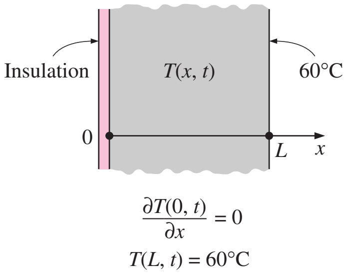


FIGURE 2–30
A plane wall with insulation and specified temperature boundary conditions.

That is, *on an insulated surface, the first derivative of temperature with respect to the space variable (the temperature gradient) in the direction normal to the insulated surface is zero.* This also means that the temperature function must be perpendicular to an insulated surface since the slope of temperature at the surface must be zero.

Another Special Case: Thermal Symmetry

Some heat transfer problems possess *thermal symmetry* as a result of the symmetry in imposed thermal conditions. For example, the two surfaces of a large hot plate of thickness L suspended vertically in air will be subjected to

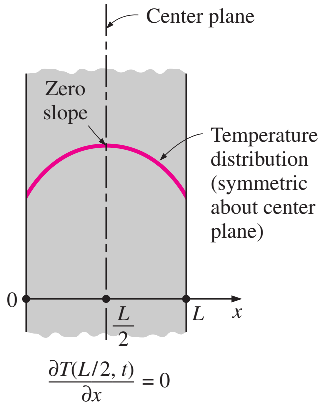


FIGURE 2–31
Thermal symmetry boundary condition at the center plane of a plane wall.

the same thermal conditions, and thus the temperature distribution in one half of the plate will be the same as that in the other half. That is, the heat transfer problem in this plate will possess thermal symmetry about the center plane at $x = L/2$. Also, the direction of heat flow at any point in the plate will be toward the surface closer to the point, and there will be no heat flow across the center plane. Therefore, the center plane can be viewed as an insulated surface, and the thermal condition at this plane of symmetry can be expressed as (Fig. 2–31)

$$\frac{\partial T(L/2, t)}{\partial x} = 0 \tag{2-50}$$

which resembles the *insulation* or *zero heat flux* boundary condition. This result can also be deduced from a plot of temperature distribution with a maximum, and thus zero slope, at the center plane.

In the case of cylindrical (or spherical) bodies having thermal symmetry about the center line (or midpoint), the thermal symmetry boundary condition requires that the first derivative of temperature with respect to r (the radial variable) be zero at the centerline (or the midpoint).

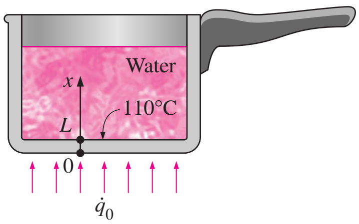


FIGURE 2–32
Schematic for Example 2–7.

EXAMPLE 2–7 Heat Flux Boundary Condition

Consider an aluminum pan used to cook beef stew on top of an electric range. The bottom section of the pan is $L = 0.3$ cm thick and has a diameter of $D = 20$ cm. The electric heating unit on the range top consumes 800 W of power during cooking, and 90 percent of the heat generated in the heating element is transferred to the pan. During steady operation, the temperature of the inner surface of the pan is measured to be 110°C. Express the boundary conditions for the bottom section of the pan during this cooking process.

SOLUTION The heat transfer through the bottom section of the pan is from the bottom surface toward the top and can reasonably be approximated as being one-dimensional. We take the direction normal to the bottom surfaces of the pan as the x axis with the origin at the outer surface, as shown in Figure 2–32. Then the inner and outer surfaces of the bottom section of the pan can be represented by $x = 0$ and $x = L$, respectively. During steady operation, the temperature will depend on x only and thus $T = T(x)$.

The boundary condition on the outer surface of the bottom of the pan at $x = 0$ can be approximated as being specified heat flux since it is stated that 90 percent of the 800 W (i.e., 720 W) is transferred to the pan at that surface. Therefore,

$$-k\frac{dT(0)}{dx} = \dot{q}_0$$

where

$$\dot{q}_0 = \frac{\text{Heat transfer rate}}{\text{Bottom surface area}} = \frac{0.720\text{ kW}}{\pi(0.1\text{ m})^2} = 22.9\text{ kW/m}^2$$

The temperature at the inner surface of the bottom of the pan is specified to be 110°C. Then the boundary condition on this surface can be expressed as

$$T(L) = 110°\text{C}$$

where $L = 0.003$ m. Note that the determination of the boundary conditions may require some reasoning and approximations.

3 Convection Boundary Condition

Convection is probably the most common boundary condition encountered in practice since most heat transfer surfaces are exposed to an environment at a specified temperature. The convection boundary condition is based on a *surface energy balance* expressed as

$$\begin{pmatrix}\text{Heat conduction}\\ \text{at the surface in a}\\ \text{selected direction}\end{pmatrix} = \begin{pmatrix}\text{Heat convection}\\ \text{at the surface in}\\ \text{the same direction}\end{pmatrix}$$

For one-dimensional heat transfer in the x-direction in a plate of thickness L, the convection boundary conditions on both surfaces can be expressed as

$$-k\frac{\partial T(0, t)}{\partial x} = h_1[T_{\infty 1} - T(0, t)] \quad \textbf{(2-51a)}$$

and

$$-k\frac{\partial T(L, t)}{\partial x} = h_2[\text{T}(L, t) - T_{\infty 2}] \quad \textbf{(2-51b)}$$

where h_1 and h_2 are the convection heat transfer coefficients and $T_{\infty 1}$ and $T_{\infty 2}$ are the temperatures of the surrounding mediums on the two sides of the plate, as shown in Figure 2–33.

In writing Eqs. 2–51 for convection boundary conditions, we have selected the direction of heat transfer to be the positive x-direction at both surfaces. But those expressions are equally applicable when heat transfer is in the opposite direction at one or both surfaces since reversing the direction of heat transfer at a surface simply reverses the signs of *both* conduction and convection terms at that surface. This is equivalent to multiplying an equation by -1, which has no effect on the equality (Fig. 2–34). Being able to select either direction as the direction of heat transfer is certainly a relief since often we do not know the surface temperature and thus the direction of heat transfer at a surface in advance. This argument is also valid for other boundary conditions such as the radiation and combined boundary conditions discussed shortly.

Note that a surface has zero thickness and thus no mass, and it cannot store any energy. Therefore, the entire net heat entering the surface from one side must leave the surface from the other side. The convection boundary condition simply states that heat continues to flow from a body to the surrounding medium at the same rate, and it just changes vehicles at the surface from conduction to convection (or vice versa in the other direction). This is analogous to people traveling on buses on land and transferring to the ships at the shore.

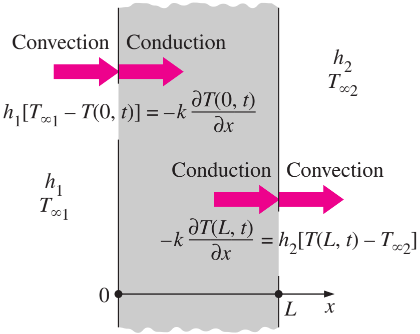


FIGURE 2–33
Convection boundary conditions on the two surfaces of a plane wall.

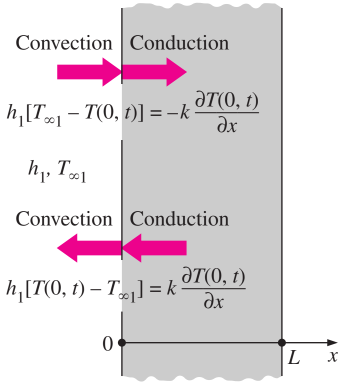


FIGURE 2–34
The assumed direction of heat transfer at a boundary has no effect on the boundary condition expression.

If the passengers are not allowed to wander around at the shore, then the rate at which the people are unloaded at the shore from the buses must equal the rate at which they board the ships. We may call this the conservation of "people" principle.

Also note that the surface temperatures $T(0, t)$ and $T(L, t)$ are not known (if they were known, we would simply use them as the specified temperature boundary condition and not bother with convection). But a surface temperature can be determined once the solution $T(x, t)$ is obtained by substituting the value of x at that surface into the solution.

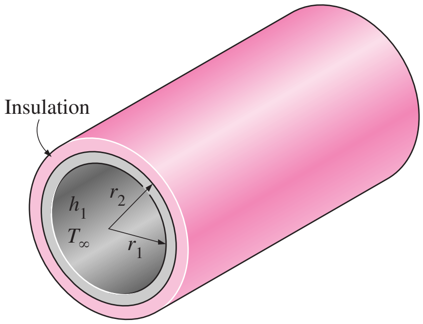


FIGURE 2–35
Schematic for Example 2–8.

EXAMPLE 2–8 Convection and Insulation Boundary Conditions

Steam flows through a pipe shown in Figure 2–35 at an average temperature of $T_\infty = 200°C$. The inner and outer radii of the pipe are $r_1 = 8$ cm and $r_2 = 8.5$ cm, respectively, and the outer surface of the pipe is heavily insulated. If the convection heat transfer coefficient on the inner surface of the pipe is $h = 65\ W/m^2 \cdot °C$, express the boundary conditions on the inner and outer surfaces of the pipe during transient periods.

SOLUTION During initial transient periods, heat transfer through the pipe material will predominantly be in the radial direction, and thus can be approximated as being one-dimensional. Then the temperature within the pipe material will change with the radial distance r and the time t. That is, $T = T(r, t)$.

It is stated that heat transfer between the steam and the pipe at the inner surface is by convection. Then taking the direction of heat transfer to be the positive r direction, the boundary condition on that surface can be expressed as

$$-k\frac{\partial T(r_1, t)}{\partial r} = h[T_\infty - T(r_1)]$$

The pipe is said to be well insulated on the outside, and thus heat loss through the outer surface of the pipe can be assumed to be negligible. Then the boundary condition at the outer surface can be expressed as

$$\frac{\partial T(r_2, t)}{\partial r} = 0$$

That is, the temperature gradient must be zero on the outer surface of the pipe at all times.

4 Radiation Boundary Condition

In some cases, such as those encountered in space and cryogenic applications, a heat transfer surface is surrounded by an evacuated space and thus there is no convection heat transfer between a surface and the surrounding medium. In such cases, *radiation* becomes the only mechanism of heat transfer between the surface under consideration and the surroundings. Using an energy balance, the radiation boundary condition on a surface can be expressed as

$$\begin{pmatrix}\text{Heat conduction}\\ \text{at the surface in a}\\ \text{selected direction}\end{pmatrix} = \begin{pmatrix}\text{Radiation exchange}\\ \text{at the surface in}\\ \text{the same direction}\end{pmatrix}$$

For one-dimensional heat transfer in the x-direction in a plate of thickness L, the radiation boundary conditions on both surfaces can be expressed as (Fig. 2–36)

$$-k\frac{\partial T(0, t)}{\partial x} = \varepsilon_1\sigma[T^4_{\text{surr, 1}} - T(0, t)^4] \tag{2-52a}$$

and

$$-k\frac{\partial T(L, t)}{\partial x} = \varepsilon_2\sigma[T(L, t)^4 - T^4_{\text{surr, 2}}] \tag{2-52b}$$

where ε_1 and ε_2 are the emissivities of the boundary surfaces, $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$ K^4 is the Stefan–Boltzmann constant, and $T_{\text{surr, 1}}$ and $T_{\text{surr, 2}}$ are the average temperatures of the surfaces surrounding the two sides of the plate, respectively. Note that the temperatures in radiation calculations must be expressed in K or R (not in °C or °F).

The radiation boundary condition involves the fourth power of temperature, and thus it is a *nonlinear* condition. As a result, the application of this boundary condition results in powers of the unknown coefficients, which makes it difficult to determine them. Therefore, it is tempting to ignore radiation exchange at a surface during a heat transfer analysis in order to avoid the complications associated with nonlinearity. This is especially the case when heat transfer at the surface is dominated by convection, and the role of radiation is minor.

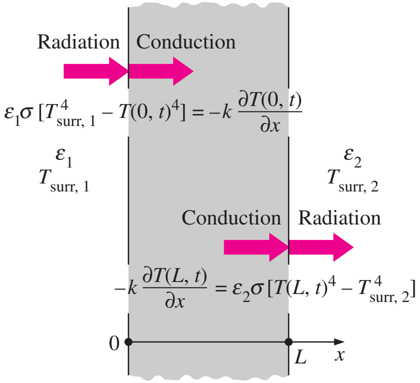


FIGURE 2–36
Radiation boundary conditions on both surfaces of a plane wall.

5 Interface Boundary Conditions

Some bodies are made up of layers of different materials, and the solution of a heat transfer problem in such a medium requires the solution of the heat transfer problem in each layer. This, in turn, requires the specification of the boundary conditions at each *interface*.

The boundary conditions at an interface are based on the requirements that (1) two bodies in contact must have the *same temperature* at the area of contact and (2) an interface (which is a surface) cannot store any energy, and thus the *heat flux* on the two sides of an interface *must be the same*. The boundary conditions at the interface of two bodies A and B in perfect contact at $x = x_0$ can be expressed as (Fig. 2–37)

$$T_A(x_0, t) = T_B(x_0, t) \tag{2-53}$$

and

$$-k_A\frac{\partial T_A(x_0, t)}{\partial x} = -k_B\frac{\partial T_B(x_0, t)}{\partial x} \tag{2-54}$$

where k_A and k_B are the thermal conductivities of the layers A and B, respectively. The case of imperfect contact results in thermal contact resistance, which is considered in the next chapter.

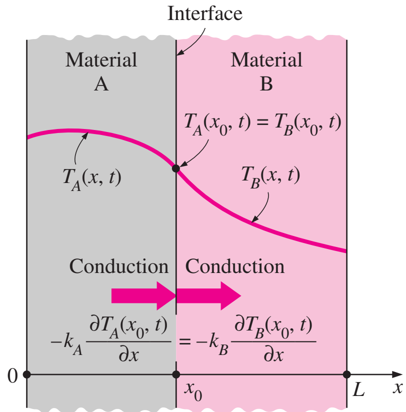


FIGURE 2–37
Boundary conditions at the interface of two bodies in perfect contact.

6 Generalized Boundary Conditions

So far we have considered surfaces subjected to *single mode* heat transfer, such as the specified heat flux, convection, or radiation for simplicity. In general, however, a surface may involve convection, radiation, *and* specified heat flux simultaneously. The boundary condition in such cases is again obtained from a surface energy balance, expressed as

$$\begin{pmatrix}\text{Heat transfer}\\ \text{to the surface}\\ \text{in all modes}\end{pmatrix} = \begin{pmatrix}\text{Heat transfer}\\ \text{from the surface}\\ \text{in all modes}\end{pmatrix} \tag{2-55}$$

This is illustrated in Examples 2–9 and 2–10.

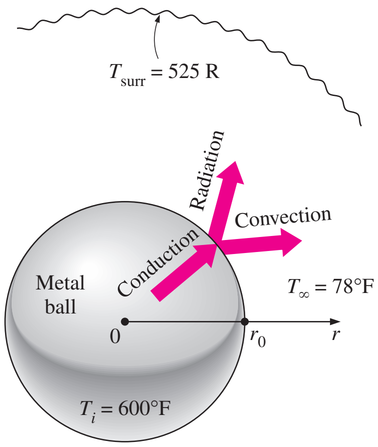


FIGURE 2–38
Schematic for Example 2–9.

EXAMPLE 2–9 Combined Convection and Radiation Condition

A spherical metal ball of radius r_0 is heated in an oven to a temperature of 600°F throughout and is then taken out of the oven and allowed to cool in ambient air at T_∞ = 78°F, as shown in Figure 2–38. The thermal conductivity of the ball material is k = 8.3 Btu/h · ft · °F, and the average convection heat transfer coefficient on the outer surface of the ball is evaluated to be h = 4.5 Btu/h · ft^2 · °F. The emissivity of the outer surface of the ball is ε = 0.6, and the average temperature of the surrounding surfaces is T_{surr} = 525 R. Assuming the ball is cooled uniformly from the entire outer surface, express the initial and boundary conditions for the cooling process of the ball.

SOLUTION The ball is initially at a uniform temperature and is cooled uniformly from the entire outer surface. Therefore, this is a one-dimensional transient heat transfer problem since the temperature within the ball will change with the radial distance r and the time t. That is, $T = T(r, t)$. Taking the moment the ball is removed from the oven to be $t = 0$, the initial condition can be expressed as

$$T(r, 0) = T_i = 600°\text{F}$$

The problem possesses symmetry about the midpoint (r = 0) since the isotherms in this case will be concentric spheres, and thus no heat will be crossing the midpoint of the ball. Then the boundary condition at the midpoint can be expressed as

$$\frac{\partial T(0, t)}{\partial r} = 0$$

The heat conducted to the outer surface of the ball is lost to the environment by convection and radiation. Then taking the direction of heat transfer to be the positive r direction, the boundary condition on the outer surface can be expressed as

$$-k\frac{\partial T(r_0, t)}{\partial r} = h[T(r_0) - T_\infty] + \varepsilon\sigma[T(r_0)^4 - T_{surr}^4]$$

All the quantities in the above relations are known except the temperatures and their derivatives at $r = 0$ and r_0. Also, the radiation part of the boundary condition is often ignored for simplicity by modifying the convection heat transfer coefficient to account for the contribution of radiation. The convection coefficient h in that case becomes the combined heat transfer coefficient.

EXAMPLE 2–10 Combined Convection, Radiation, and Heat Flux

Consider the south wall of a house that is $L = 0.2$ m thick. The outer surface of the wall is exposed to solar radiation and has an absorptivity of $\alpha = 0.5$ for solar energy. The interior of the house is maintained at $T_{\infty 1} = 20°C$, while the ambient air temperature outside remains at $T_{\infty 2} = 5°C$. The sky, the ground, and the surfaces of the surrounding structures at this location can be modeled as a surface at an effective temperature of $T_{sky} = 255$ K for radiation exchange on the outer surface. The radiation exchange between the inner surface of the wall and the surfaces of the walls, floor, and ceiling it faces is negligible. The convection heat transfer coefficients on the inner and the outer surfaces of the wall are $h_1 = 6$ W/m² · °C and $h_2 = 25$ W/m² · °C, respectively. The thermal conductivity of the wall material is $k = 0.7$ W/m · °C, and the emissivity of the outer surface is $\varepsilon_2 = 0.9$. Assuming the heat transfer through the wall to be steady and one-dimensional, express the boundary conditions on the inner and the outer surfaces of the wall.

SOLUTION We take the direction normal to the wall surfaces as the x-axis with the origin at the inner surface of the wall, as shown in Figure 2–39. The heat transfer through the wall is given to be steady and one-dimensional, and thus the temperature depends on x only and not on time. That is, $T = T(x)$.

The boundary condition on the inner surface of the wall at $x = 0$ is a typical convection condition since it does not involve any radiation or specified heat flux. Taking the direction of heat transfer to be the positive x-direction, the boundary condition on the inner surface can be expressed as

$$-k\frac{dT(0)}{dx} = h_1[T_{\infty 1} - T(0)]$$

The boundary condition on the outer surface at $x = 0$ is quite general as it involves conduction, convection, radiation, and specified heat flux. Again taking the direction of heat transfer to be the positive x-direction, the boundary condition on the outer surface can be expressed as

$$-k\frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}] + \varepsilon_2\sigma[T(L)^4 - T_{sky}^4] - \alpha\dot{q}_{solar}$$

where $\dot{q}_{solar}$ is the incident solar heat flux. Assuming the opposite direction for heat transfer would give the same result multiplied by -1, which is equivalent to the relation here. All the quantities in these relations are known except the temperatures and their derivatives at the two boundaries.

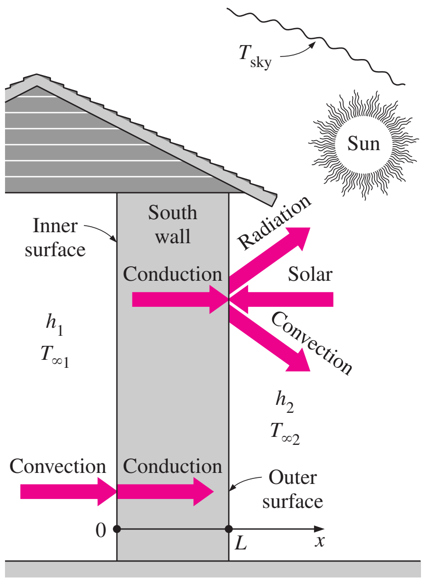


FIGURE 2–39
Schematic for Example 2–10.

Note that a heat transfer problem may involve different kinds of boundary conditions on different surfaces. For example, a plate may be subject to *heat flux* on one surface while losing or gaining heat by *convection* from the other surface. Also, the two boundary conditions in a direction may be specified *at the same boundary,* while no condition is imposed on the other boundary. For example, specifying the temperature and heat flux at $x = 0$ of a plate of thickness L will result in a unique solution for the one-dimensional steady temperature distribution in the plate, including the value of temperature at the surface $x = L$. Although not necessary, there is nothing wrong with specifying more than two boundary conditions in a specified direction, provided that there is no contradiction. The extra conditions in this case can be used to verify the results.

2–5 ▪ SOLUTION OF STEADY ONE-DIMENSIONAL HEAT CONDUCTION PROBLEMS

So far we have derived the differential equations for heat conduction in various coordinate systems and discussed the possible boundary conditions. A heat conduction problem can be formulated by specifying the applicable differential equation and a set of proper boundary conditions.

In this section we will solve a wide range of heat conduction problems in rectangular, cylindrical, and spherical geometries. We will limit our attention to problems that result in *ordinary differential equations* such as the *steady one-dimensional* heat conduction problems. We will also assume *constant thermal conductivity,* but will consider variable conductivity later in this chapter. If you feel rusty on differential equations or haven't taken differential equations yet, no need to panic. *Simple integration* is all you need to solve the steady one-dimensional heat conduction problems.

The solution procedure for solving heat conduction problems can be summarized as (1) *formulate* the problem by obtaining the applicable differential equation in its simplest form and specifying the boundary conditions, (2) obtain the *general solution* of the differential equation, and (3) apply the *boundary conditions* and determine the arbitrary constants in the general solution (Fig. 2–40). This is demonstrated below with examples.

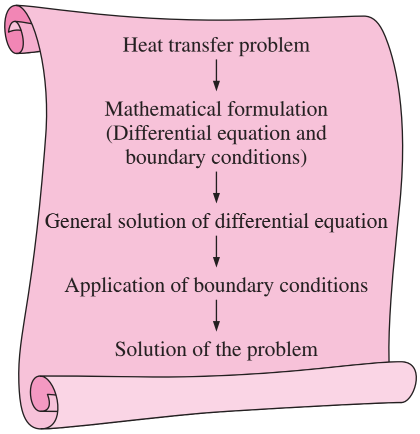


FIGURE 2–40
Basic steps involved in the solution of heat transfer problems.

EXAMPLE 2–11 Heat Conduction in a Plane Wall

Consider a large plane wall of thickness $L = 0.2$ m, thermal conductivity $k = 1.2$ W/m · °C, and surface area $A = 15$ m^2. The two sides of the wall are maintained at constant temperatures of $T_1 = 120$°C and $T_2 = 50$°C, respectively, as shown in Figure 2–41. Determine (*a*) the variation of temperature within the wall and the value of temperature at $x = 0.1$ m and (*b*) the rate of heat conduction through the wall under steady conditions.

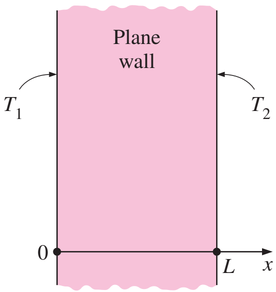


FIGURE 2–41
Schematic for Example 2–11.

SOLUTION A plane wall with specified surface temperatures is given. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat conduction is steady. **2** Heat conduction is one-dimensional since the wall is large relative to its thickness and the thermal conditions on both sides are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 1.2$ W/m · °C.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x*-direction, the differential equation for this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

with boundary conditions

$$T(0) = T_1 = 120°\text{C}$$
$$T(L) = T_2 = 50°\text{C}$$

The differential equation is linear and second order, and a quick inspection of it reveals that it has a single term involving derivatives and no terms involving the unknown function T as a factor. Thus, it can be solved by direct integration. Noting that an integration reduces the order of a derivative by one, the general solution of the differential equation above can be obtained by two simple successive integrations, each of which introduces an integration constant.

Integrating the differential equation once with respect to x yields

$$\frac{dT}{dx} = C_1$$

where C_1 is an arbitrary constant. Notice that the order of the derivative went down by one as a result of integration. As a check, if we take the derivative of this equation, we will obtain the original differential equation. This equation is not the solution yet since it involves a derivative.

Integrating one more time, we obtain

$$T(x) = C_1x + C_2$$

which is the general solution of the differential equation (Fig. 2–42). The general solution in this case resembles the general formula of a straight line whose slope is C_1 and whose value at $x = 0$ is C_2. This is not surprising since the second derivative represents the change in the slope of a function, and a zero second derivative indicates that the slope of the function remains constant. Therefore, *any straight line* is a solution of this differential equation.

The general solution contains two unknown constants C_1 and C_2, and thus we need two equations to determine them uniquely and obtain the specific solution. These equations are obtained by forcing the general solution to satisfy the specified boundary conditions. The application of each condition yields one equation, and thus we need to specify two conditions to determine the constants C_1 and C_2.

When applying a boundary condition to an equation, *all occurrences of the dependent and independent variables and any derivatives are replaced by the specified values.* Thus the only unknowns in the resulting equations are the arbitrary constants.

The first boundary condition can be interpreted as *in the general solution, replace all the x's by zero and $T(x)$ by T_1*. That is (Fig. 2–43),

$$T(0) = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_1$$

Differential equation:

$$\frac{d^2T}{dx^2} = 0$$

Integrate:

$$\frac{dT}{dx} = C_1$$

Integrate again:

$$T(x) = C_1x + C_2$$

General solution   Arbitrary constants

FIGURE 2–42
Obtaining the general solution of a simple second order differential equation by integration.

Boundary condition:

$$T(0) = T_1$$

General solution:

$$T(x) = C_1x + C_2$$

Applying the boundary condition:

$$\underbrace{T(\overset{\uparrow}{0})}_{T_1} = C_1 \overset{\uparrow}{0} + C_2$$

Substituting:

$$T_1 = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_1$$

It cannot involve x or $T(x)$ after the boundary condition is applied.

FIGURE 2–43
When applying a boundary condition to the general solution at a specified point, all occurrences of the dependent and independent variables should be replaced by their specified values at that point.

The second boundary condition can be interpreted as *in the general solution, replace all the x's by L and T(x) by T_2*. That is,

$$T(L) = C_1L + C_2 \quad \rightarrow \quad T_2 = C_1L + T_1 \quad \rightarrow \quad C_1 = \frac{T_2 - T_1}{L}$$

Substituting the C_1 and C_2 expressions into the general solution, we obtain

$$T(x) = \frac{T_2 - T_1}{L}x + T_1 \tag{2-56}$$

which is the desired solution since it satisfies not only the differential equation but also the two specified boundary conditions. That is, differentiating Eq. 2–56 with respect to x twice will give d^2T/dx^2, which is the given differential equation, and substituting $x = 0$ and $x = L$ into Eq. 2–56 gives $T(0) = T_1$ and $T(L) = T_2$, respectively, which are the specified conditions at the boundaries.

Substituting the given information, the value of the temperature at $x = 0.1$ m is determined to be

$$T(0.1 \text{ m}) = \frac{(50 - 120)°\text{C}}{0.2 \text{ m}}(0.1 \text{ m}) + 120°\text{C} = \mathbf{85°C}$$

(*b*) The rate of heat conduction anywhere in the wall is determined from Fourier's law to be

$$\dot{Q}_{\text{wall}} = -kA\frac{dT}{dx} = -kAC_1 = -kA\frac{T_2 - T_1}{L} = kA\frac{T_1 - T_2}{L} \tag{2-57}$$

The numerical value of the rate of heat conduction through the wall is determined by substituting the given values to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (1.2 \text{ W/m} \cdot °\text{C})(15 \text{ m}^2)\frac{(120 - 50)°\text{C}}{0.2 \text{ m}} = \mathbf{6300 \text{ W}}$$

Discussion Note that under steady conditions, the rate of heat conduction through a plane wall is constant.

EXAMPLE 2–12 A Wall with Various Sets of Boundary Conditions

Consider steady one-dimensional heat conduction in a large plane wall of thickness L and constant thermal conductivity k with no heat generation. Obtain expressions for the variation of temperature within the wall for the following pairs of boundary conditions (Fig. 2–44):

$$(a) \quad -k\frac{dT(0)}{dx} = \dot{q}_0 = 40 \text{ W/cm}^2 \quad \text{and} \quad T(0) = T_0 = 15°\text{C}$$

$$(b) \quad -k\frac{dT(0)}{dx} = \dot{q}_0 = 40 \text{ W/cm}^2 \quad \text{and} \quad -k\frac{dT(L)}{dx} = \dot{q}_L = -25 \text{ W/cm}^2$$

$$(c) \quad -k\frac{dT(0)}{dx} = \dot{q}_0 = 40 \text{ W/cm}^2 \quad \text{and} \quad -k\frac{dT(L)}{dx} = \dot{q}_0 = 40 \text{ W/cm}^2$$

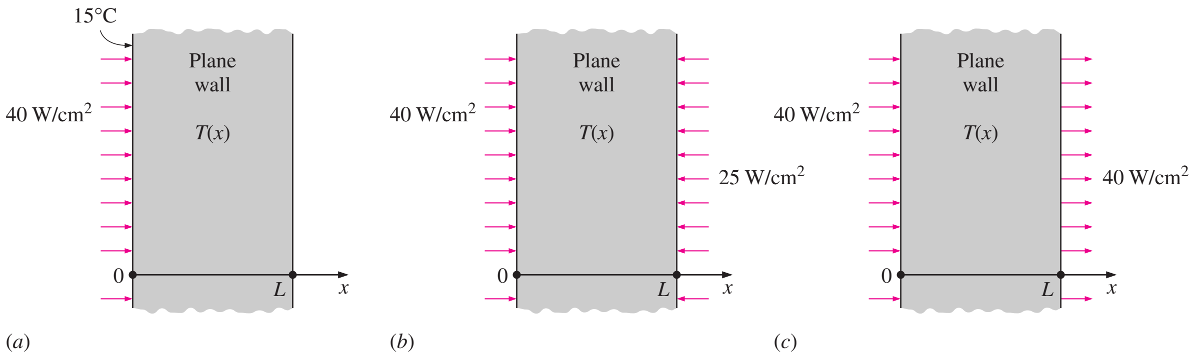


FIGURE 2–44
Schematic for Example 2–12.

SOLUTION This is a steady one-dimensional heat conduction problem with constant thermal conductivity and no heat generation in the medium, and the heat conduction equation in this case can be expressed as (Eq. 2–17)

$$\frac{d^2T}{dx^2} = 0$$

whose general solution was determined in the previous example by direct integration to be

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are two arbitrary integration constants. The specific solutions corresponding to each specified pair of boundary conditions are determined as follows.

(*a*) In this case, both boundary conditions are specified at the same boundary at $x = 0$, and no boundary condition is specified at the other boundary at $x = L$. Noting that

$$\frac{dT}{dx} = C_1$$

the application of the boundary conditions gives

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \quad \rightarrow \quad -kC_1 = \dot{q}_0 \quad \rightarrow \quad C_1 = -\frac{\dot{q}_0}{k}$$

and

$$T(0) = T_0 \quad \rightarrow \quad T_0 = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_0$$

Substituting, the specific solution in this case is determined to be

$$T(x) = -\frac{\dot{q}_0}{k} + T_0$$

Therefore, the two boundary conditions can be specified at the same boundary, and it is not necessary to specify them at different locations. In fact, the fundamental theorem of linear ordinary differential equations guarantees that a

Differential equation:

$$T''(x) = 0$$

General solution:

$$T(x) = C_1x + C_2$$

(*a*) *Unique solution:*

$$\left.\begin{aligned}-kT'(0) &= \dot{q}_0 \\ T(0) &= T_0\end{aligned}\right\} T(x) = -\frac{\dot{q}_0}{k}x + T_0$$

(*b*) *No solution:*

$$\left.\begin{aligned}-kT'(0) &= \dot{q}_0 \\ -kT'(L) &= \dot{q}_L\end{aligned}\right\} T(x) = \text{None}$$

(*c*) *Multiple solutions:*

$$\left.\begin{aligned}-kT'(0) &= \dot{q}_0 \\ -kT'(L) &= \dot{q}_0\end{aligned}\right\} T(x) = -\frac{\dot{q}_0}{k}x + \underset{\substack{\uparrow \\ \text{Arbitrary}}}{C_2}$$

FIGURE 2–45
A boundary-value problem may have a unique solution, infinitely many solutions, or no solutions at all.

unique solution exists when both conditions are specified at the same location. But no such guarantee exists when the two conditions are specified at different boundaries, as you will see below.

(*b*) In this case different heat fluxes are specified at the two boundaries. The application of the boundary conditions gives

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \quad \rightarrow \quad -kC_1 = \dot{q}_0 \quad \rightarrow \quad C_1 = -\frac{\dot{q}_0}{k}$$

and

$$-k\frac{dT(L)}{dx} = \dot{q}_L \quad \rightarrow \quad -kC_1 = \dot{q}_L \quad \rightarrow \quad C_1 = -\frac{\dot{q}_L}{k}$$

Since $\dot{q}_0 \neq \dot{q}_L$ and the constant C_1 cannot be equal to two different things at the same time, there is no solution in this case. This is not surprising since this case corresponds to supplying heat to the plane wall from both sides and expecting the temperature of the wall to remain steady (not to change with time). This is impossible.

(*c*) In this case, the same values for heat flux are specified at the two boundaries. The application of the boundary conditions gives

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \quad \rightarrow \quad -kC_1 = \dot{q}_0 \quad \rightarrow \quad C_1 = -\frac{\dot{q}_0}{k}$$

and

$$-k\frac{dT(L)}{dx} = \dot{q}_0 \quad \rightarrow \quad -kC_1 = \dot{q}_0 \quad \rightarrow \quad C_1 = -\frac{\dot{q}_0}{k}$$

Thus, both conditions result in the same value for the constant C_1, but no value for C_2. Substituting, the specific solution in this case is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + C_2$$

which is not a unique solution since C_2 is arbitrary. This solution represents a family of straight lines whose slope is $-\dot{q}_0/k$. Physically, this problem corresponds to requiring the rate of heat supplied to the wall at $x = 0$ be equal to the rate of heat removal from the other side of the wall at $x = L$. But this is a consequence of the heat conduction through the wall being steady, and thus the second boundary condition does not provide any new information. So it is not surprising that the solution of this problem is not unique. The three cases discussed above are summarized in Figure 2–45.

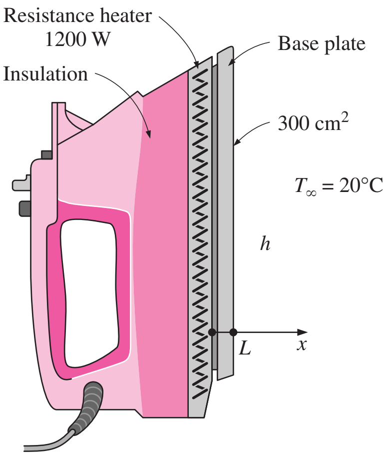


FIGURE 2–46
Schematic for Example 2–13.

EXAMPLE 2–13 Heat Conduction in the Base Plate of an Iron

Consider the base plate of a 1200-W household iron that has a thickness of $L = 0.5$ cm, base area of $A = 300$ cm^2, and thermal conductivity of $k = 15$ W/m · °C. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside, and the outer surface loses heat to the surroundings at $T_\infty = 20$°C by convection, as shown in Figure 2–46.

Taking the convection heat transfer coefficient to be $h = 80\ \text{W/m}^2 \cdot °\text{C}$ and disregarding heat loss by radiation, obtain an expression for the variation of temperature in the base plate, and evaluate the temperatures at the inner and the outer surfaces.

SOLUTION The base plate of an iron is considered. The variation of temperature in the plate and the surface temperatures are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since the surface area of the base plate is large relative to its thickness, and the thermal conditions on both sides are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation in the medium. **5** Heat transfer by radiation is negligible. **6** The upper part of the iron is well insulated so that the entire heat generated in the resistance wires is transferred to the base plate through its inner surface.

Properties The thermal conductivity is given to be $k = 15\ \text{W/m} \cdot °\text{C}$.

Analysis The inner surface of the base plate is subjected to uniform heat flux at a rate of

$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{1200\ \text{W}}{0.03\ \text{m}^2} = 40{,}000\ \text{W/m}^2$$

The outer side of the plate is subjected to the convection condition. Taking the direction normal to the surface of the wall as the x-direction with its origin on the inner surface, the differential equation for this problem can be expressed as (Fig. 2–47)

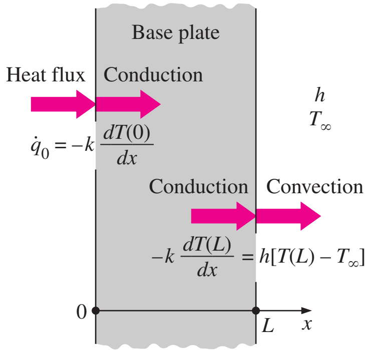


FIGURE 2–47
The boundary conditions on the base plate of the iron discussed in Example 2–13.

$$\frac{d^2T}{dx^2} = 0$$

with the boundary conditions

$$-k\frac{dT(0)}{dx} = \dot{q}_0 = 40{,}000\ \text{W/m}^2$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

The general solution of the differential equation is again obtained by two successive integrations to be

$$\frac{dT}{dx} = C_1$$

and

$$T(x) = C_1x + C_2 \qquad \textbf{(a)}$$

where C_1 and C_2 are arbitrary constants. Applying the first boundary condition,

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \quad \rightarrow \quad -kC_1 = \dot{q}_0 \quad \rightarrow \quad C_1 = -\frac{\dot{q}_0}{k}$$

Noting that $dT/dx = C_1$ and $T(L) = C_1L + C_2$, the application of the second boundary condition gives

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty] \quad \rightarrow \quad -kC_1 = h[(C_1L + C_2) - T_\infty]$$

Substituting $C_1 = -\dot{q}_0/k$ and solving for C_2, we obtain

$$C_2 = T_\infty + \frac{\dot{q}_0}{h} + \frac{\dot{q}_0}{k}L$$

Now substituting C_1 and C_2 into the general solution (*a*) gives

$$T(x) = T_\infty + \dot{q}_0\left(\frac{L-x}{k} + \frac{1}{h}\right) \qquad \textbf{(b)}$$

which is the solution for the variation of the temperature in the plate. The temperatures at the inner and outer surfaces of the plate are determined by substituting $x = 0$ and $x = L$, respectively, into the relation (*b*):

$$\begin{aligned} T(0) &= T_\infty + \dot{q}_0\left(\frac{L}{k} + \frac{1}{h}\right) \\ &= 20°\text{C} + (40{,}000 \text{ W/m}^2)\left(\frac{0.005 \text{ m}}{15 \text{ W/m}\cdot°\text{C}} + \frac{1}{80 \text{ W/m}^2\cdot°\text{C}}\right) = \mathbf{533°C} \end{aligned}$$

and

$$T(L) = T_\infty + \dot{q}_0\left(0 + \frac{1}{h}\right) = 20°\text{C} + \frac{40{,}000 \text{ W/m}^2}{80 \text{ W/m}^2\cdot°\text{C}} = \mathbf{520°C}$$

Discussion Note that the temperature of the inner surface of the base plate will be 13°C higher than the temperature of the outer surface when steady operating conditions are reached. Also note that this heat transfer analysis enables us to calculate the temperatures of surfaces that we cannot even reach. This example demonstrates how the heat flux and convection boundary conditions are applied to heat transfer problems.

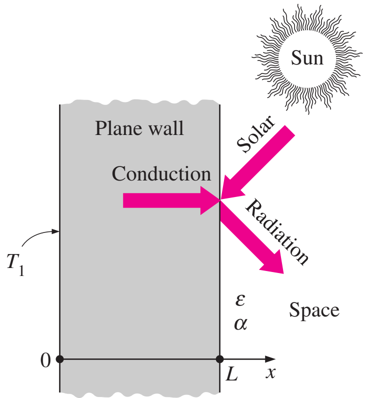


FIGURE 2–48
Schematic for Example 2–14.

EXAMPLE 2–14 Heat Conduction in a Solar Heated Wall

Consider a large plane wall of thickness $L = 0.06$ m and thermal conductivity $k = 1.2$ W/m · °C in space. The wall is covered with white porcelain tiles that have an emissivity of $\varepsilon = 0.85$ and a solar absorptivity of $\alpha = 0.26$, as shown in Figure 2–48. The inner surface of the wall is maintained at $T_1 = 300$ K at all times, while the outer surface is exposed to solar radiation that is incident at a rate of $\dot{q}_{\text{solar}} = 800$ W/m^2. The outer surface is also losing heat by radiation to deep space at 0 K. Determine the temperature of the outer surface of the wall and the rate of heat transfer through the wall when steady operating conditions are reached. What would your response be if no solar radiation was incident on the surface?

SOLUTION A plane wall in space is subjected to specified temperature on one side and solar radiation on the other side. The outer surface temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness, and the thermal conditions on both sides are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 1.2$ W/m · °C.

Analysis Taking the direction normal to the surface of the wall as the x-direction with its origin on the inner surface, the differential equation for this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

with boundary conditions

$$T(0) = T_1 = 300 \text{ K}$$

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma[T(L)^4 - T_{\text{space}}^4] - \alpha\dot{q}_{\text{solar}}$$

where $T_{\text{space}} = 0$. The general solution of the differential equation is again obtained by two successive integrations to be

$$T(x) = C_1x + C_2 \tag{a}$$

where C_1 and C_2 are arbitrary constants. Applying the first boundary condition yields

$$T(0) = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_1$$

Noting that $dT/dx = C_1$ and $T(L) = C_1L + C_2 = C_1L + T_1$, the application of the second boundary conditions gives

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma T(L)^4 - \alpha\dot{q}_{\text{solar}} \quad \rightarrow \quad -kC_1 = \varepsilon\sigma(C_1L + T_1)^4 - \alpha\dot{q}_{\text{solar}}$$

Although C_1 is the only unknown in this equation, we cannot get an explicit expression for it because the equation is nonlinear, and thus we cannot get a closed-form expression for the temperature distribution. This should explain why we do our best to avoid nonlinearities in the analysis, such as those associated with radiation.

Let us back up a little and denote the outer surface temperature by $T(L) = T_L$ instead of $T(L) = C_1L + T_1$. The application of the second boundary condition in this case gives

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma T(L)^4 - \alpha\dot{q}_{\text{solar}} \quad \rightarrow \quad -kC_1 = \varepsilon\sigma T_L^4 - \alpha\dot{q}_{\text{solar}}$$

Solving for C_1 gives

$$C_1 = \frac{\alpha\dot{q}_{\text{solar}} - \varepsilon\sigma T_L^4}{k} \tag{b}$$

Now substituting C_1 and C_2 into the general solution (a), we obtain

$$T(x) = \frac{\alpha\dot{q}_{\text{solar}} - \varepsilon\sigma T_L^4}{k}x + T_1 \tag{c}$$

(1) *Rearrange the equation to be solved:*

$$T_L = 310.4 - 0.240975\left(\frac{T_L}{100}\right)^4$$

The equation is in the proper form since the left side consists of T_L only.

(2) *Guess the value of T_L, say 300 K, and substitute into the right side of the equation. It gives*

$$T_L = 290.2 \text{ K}$$

(3) *Now substitute this value of T_L into the right side of the equation and get*

$$T_L = 293.1 \text{ K}$$

(4) *Repeat step (3) until convergence to desired accuracy is achieved. The subsequent iterations give*

$$T_L = 292.6 \text{ K}$$
$$T_L = 292.7 \text{ K}$$
$$T_L = 292.7 \text{ K}$$

Therefore, the solution is $T_L = 292.7$ K. The result is independent of the initial guess.

FIGURE 2–49
A simple method of solving a nonlinear equation is to arrange the equation such that the unknown is alone on the left side while everything else is on the right side, and to iterate after an initial guess until convergence.

which is the solution for the variation of the temperature in the wall in terms of the unknown outer surface temperature T_L. At $x = L$ it becomes

$$T_L = \frac{\alpha \dot{q}_{\text{solar}} - \varepsilon\sigma T_L^4}{k} L + T_1 \qquad (d)$$

which is an implicit relation for the outer surface temperature T_L. Substituting the given values, we get

$$T_L = \frac{0.26 \times (800 \text{ W/m}^2) - 0.85 \times (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)\, T_L^4}{1.2 \text{ W/m} \cdot \text{K}} (0.06 \text{ m}) + 300 \text{ K}$$

which simplifies to

$$T_L = 310.4 - 0.240975\left(\frac{T_L}{100}\right)^4$$

This equation can be solved by one of the several nonlinear equation solvers available (or by the old fashioned trial-and-error method) to give (Fig. 2–49)

$$T_L = \mathbf{292.7\ K}$$

Knowing the outer surface temperature and knowing that it must remain constant under steady conditions, the temperature distribution in the wall can be determined by substituting the T_L value above into Eq. (*c*):

$$T(x) = \frac{0.26 \times (800 \text{ W/m}^2) - 0.85 \times (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(292.7 \text{ K})^4}{1.2 \text{ W/m} \cdot \text{K}} x + 300 \text{ K}$$

which simplifies to

$$T(x) = (-121.5 \text{ K/m})x + 300 \text{ K}$$

Note that the outer surface temperature turned out to be lower than the inner surface temperature. Therefore, the heat transfer through the wall will be toward the outside despite the absorption of solar radiation by the outer surface. Knowing both the inner and outer surface temperatures of the wall, the steady rate of heat conduction through the wall can be determined from

$$\dot{q} = k\frac{T_0 - T_L}{L} = (1.2 \text{ W/m} \cdot \text{K})\frac{(300 - 292.7) \text{ K}}{0.06 \text{ m}} = \mathbf{146\ W/m^2}$$

Discussion In the case of no incident solar radiation, the outer surface temperature, determined from Eq. (*d*) by setting $\dot{q}_{\text{solar}} = 0$, will be $T_L =$ **284.3 K.** It is interesting to note that the solar energy incident on the surface causes the surface temperature to increase by about 8 K only when the inner surface temperature of the wall is maintained at 300 K.

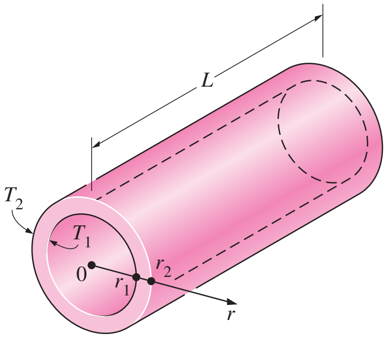


FIGURE 2–50
Schematic for Example 2–15.

EXAMPLE 2–15 Heat Loss through a Steam Pipe

Consider a steam pipe of length $L = 20$ m, inner radius $r_1 = 6$ cm, outer radius $r_2 = 8$ cm, and thermal conductivity $k = 20$ W/m · °C, as shown in Figure 2–50. The inner and outer surfaces of the pipe are maintained at average temperatures of $T_1 = 150$°C and $T_2 = 60$°C, respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions, and determine the rate of heat loss from the steam through the pipe.

SOLUTION A steam pipe is subjected to specified temperatures on its surfaces. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction, and thus $T = T(r)$. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 20$ W/m · °C.

Analysis The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 150°\text{C}$$
$$T(r_2) = T_2 = 60°\text{C}$$

Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

where C_1 is an arbitrary constant. We now divide both sides of this equation by r to bring it to a readily integrable form,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Again integrating with respect to r gives (Fig. 2–51)

$$T(r) = C_1 \ln r + C_2 \quad \textbf{(a)}$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in Eq. (a) with the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad C_1 \ln r_1 + C_2 = T_1$$
$$T(r_2) = T_2 \quad \rightarrow \quad C_1 \ln r_2 + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = \frac{T_2 - T_1}{\ln(r_2/r_1)} \qquad \text{and} \qquad C_2 = T_1 - \frac{T_2 - T_1}{\ln(r_2/r_1)}\ln r_1$$

Substituting them into Eq. (a) and rearranging, the variation of temperature within the pipe is determined to be

$$T(r) = \left(\frac{\ln(r/r_1)}{\ln(r_2/r_1)}\right)(T_2 - T_1) + T_1 \quad \textbf{(2-58)}$$

The rate of heat loss from the steam is simply the total rate of heat conduction through the pipe, and is determined from Fourier's law to be

Differential equation:

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrate:

$$r\frac{dT}{dr} = C_1$$

Divide by r $(r \neq 0)$:

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Integrate again:

$$T(r) = C_1 \ln r + C_2$$

which is the general solution.

FIGURE 2–51
Basic steps involved in the solution of the steady one-dimensional heat conduction equation in cylindrical coordinates.

$$\dot{Q}_{\text{cylinder}} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi kLC_1 = 2\pi kL\frac{T_1 - T_2}{\ln(r_2/r_1)} \quad \textbf{(2-59)}$$

The numerical value of the rate of heat conduction through the pipe is determined by substituting the given values

$$\dot{Q} = 2\pi(20\ \text{W/m}\cdot{}^\circ\text{C})(20\ \text{m})\frac{(150-60)^\circ\text{C}}{\ln(0.08/0.06)} = \mathbf{786\ kW}$$

DISCUSSION Note that the total rate of heat transfer through a pipe is constant, but the heat flux is not since it decreases in the direction of heat transfer with increasing radius since $\dot{q} = \dot{Q}/(2\pi rL)$.

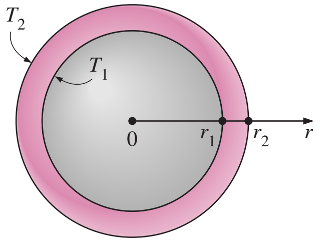


FIGURE 2–52
Schematic for Example 2–16.

EXAMPLE 2–16 Heat Conduction through a Spherical Shell

Consider a spherical container of inner radius $r_1 = 8$ cm, outer radius $r_2 = 10$ cm, and thermal conductivity $k = 45$ W/m · °C, as shown in Figure 2–52. The inner and outer surfaces of the container are maintained at constant temperatures of $T_1 = 200$°C and $T_2 = 80$°C, respectively, as a result of some chemical reactions occurring inside. Obtain a general relation for the temperature distribution inside the shell under steady conditions, and determine the rate of heat loss from the container.

SOLUTION A spherical container is subjected to specified temperatures on its surfaces. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint, and thus $T = T(r)$. **3** Thermal conductivity is constant. **4** There is no heat generation.

Properties The thermal conductivity is given to be $k = 45$ W/m · °C.

Analysis The mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 200^\circ\text{C}$$
$$T(r_2) = T_2 = 80^\circ\text{C}$$

Integrating the differential equation once with respect to r yields

$$r^2\frac{dT}{dr} = C_1$$

where C_1 is an arbitrary constant. We now divide both sides of this equation by r^2 to bring it to a readily integrable form,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

Again integrating with respect to r gives

$$T(r) = -\frac{C_1}{r} + C_2 \qquad (a)$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in the relation above by the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad -\frac{C_1}{r_1} + C_2 = T_1$$

$$T(r_2) = T_2 \quad \rightarrow \quad -\frac{C_1}{r_2} + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = -\frac{r_1 r_2}{r_2 - r_1}(T_1 - T_2) \qquad \text{and} \qquad C_2 = \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$$

Substituting into Eq. (a), the variation of temperature within the spherical shell is determined to be

$$T(r) = \frac{r_1 r_2}{r(r_2 - r_1)}(T_1 - T_2) + \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1} \qquad \textbf{(2-60)}$$

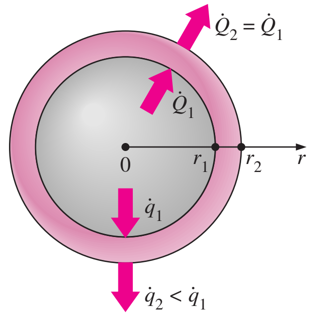
The rate of heat loss from the container is simply the total rate of heat conduction through the container wall and is determined from Fourier's law

$$\dot{Q}_{\text{sphere}} = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{C_1}{r^2} = -4\pi k C_1 = 4\pi k r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} \qquad \textbf{(2-61)}$$

The numerical value of the rate of heat conduction through the wall is determined by substituting the given values to be

$$\dot{Q} = 4\pi(45 \text{ W/m}\cdot{}^\circ\text{C})(0.08 \text{ m})(0.10 \text{ m})\frac{(200 - 80)^\circ\text{C}}{(0.10 - 0.08) \text{ m}} = \mathbf{27{,}140 \text{ W}}$$

Discussion Note that the total rate of heat transfer through a spherical shell is constant, but the heat flux, $\dot{q} = \dot{Q}/4\pi r^2$, is not since it decreases in the direction of heat transfer with increasing radius as shown in Figure 2–53.



$$\dot{q}_1 = \frac{\dot{Q}_1}{A_1} = \frac{27.14 \text{ kW}}{4\pi(0.08 \text{ m})^2} = 337.5 \text{ kW/m}^2$$

$$\dot{q}_2 = \frac{\dot{Q}_2}{A_2} = \frac{27.14 \text{ kW}}{4\pi(0.10 \text{ m})^2} = 216.0 \text{ kW/m}^2$$

FIGURE 2–53
During steady one-dimensional heat conduction in a spherical (or cylindrical) container, the total rate of heat transfer remains constant, but the heat flux decreases with increasing radius.

2–6 HEAT GENERATION IN A SOLID

Many practical heat transfer applications involve the conversion of some form of energy into *thermal* energy in the medium. Such mediums are said to involve internal *heat generation,* which manifests itself as a rise in temperature throughout the medium. Some examples of heat generation are *resistance heating* in wires, exothermic *chemical reactions* in a solid, and *nuclear reactions* in nuclear fuel rods where electrical, chemical, and nuclear energies are converted to heat, respectively (Fig. 2–54). The absorption of radiation throughout the volume of a semitransparent medium such as water can also be considered as heat generation within the medium, as explained earlier.

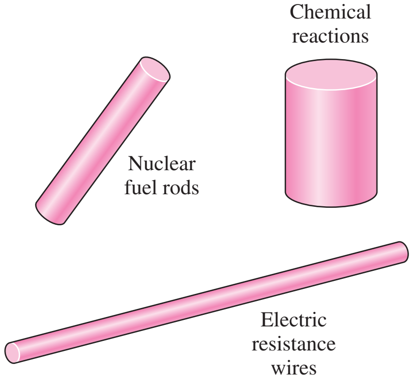


FIGURE 2–54
Heat generation in solids is commonly encountered in practice.

Heat generation is usually expressed *per unit volume* of the medium, and is denoted by $\dot{g}$, whose unit is W/m^3. For example, heat generation in an electrical wire of outer radius r_0 and length L can be expressed as

$$\dot{g} = \frac{\dot{E}_{\text{g.electric}}}{V_{\text{wire}}} = \frac{I^2 R_e}{\pi r_o^2 L} \qquad (\text{W/m}^3) \tag{2-62}$$

where I is the electric current and R_e is the electrical resistance of the wire.

The temperature of a medium *rises* during heat generation as a result of the absorption of the generated heat by the medium during transient start-up period. As the temperature of the medium increases, so does the heat transfer from the medium to its surroundings. This continues until steady operating conditions are reached and the rate of heat generation equals the rate of heat transfer to the surroundings. Once steady operation has been established, the temperature of the medium at any point no longer changes.

The *maximum temperature* $T_{\max}$ in a solid that involves uniform heat generation will occur at a location *farthest away* from the outer surface when the outer surface of the solid is maintained at a constant temperature T_s. For example, the maximum temperature occurs at the *midplane* in a plane wall, at the *centerline* in a long cylinder, and at the *midpoint* in a sphere. The temperature distribution within the solid in these cases will be *symmetrical* about the center of symmetry.

The quantities of major interest in a medium with heat generation are the surface temperature T_s and the maximum temperature $T_{\max}$ that occurs in the medium in *steady* operation. Below we develop expressions for these two quantities for common geometries for the case of *uniform* heat generation ($\dot{g}$ = constant) within the medium.

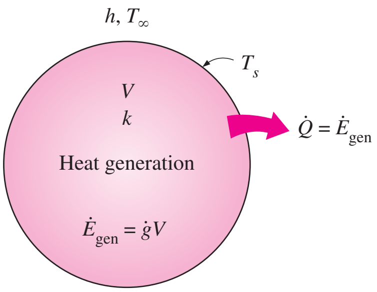


FIGURE 2–55
At steady conditions, the entire heat generated in a solid must leave the solid through its outer surface.

Consider a solid medium of surface area A_s, volume V, and constant thermal conductivity k, where heat is generated at a constant rate of $\dot{g}$ per unit volume. Heat is transferred from the solid to the surrounding medium at T_∞, with a constant heat transfer coefficient of h. All the surfaces of the solid are maintained at a common temperature T_s. Under *steady* conditions, the energy balance for this solid can be expressed as (Fig. 2–55)

$$\left(\begin{array}{c}\text{Rate of}\\ \textit{heat transfer}\\ \text{from the solid}\end{array}\right) = \left(\begin{array}{c}\text{Rate of}\\ \textit{energy generation}\\ \text{within the solid}\end{array}\right) \tag{2-63}$$

or

$$\dot{Q} = \dot{g}V \qquad (\text{W}) \tag{2-64}$$

Disregarding radiation (or incorporating it in the heat transfer coefficient h), the heat transfer rate can also be expressed from Newton's law of cooling as

$$\dot{Q} = hA_s(T_s - T_\infty) \qquad (\text{W}) \tag{2-65}$$

Combining Eqs. 2–64 and 2–65 and solving for the surface temperature T_s gives

$$T_s = T_\infty + \frac{\dot{g}V}{hA_s} \tag{2-66}$$

For a large *plane wall* of thickness $2L$ ($A_s = 2A_{\text{wall}}$ and $V = 2LA_{\text{wall}}$), a long solid *cylinder* of radius r_o ($A_s = 2\pi r_o L$ and $V = \pi r_o^2 L$), and a solid *sphere* of radius r_0 ($A_s = 4\pi r_o^2$ and $V = \frac{4}{3}\pi r_o^3$), Eq. 2–66 reduces to

$$T_{s,\text{ plane wall}} = T_\infty + \frac{\dot{g}L}{h} \tag{2-67}$$

$$T_{s,\text{ cylinder}} = T_\infty + \frac{\dot{g}r_o}{2h} \tag{2-68}$$

$$T_{s,\text{ sphere}} = T_\infty + \frac{\dot{g}r_o}{3h} \tag{2-69}$$

Note that the rise in surface temperature T_s is due to heat generation in the solid.

Reconsider heat transfer from a long solid cylinder with heat generation. We mentioned above that, under *steady* conditions, the entire heat generated within the medium is conducted through the outer surface of the cylinder. Now consider an imaginary inner cylinder of radius r within the cylinder (Fig. 2–56). Again the *heat generated* within this inner cylinder must be equal to the *heat conducted* through the outer surface of this inner cylinder. That is, from Fourier's law of heat conduction,

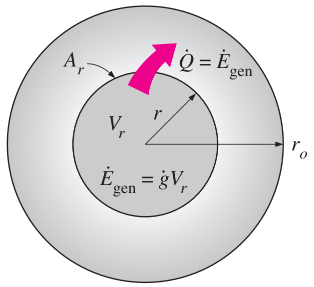


FIGURE 2–56
Heat conducted through a cylindrical shell of radius r is equal to the heat generated within a shell.

$$-kA_r\frac{dT}{dr} = \dot{g}V_r \tag{2-70}$$

where $A_r = 2\pi rL$ and $V_r = \pi r^2 L$ at any location r. Substituting these expressions into Eq. 2–70 and separating the variables, we get

$$-k(2\pi rL)\frac{dT}{dr} = \dot{g}(\pi r^2 L) \quad \rightarrow \quad dT = -\frac{\dot{g}}{2k}rdr$$

Integrating from $r = 0$ where $T(0) = T_0$ to $r = r_o$ where $T(r_o) = T_s$ yields

$$\Delta T_{\text{max, cylinder}} = T_o - T_s = \frac{\dot{g}r_o^2}{4k} \tag{2-71}$$

where T_o is the centerline temperature of the cylinder, which is the *maximum temperature,* and ΔT_{max} is the difference between the centerline and the surface temperatures of the cylinder, which is the *maximum temperature rise* in the cylinder above the surface temperature. Once ΔT_{max} is available, the centerline temperature can easily be determined from (Fig. 2–57)

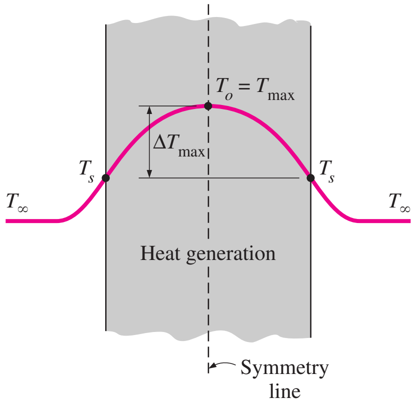


FIGURE 2–57
The maximum temperature in a symmetrical solid with uniform heat generation occurs at its center.

$$T_{\text{center}} = T_o = T_s + \Delta T_{\text{max}} \tag{2-72}$$

The approach outlined above can also be used to determine the *maximum temperature rise* in a plane wall of thickness $2L$ and a solid sphere of radius r_0, with these results:

$$\Delta T_{\text{max, plane wall}} = \frac{\dot{g}L^2}{2k} \tag{2-73}$$

$$\Delta T_{\text{max, sphere}} = \frac{\dot{g}r_o^2}{6k} \tag{2-74}$$

Again the maximum temperature at the center can be determined from Eq. 2–72 by adding the maximum temperature rise to the surface temperature of the solid.

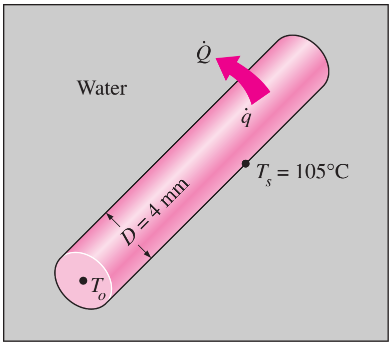


FIGURE 2–58
Schematic for Example 2–17.

EXAMPLE 2–17 Centerline Temperature of a Resistance Heater

A 2-kW resistance heater wire whose thermal conductivity is $k = 15$ W/m · °C has a diameter of $D = 4$ mm and a length of $L = 0.5$ m, and is used to boil water (Fig. 2–58). If the outer surface temperature of the resistance wire is $T_s = 105°C$, determine the temperature at the center of the wire.

SOLUTION The surface temperature of a resistance heater submerged in water is to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.

Properties The thermal conductivity is given to be $k = 15$ W/m · °C.

Analysis The 2-kW resistance heater converts electric energy into heat at a rate of 2 kW. The heat generation per unit volume of the wire is

$$\dot{g} = \frac{\dot{Q}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{Q}_{\text{gen}}}{\pi r_o^2 L} = \frac{2000 \text{ W}}{\pi(0.002 \text{ m})^2(0.5 \text{ m})} = 0.318 \times 10^9 \text{ W/m}^3$$

Then the center temperature of the wire is determined from Eq. 2–71 to be

$$T_o = T_s + \frac{\dot{g} r_o^2}{4k} = 105°\text{C} + \frac{(0.318 \times 10^9 \text{ W/m}^3)(0.002 \text{ m})^2}{4 \times (15 \text{ W/m} \cdot °\text{C})} = \mathbf{126°C}$$

Discussion Note that the temperature difference between the center and the surface of the wire is 21°C.

We have developed these relations using the intuitive *energy balance* approach. However, we could have obtained the same relations by setting up the appropriate *differential equations* and solving them, as illustrated in Examples 2–18 and 2–19.

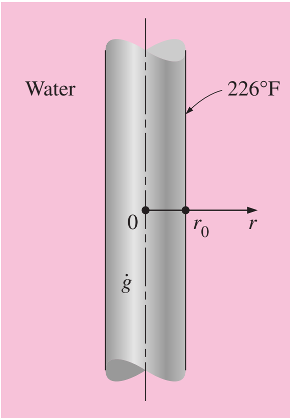


FIGURE 2–59
Schematic for Example 2–18.

EXAMPLE 2–18 Variation of Temperature in a Resistance Heater

A long homogeneous resistance wire of radius $r_0 = 0.2$ in. and thermal conductivity $k = 7.8$ Btu/h · ft · °F is being used to boil water at atmospheric pressure by the passage of electric current, as shown in Figure 2–59. Heat is generated in the wire uniformly as a result of resistance heating at a rate of $\dot{g} = 2400$ Btu/h · in^3. If the outer surface temperature of the wire is measured to be $T_s = 226°F$, obtain a relation for the temperature distribution, and determine the temperature at the centerline of the wire when steady operating conditions are reached.

SOLUTION This heat transfer problem is similar to the problem in Example 2–17, except that we need to obtain a relation for the variation of temperature within the wire with *r*. Differential equations are well suited for this purpose.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is no thermal symmetry about the centerline and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be $k = 7.8$ Btu/h · ft · °F.

Analysis The differential equation which governs the variation of temperature in the wire is simply Eq. 2–27,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

This is a second-order linear ordinary differential equation, and thus its general solution will contain two arbitrary constants. The determination of these constants requires the specification of two boundary conditions, which can be taken to be

$$T(r_0) = T_s = 226°\text{F}$$

and

$$\frac{dT(0)}{dr} = 0$$

The first boundary condition simply states that the temperature of the outer surface of the wire is 226°F. The second boundary condition is the symmetry condition at the centerline, and states that the maximum temperature in the wire will occur at the centerline, and thus the slope of the temperature at $r = 0$ must be zero (Fig. 2–60). This completes the mathematical formulation of the problem.

Although not immediately obvious, the differential equation is in a form that can be solved by direct integration. Multiplying both sides of the equation by *r* and rearranging, we obtain

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{g}}{k}\,r$$

Integrating with respect to *r* gives

$$r\frac{dT}{dr} = -\frac{\dot{g}}{k}\frac{r^2}{2} + C_1 \qquad \textbf{(a)}$$

since the heat generation is constant, and the integral of a derivative of a function is the function itself. That is, integration removes a derivative. It is convenient at this point to apply the second boundary condition, since it is related to the first derivative of the temperature, by replacing all occurrences of *r* and *dT*/*dr* in Eq. (*a*) by zero. It yields

$$0 \times \frac{dT(0)}{dr} = -\frac{\dot{g}}{2k} \times 0 + C_1 \quad \rightarrow \quad C_1 = 0$$

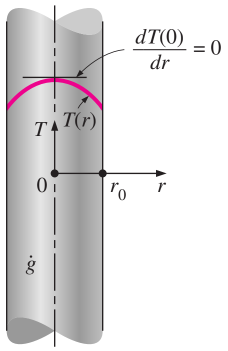


FIGURE 2–60
The thermal symmetry condition at the centerline of a wire in which heat is generated uniformly.

Thus C_1 cancels from the solution. We now divide Eq. (*a*) by r to bring it to a readily integrable form,

$$\frac{dT}{dr} = -\frac{\dot{g}}{2k}\, r$$

Again integrating with respect to r gives

$$T(r) = -\frac{\dot{g}}{4k}\, r^2 + C_2 \qquad (b)$$

We now apply the first boundary condition by replacing all occurrences of r by r_0 and all occurrences of T by T_s. We get

$$T_s = -\frac{\dot{g}}{4k}\, r_0^2 + C_2 \quad \rightarrow \quad C_2 = T_s + \frac{\dot{g}}{4k}\, r_0^2$$

Substituting this C_2 relation into Eq. (*b*) and rearranging give

$$T(r) = T_s + \frac{\dot{g}}{4k}\,(r_0^2 - r^2) \qquad (c)$$

which is the desired solution for the temperature distribution in the wire as a function of r. The temperature at the centerline ($r = 0$) is obtained by replacing r in Eq. (*c*) by zero and substituting the known quantities,

$$T(0) = T_s + \frac{\dot{g}}{4k}\, r_0^2 = 226°\text{F} + \frac{2400 \text{ Btu/h} \cdot \text{in}^3}{4 \times (7.8 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F})}\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)(0.2 \text{ in.})^2 = \mathbf{263°F}$$

Discussion The temperature of the centerline will be 37°F above the temperature of the outer surface of the wire. Note that the expression above for the centerline temperature is identical to Eq. 2–71, which was obtained using an energy balance on a control volume.

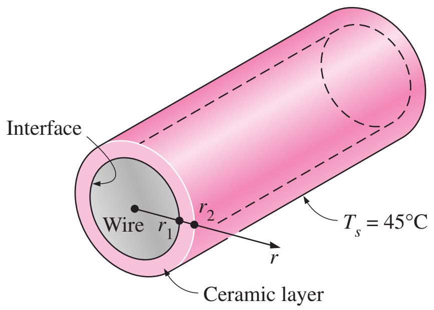


FIGURE 2–61
Schematic for Example 2–19.

EXAMPLE 2–19 Heat Conduction in a Two-Layer Medium

Consider a long resistance wire of radius $r_1 = 0.2$ cm and thermal conductivity $k_{\text{wire}} = 15$ W/m · °C in which heat is generated uniformly as a result of resistance heating at a constant rate of $\dot{g} = 50$ W/cm^3 (Fig. 2–61). The wire is embedded in a 0.5-cm-thick layer of ceramic whose thermal conductivity is $k_{\text{ceramic}} = 1.2$ W/m · °C. If the outer surface temperature of the ceramic layer is measured to be $T_s = 45°$C, determine the temperatures at the center of the resistance wire and the interface of the wire and the ceramic layer under steady conditions.

SOLUTION The surface and interface temperatures of a resistance wire covered with a ceramic layer are to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since this two-layer heat transfer problem possesses symmetry about the centerline and involves no change in the axial direction, and thus $T = T(r)$. **3** Thermal conductivities are constant. **4** Heat generation in the wire is uniform.

Properties It is given that $k_{\text{wire}} = 15$ W/m · °C and $k_{\text{ceramic}} = 1.2$ W/m · ° C.

Analysis Letting T_I denote the unknown interface temperature, the heat transfer problem in the wire can be formulated as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)+\frac{\dot{g}}{k}=0$$

with

$$T_{\text{wire}}(r_1)=T_I$$
$$\frac{dT_{\text{wire}}(0)}{dr}=0$$

This problem was solved in Example 2–18, and its solution was determined to be

$$T_{\text{wire}}(r)=T_I+\frac{\dot{g}}{4k_{\text{wire}}}(r_1^2-r^2) \quad \textbf{(a)}$$

Noting that the ceramic layer does not involve any heat generation and its outer surface temperature is specified, the heat conduction problem in that layer can be expressed as

$$\frac{d}{dr}\left(r\frac{dT_{\text{ceramic}}}{dr}\right)=0$$

with

$$T_{\text{ceramic}}(r_1)=T_I$$
$$T_{\text{ceramic}}(r_2)=T_s=45^\circ\text{C}$$

This problem was solved in Example 2–15, and its solution was determined to be

$$T_{\text{ceramic}}(r)=\frac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_s-T_I)+T_I \quad \textbf{(b)}$$

We have already utilized the first interface condition by setting the wire and ceramic layer temperatures equal to T_I at the interface $r = r_1$. The interface temperature T_I is determined from the second interface condition that the heat flux in the wire and the ceramic layer at $r = r_1$ must be the same:

$$-k_{\text{wire}}\frac{dT_{\text{wire}}(r_1)}{dr}=-k_{\text{ceramic}}\frac{dT_{\text{ceramic}}(r_1)}{dr} \quad \rightarrow \quad \frac{\dot{g}r_1}{2}=-k_{\text{ceramic}}\frac{T_s-T_I}{\ln(r_2/r_1)}\left(\frac{1}{r_1}\right)$$

Solving for T_I and substituting the given values, the interface temperature is determined to be

$$\begin{aligned}T_I&=\frac{\dot{g}r_1^2}{2k_{\text{ceramic}}}\ln\frac{r_2}{r_1}+T_s\\&=\frac{(50\times10^6\text{ W/m}^3)(0.002\text{ m})^2}{2(1.2\text{ W/m}\cdot{}^\circ\text{C})}\ln\frac{0.007\text{ m}}{0.002\text{ m}}+45^\circ\text{ C}=\mathbf{149.4^\circ C}\end{aligned}$$

Knowing the interface temperature, the temperature at the centerline ($r = 0$) is obtained by substituting the known quantities into Eq. (*a*),

$$T_{\text{wire}}(0)=T_I+\frac{\dot{g}r_1^2}{4k_{\text{wire}}}=149.4^\circ\text{C}+\frac{(50\times10^6\text{ W/m}^3)(0.002\text{ m})^2}{4\times(15\text{ W/m}\cdot{}^\circ\text{C})}=\mathbf{152.7^\circ C}$$

Thus the temperature of the centerline will be slightly above the interface temperature.

Discussion This example demonstrates how steady one-dimensional heat conduction problems in composite media can be solved. We could also solve this problem by determining the heat flux at the interface by dividing the total heat generated in the wire by the surface area of the wire, and then using this value as the specifed heat flux boundary condition for both the wire and the ceramic layer. This way the two problems are decoupled and can be solved separately.

2–7 ▪ VARIABLE THERMAL CONDUCTIVITY, *k*(*T*)

You will recall from Chapter 1 that the thermal conductivity of a material, in general, varies with temperature (Fig. 2–62). However, this variation is mild for many materials in the range of practical interest and can be disregarded. In such cases, we can use an average value for the thermal conductivity and treat it as a constant, as we have been doing so far. This is also common practice for other temperature-dependent properties such as the density and specific heat.

When the variation of thermal conductivity with temperature in a specified temperature interval is large, however, it may be necessary to account for this variation to minimize the error. Accounting for the variation of the thermal conductivity with temperature, in general, complicates the analysis. But in the case of simple one-dimensional cases, we can obtain heat transfer relations in a straightforward manner.

When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$k_{\text{ave}} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1} \tag{2-75}$$

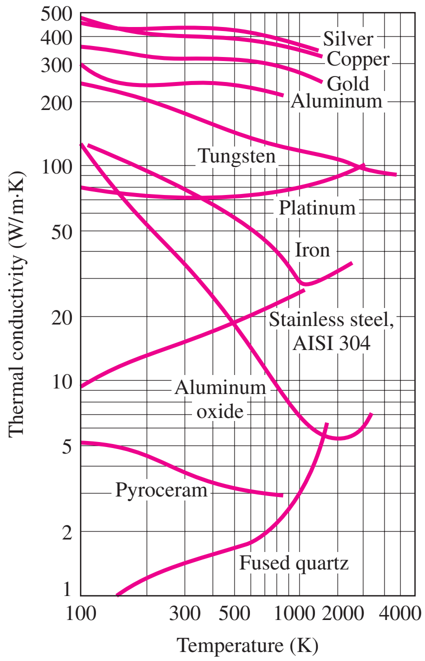


FIGURE 2–62
Variation of the thermal conductivity of some solids with temperature.

This relation is based on the requirement that the rate of heat transfer through a medium with constant average thermal conductivity k_{ave} equals the rate of heat transfer through the same medium with variable conductivity $k(T)$. Note that in the case of constant thermal conductivity $k(T) = k$, Eq. 2–75 reduces to $k_{\text{ave}} = k$, as expected.

Then the rate of steady heat transfer through a plane wall, cylindrical layer, or spherical layer for the case of variable thermal conductivity can be determined by replacing the constant thermal conductivity k in Eqs. 2–57, 2–59, and 2–61 by the k_{ave} expression (or value) from Eq. 2–75:

$$\dot{Q}_{\text{plane wall}} = k_{\text{ave}} A \frac{T_1 - T_2}{L} = \frac{A}{L}\int_{T_2}^{T_1} k(T)dT \tag{2-76}$$

$$\dot{Q}_{\text{cylinder}} = 2\pi k_{\text{ave}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = \frac{2\pi L}{\ln(r_2/r_1)}\int_{T_2}^{T_1} k(T)dT \tag{2-77}$$

$$\dot{Q}_{\text{sphere}} = 4\pi k_{\text{ave}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = \frac{4\pi r_1 r_2}{r_2 - r_1}\int_{T_2}^{T_1} k(T)dT \tag{2-78}$$

The variation in thermal conductivity of a material with temperature in the temperature range of interest can often be approximated as a linear function and expressed as

$$k(T) = k_0(1 + \beta T) \tag{2-79}$$

where β is called the **temperature coefficient of thermal conductivity.** The *average* value of thermal conductivity in the temperature range T_1 to T_2 in this case can be determined from

$$k_{\text{ave}} = \frac{\int_{T_1}^{T_2} k_0(1 + \beta T)dT}{T_2 - T_1} = k_0\left(1 + \beta \frac{T_2 + T_1}{2}\right) = k(T_{\text{ave}}) \tag{2-80}$$

Note that the *average thermal conductivity* in this case is equal to the thermal conductivity value at the *average temperature.*

We have mentioned earlier that in a plane wall the temperature varies linearly during steady one-dimensional heat conduction when the thermal conductivity is constant. But this is no longer the case when the thermal conductivity changes with temperature, even linearly, as shown in Figure 2–63.

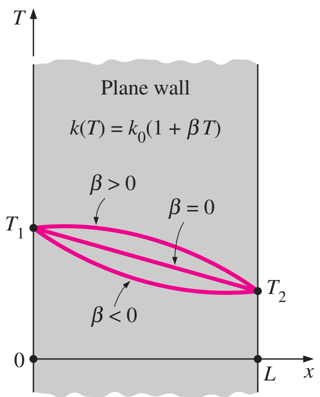


FIGURE 2–63
The variation of temperature in a plane wall during steady one-dimensional heat conduction for the cases of constant and variable thermal conductivity.

EXAMPLE 2–20 Variation of Temperature in a Wall with $k(T)$

Consider a plane wall of thickness L whose thermal conductivity varies linearly in a specified temperature range as $k(T) = k_0(1 + \beta T)$ where k_0 and β are constants. The wall surface at $x = 0$ is maintained at a constant temperature of T_1 while the surface at $x = L$ is maintained at T_2, as shown in Figure 2–64. Assuming steady one-dimensional heat transfer, obtain a relation for (*a*) the heat transfer rate through the wall and (*b*) the temperature distribution $T(x)$ in the wall.

SOLUTION A plate with variable conductivity is subjected to specified temperatures on both sides. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies linearly. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis (*a*) The rate of heat transfer through the wall can be determined from

$$\dot{Q} = k_{\text{ave}} A \frac{T_1 - T_2}{L}$$

where A is the heat conduction area of the wall and

$$k_{\text{ave}} = k(T_{\text{ave}}) = k_0\left(1 + \beta \frac{T_2 + T_1}{2}\right)$$

is the average thermal conductivity (Eq. 2–80).

(*b*) To determine the temperature distribution in the wall, we begin with Fourier's law of heat conduction, expressed as

$$\dot{Q} = -k(T)\, A \frac{dT}{dx}$$

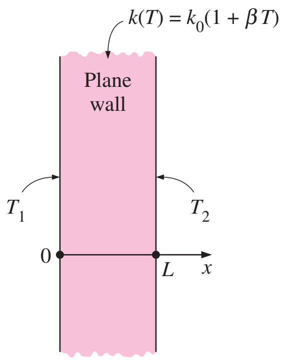


FIGURE 2–64
Schematic for Example 2–20.

where the rate of conduction heat transfer $\dot{Q}$ and the area A are constant. Separating variables and integrating from $x = 0$ where $T(0) = T_1$ to any x where $T(x) = T$, we get

$$\int_0^x \dot{Q}\,dx = -A\int_{T_1}^{T} k(T)dT$$

Substituting $k(T) = k_0(1 + \beta T)$ and performing the integrations we obtain

$$\dot{Q}x = -Ak_0[(T - T_1) + \beta(T^2 - T_1^2)/2]$$

Substituting the $\dot{Q}$ expression from part (*a*) and rearranging give

$$T^2 + \frac{2}{\beta}T + \frac{2k_{\text{ave}}}{\beta k_0}\frac{x}{L}(T_1 - T_2) - T_1^2 - \frac{2}{\beta}T_1 = 0$$

which is a *quadratic* equation in the unknown temperature T. Using the quadratic formula, the temperature distribution $T(x)$ in the wall is determined to be

$$T(x) = -\frac{1}{\beta} \pm \sqrt{\frac{1}{\beta^2} - \frac{2k_{\text{ave}}}{\beta k_0}\frac{x}{L}(T_1 - T_2) + T_1^2 + \frac{2}{\beta}T_1}$$

The proper sign of the square root term (+ or −) is determined from the requirement that the temperature at any point within the medium must remain between T_1 and T_2. This result explains why the temperature distribution in a plane wall is no longer a straight line when the thermal conductivity varies with temperature.

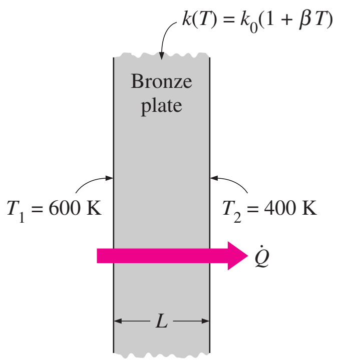


FIGURE 2–65
Schematic for Example 2–21.

EXAMPLE 2–21 Heat Conduction through a Wall with $k(T)$

Consider a 2-m-high and 0.7-m-wide bronze plate whose thickness is 0.1 m. One side of the plate is maintained at a constant temperature of 600 K while the other side is maintained at 400 K, as shown in Figure 2–65. The thermal conductivity of the bronze plate can be assumed to vary linearly in that temperature range as $k(T) = k_0(1 + \beta T)$ where $k_0 = 38$ W/m · K and $\beta = 9.21 \times 10^{-4}$ K^{-1}. Disregarding the edge effects and assuming steady one-dimensional heat transfer, determine the rate of heat conduction through the plate.

SOLUTION A plate with variable conductivity is subjected to specified temperatures on both sides. The rate of heat transfer is to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies linearly. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The average thermal conductivity of the medium in this case is simply the value at the average temperature and is determined from

$$\begin{aligned} k_{\text{ave}} &= k(T_{\text{ave}}) = k_0\left(1 + \beta\,\frac{T_2 + T_1}{2}\right) \\ &= (38\text{ W/m}\cdot\text{K})\left[1 + (9.21 \times 10^{-4}\text{ K}^{-1})\,\frac{(600 + 400)\text{ K}}{2}\right] \\ &= 55.5\text{ W/m}\cdot\text{K} \end{aligned}$$

Then the rate of heat conduction through the plate can be determined from Eq. 2–76 to be

$$\dot{Q} = k_{\text{ave}} A \frac{T_1 - T_2}{L}$$

$$= (55.5\ \text{W/m}\cdot\text{K})(2\ \text{m} \times 0.7\ \text{m}) \frac{(600 - 400)\text{K}}{0.1\ \text{m}} = \mathbf{155{,}400\ W}$$

Discussion We would have obtained the same result by substituting the given $k(T)$ relation into the second part of Eq. 2–76 and performing the indicated integration.

TOPIC OF SPECIAL INTEREST

*A Brief Review of Differential Equations**

As we mentioned in Chapter 1, the description of most scientific problems involves relations that involve changes in some key variables with respect to each other. Usually the smaller the increment chosen in the changing variables, the more general and accurate the description. In the limiting case of infinitesimal or differential changes in variables, we obtain *differential equations,* which provide precise mathematical formulations for the physical principles and laws by representing the rates of change as *derivatives.* Therefore, differential equations are used to investigate a wide variety of problems in science and engineering, including heat transfer.

Differential equations arise when relevant *physical laws* and *principles* are applied to a problem by considering infinitesimal changes in the variables of interest. Therefore, obtaining the governing differential equation for a specific problem requires an adequate knowledge of the nature of the problem, the variables involved, appropriate simplifying assumptions, and the applicable physical laws and principles involved, as well as a careful analysis (Fig. 2–66).

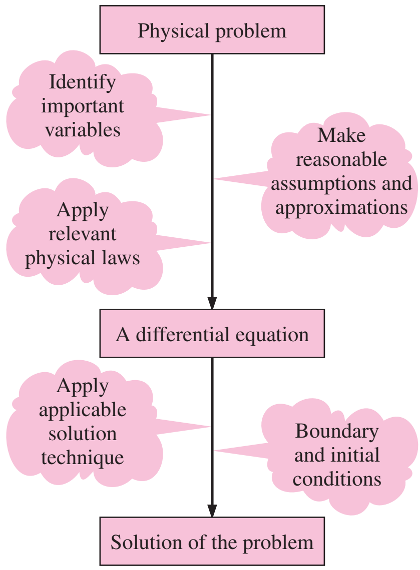


FIGURE 2–66
Mathematical modeling of physical problems.

An equation, in general, may involve one or more variables. As the name implies, a **variable** is a quantity that may assume various values during a study. A quantity whose value is fixed during a study is called a **constant.** Constants are usually denoted by the earlier letters of the alphabet such as a, b, c, and d, whereas variables are usually denoted by the later ones such as t, x, y, and z. A variable whose value can be changed arbitrarily is called an **independent variable** (or argument). A variable whose value depends on the value of other variables and thus cannot be varied independently is called a **dependent variable** (or a function).

A dependent variable y that depends on a variable x is usually denoted as $y(x)$ for clarity. However, this notation becomes very inconvenient and cumbersome when y is repeated several times in an expression. In such cases it is desirable to denote $y(x)$ simply as y when it is clear that y is a function of x. This shortcut in notation improves the appearance and the

*This section can be skipped if desired without a loss in continuity.

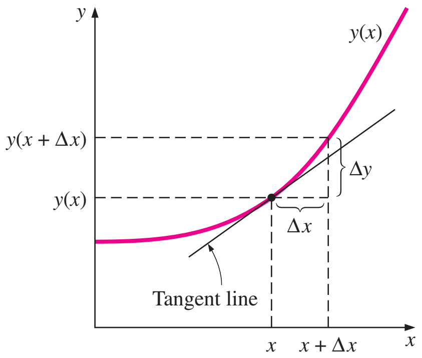


FIGURE 2–67
The derivative of a function at a point represents the slope of the tangent line of the function at that point.

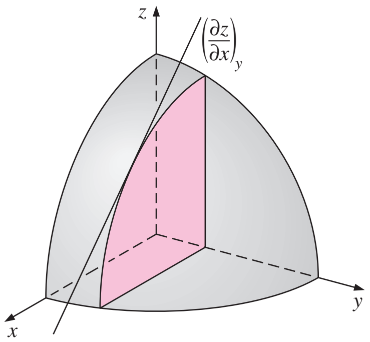


FIGURE 2–68
Graphical representation of partial derivative $\partial z/\partial x$.

readability of the equations. The value of y at a fixed number a is denoted by $y(a)$.

The **derivative** of a function $y(x)$ at a point is equivalent to the *slope* of the tangent line to the graph of the function at that point and is defined as (Fig. 2–67)

$$y'(x) = \frac{dy(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y(x + \Delta x) - y(x)}{\Delta x} \tag{2-81}$$

Here Δx represents a (small) change in the independent variable x and is called an *increment* of x. The corresponding change in the function y is called an increment of y and is denoted by Δy. Therefore, the derivative of a function can be viewed as the ratio of the increment Δy of the function to the increment Δx of the independent variable for very small Δx. Note that Δy and thus $y'(x)$ will be zero if the function y does not change with x.

Most problems encountered in practice involve quantities that change with time t, and their first derivatives with respect to time represent the rate of change of those quantities with time. For example, if $N(t)$ denotes the population of a bacteria colony at time t, then the first derivative $N' = dN/dt$ represents the rate of change of the population, which is the amount the population increases or decreases per unit time.

The derivative of the first derivative of a function y is called the *second derivative* of y, and is denoted by y'' or d^2y/dx^2. In general, the derivative of the $(n - 1)$st derivative of y is called the nth derivative of y and is denoted by $y^{(n)}$ or d^ny/dx^n. Here, n is a positive integer and is called the *order* of the derivative. The order n should not be confused with the *degree* of a derivative. For example, y''' is the third-order derivative of y, but $(y')^3$ is the third degree of the first derivative of y. Note that the first derivative of a function represents the *slope* or the *rate of change* of the function with the independent variable, and the second derivative represents the *rate of change of the slope* of the function with the independent variable.

When a function y depends on two or more independent variables such as x and t, it is sometimes of interest to examine the dependence of the function on one of the variables only. This is done by taking the derivative of the function with respect to that variable while holding the other variables constant. Such derivatives are called **partial derivatives.** The first partial derivatives of the function $y(x, t)$ with respect to x and t are defined as (Fig. 2–68)

$$\frac{\partial y}{\partial x} = \lim_{\Delta x \to 0} \frac{y(x + \Delta x, t) - y(x, t)}{\Delta x} \tag{2-82}$$

$$\frac{\partial y}{\partial t} = \lim_{\Delta t \to 0} \frac{y(x, t + \Delta t) - y(x, t)}{\Delta t} \tag{2-83}$$

Note that when finding $\partial y/\partial x$ we treat t as a constant and differentiate y with respect to x. Likewise, when finding $\partial y/\partial t$ we treat x as a constant and differentiate y with respect to t.

Integration can be viewed as the inverse process of differentiation. Integration is commonly used in solving differential equations since solving a differential equation is essentially a process of removing the derivatives from the equation. Differentiation is the process of finding $y'(x)$ when a function $y(x)$ is given, whereas integration is the process of finding the function $y(x)$ when its derivative $y'(x)$ is given. The integral of this derivative is expressed as

$$\int y'(x)dx = \int dy = y(x) + C \tag{2-84}$$

since $y'(x)dx = dy$ and the integral of the differential of a function is the function itself (plus a constant, of course). In Eq. 2–84, x is the integration variable and C is an arbitrary constant called the **integration constant.**

The derivative of $y(x) + C$ is $y'(x)$ no matter what the value of the constant C is. Therefore, two functions that differ by a constant have the same derivative, and we always add a constant C during integration to recover this constant that is lost during differentiation. The integral in Eq. 2–84 is called an **indefinite integral** since the value of the arbitrary constant C is indefinite. The described procedure can be extended to higher-order derivatives (Fig. 2–69). For example,

$$\int y''(x)dx = y'(x) + C \tag{2-85}$$

This can be proved by defining a new variable $u(x) = y'(x)$, differentiating it to obtain $u'(x) = y''(x)$, and then applying Eq. 2–84. Therefore, the order of a derivative decreases by one each time it is integrated.

$$\int dy = y + C$$
$$\int y'\,dx = y + C$$
$$\int y''\,dx = y' + C$$
$$\int y'''\,dx = y'' + C$$
$$\int y^{(n)}\,dx = y^{(n-1)} + C$$

FIGURE 2–69
Some indefinite integrals that involve derivatives.

Classification of Differential Equations

A differential equation that involves only ordinary derivatives is called an **ordinary differential equation,** and a differential equation that involves partial derivatives is called a **partial differential equation.** Then it follows that problems that involve a single independent variable result in ordinary differential equations, and problems that involve two or more independent variables result in partial differential equations. A differential equation may involve several derivatives of various orders of an unknown function. The order of the highest derivative in a differential equation is the order of the equation. For example, the order of $y''' + (y'')^4 = 7x^5$ is 3 since it contains no fourth or higher order derivatives.

You will remember from algebra that the equation $3x - 5 = 0$ is much easier to solve than the equation $x^4 + 3x - 5 = 0$ because the first equation is linear whereas the second one is nonlinear. This is also true for differential equations. Therefore, before we start solving a differential equation, we usually check for linearity. A differential equation is said to be **linear** if the dependent variable and all of its derivatives are of the first degree and their coefficients depend on the independent variable only. In other words, a differential equation is linear if it can be written in a form that does not involve (1) any powers of the dependent variable or its derivatives such as y^3 or $(y')^2$, (2) any products of the dependent variable or its derivatives such as yy' or $y'y'''$, and (3) any other nonlinear functions of the dependent variable such as sin y or e^y. If any of these conditions apply, it is **nonlinear** (Fig. 2–70).

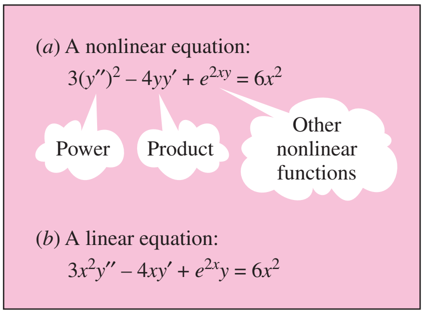


FIGURE 2–70
A differential equation that is (*a*) nonlinear and (*b*) linear. When checking for linearity, we examine the dependent variable only.

(a) *With constant coefficients:*

$$y'' + 6y' - 2y = xe^{-2x}$$

Constant

(b) *With variable coefficients:*

$$y'' - 6x^2y' - \frac{2}{x-1}y = xe^{-2x}$$

Variable

FIGURE 2–71
A differential equation with (*a*) constant coefficients and (*b*) variable coefficients.

A linear differential equation, however, may contain (1) powers or nonlinear functions of the independent variable, such as x^2 and cos x and (2) products of the dependent variable (or its derivatives) and functions of the independent variable, such as x^3y', x^2y, and $e^{-2x}y''$. A linear differential equation of order n can be expressed in the most general form as

$$y^{(n)} + f_1(x)y^{(n-1)} + \cdots + f_{n-1}(x)y' + f_n(x)y = R(x) \qquad \textbf{(2-86)}$$

A differential equation that cannot be put into this form is nonlinear. A linear differential equation in y is said to be **homogeneous** as well if $R(x) = 0$. Otherwise, it is nonhomogeneous. That is, each term in a linear homogeneous equation contains the dependent variable or one of its derivatives after the equation is cleared of any common factors. The term $R(x)$ is called the *nonhomogeneous term.*

Differential equations are also classified by the nature of the coefficients of the dependent variable and its derivatives. A differential equation is said to have **constant coefficients** if the coefficients of all the terms that involve the dependent variable or its derivatives are constants. If, after clearing any common factors, any of the terms with the dependent variable or its derivatives involve the independent variable as a coefficient, that equation is said to have **variable coefficients** (Fig. 2–71). Differential equations with constant coefficients are usually much easier to solve than those with variable coefficients.

Solutions of Differential Equations

Solving a differential equation can be as easy as performing one or more integrations; but such simple differential equations are usually the exception rather than the rule. There is no single general solution method applicable to all differential equations. There are different solution techniques, each being applicable to different classes of differential equations. Sometimes solving a differential equation requires the use of two or more techniques as well as ingenuity and mastery of solution methods. Some differential equations can be solved only by using some very clever tricks. Some cannot be solved analytically at all.

In algebra, we usually seek discrete values that satisfy an algebraic equation such as $x^2 - 7x - 10 = 0$. When dealing with differential equations, however, we seek functions that satisfy the equation in a specified interval. For example, the algebraic equation $x^2 - 7x - 10 = 0$ is satisfied by two numbers only: 2 and 5. But the differential equation $y' - 7y = 0$ is satisfied by the function e^{7x} for any value of x (Fig. 2–72).

(a) *An algebraic equation:*

$$y^2 - 7y - 10 = 0$$

Solution: $y = 2$ and $y = 5$

(b) *A differential equation:*

$$y' - 7y = 0$$

Solution: $y = e^{7x}$

FIGURE 2–72
Unlike those of algebraic equations, the solutions of differential equations are typically functions instead of discrete values.

Consider the algebraic equation $x^3 - 6x^2 + 11x - 6 = 0$. Obviously, $x = 1$ satisfies this equation, and thus it is a solution. However, it is not the only solution of this equation. We can easily show by direct substitution that $x = 2$ and $x = 3$ also satisfy this equation, and thus they are solutions as well. But there are no other solutions to this equation. Therefore, we say that the set 1, 2, and 3 forms the complete solution to this algebraic equation.

The same line of reasoning also applies to differential equations. Typically, differential equations have multiple solutions that contain at least one arbitrary constant. Any function that satisfies the differential equation on an interval is called a *solution* of that differential equation in that interval. A solution that involves one or more arbitrary constants represents a family of functions that satisfy the differential equation and is called **a general solution** of that equation. Not surprisingly, a differential equation may have more than one general solution. A general solution is usually referred to as **the general solution** or the **complete solution** if every solution of the equation can be obtained from it as a special case. A solution that can be obtained from a general solution by assigning particular values to the arbitrary constants is called a **specific solution.**

You will recall from algebra that a number is a solution of an algebraic equation if it satisfies the equation. For example, 2 is a solution of the equation $x^3 - 8 = 0$ because the substitution of 2 for x yields identically zero. Likewise, a function is a solution of a differential equation if that function satisfies the differential equation. In other words, a solution function yields identity when substituted into the differential equation. For example, it can be shown by direct substitution that the function $3e^{-2x}$ is a solution of $y'' - 4y = 0$ (Fig. 2–73).

Function: $f = 3e^{-2x}$

Differential equation: $y'' - 4y = 0$

Derivatives of f:

$$f' = -6e^{-2x}$$
$$f'' = 12e^{-2x}$$

Substituting into $y'' - 4y = 0$:

$$f'' - 4f \stackrel{?}{=} 0$$
$$12e^{-2x} - 4 \times 3e^{-2x} \stackrel{?}{=} 0$$
$$0 = 0$$

Therefore, the function $3e^{-2x}$ is a solution of the differential equation $y'' - 4y = 0$.

FIGURE 2–73
Verifying that a given function is a solution of a differential equation.

SUMMARY

In this chapter we have studied the heat conduction equation and its solutions. Heat conduction in a medium is said to be *steady* when the temperature does not vary with time and *unsteady* or *transient* when it does. Heat conduction in a medium is said to be *one-dimensional* when conduction is significant in one dimension only and negligible in the other two dimensions. It is said to be *two-dimensional* when conduction in the third dimension is negligible and *three-dimensional* when conduction in all dimensions is significant. In heat transfer analysis, the conversion of electrical, chemical, or nuclear energy into heat (or thermal) energy is characterized as *heat generation.*

The heat conduction equation can be derived by performing an energy balance on a differential volume element. The one-dimensional heat conduction equation in rectangular, cylindrical, and spherical coordinate systems for the case of constant thermal conductivities are expressed as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where the property $\alpha = k/\rho C$ is the *thermal diffusivity* of the material.

The solution of a heat conduction problem depends on the conditions at the surfaces, and the mathematical expressions for the thermal conditions at the boundaries are called the *boundary conditions.* The solution of transient heat conduction problems also depends on the condition of the medium at the beginning of the heat conduction process. Such a condition, which is usually specified at time $t = 0$, is called the *initial condition,* which is a mathematical expression for the temperature distribution of the medium initially. Complete mathematical description of a heat conduction problem requires the specification of two boundary conditions for each dimension along which heat conduction is significant, and an initial condition when the problem is transient. The most common boundary conditions are the *specified temperature, specified heat flux, convection,* and *radiation* boundary conditions. A boundary surface, in general, may involve specified heat flux, convection, and radiation at the same time.

For steady one-dimensional heat transfer through a plate of thickness L, the various types of boundary conditions at the surfaces at $x = 0$ and $x = L$ can be expressed as

Specified temperature:

$$T(0) = T_1 \qquad \text{and} \qquad T(L) = T_2$$

where T_1 and T_2 are the specified temperatures at surfaces at $x = 0$ and $x = L$.

Specified heat flux:

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \qquad \text{and} \qquad -k\frac{dT(L)}{dx} = \dot{q}_L$$

where $\dot{q}_0$ and $\dot{q}_L$ are the specified heat fluxes at surfaces at $x = 0$ and $x = L$.

Insulation or thermal symmetry:

$$\frac{dT(0)}{dx} = 0 \qquad \text{and} \qquad \frac{dT(L)}{dx} = 0$$

Convection:

$$-k\frac{dT(0)}{dx} = h_1[T_{\infty 1} - T(0)] \quad \text{and} \quad -k\frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}]$$

where h_1 and h_2 are the convection heat transfer coefficients and $T_{\infty 1}$ and $T_{\infty 2}$ are the temperatures of the surrounding mediums on the two sides of the plate.

Radiation:

$$-k\frac{dT(0)}{dx} = \varepsilon_1\sigma[T^4_{\text{surr, 1}} - T(0)^4] \qquad \text{and}$$

$$-k\frac{dT(L)}{dx} = \varepsilon_2\sigma[T(L)^4 - T^4_{\text{surr, 2}}]$$

where ε_1 and ε_2 are the emissivities of the boundary surfaces, $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$ K^4 is the Stefan–Boltzmann constant, and $T_{\text{surr, 1}}$ and $T_{\text{surr, 2}}$ are the average temperatures of the surfaces surrounding the two sides of the plate. In radiation calculations, the temperatures must be in K or R.

Interface of two bodies A *and* B *in perfect contact at* $x = x_0$*:*

$$T_A(x_0) = T_B(x_0) \qquad \text{and} \qquad -k_A\frac{dT_A(x_0)}{dx} = -k_B\frac{dT_B(x_0)}{dx}$$

where k_A and k_B are the thermal conductivities of the layers A and B.

Heat generation is usually expressed *per unit volume* of the medium and is denoted by $\dot{g}$, whose unit is W/m^3. Under steady conditions, the surface temperature T_s of a plane wall of thickness $2L$, a cylinder of outer radius r_o, and a sphere of radius r_o in which heat is generated at a constant rate of $\dot{g}$ per unit volume in a surrounding medium at T_∞ can be expressed as

$$T_{s,\text{ plane wall}} = T_\infty + \frac{\dot{g}L}{h}$$

$$T_{s,\text{ cylinder}} = T_\infty + \frac{\dot{g}r_o}{2h}$$

$$T_{s,\text{ sphere}} = T_\infty + \frac{\dot{g}r_o}{3h}$$

where h is the convection heat transfer coefficient. The maximum temperature rise between the surface and the midsection of a medium is given by

$$\Delta T_{\text{max, plane wall}} = \frac{\dot{g}L^2}{2k}$$

$$\Delta T_{\text{max, cylinder}} = \frac{\dot{g}r_o^2}{4k}$$

$$\Delta T_{\text{max, sphere}} = \frac{\dot{g}r_o^2}{6k}$$

When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$k_{\text{ave}} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1}$$

Then the rate of steady heat transfer through a plane wall, cylindrical layer, or spherical layer can be expressed as

$$\dot{Q}_{\text{plane wall}} = k_{\text{ave}}A\frac{T_1 - T_2}{L} = \frac{A}{L}\int_{T_2}^{T_1} k(T)dT$$

$$\dot{Q}_{\text{cylinder}} = 2\pi k_{\text{ave}}L\frac{T_1 - T_2}{\ln(r_2/r_1)} = \frac{2\pi L}{\ln(r_2/r_1)}\int_{T_2}^{T_1} k(T)dT$$

$$\dot{Q}_{\text{sphere}} = 4\pi k_{\text{ave}}r_1r_2\frac{T_1 - T_2}{r_2 - r_1} = \frac{4\pi r_1r_2}{r_2 - r_1}\int_{T_2}^{T_1} k(T)dT$$

The variation of thermal conductivity of a material with temperature can often be approximated as a linear function and expressed as

$$k(T) = k_0(1 + \beta T)$$

where β is called the *temperature coefficient of thermal conductivity.*

REFERENCES AND SUGGESTED READING

1. W. E. Boyce and R. C. Diprima. *Elementary Differential Equations and Boundary Value Problems.* 4th ed. New York: John Wiley & Sons, 1986.
2. J. P. Holman. *Heat Transfer.* 9th ed. New York: McGraw-Hill, 2002.

3. F. P. Incropera and D. P. DeWitt. *Introduction to Heat Transfer.* 4th ed. New York: John Wiley & Sons, 2002.

4. S. S. Kutateladze. *Fundamentals of Heat Transfer.* New York: Academic Press, 1963.

5. M. N. Ozisik. *Heat Transfer—A Basic Approach.* New York: McGraw-Hill, 1985.

6. F. M. White. *Heat and Mass Transfer.* Reading, MA: Addison-Wesley, 1988.

PROBLEMS*

Introduction

2–1C Is heat transfer a scalar or vector quantity? Explain. Answer the same question for temperature.

2–2C How does transient heat transfer differ from steady heat transfer? How does one-dimensional heat transfer differ from two-dimensional heat transfer?

2–3C Consider a cold canned drink left on a dinner table. Would you model the heat transfer to the drink as one-, two-, or three-dimensional? Would the heat transfer be steady or transient? Also, which coordinate system would you use to analyze this heat transfer problem, and where would you place the origin? Explain.

2–4C Consider a round potato being baked in an oven. Would you model the heat transfer to the potato as one-, two-, or three-dimensional? Would the heat transfer be steady or transient? Also, which coordinate system would you use to solve this problem, and where would you place the origin? Explain.

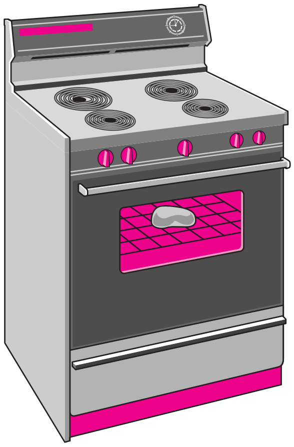


FIGURE P2–4

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

2–5C Consider an egg being cooked in boiling water in a pan. Would you model the heat transfer to the egg as one-, two-, or three-dimensional? Would the heat transfer be steady or transient? Also, which coordinate system would you use to solve this problem, and where would you place the origin? Explain.

2–6C Consider a hot dog being cooked in boiling water in a pan. Would you model the heat transfer to the hot dog as one-, two-, or three-dimensional? Would the heat transfer be steady or transient? Also, which coordinate system would you use to solve this problem, and where would you place the origin? Explain.

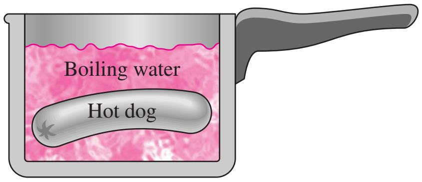


FIGURE P2–6

2–7C Consider the cooking process of a roast beef in an oven. Would you consider this to be a steady or transient heat transfer problem? Also, would you consider this to be one-, two-, or three-dimensional? Explain.

2–8C Consider heat loss from a 200-L cylindrical hot water tank in a house to the surrounding medium. Would you consider this to be a steady or transient heat transfer problem? Also, would you consider this heat transfer problem to be one-, two-, or three-dimensional? Explain.

2–9C Does a heat flux vector at a point P on an isothermal surface of a medium have to be perpendicular to the surface at that point? Explain.

2–10C From a heat transfer point of view, what is the difference between isotropic and unisotropic materials?

2–11C What is heat generation in a solid? Give examples.

2–12C Heat generation is also referred to as energy generation or thermal energy generation. What do you think of these phrases?

2–13C In order to determine the size of the heating element of a new oven, it is desired to determine the rate of heat transfer through the walls, door, and the top and bottom section of the oven. In your analysis, would you consider this to be a steady or transient heat transfer problem? Also, would you consider the heat transfer to be one-dimensional or multidimensional? Explain.

2–14E The resistance wire of a 1000-W iron is 15 in. long and has a diameter of $D = 0.08$ in. Determine the rate of heat generation in the wire per unit volume, in Btu/h · ft^3, and the heat flux on the outer surface of the wire, in Btu/h · ft^2, as a result of this heat generation.

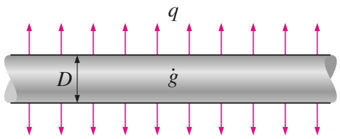


FIGURE P2–14E

2–15E  Reconsider Problem 2–14E. Using EES (or other) software, evaluate and plot the surface heat flux as a function of wire diameter as the diameter varies from 0.02 to 0.20 in. Discuss the results.

2–16 In a nuclear reactor, heat is generated uniformly in the 5-cm-diameter cylindrical uranium rods at a rate of 7×10^7 W/m^3. If the length of the rods is 1 m, determine the rate of heat generation in each rod. *Answer:* 137.4 kW

2–17 In a solar pond, the absorption of solar energy can be modeled as heat generation and can be approximated by $\dot{g} = \dot{g}_0 e^{-bx}$, where g_0 is the rate of heat absorption at the top surface per unit volume and b is a constant. Obtain a relation for the total rate of heat generation in a water layer of surface area A and thickness L at the top of the pond.

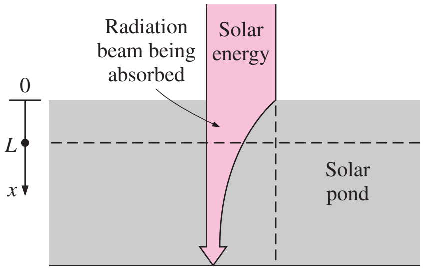


FIGURE P2–17

2–18 Consider a large 3-cm-thick stainless steel plate in which heat is generated uniformly at a rate of 5×10^6 W/m^3. Assuming the plate is losing heat from both sides, determine the heat flux on the surface of the plate during steady operation. *Answer:* 75,000 W/m^2

Heat Conduction Equation

2–19 Write down the one-dimensional transient heat conduction equation for a plane wall with constant thermal conductivity and heat generation in its simplest form, and indicate what each variable represents.

2–20 Write down the one-dimensional transient heat conduction equation for a long cylinder with constant thermal conductivity and heat generation, and indicate what each variable represents.

2–21 Starting with an energy balance on a rectangular volume element, derive the one-dimensional transient heat conduction equation for a plane wall with constant thermal conductivity and no heat generation.

2–22 Starting with an energy balance on a cylindrical shell volume element, derive the steady one-dimensional heat conduction equation for a long cylinder with constant thermal conductivity in which heat is generated at a rate of $\dot{g}$.

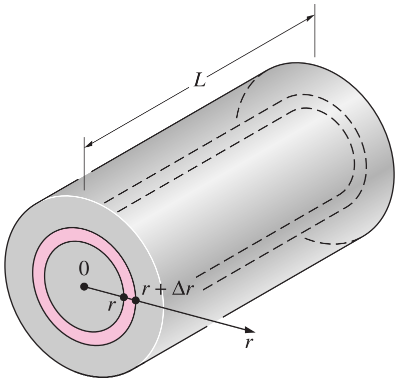


FIGURE P2–22

2–23 Starting with an energy balance on a spherical shell volume element, derive the one-dimensional transient heat conduction equation for a sphere with constant thermal conductivity and no heat generation.

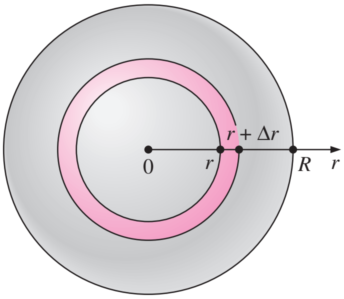


FIGURE P2–23

2–24 Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

2–25 Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right) + \dot{g} = 0$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

2–26 Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

2–27 Consider a medium in which the heat conduction equation is given in its simplest form as

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

2–28 Starting with an energy balance on a volume element, derive the two-dimensional transient heat conduction equation in rectangular coordinates for $T(x, y, t)$ for the case of constant thermal conductivity and no heat generation.

2–29 Starting with an energy balance on a ring-shaped volume element, derive the two-dimensional steady heat conduction equation in cylindrical coordinates for $T(r, z)$ for the case of constant thermal conductivity and no heat generation.

2–30 Starting with an energy balance on a disk volume element, derive the one-dimensional transient heat conduction equation for $T(z, t)$ in a cylinder of diameter D with an insulated side surface for the case of constant thermal conductivity with heat generation.

2–31 Consider a medium in which the heat conduction equation is given in its simplest form as

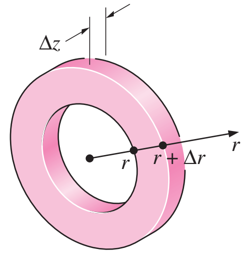


FIGURE P2–29

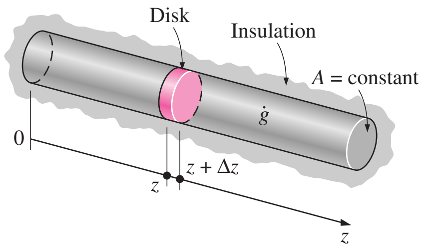


FIGURE P2–30

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

2–32 Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = 0$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

2–33 Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial t}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?

Boundary and Initial Conditions; Formulation of Heat Conduction Problems

2–34C What is a boundary condition? How many boundary conditions do we need to specify for a two-dimensional heat transfer problem?

2–35C What is an initial condition? How many initial conditions do we need to specify for a two-dimensional heat transfer problem?

2–36C What is a thermal symmetry boundary condition? How is it expressed mathematically?

2–37C How is the boundary condition on an insulated surface expressed mathematically?

2–38C It is claimed that the temperature profile in a medium must be perpendicular to an insulated surface. Is this a valid claim? Explain.

2–39C Why do we try to avoid the radiation boundary conditions in heat transfer analysis?

2–40 Consider a spherical container of inner radius r_1, outer radius r_2, and thermal conductivity k. Express the boundary condition on the inner surface of the container for steady one-dimensional conduction for the following cases: (*a*) specified temperature of 50°C, (*b*) specified heat flux of 30 W/m^2 toward the center, (*c*) convection to a medium at T_∞ with a heat transfer coefficient of h.

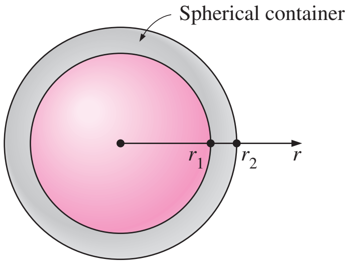


FIGURE P2–40

2–41 Heat is generated in a long wire of radius r_0 at a constant rate of $\dot{g}_0$ per unit volume. The wire is covered with a plastic insulation layer. Express the heat flux boundary condition at the interface in terms of the heat generated.

2–42 Consider a long pipe of inner radius r_1, outer radius r_2, and thermal conductivity k. The outer surface of the pipe is subjected to convection to a medium at T_∞ with a heat transfer coefficient of h, but the direction of heat transfer is not known. Express the convection boundary condition on the outer surface of the pipe.

2–43 Consider a spherical shell of inner radius r_1, outer radius r_2, thermal conductivity k, and emissivity ε. The outer surface of the shell is subjected to radiation to surrounding surfaces at T_{surr}, but the direction of heat transfer is not known. Express the radiation boundary condition on the outer surface of the shell.

2–44 A container consists of two spherical layers, A and B, that are in perfect contact. If the radius of the interface is r_0, express the boundary conditions at the interface.

2–45 Consider a steel pan used to boil water on top of an electric range. The bottom section of the pan is $L = 0.5$ cm thick and has a diameter of $D = 20$ cm. The electric heating unit on the range top consumes 1000 W of power during cooking, and 85 percent of the heat generated in the heating element is transferred uniformly to the pan. Heat transfer from the top surface of the bottom section to the water is by convection with a heat transfer coefficient of h. Assuming constant thermal conductivity and one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem during steady operation. Do not solve.

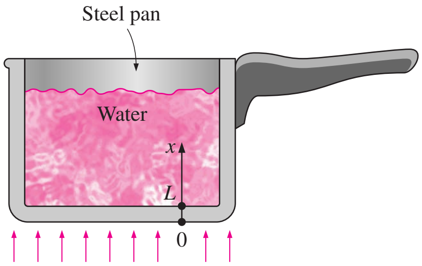


FIGURE P2–45

2–46E A 2-kW resistance heater wire whose thermal conductivity is $k = 10.4$ Btu/h · ft · °F has a radius of $r_0 = 0.06$ in. and a length of $L = 15$ in., and is used for space heating. Assuming constant thermal conductivity and one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem during steady operation. Do not solve.

2–47 Consider an aluminum pan used to cook stew on top of an electric range. The bottom section of the pan is $L = 0.25$ cm thick and has a diameter of $D = 18$ cm. The electric heating unit on the range top consumes 900 W of power during cooking, and 90 percent of the heat generated in the heating element

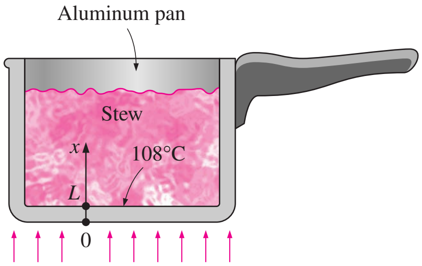


FIGURE P2–47

is transferred to the pan. During steady operation, the temperature of the inner surface of the pan is measured to be 108°C. Assuming temperature-dependent thermal conductivity and one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem during steady operation. Do not solve.

2–48 Water flows through a pipe at an average temperature of $T_\infty = 50°C$. The inner and outer radii of the pipe are $r_1 = 6$ cm and $r_2 = 6.5$ cm, respectively. The outer surface of the pipe is wrapped with a thin electric heater that consumes 300 W per m length of the pipe. The exposed surface of the heater is heavily insulated so that the entire heat generated in the heater is transferred to the pipe. Heat is transferred from the inner surface of the pipe to the water by convection with a heat transfer coefficient of $h = 55 \text{ W/m}^2 \cdot °C$. Assuming constant thermal conductivity and one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of the heat conduction in the pipe during steady operation. Do not solve.

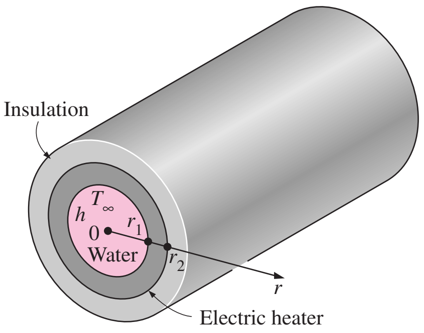


FIGURE P2–48

2–49 A spherical metal ball of radius r_0 is heated in an oven to a temperature of T_i throughout and is then taken out of the oven and dropped into a large body of water at T_∞ where it is cooled by convection with an average convection heat transfer coefficient of h. Assuming constant thermal conductivity and transient one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem. Do not solve.

2–50 A spherical metal ball of radius r_0 is heated in an oven to a temperature of T_i throughout and is then taken out of the oven and allowed to cool in ambient air at T_∞ by convection and radiation. The emissivity of the outer surface of the cylinder is ε, and the temperature of the surrounding surfaces is T_{surr}. The average convection heat transfer coefficient is estimated to be h. Assuming variable thermal conductivity and transient one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem. Do not solve.

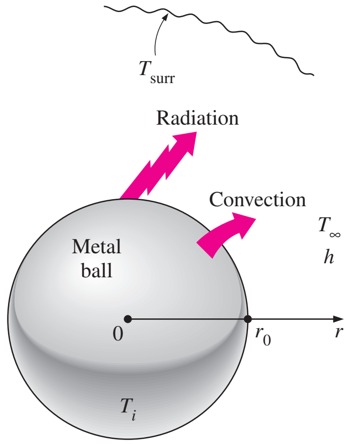


FIGURE P2–50

2–51 Consider the north wall of a house of thickness L. The outer surface of the wall exchanges heat by both convection and radiation. The interior of the house is maintained at $T_{\infty 1}$, while the ambient air temperature outside remains at $T_{\infty 2}$. The sky, the ground, and the surfaces of the surrounding structures at this location can be modeled as a surface at an effective temperature of T_{sky} for radiation exchange on the outer surface. The radiation exchange between the inner surface of the wall and the surfaces of the walls, floor, and ceiling it faces is negligible. The convection heat transfer coefficients on the inner and outer surfaces of the wall are h_1 and h_2, respectively. The thermal conductivity of the wall material is k and the emissivity of the outer surface is ε_2. Assuming the heat transfer through the wall to be steady and one-dimensional, express the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem. Do not solve.

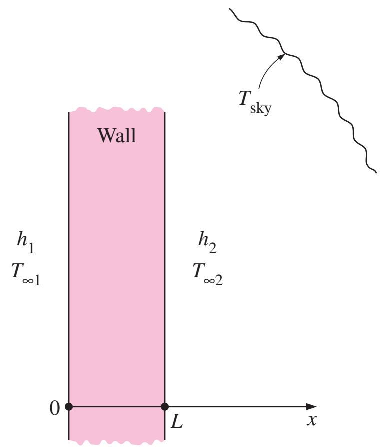


FIGURE P2–51

Solution of Steady One-Dimensional Heat Conduction Problems

2–52C Consider one-dimensional heat conduction through a large plane wall with no heat generation that is perfectly insulated on one side and is subjected to convection and radiation on the other side. It is claimed that under steady conditions, the temperature in a plane wall must be uniform (the same everywhere). Do you agree with this claim? Why?

2–53C It is stated that the temperature in a plane wall with constant thermal conductivity and no heat generation varies linearly during steady one-dimensional heat conduction. Will this still be the case when the wall loses heat by radiation from its surfaces?

2–54C Consider a solid cylindrical rod whose ends are maintained at constant but different temperatures while the side surface is perfectly insulated. There is no heat generation. It is claimed that the temperature along the axis of the rod varies linearly during steady heat conduction. Do you agree with this claim? Why?

2–55C Consider a solid cylindrical rod whose side surface is maintained at a constant temperature while the end surfaces are perfectly insulated. The thermal conductivity of the rod material is constant and there is no heat generation. It is claimed that the temperature in the radial direction within the rod will not vary during steady heat conduction. Do you agree with this claim? Why?

2–56 Consider a large plane wall of thickness $L = 0.4$ m, thermal conductivity $k = 2.3$ W/m · °C, and surface area $A = 20$ m^2. The left side of the wall is maintained at a constant temperature of $T_1 = 80$°C while the right side loses heat by convection to the surrounding air at $T_\infty = 15$°C with a heat transfer coefficient of $h = 24$ W/m^2 · °C. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the rate of heat transfer through the wall. *Answer:* (*c*) 6030 W

2–57 Consider a solid cylindrical rod of length 0.15 m and diameter 0.05 m. The top and bottom surfaces of the rod are maintained at constant temperatures of 20°C and 95°C, respectively, while the side surface is perfectly insulated. Determine the rate of heat transfer through the rod if it is made of (*a*) copper, $k = 380$ W/m · °C, (*b*) steel, $k = 18$ W/m · °C, and (*c*) granite, $k = 1.2$ W/m · °C.

2–58 Reconsider Problem 2–57. Using EES (or other) software, plot the rate of heat transfer as a function of the thermal conductivity of the rod in the range of 1 W/m · °C to 400 W/m · °C. Discuss the results.

2–59 Consider the base plate of a 800-W household iron with a thickness of $L = 0.6$ cm, base area of $A = 160$ cm^2, and thermal conductivity of $k = 20$ W/m · °C. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside. When steady operating conditions are reached, the outer surface temperature of the plate is measured to be 85°C. Disregarding any heat loss through the upper part of the iron, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the plate, (*b*) obtain a relation for the variation of temperature in the base plate by solving the differential equation, and (*c*) evaluate the inner surface temperature.
Answer: (*c*) 100°C

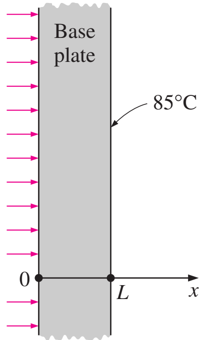


FIGURE P2–59

2–60 Repeat Problem 2–59 for a 1200-W iron.

2–61 Reconsider Problem 2–59. Using the relation obtained for the variation of temperature in the base plate, plot the temperature as a function of the distance x in the range of $x = 0$ to $x = L$, and discuss the results. Use the EES (or other) software.

2–62E Consider a steam pipe of length $L = 15$ ft, inner radius $r_1 = 2$ in., outer radius $r_2 = 2.4$ in., and thermal conductivity $k = 7.2$ Btu/h · ft · °F. Steam is flowing through the pipe at an average temperature of 250°F, and the average convection heat transfer coefficient on the inner surface is given to be $h = 1.25$ Btu/h · ft^2 · °F . If the average temperature on the outer

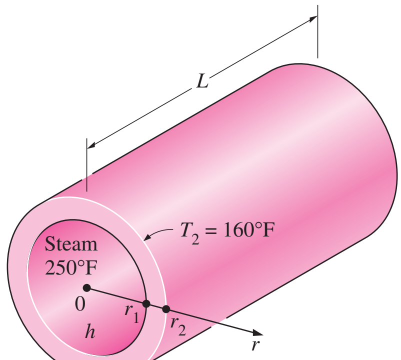


FIGURE P2–62E

surfaces of the pipe is $T_2 = 160°F$, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the pipe, (*b*) obtain a relation for the variation of temperature in the pipe by solving the differential equation, and (*c*) evaluate the rate of heat loss from the steam through the pipe. *Answer:* (*c*) 16,800 Btu/h

2–63 A spherical container of inner radius $r_1 = 2$ m, outer radius $r_2 = 2.1$ m, and thermal conductivity $k = 30$ W/m · °C is filled with iced water at 0°C. The container is gaining heat by convection from the surrounding air at $T_\infty = 25°C$ with a heat transfer coefficient of $h = 18$ W/m^2 · °C. Assuming the inner surface temperature of the container to be 0°C, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the container, (*b*) obtain a relation for the variation of temperature in the container by solving the differential equation, and (*c*) evaluate the rate of heat gain to the iced water.

2–64 Consider a large plane wall of thickness $L = 0.3$ m, thermal conductivity $k = 2.5$ W/m · °C, and surface area $A = 12$ m^2. The left side of the wall at $x = 0$ is subjected to a net heat flux of $\dot{q}_0 = 700$ W/m^2 while the temperature at that surface is measured to be $T_1 = 80°C$. Assuming constant thermal conductivity and no heat generation in the wall, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the temperature of the right surface of the wall at $x = L$. *Answer:* (*c*) −4°C

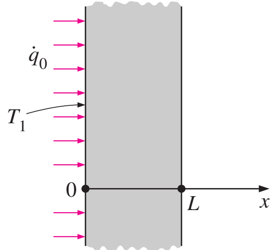


FIGURE P2–64

2–65 Repeat Problem 2–64 for a heat flux of 950 W/m^2 and a surface temperature of 85°C at the left surface at $x = 0$.

2–66E A large steel plate having a thickness of $L = 4$ in., thermal conductivity of $k = 7.2$ Btu/h · ft · °F, and an emissivity of $\varepsilon = 0.6$ is lying on the ground. The exposed surface of the plate at $x = L$ is known to exchange heat by convection with the ambient air at $T_\infty = 90°F$ with an average heat transfer coefficient of $h = 12$ Btu/h · ft^2 · °F as well as by radiation with the open sky with an equivalent sky temperature of $T_{sky} = 510$ R. Also, the temperature of the upper surface of the plate is measured to be 75°F. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the plate, (*b*) obtain a relation for the variation of temperature in the plate by solving

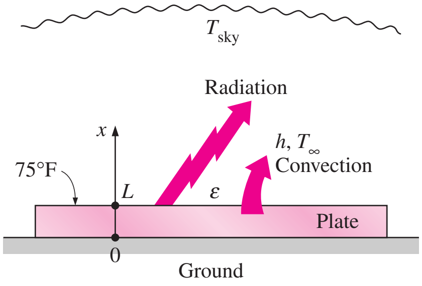


FIGURE P2–66E

the differential equation, and (*c*) determine the value of the lower surface temperature of the plate at $x = 0$.

2–67E Repeat Problem 2–66E by disregarding radiation heat transfer.

2–68 When a long section of a compressed air line passes through the outdoors, it is observed that the moisture in the compressed air freezes in cold weather, disrupting and even completely blocking the air flow in the pipe. To avoid this problem, the outer surface of the pipe is wrapped with electric strip heaters and then insulated.

Consider a compressed air pipe of length $L = 6$ m, inner radius $r_1 = 3.7$ cm, outer radius $r_2 = 4.0$ cm, and thermal conductivity $k = 14$ W/m · °C equipped with a 300-W strip heater. Air is flowing through the pipe at an average temperature of −10°C, and the average convection heat transfer coefficient on the inner surface is $h = 30$ W/m^2 · °C. Assuming 15 percent of the heat generated in the strip heater is lost through the insulation, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the pipe, (*b*) obtain a relation for the variation of temperature in the pipe material by solving the differential equation, and (*c*) evaluate the inner and outer surface temperatures of the pipe. *Answers:* (*c*) −3.91°C, −3.87°C

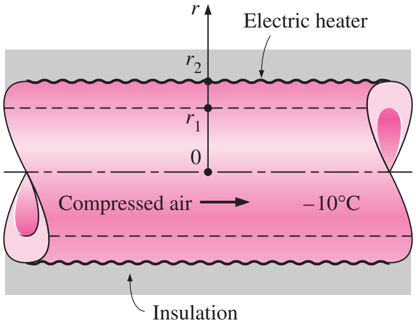


FIGURE P2–68

2–69 Reconsider Problem 2–68. Using the relation obtained for the variation of temperature in the pipe material, plot the temperature as a function of the radius r in the range of $r = r_1$ to $r = r_2$, and discuss the results. Use the EES (or other) software.

2–70 In a food processing facility, a spherical container of inner radius $r_1 = 40$ cm, outer radius $r_2 = 41$ cm, and thermal conductivity $k = 1.5$ W/m · °C is used to store hot water and to keep it at 100°C at all times. To accomplish this, the outer surface of the container is wrapped with a 500-W electric strip heater and then insulated. The temperature of the inner surface of the container is observed to be nearly 100°C at all times. Assuming 10 percent of the heat generated in the heater is lost through the insulation, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the container, (*b*) obtain a relation for the variation of temperature in the container material by solving the differential equation, and (*c*) evaluate the outer surface temperature of the container. Also determine how much water at 100°C this tank can supply steadily if the cold water enters at 20°C.

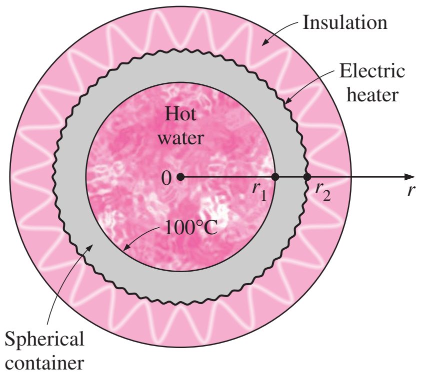


FIGURE P2–70

2–71 Reconsider Problem 2–70. Using the relation obtained for the variation of temperature in the container material, plot the temperature as a function of the radius r in the range of $r = r_1$ to $r = r_2$, and discuss the results. Use the EES (or other) software.

Heat Generation in a Solid

2–72C Does heat generation in a solid violate the first law of thermodynamics, which states that energy cannot be created or destroyed? Explain.

2–73C What is heat generation? Give some examples.

2–74C An iron is left unattended and its base temperature rises as a result of resistance heating inside. When will the rate of heat generation inside the iron be equal to the rate of heat loss from the iron?

2–75C Consider the uniform heating of a plate in an environment at a constant temperature. Is it possible for part of the heat generated in the left half of the plate to leave the plate through the right surface? Explain.

2–76C Consider uniform heat generation in a cylinder and a sphere of equal radius made of the same material in the same environment. Which geometry will have a higher temperature at its center? Why?

2–77 A 2-kW resistance heater wire with thermal conductivity of $k = 20$ W/m · °C, a diameter of $D = 5$ mm, and a length of $L = 0.7$ m is used to boil water. If the outer surface temperature of the resistance wire is $T_s = 110$°C, determine the temperature at the center of the wire.

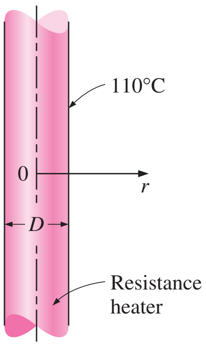


FIGURE P2–77

2–78 Consider a long solid cylinder of radius $r_0 = 4$ cm and thermal conductivity $k = 25$ W/m · °C. Heat is generated in the cylinder uniformly at a rate of $\dot{g}_0 = 35$ W/cm^3. The side surface of the cylinder is maintained at a constant temperature of $T_s = 80$°C. The variation of temperature in the cylinder is given by

$$T(r) = \frac{\dot{g}\, r_0^2}{k}\left[1 - \left(\frac{r}{r_0}\right)^2\right] + T_s$$

Based on this relation, determine (*a*) if the heat conduction is steady or transient, (*b*) if it is one-, two-, or three-dimensional, and (*c*) the value of heat flux on the side surface of the cylinder at $r = r_0$.

2–79 Reconsider Problem 2–78. Using the relation obtained for the variation of temperature in the cylinder, plot the temperature as a function of the radius r in the range of $r = 0$ to $r = r_0$, and discuss the results. Use the EES (or other) software.

2–80E A long homogeneous resistance wire of radius $r_0 =$ 0.25 in. and thermal conductivity $k = 8.6$ Btu/h · ft · °F is being used to boil water at atmospheric pressure by the passage of

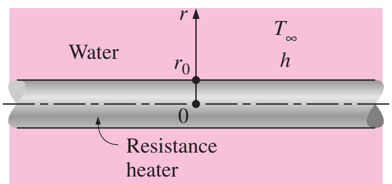


FIGURE P2–80E

electric current. Heat is generated in the wire uniformly as a result of resistance heating at a rate of $\dot{g}$ = 1800 Btu/h · in^3. The heat generated is transferred to water at 212°F by convection with an average heat transfer coefficient of h = 820 Btu/h · ft^2 · °F. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the wire, (*b*) obtain a relation for the variation of temperature in the wire by solving the differential equation, and (*c*) determine the temperature at the centerline of the wire. *Answer:* (*c*) 290.8°F

2–81E Reconsider Problem 2–80E. Using the relation obtained for the variation of temperature in the wire, plot the temperature at the centerline of the wire as a function of the heat generation $\dot{g}$ in the range of 400 Btu/h · in^3 to 2400 Btu/h · in^3, and discuss the results. Use the EES (or other) software.

2–82 In a nuclear reactor, 1-cm-diameter cylindrical uranium rods cooled by water from outside serve as the fuel. Heat is generated uniformly in the rods (k = 29.5 W/m · °C) at a rate of 7 × 10^7 W/m^3. If the outer surface temperature of rods is 175°C, determine the temperature at their center.

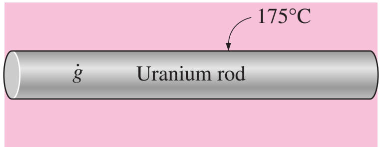


FIGURE P2–82

2–83 Consider a large 3-cm-thick stainless steel plate (k = 15.1 W/m · °C) in which heat is generated uniformly at a rate of 5 × 10^5 W/m^3. Both sides of the plate are exposed to an environment at 30°C with a heat transfer coefficient of 60 W/m^2 · °C. Explain where in the plate the highest and the lowest temperatures will occur, and determine their values.

2–84 Consider a large 5-cm-thick brass plate (k = 111 W/m · °C) in which heat is generated uniformly at a rate of 2 × 10^5 W/m^3. One side of the plate is insulated while the other side is exposed to an environment at 25°C with a heat transfer coefficient of 44 W/m^2 · °C. Explain where in the plate the highest and the lowest temperatures will occur, and determine their values.

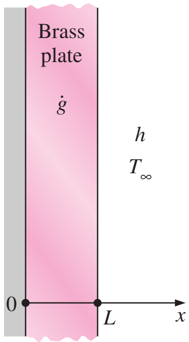


FIGURE P2–84

2–85 Reconsider Problem 2–84. Using EES (or other) software, investigate the effect of the heat transfer coefficient on the highest and lowest temperatures in the plate. Let the heat transfer coefficient vary from 20 W/m^2 · °C to 100 W/m^2 · °C. Plot the highest and lowest temperatures as a function of the heat transfer coefficient, and discuss the results.

2–86 A 6-m-long 2-kW electrical resistance wire is made of 0.2-cm-diameter stainless steel (k = 15.1 W/m · °C). The resistance wire operates in an environment at 30°C with a heat transfer coefficient of 140 W/m^2 · °C at the outer surface. Determine the surface temperature of the wire (*a*) by using the applicable relation and (*b*) by setting up the proper differential equation and solving it. *Answers:* (*a*) 409°C, (*b*) 409°C

2–87E Heat is generated uniformly at a rate of 3 kW per ft length in a 0.08-in.-diameter electric resistance wire made of nickel steel (k = 5.8 Btu/h · ft · °F). Determine the temperature difference between the centerline and the surface of the wire.

2–88E Repeat Problem 2–87E for a manganese wire (k = 4.5 Btu/h · ft · °F).

2–89 Consider a homogeneous spherical piece of radioactive material of radius r_0 = 0.04 m that is generating heat at a constant rate of $\dot{g}$ = 4 × 10^7 W/m^3. The heat generated is dissipated to the environment steadily. The outer surface of the sphere is maintained at a uniform temperature of 80°C and the thermal conductivity of the sphere is k = 15 W/m · °C. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the sphere, (*b*) obtain a relation for the variation of temperature in the sphere by solving the differential equation, and (*c*) determine the temperature at the center of the sphere.

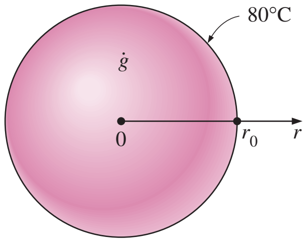


FIGURE P2–89

2–90 Reconsider Problem 2–89. Using the relation obtained for the variation of temperature in the sphere, plot the temperature as a function of the radius r in the range of r = 0 to r = r_0. Also, plot the center temperature of the sphere as a function of the thermal conductivity in the range of 10 W/m · °C to 400 W/m · °C. Discuss the results. Use the EES (or other) software.

2–91 A long homogeneous resistance wire of radius r_0 = 5 mm is being used to heat the air in a room by the passage of electric current. Heat is generated in the wire uniformly at a rate of $\dot{g} = 5 \times 10^7$ W/m^3 as a result of resistance heating. If the temperature of the outer surface of the wire remains at 180°C, determine the temperature at r = 2 mm after steady operation conditions are reached. Take the thermal conductivity of the wire to be k = 8 W/m · °C. *Answer:* 212.8°C

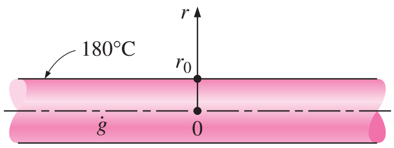


FIGURE P2–91

2–92 Consider a large plane wall of thickness L = 0.05 m. The wall surface at x = 0 is insulated, while the surface at x = L is maintained at a temperature of 30°C. The thermal conductivity of the wall is k = 30 W/m · °C, and heat is generated in the wall at a rate of $\dot{g} = \dot{g}_0 e^{-0.5x/L}$ W/m^3 where $\dot{g}_0 = 8 \times 10^6$ W/m^3. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) determine the temperature of the insulated surface of the wall. *Answer:* (*c*) 314°C

2–93 Reconsider Problem 2–92. Using the relation given for the heat generation in the wall, plot the heat generation as a function of the distance x in the range of x = 0 to x = L, and discuss the results. Use the EES (or other) software.

Variable Thermal Conductivity, $k(T)$

2–94C Consider steady one-dimensional heat conduction in a plane wall, long cylinder, and sphere with constant thermal conductivity and no heat generation. Will the temperature in any of these mediums vary linearly? Explain.

2–95C Is the thermal conductivity of a medium, in general, constant or does it vary with temperature?

2–96C Consider steady one-dimensional heat conduction in a plane wall in which the thermal conductivity varies linearly. The error involved in heat transfer calculations by assuming constant thermal conductivity at the average temperature is (*a*) none, (*b*) small, or (*c*) significant.

2–97C The temperature of a plane wall during steady one-dimensional heat conduction varies linearly when the thermal conductivity is constant. Is this still the case when the thermal conductivity varies linearly with temperature?

2–98C When the thermal conductivity of a medium varies linearly with temperature, is the average thermal conductivity always equivalent to the conductivity value at the average temperature?

2–99 Consider a plane wall of thickness L whose thermal conductivity varies in a specified temperature range as $k(T) = k_0(1 + \beta T^2)$ where k_0 and β are two specified constants. The wall surface at x = 0 is maintained at a constant temperature of T_1, while the surface at x = L is maintained at T_2. Assuming steady one-dimensional heat transfer, obtain a relation for the heat transfer rate through the wall.

2–100 Consider a cylindrical shell of length L, inner radius r_1, and outer radius r_2 whose thermal conductivity varies linearly in a specified temperature range as $k(T) = k_0(1 + \beta T)$ where k_0 and β are two specified constants. The inner surface of the shell is maintained at a constant temperature of T_1, while the outer surface is maintained at T_2. Assuming steady one-dimensional heat transfer, obtain a relation for (*a*) the heat transfer rate through the wall and (*b*) the temperature distribution $T(r)$ in the shell.

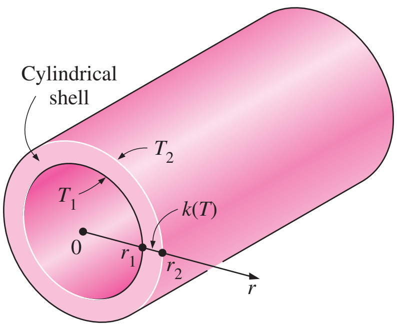


FIGURE P2–100

2–101 Consider a spherical shell of inner radius r_1 and outer radius r_2 whose thermal conductivity varies linearly in a specified temperature range as $k(T) = k_0(1 + \beta T)$ where k_0 and β are two specified constants. The inner surface of the shell is maintained at a constant temperature of T_1 while the outer surface is maintained at T_2. Assuming steady one-dimensional heat transfer, obtain a relation for (*a*) the heat transfer rate through the shell and (*b*) the temperature distribution $T(r)$ in the shell.

2–102 Consider a 1.5-m-high and 0.6-m-wide plate whose thickness is 0.15 m. One side of the plate is maintained at a constant temperature of 500 K while the other side is maintained at 350 K. The thermal conductivity of the plate can be assumed to vary linearly in that temperature range as $k(T) = k_0(1 + \beta T)$ where k_0 = 25 W/m · K and $\beta = 8.7 \times 10^{-4}$ K^{-1}. Disregarding the edge effects and assuming steady one-dimensional heat transfer, determine the rate of heat conduction through the plate. *Answer:* 30,800 W

2–103 Reconsider Problem 2–102. Using EES (or other) software, plot the rate of heat conduction through the plate as a function of the temperature of the hot side of the plate in the range of 400 K to 700 K. Discuss the results.

Special Topic: Review of Differential Equations

2–104C Why do we often utilize simplifying assumptions when we derive differential equations?

2–105C What is a variable? How do you distinguish a dependent variable from an independent one in a problem?

2–106C Can a differential equation involve more than one independent variable? Can it involve more than one dependent variable? Give examples.

2–107C What is the geometrical interpretation of a derivative? What is the difference between partial derivatives and ordinary derivatives?

2–108C What is the difference between the degree and the order of a derivative?

2–109C Consider a function $f(x, y)$ and its partial derivative $\partial f/\partial x$. Under what conditions will this partial derivative be equal to the ordinary derivative df/dx?

2–110C Consider a function $f(x)$ and its derivative df/dx. Does this derivative have to be a function of x?

2–111C How is integration related to derivation?

2–112C What is the difference between an algebraic equation and a differential equation?

2–113C What is the difference between an ordinary differential equation and a partial differential equation?

2–114C How is the order of a differential equation determined?

2–115C How do you distinguish a linear differential equation from a nonlinear one?

2–116C How do you recognize a linear homogeneous differential equation? Give an example and explain why it is linear and homogeneous.

2–117C How do differential equations with constant coefficients differ from those with variable coefficients? Give an example for each type.

2–118C What kind of differential equations can be solved by direct integration?

2–119C Consider a third order linear and homogeneous differential equation. How many arbitrary constants will its general solution involve?

Review Problems

2–120 Consider a small hot metal object of mass m and specific heat C that is initially at a temperature of T_i. Now the object is allowed to cool in an environment at T_∞ by convection with a heat transfer coefficient of h. The temperature of the metal object is observed to vary uniformly with time during cooling. Writing an energy balance on the entire metal object, derive the differential equation that describes the variation of temperature of the ball with time, $T(t)$. Assume constant thermal conductivity and no heat generation in the object. Do not solve.

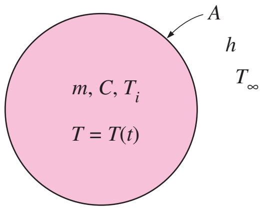


FIGURE P2–120

2–121 Consider a long rectangular bar of length a in the x-direction and width b in the y-direction that is initially at a uniform temperature of T_i. The surfaces of the bar at $x = 0$ and $y = 0$ are insulated, while heat is lost from the other two surfaces by convection to the surrounding medium at temperature T_∞ with a heat transfer coefficient of h. Assuming constant thermal conductivity and transient two-dimensional heat transfer with no heat generation, express the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem. Do not solve.

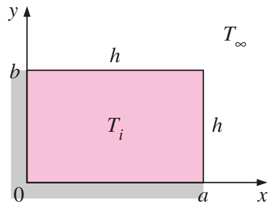


FIGURE P2–121

2–122 Consider a short cylinder of radius r_0 and height H in which heat is generated at a constant rate of $\dot{g}_0$. Heat is lost from the cylindrical surface at $r = r_0$ by convection to the surrounding medium at temperature T_∞ with a heat transfer coefficient of h. The bottom surface of the cylinder at $z = 0$ is insulated, while the top surface at $z = H$ is subjected to uniform heat flux $\dot{q}_h$. Assuming constant thermal conductivity and steady two-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem. Do not solve.

2–123E Consider a large plane wall of thickness $L = 0.5$ ft and thermal conductivity $k = 1.2$ Btu/h · ft · °F. The wall is covered with a material that has an emissivity of $\varepsilon = 0.80$ and a solar absorptivity of $\alpha = 0.45$. The inner surface of the wall is maintained at $T_1 = 520$ R at all times, while the outer surface is exposed to solar radiation that is incident at a rate of $\dot{q}_{solar} = 300$ Btu/h · ft^2. The outer surface is also losing heat by

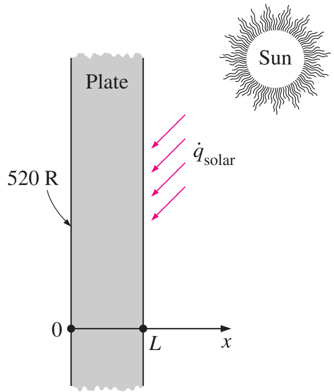


FIGURE P2–123E

radiation to deep space at 0 K. Determine the temperature of the outer surface of the wall and the rate of heat transfer through the wall when steady operating conditions are reached. *Answers:* 530.9 R, 26.2 Btu/h · ft²

2–124E Repeat Problem 2–123E for the case of no solar radiation incident on the surface.

2–125 Consider a steam pipe of length L, inner radius r_1, outer radius r_2, and constant thermal conductivity k. Steam flows inside the pipe at an average temperature of T_i with a convection heat transfer coefficient of h_i. The outer surface of the pipe is exposed to convection to the surrounding air at a temperature of T_0 with a heat transfer coefficient of h_o. Assuming steady one-dimensional heat conduction through the pipe, (*a*) express the differential equation and the boundary conditions for heat conduction through the pipe material, (*b*) obtain a relation for the variation of temperature in the pipe material by solving the differential equation, and (*c*) obtain a relation for the temperature of the outer surface of the pipe.

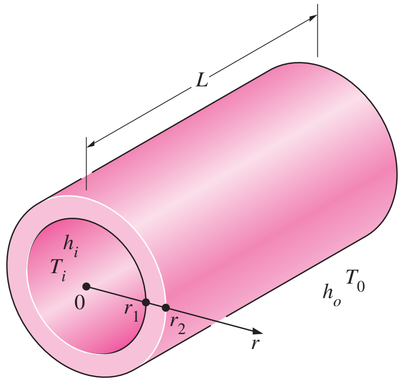


FIGURE P2–125

2–126 The boiling temperature of nitrogen at atmospheric pressure at sea level (1 atm pressure) is −196°C. Therefore, nitrogen is commonly used in low temperature scientific studies since the temperature of liquid nitrogen in a tank open to the atmosphere will remain constant at −196°C until the liquid nitrogen in the tank is depleted. Any heat transfer to the tank will result in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg and a density of 810 kg/m³ at 1 atm.

Consider a thick-walled spherical tank of inner radius r_1 = 2 m, outer radius r_2 = 2.1 m , and constant thermal conductivity k = 18 W/m · °C. The tank is initially filled with liquid nitrogen at 1 atm and −196°C, and is exposed to ambient air at T_∞ = 20°C with a heat transfer coefficient of h = 25 W/m² · °C. The inner surface temperature of the spherical tank is observed to be almost the same as the temperature of the nitrogen inside. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the tank, (*b*) obtain a relation for the variation of temperature in the tank material by solving the differential equation, and (*c*) determine the rate of evaporation of the liquid nitrogen in the tank as a result of the heat transfer from the ambient air. *Answer:* (*c*) 1.32 kg/s

2–127 Repeat Problem 2–126 for liquid oxygen, which has a boiling temperature of −183°C, a heat of vaporization of 213 kJ/kg, and a density of 1140 kg/m³ at 1 atm.

2–128 Consider a large plane wall of thickness L = 0.4 m and thermal conductivity k = 8.4 W/m · °C. There is no access to the inner side of the wall at x = 0 and thus the thermal conditions on that surface are not known. However, the outer surface of the wall at x = L, whose emissivity is ε = 0.7, is known to exchange heat by convection with ambient air at T_∞ = 25°C with an average heat transfer coefficient of h = 14 W/m² · °C as well as by radiation with the surrounding surfaces at an average temperature of T_{surr} = 290 K. Further, the temperature of the outer surface is measured to be T_2 = 45°C. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the plate, (*b*) obtain a relation for the temperature of the outer surface of the plate by solving the differential equation, and (*c*) evaluate the inner surface temperature of the wall at x = 0. *Answer:* (*c*) 64.3°C

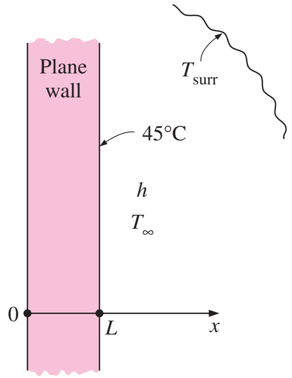


FIGURE P2–128

2–129 A 1000-W iron is left on the iron board with its base exposed to ambient air at 20°C. The base plate of the iron has a thickness of $L = 0.5$ cm, base area of $A = 150$ cm^2, and thermal conductivity of $k = 18$ W/m · °C. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside. The outer surface of the base plate whose emissivity is $\varepsilon = 0.7$, loses heat by convection to ambient air at $T_\infty = 22°$ C with an average heat transfer coefficient of $h = 30$ W/m^2 · °C as well as by radiation to the surrounding surfaces at an average temperature of $T_{surr} = 290$ K. Disregarding any heat loss through the upper part of the iron, (*a*) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the plate, (*b*) obtain a relation for the temperature of the outer surface of the plate by solving the differential equation, and (*c*) evaluate the outer surface temperature.

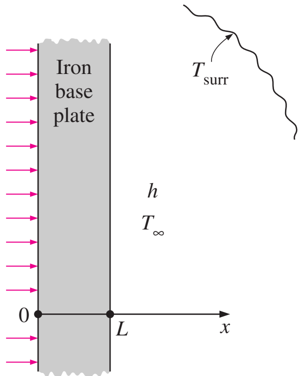


FIGURE P2–129

2–130 Repeat Problem 2–129 for a 1500-W iron.

2–131E The roof of a house consists of a 0.8-ft-thick concrete slab ($k = 1.1$ Btu/h · ft · °F) that is 25 ft wide and 35 ft long. The emissivity of the outer surface of the roof is 0.8, and the convection heat transfer coefficient on that surface is estimated to be 3.2 Btu/h · ft^2 · °F. On a clear winter night, the ambient air is reported to be at 50°F, while the night sky temperature for radiation heat transfer is 310 R. If the inner surface temperature of the roof is $T_1 = 62$°F, determine the outer surface temperature of the roof and the rate of heat loss through the roof when steady operating conditions are reached.

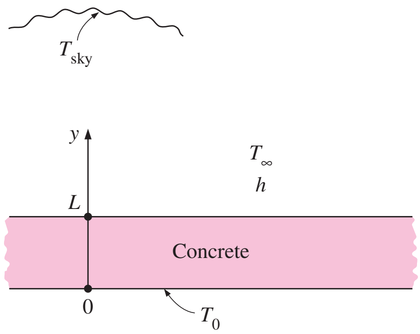


FIGURE P2–131E

2–132 Consider a long resistance wire of radius $r_1 = 0.3$ cm and thermal conductivity $k_{wire} = 18$ W/m · °C in which heat is generated uniformly at a constant rate of $\dot{g} = 1.5$ W/cm^3 as a result of resistance heating. The wire is embedded in a 0.4-cm-thick layer of plastic whose thermal conductivity is $k_{plastic} = 1.8$ W/m · °C. The outer surface of the plastic cover loses heat by convection to the ambient air at $T_\infty = 25$°C with an average combined heat transfer coefficient of $h = 14$ W/m^2 · °C. Assuming one-dimensional heat transfer, determine the temperatures at the center of the resistance wire and the wire-plastic layer interface under steady conditions.
Answers: 97.1°C, 97.3°C

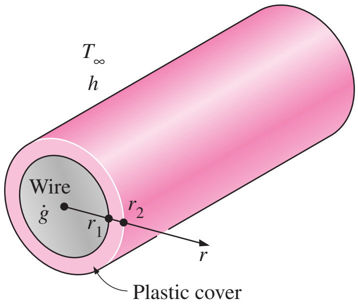


FIGURE P2–132

2–133 Consider a cylindrical shell of length L, inner radius r_1, and outer radius r_2 whose thermal conductivity varies in a specified temperature range as $k(T) = k_0(1 + \beta T^2)$ where k_0 and β are two specified constants. The inner surface of the shell is maintained at a constant temperature of T_1 while the outer surface is maintained at T_2. Assuming steady one-dimensional heat transfer, obtain a relation for the heat transfer rate through the shell.

2–134 In a nuclear reactor, heat is generated in 1-cm-diameter cylindrical uranium fuel rods at a rate of 4×10^7 W/m^3. Determine the temperature difference between the center and the surface of the fuel rod. *Answer:* 9.0°C

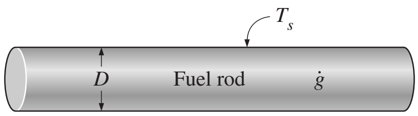


FIGURE P2–134

2–135 Consider a 20-cm-thick large concrete plane wall ($k = 0.77$ W/m · °C) subjected to convection on both sides with $T_{\infty 1} = 27$°C and $h_1 = 5$ W/m^2 · °C on the inside, and $T_{\infty 2} = 8$°C and $h_2 = 12$ W/m^2 · °C on the outside. Assuming constant thermal conductivity with no heat generation and negligible radiation, (*a*) express the differential equations and the boundary conditions for steady one-dimensional heat conduction through the wall, (*b*) obtain a relation for the variation of temperature in the wall by solving the differential equation, and (*c*) evaluate the temperatures at the inner and outer surfaces of the wall.

2–136 Consider a water pipe of length $L = 12$ m, inner radius $r_1 = 15$ cm, outer radius $r_2 = 20$ cm, and thermal conductivity $k = 20$ W/m · °C. Heat is generated in the pipe material uniformly by a 25-kW electric resistance heater. The inner and outer surfaces of the pipe are at $T_1 = 60$°C and $T_2 = 80$°C, respectively. Obtain a general relation for temperature distribution inside the pipe under steady conditions and determine the temperature at the center plane of the pipe.

2–137 Heat is generated uniformly at a rate of 2.6×10^6 W/m^3 in a spherical ball ($k = 45$ W/m · °C) of diameter 30 cm. The ball is exposed to iced-water at 0°C with a heat transfer coefficient of 1200 W/m^2 · °C. Determine the temperatures at the center and the surface of the ball.

Computer, Design, and Essay Problems

2–138 Write an essay on heat generation in nuclear fuel rods. Obtain information on the ranges of heat generation, the variation of heat generation with position in the rods, and the absorption of emitted radiation by the cooling medium.

2–139 Write an interactive computer program to calculate the heat transfer rate and the value of temperature anywhere in the medium for steady one-dimensional heat conduction in a long cylindrical shell for any combination of specified temperature, specified heat flux, and convection boundary conditions. Run the program for five different sets of specified boundary conditions.

2–140 Write an interactive computer program to calculate the heat transfer rate and the value of temperature anywhere in the medium for steady one-dimensional heat conduction in a spherical shell for any combination of specified temperature, specified heat flux, and convection boundary conditions. Run the program for five different sets of specified boundary conditions.

2–141 Write an interactive computer program to calculate the heat transfer rate and the value of temperature anywhere in the medium for steady one-dimensional heat conduction in a plane wall whose thermal conductivity varies linearly as $k(T) = k_0(1 + \beta T)$ where the constants k_0 and β are specified by the user for specified temperature boundary conditions.

CHAPTER

3

STEADY HEAT CONDUCTION

In heat transfer analysis, we are often interested in the rate of heat transfer through a medium under steady conditions and surface temperatures. Such problems can be solved easily without involving any differential equations by the introduction of *thermal resistance concepts* in an analogous manner to electrical circuit problems. In this case, the thermal resistance corresponds to electrical resistance, temperature difference corresponds to voltage, and the heat transfer rate corresponds to electric current.

We start this chapter with *one-dimensional steady heat conduction* in a plane wall, a cylinder, and a sphere, and develop relations for *thermal resistances* in these geometries. We also develop thermal resistance relations for convection and radiation conditions at the boundaries. We apply this concept to heat conduction problems in *multilayer* plane walls, cylinders, and spheres and generalize it to systems that involve heat transfer in two or three dimensions. We also discuss the *thermal contact resistance* and the *overall heat transfer coefficient* and develop relations for the critical radius of insulation for a cylinder and a sphere. Finally, we discuss steady heat transfer from *finned surfaces* and some complex geometrics commonly encountered in practice through the use of *conduction shape factors.*

CONTENTS

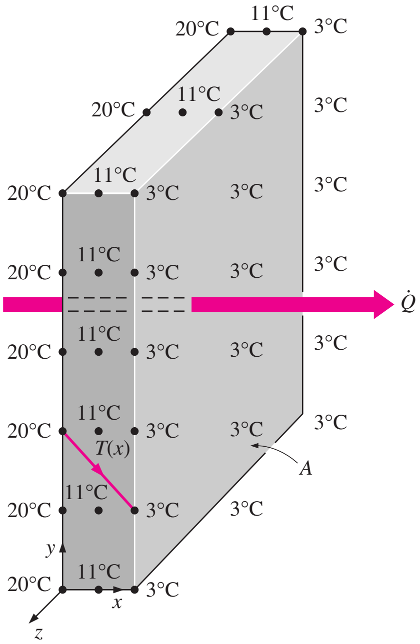


FIGURE 3–1
Heat flow through a wall is one-dimensional when the temperature of the wall varies in one direction only.

3–1 ▪ STEADY HEAT CONDUCTION IN PLANE WALLS

Consider steady heat conduction through the walls of a house during a winter day. We know that heat is continuously lost to the outdoors through the wall. We intuitively feel that heat transfer through the wall is in the *normal direction* to the wall surface, and no significant heat transfer takes place in the wall in other directions (Fig. 3–1).

Recall that heat transfer in a certain direction is driven by the *temperature gradient* in that direction. There will be no heat transfer in a direction in which there is no change in temperature. Temperature measurements at several locations on the inner or outer wall surface will confirm that a wall surface is nearly *isothermal.* That is, the temperatures at the top and bottom of a wall surface as well as at the right or left ends are almost the same. Therefore, there will be no heat transfer through the wall from the top to the bottom, or from left to right, but there will be considerable temperature difference between the inner and the outer surfaces of the wall, and thus significant heat transfer in the direction from the inner surface to the outer one.

The small thickness of the wall causes the temperature gradient in that direction to be large. Further, if the air temperatures in and outside the house remain constant, then heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional.* The temperature of the wall in this case will depend on one direction only (say the x-direction) and can be expressed as $T(x)$.

Noting that heat transfer is the only energy interaction involved in this case and there is no heat generation, the *energy balance* for the wall can be expressed as

$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

or

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt} \tag{3-1}$$

But $dE_{\text{wall}}/dt = 0$ for *steady* operation, since there is no change in the temperature of the wall with time at any point. Therefore, the rate of heat transfer into the wall must be equal to the rate of heat transfer out of it. In other words, *the rate of heat transfer through the wall must be constant,* $\dot{Q}_{\text{cond, wall}} =$ constant.

Consider a plane wall of thickness L and average thermal conductivity k. The two surfaces of the wall are maintained at constant temperatures of T_1 and T_2. For one-dimensional steady heat conduction through the wall, we have $T(x)$. Then Fourier's law of heat conduction for the wall can be expressed as

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx} \qquad \text{(W)} \tag{3-2}$$

where the rate of conduction heat transfer $\dot{Q}_{\text{cond wall}}$ and the wall area A are constant. Thus we have $dT/dx =$ constant, which means that *the temperature* *through the wall varies linearly with x.* That is, the temperature distribution in the wall under steady conditions is a *straight line* (Fig. 3–2).

Separating the variables in the above equation and integrating from $x = 0$, where $T(0) = T_1$, to $x = L$, where $T(L) = T_2$, we get

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)} \tag{3-3}$$

which is identical to Eq. 3–1. Again, *the rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.* Also, once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 in Eq. 3–3 by T, and L by x.

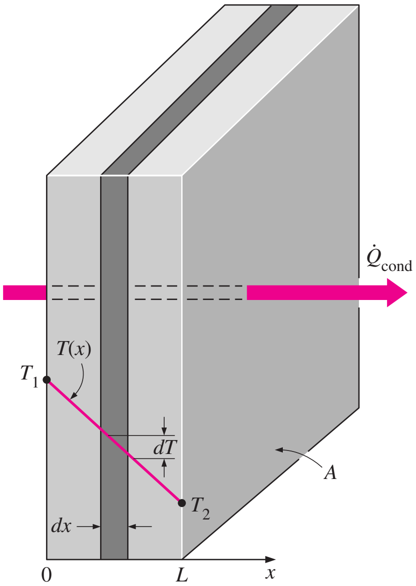


FIGURE 3–2
Under steady conditions, the temperature distribution in a plane wall is a straight line.

The Thermal Resistance Concept

Equation 3–3 for heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)} \tag{3-4}$$

where

$$R_{\text{wall}} = \frac{L}{kA} \qquad (\text{°C/W}) \tag{3-5}$$

is the *thermal resistance* of the wall against heat conduction or simply the **conduction resistance** of the wall. Note that the thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

The equation above for heat flow is analogous to the relation for *electric current flow I,* expressed as

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e} \tag{3-6}$$

where $R_e = L/\sigma_e A$ is the *electric resistance* and $\mathbf{V}_1 - \mathbf{V}_2$ is the *voltage difference* across the resistance (σ_e is the electrical conductivity). Thus, the *rate of heat transfer* through a layer corresponds to the *electric current,* the *thermal resistance* corresponds to *electrical resistance,* and the *temperature difference* corresponds to *voltage difference* across the layer (Fig. 3–3).

Consider convection heat transfer from a solid surface of area A_s and temperature T_s to a fluid whose temperature sufficiently far from the surface is T_∞, with a convection heat transfer coefficient h. Newton's law of cooling for convection heat transfer rate $\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$ can be rearranged as

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \qquad \text{(W)} \tag{3-7}$$

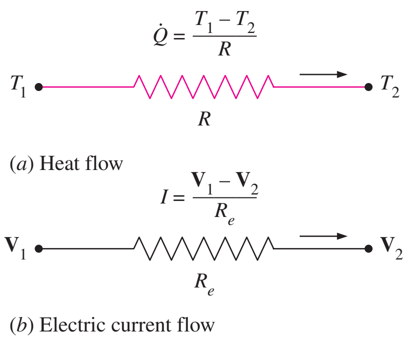


FIGURE 3–3
Analogy between thermal and electrical resistance concepts.

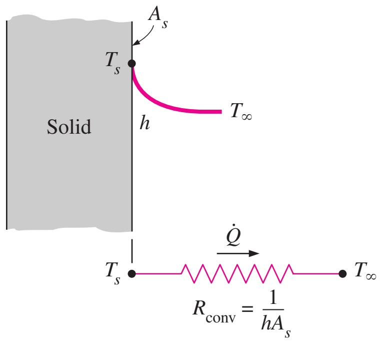


FIGURE 3–4
Schematic for convection resistance at a surface.

where

$$R_{\text{conv}} = \frac{1}{hA_s} \qquad (°\text{C/W}) \tag{3-8}$$

is the *thermal resistance* of the surface against heat convection, or simply the **convection resistance** of the surface (Fig. 3–4). Note that when the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes *zero* and $T_s \approx T_\infty$. That is, the surface offers *no resistance to convection,* and thus it does not slow down the heat transfer process. This situation is approached in practice at surfaces where boiling and condensation occur. Also note that the surface does not have to be a plane surface. Equation 3–8 for convection resistance is valid for surfaces of any shape, provided that the assumption of $h =$ constant and uniform is reasonable.

When the wall is surrounded by a gas, the *radiation effects,* which we have ignored so far, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity ε and area A_s at temperature T_s and the surrounding surfaces at some average temperature T_{surr} can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s (T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}} A_s (T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}} \qquad (\text{W}) \tag{3-9}$$

where

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_s} \qquad (\text{K/W}) \tag{3-10}$$

is the *thermal resistance* of a surface against radiation, or the *radiation resistance,* and

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s(T_s - T_{\text{surr}})} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \qquad (\text{W/m}^2 \cdot \text{K}) \tag{3-11}$$

is the **radiation heat transfer coefficient.** Note that both T_s and T_{surr} *must* be in K in the evaluation of h_{rad}. The definition of the radiation heat transfer coefficient enables us to express radiation conveniently in an analogous manner to convection in terms of a temperature difference. But h_{rad} depends strongly on temperature while h_{conv} usually does not.

A surface exposed to the surrounding air involves convection and radiation simultaneously, and the total heat transfer at the surface is determined by adding (or subtracting, if in the opposite direction) the radiation and convection components. The convection and radiation resistances are parallel to each other, as shown in Fig. 3–5, and may cause some complication in the thermal resistance network. When $T_{\text{surr}} \approx T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation by

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} \qquad (\text{W/m}^2 \cdot \text{K}) \tag{3-12}$$

where h_{combined} is the **combined heat transfer coefficient.** This way all the complications associated with radiation are avoided.

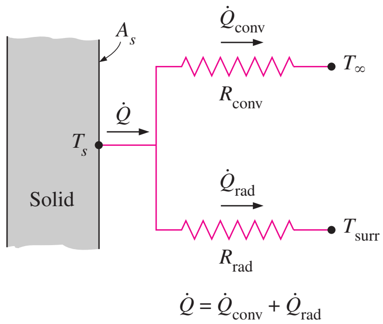


FIGURE 3–5
Schematic for convection and radiation resistances at a surface.

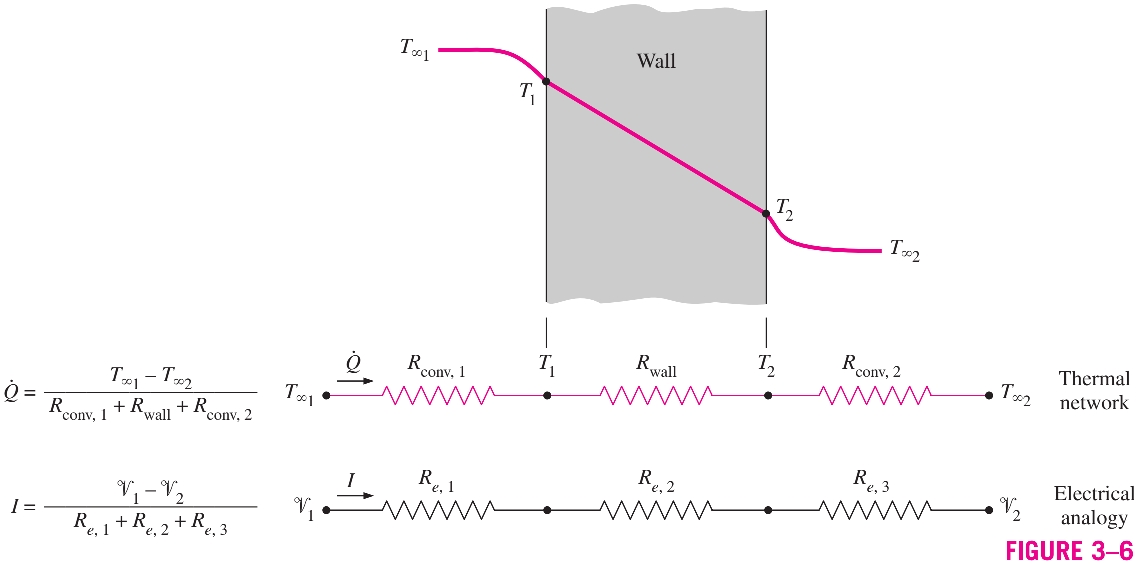


FIGURE 3–6
The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

Thermal Resistance Network

Now consider steady one-dimensional heat flow through a plane wall of thickness L, area A, and thermal conductivity k that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively, as shown in Fig. 3–6. Assuming $T_{\infty 2} < T_{\infty 1}$, the variation of temperature will be as shown in the figure. Note that the temperature varies linearly in the wall, and asymptotically approaches $T_{\infty 1}$ and $T_{\infty 2}$ in the fluids as we move away from the wall.

Under steady conditions we have

$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2}) \qquad \textbf{(3-13)}$$

which can be rearranged as

$$\begin{aligned}\dot{Q} &= \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}\\ &= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}\end{aligned} \qquad \textbf{(3-14)}$$

Adding the numerators and denominators yields (Fig. 3–7)

$$\dot{Q} = \frac{T_{\infty} - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)} \qquad \textbf{(3-15)}$$

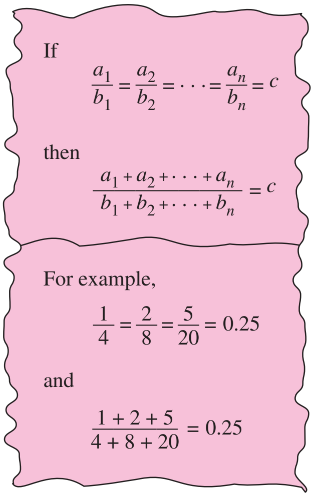


FIGURE 3–7
A useful mathematical identity.

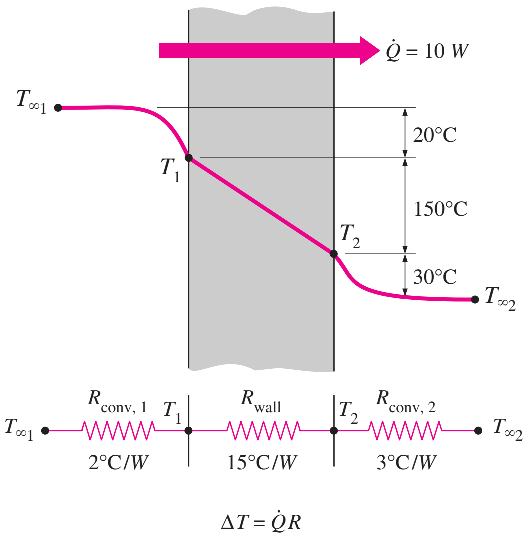


FIGURE 3–8
The temperature drop across a layer is proportional to its thermal resistance.

where

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°\text{C/W}) \tag{3-16}$$

Note that the heat transfer area A is constant for a plane wall, and the rate of heat transfer through a wall separating two mediums is equal to the temperature difference divided by the total thermal resistance between the mediums. Also note that the thermal resistances are in *series,* and the equivalent thermal resistance is determined by simply *adding* the individual resistances, just like the electrical resistances connected in series. Thus, the electrical analogy still applies. We summarize this as *the rate of steady heat transfer between two surfaces is equal to the temperature difference divided by the total thermal resistance between those two surfaces.*

Another observation that can be made from Eq. 3–15 is that the ratio of the temperature drop to the thermal resistance across any layer is constant, and thus the temperature drop across any layer is proportional to the thermal resistance of the layer. The larger the resistance, the larger the temperature drop. In fact, the equation $\dot{Q} = \Delta T/R$ can be rearranged as

$$\Delta T = \dot{Q}R \qquad (°\text{C}) \tag{3-17}$$

which indicates that the *temperature drop* across any layer is equal to the *rate of heat transfer* times the *thermal resistance* across that layer (Fig. 3–8). You may recall that this is also true for voltage drop across an electrical resistance when the electric current is constant.

It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA\,\Delta T \qquad (\text{W}) \tag{3-18}$$

where U is the **overall heat transfer coefficient.** A comparison of Eqs. 3–15 and 3–18 reveals that

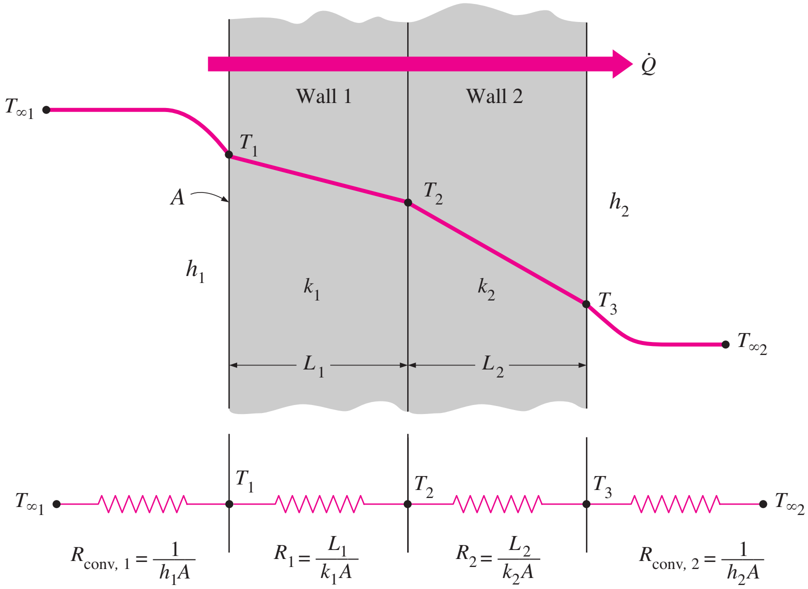


FIGURE 3–9
The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

$$UA = \frac{1}{R_{\text{total}}} \tag{3-19}$$

Therefore, for a unit area, the overall heat transfer coefficient is equal to the inverse of the total thermal resistance.

Note that we do not need to know the surface temperatures of the wall in order to evaluate the rate of steady heat transfer through it. All we need to know is the convection heat transfer coefficients and the fluid temperatures on both sides of the wall. The *surface temperature* of the wall can be determined as described above using the thermal resistance concept, but by taking the surface at which the temperature is to be determined as one of the terminal surfaces. For example, once $\dot{Q}$ is evaluated, the surface temperature T_1 can be determined from

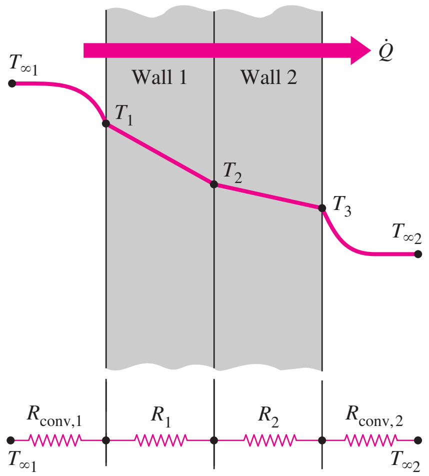
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_{\infty 1} - T_1}{1/h_1 A} \tag{3-20}$$

Multilayer Plane Walls

In practice we often encounter plane walls that consist of several layers of different materials. The thermal resistance concept can still be used to determine the rate of steady heat transfer through such *composite* walls. As you may have already guessed, this is done by simply noting that the conduction resistance of each wall is L/kA connected in series, and using the electrical analogy. That is, by dividing the *temperature difference* between two surfaces at known temperatures by the *total thermal resistance* between them.

Consider a plane wall that consists of two layers (such as a brick wall with a layer of insulation). The rate of steady heat transfer through this two-layer composite wall can be expressed as (Fig. 3–9)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \tag{3-21}$$



To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{conv,1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{conv,1} + R_1}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{conv,2}}$

FIGURE 3–10
The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

where R_{total} is the *total thermal resistance,* expressed as

$$R_{total} = R_{conv,\,1} + R_{wall,\,1} + R_{wall,\,2} + R_{conv,\,2}$$
$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \tag{3-22}$$

The subscripts 1 and 2 in the R_{wall} relations above indicate the first and the second layers, respectively. We could also obtain this result by following the approach used above for the single-layer case by noting that the rate of steady heat transfer $\dot{Q}$ through a multilayer medium is constant, and thus it must be the same through each layer. Note from the thermal resistance network that the resistances are *in series,* and thus the *total thermal resistance* is simply the *arithmetic sum* of the individual thermal resistances in the path of heat flow.

This result for the *two-layer* case is analogous to the *single-layer* case, except that an *additional resistance* is added for the *additional layer.* This result can be extended to plane walls that consist of *three* or *more layers* by adding an *additional resistance* for each *additional layer.*

Once $\dot{Q}$ is *known,* an unknown surface temperature T_j at any surface or interface j can be determined from

$$\dot{Q} = \frac{T_i - T_j}{R_{total,\,i-j}} \tag{3-23}$$

where T_i is a *known* temperature at location i and $R_{total,\,i-j}$ is the total thermal resistance between locations i and j. For example, when the fluid temperatures $T_{\infty 1}$ and $T_{\infty 2}$ for the two-layer case shown in Fig. 3–9 are available and $\dot{Q}$ is calculated from Eq. 3–21, the interface temperature T_2 between the two walls can be determined from (Fig. 3–10)

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,\,1} + R_{wall,\,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}} \tag{3-24}$$

The temperature drop across a layer is easily determined from Eq. 3–17 by multiplying $\dot{Q}$ by the thermal resistance of that layer.

The thermal resistance concept is widely used in practice because it is intuitively easy to understand and it has proven to be a powerful tool in the solution of a wide range of heat transfer problems. But its use is limited to systems through which the rate of heat transfer $\dot{Q}$ remains *constant*; that is, to systems involving *steady* heat transfer with *no heat generation* (such as resistance heating or chemical reactions) within the medium.

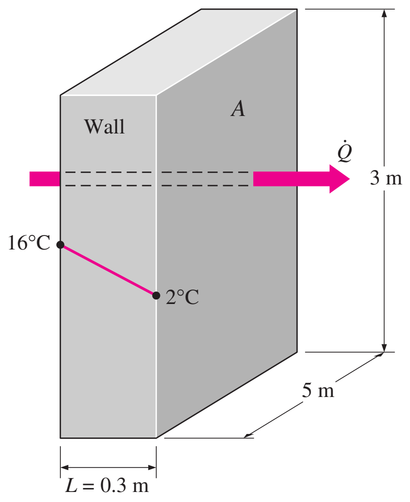


FIGURE 3–11
Schematic for Example 3–1.

EXAMPLE 3–1 **Heat Loss through a Wall**

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is $k = 0.9$ W/m · °C (Fig. 3–11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.

SOLUTION The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.9$ W/m · °C.

Analysis Noting that the heat transfer through the wall is by conduction and the area of the wall is $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$, the steady rate of heat transfer through the wall can be determined from Eq. 3–3 to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9 \text{ W/m} \cdot {}^\circ\text{C})(15 \text{ m}^2)\frac{(16 - 2)^\circ\text{C}}{0.3 \text{ m}} = \mathbf{630 \text{ W}}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3 \text{ m}}{(0.9 \text{ W/m} \cdot {}^\circ\text{C})(15 \text{ m}^2)} = 0.02222^\circ\text{C/W}$$

Substituting, we get

$$\dot{Q} = \frac{(16 - 2)^\circ\text{C}}{0.02222^\circ\text{C/W}} = 630 \text{ W}$$

Discussion This is the same result obtained earlier. Note that heat conduction through a plane wall with specified surface temperatures can be determined directly and easily without utilizing the thermal resistance concept. However, the thermal resistance concept serves as a valuable tool in more complex heat transfer problems, as you will see in the following examples.

EXAMPLE 3–2 Heat Loss through a Single-Pane Window

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 40$ W/m^2 · °C, which includes the effects of radiation.

SOLUTION Heat loss through a window glass is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

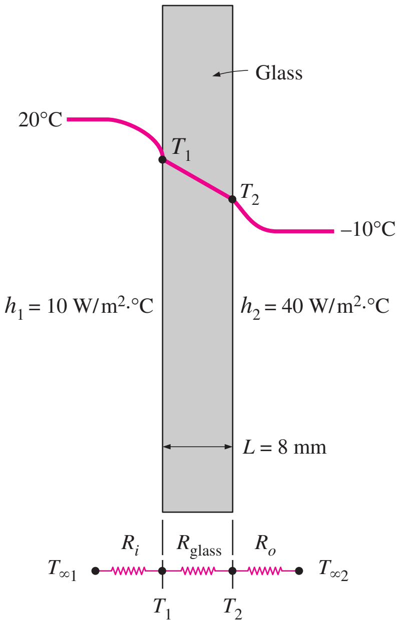


FIGURE 3–12
Schematic for Example 3–2.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer through the wall is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.78$ W/m · °C.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3–12. Noting that the area of the window is $A = 0.8\text{ m} \times 1.5\text{ m} = 1.2\text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2 \cdot {}^\circ\text{C})(1.2\text{ m}^2)} = 0.08333{}^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008\text{ m}}{(0.78\text{ W/m} \cdot {}^\circ\text{C})(1.2\text{ m}^2)} = 0.00855{}^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40\text{ W/m}^2 \cdot {}^\circ\text{C})(1.2\text{ m}^2)} = 0.02083{}^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass}} + R_{\text{conv, 2}} = 0.08333 + 0.00855 + 0.02083 \\ &= 0.1127{}^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]{}^\circ\text{C}}{0.1127{}^\circ\text{C/W}} = \mathbf{266\ W}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} \longrightarrow \begin{aligned} T_1 &= T_{\infty 1} - \dot{Q}R_{\text{conv, 1}} \\ &= 20{}^\circ\text{C} - (266\text{ W})(0.08333{}^\circ\text{C/W}) \\ &= \mathbf{-2.2{}^\circ C} \end{aligned}$$

Discussion Note that the inner surface temperature of the window glass will be −2.2°C even though the temperature of the air in the room is maintained at 20°C. Such low surface temperatures are highly undesirable since they cause the formation of fog or even frost on the inner surfaces of the glass when the humidity in the room is high.

EXAMPLE 3–3 Heat Loss through Double-Pane Windows

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass (k = 0.78 W/m · °C) separated by a 10-mm-wide stagnant air space (k = 0.026 W/m · °C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m$^2 \cdot$ °C and $h_2 = 40$ W/m$^2 \cdot$ °C, which includes the effects of radiation.

SOLUTION A double-pane window is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Analysis This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant air space. Therefore, the thermal resistance network of this problem will involve two additional conduction resistances corresponding to the two additional layers, as shown in Fig. 3–13. Noting that the area of the window is again $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot °\text{C})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m} \cdot °\text{C})(1.2 \text{ m}^2)} = 0.00427°\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m} \cdot °\text{C})(1.2 \text{ m}^2)} = 0.3205°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot °\text{C})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass, 1}} + R_{\text{air}} + R_{\text{glass, 2}} + R_{\text{conv, 2}} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332°\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{0.4332°\text{C/W}} = \mathbf{69.2 \text{ W}}$$

which is about one-fourth of the result obtained in the previous example. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q} R_{\text{conv, 1}} = 20°\text{C} - (69.2 \text{ W})(0.08333°\text{C/W}) = \mathbf{14.2°C}$$

which is considerably higher than the −2.2°C obtained in the previous example. Therefore, a double-pane window will rarely get fogged. A double-pane window will also reduce the heat gain in summer, and thus reduce the air-conditioning costs.

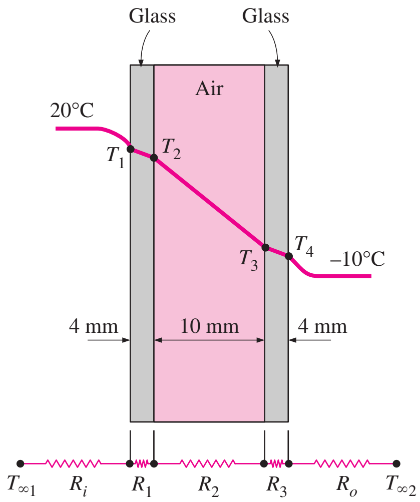
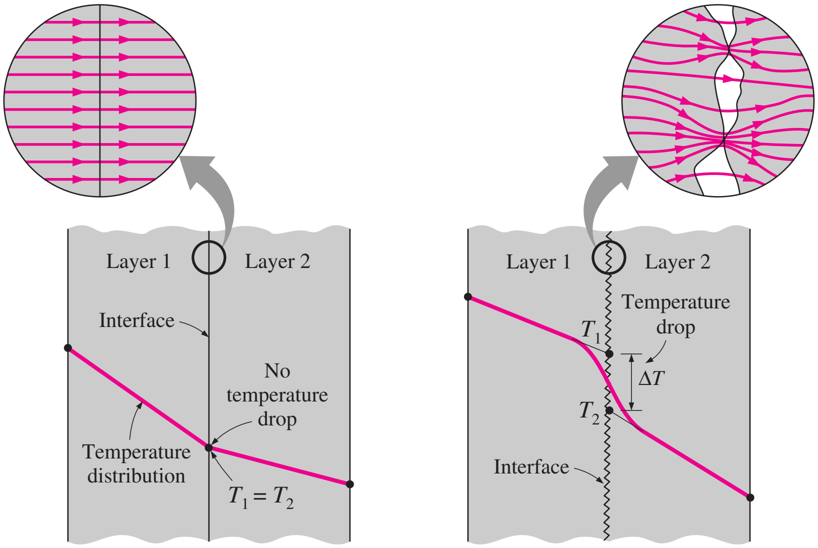


FIGURE 3–13
Schematic for Example 3–3.



(*a*) Ideal (perfect) thermal contact

(*b*) Actual (imperfect) thermal contact

FIGURE 3–14
Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

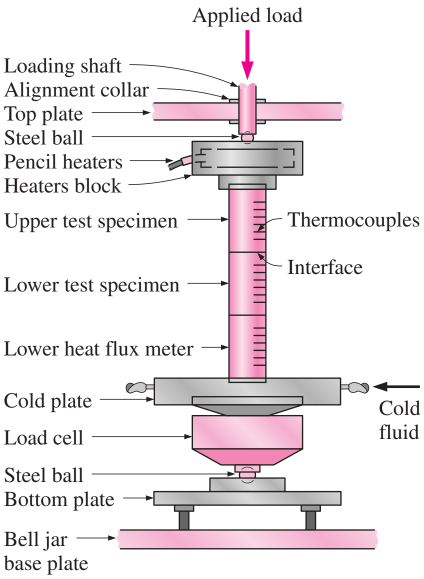


FIGURE 3–15
A typical experimental setup for the determination of thermal contact resistance (from Song et al., Ref. 11).

3–2 ▪ THERMAL CONTACT RESISTANCE

In the analysis of heat conduction through multilayer solids, we assumed "perfect contact" at the interface of two layers, and thus no temperature drop at the interface. This would be the case when the surfaces are perfectly smooth and they produce a perfect contact at each point. In reality, however, even flat surfaces that appear smooth to the eye turn out to be rather rough when examined under a microscope, as shown in Fig. 3–14, with numerous peaks and valleys. That is, a surface is *microscopically rough* no matter how smooth it appears to be.

When two such surfaces are pressed against each other, the peaks will form good material contact but the valleys will form voids filled with air. As a result, an interface will contain numerous *air gaps* of varying sizes that act as *insulation* because of the low thermal conductivity of air. Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the **thermal contact resistance,** R_c. The value of R_c is determined experimentally using a setup like the one shown in Fig. 3–15, and as expected, there is considerable scatter of data because of the difficulty in characterizing the surfaces.

Consider heat transfer through two metal rods of cross-sectional area A that are pressed against each other. Heat transfer through the interface of these two rods is the sum of the heat transfers through the *solid contact spots* and the *gaps* in the noncontact areas and can be expressed as

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}} \tag{3-25}$$

It can also be expressed in an analogous manner to Newton's law of cooling as

$$\dot{Q} = h_c A\, \Delta T_{\text{interface}} \tag{3-26}$$

where A is the apparent interface area (which is the same as the cross-sectional area of the rods) and $\Delta T_{\text{interface}}$ is the effective temperature difference at the interface. The quantity h_c, which corresponds to the convection heat transfer coefficient, is called the **thermal contact conductance** and is expressed as

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C}) \tag{3-27}$$

It is related to thermal contact resistance by

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \qquad (\text{m}^2 \cdot {}^\circ\text{C/W}) \tag{3-28}$$

That is, thermal contact resistance is the inverse of thermal contact conductance. Usually, thermal contact conductance is reported in the literature, but the concept of thermal contact resistance serves as a better vehicle for explaining the effect of interface on heat transfer. Note that R_c represents thermal contact resistance *per unit area.* The thermal resistance for the entire interface is obtained by dividing R_c by the apparent interface area A.

The thermal contact resistance can be determined from Eq. 3–28 by measuring the temperature drop at the interface and dividing it by the heat flux under steady conditions. The value of thermal contact resistance depends on the *surface roughness* and the *material properties* as well as the *temperature* and *pressure* at the interface and the *type of fluid* trapped at the interface. The situation becomes more complex when plates are fastened by bolts, screws, or rivets since the interface pressure in this case is nonuniform. The thermal contact resistance in that case also depends on the plate thickness, the bolt radius, and the size of the contact zone. Thermal contact resistance is observed to *decrease* with *decreasing surface roughness* and *increasing interface pressure,* as expected. Most experimentally determined values of the thermal contact resistance fall between 0.000005 and 0.0005 $\text{m}^2 \cdot {}^\circ\text{C/W}$ (the corresponding range of thermal contact conductance is 2000 to 200,000 $\text{W/m}^2 \cdot {}^\circ\text{C}$).

When we analyze heat transfer in a medium consisting of two or more layers, the first thing we need to know is whether the thermal contact resistance is *significant* or not. We can answer this question by comparing the magnitudes of the thermal resistances of the layers with typical values of thermal contact resistance. For example, the thermal resistance of a 1-cm-thick layer of an insulating material per unit surface area is

$$R_{c,\text{ insulation}} = \frac{L}{k} = \frac{0.01\text{ m}}{0.04\text{ W/m} \cdot {}^\circ\text{C}} = 0.25\text{ m}^2 \cdot {}^\circ\text{C/W}$$

whereas for a 1-cm-thick layer of copper, it is

$$R_{c,\text{ copper}} = \frac{L}{k} = \frac{0.01\text{ m}}{386\text{ W/m} \cdot {}^\circ\text{C}} = 0.000026\text{ m}^2 \cdot {}^\circ\text{C/W}$$

Comparing the values above with typical values of thermal contact resistance, we conclude that thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but can be

TABLE 3–1

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm (from Fried, Ref. 5)

Fluid at the Interface	Contact Conductance, h_c, W/m² · °C
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

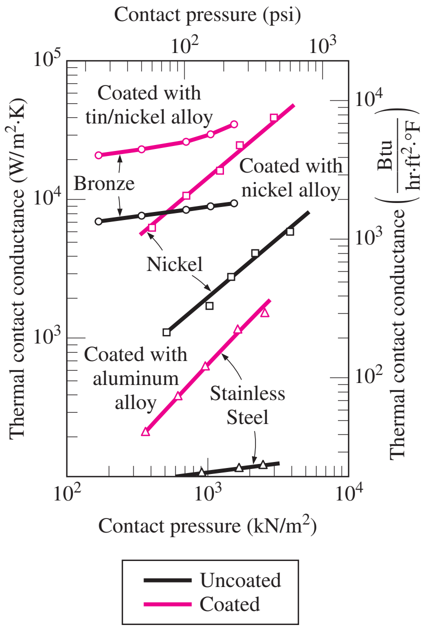


FIGURE 3–16
Effect of metallic coatings on thermal contact conductance (from Peterson, Ref. 10).

disregarded for poor heat conductors such as insulations. This is not surprising since insulating materials consist mostly of air space just like the interface itself.

The thermal contact resistance can be minimized by applying a thermally conducting liquid called a *thermal grease* such as silicon oil on the surfaces before they are pressed against each other. This is commonly done when attaching electronic components such as power transistors to heat sinks. The thermal contact resistance can also be reduced by replacing the air at the interface by a *better conducting gas* such as helium or hydrogen, as shown in Table 3–1.

Another way to minimize the contact resistance is to insert a *soft metallic foil* such as tin, silver, copper, nickel, or aluminum between the two surfaces. Experimental studies show that the thermal contact resistance can be reduced by a factor of up to 7 by a metallic foil at the interface. For maximum effectiveness, the foils must be very thin. The effect of metallic coatings on thermal contact conductance is shown in Fig. 3–16 for various metal surfaces.

There is considerable uncertainty in the contact conductance data reported in the literature, and care should be exercised when using them. In Table 3–2 some experimental results are given for the contact conductance between similar and dissimilar metal surfaces for use in preliminary design calculations. Note that the *thermal contact conductance* is *highest* (and thus the contact resistance is lowest) for *soft metals* with *smooth surfaces* at *high pressure.*

EXAMPLE 3–4 Equivalent Thickness for Contact Resistance

The thermal contact conductance at the interface of two 1-cm-thick aluminum plates is measured to be 11,000 W/m² · °C. Determine the thickness of the aluminum plate whose thermal resistance is equal to the thermal resistance of the interface between the plates (Fig. 3–17).

SOLUTION The thickness of the aluminum plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Properties The thermal conductivity of aluminum at room temperature is $k = 237$ W/m · °C (Table A-3).

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is

$$R_c = \frac{1}{h_c} = \frac{1}{11{,}000\ \text{W/m}^2 \cdot {}^\circ\text{C}} = 0.909 \times 10^{-4}\ \text{m}^2 \cdot {}^\circ\text{C/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as

$$R = \frac{L}{k}$$

where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR_c = (237\ \text{W/m} \cdot {}^\circ\text{C})(0.909 \times 10^{-4}\ \text{m}^2 \cdot {}^\circ\text{C/W}) = 0.0215\ \text{m} = \mathbf{2.15\ cm}$$

TABLE 3–2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface Condition	Roughness, μm	Temperature, °C	Pressure, MPa	h_c,* W/m^2 · °C
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.3–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.7–7	11,400
Dissimilar Metal Pairs					
Stainless steel–				10	2900
Aluminum		20–30	20	20	3600
Stainless steel–				10	16,400
Aluminum		1.0–2.0	20	20	20,800
Steel Ct-30–				10	50,000
Aluminum	Ground	1.4–2.0	20	15–35	59,000
Steel Ct-30–				10	4800
Aluminum	Milled	4.5–7.2	20	30	8300
				5	42,000
Aluminum-Copper	Ground	1.3–1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4–4.5	20	20–35	22,000

*Divide the given values by 5.678 to convert to Btu/h · ft^2 · °F.

Discussion Note that the interface between the two plates offers as much resistance to heat transfer as a 2.3-cm-thick aluminum plate. It is interesting that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

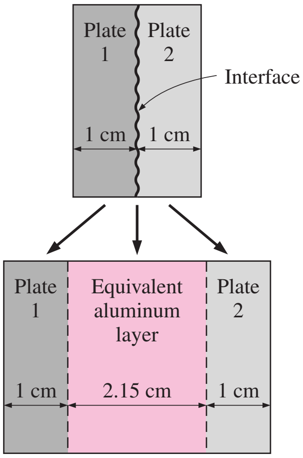


FIGURE 3–17
Schematic for Example 3–4.

EXAMPLE 3–5 Contact Resistance of Transistors

Four identical power transistors with aluminum casing are attached on one side of a 1-cm-thick 20-cm × 20-cm square copper plate (k = 386 W/m · °C) by screws that exert an average pressure of 6 MPa (Fig. 3–18). The base area of each transistor is 8 cm^2, and each transistor is placed at the center of a 10-cm × 10-cm quarter section of the plate. The interface roughness is estimated to be about 1.5 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 20°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 25 W/m^2 · °C. If the case temperature of

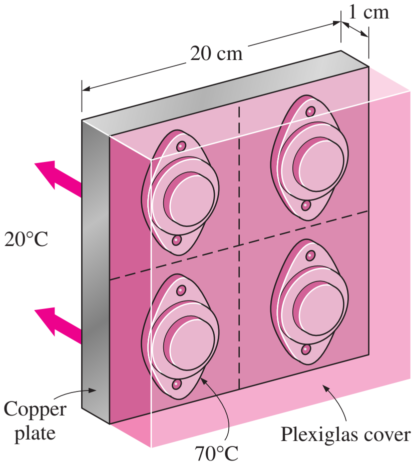


FIGURE 3–18
Schematic for Example 3–5.

the transistor is not to exceed 70°C, determine the maximum power each transistor can dissipate safely, and the temperature jump at the case-plate interface.

SOLUTION Four identical power transistors are attached on a copper plate. For a maximum case temperature of 70°C, the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. **3** All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick Plexiglas layer. **4** Thermal conductivities are constant.

Properties The thermal conductivity of copper is given to be $k = 386$ W/m · °C. The contact conductance is obtained from Table 3–2 to be $h_c = 42{,}000$ W/m^2 · °C, which corresponds to copper-aluminum interface for the case of 1.3–1.4 μm roughness and 5 MPa pressure, which is sufficiently close to what we have.

Analysis The contact area between the case and the plate is given to be 8 cm^2, and the plate area for each transistor is 100 cm^2. The thermal resistance network of this problem consists of three resistances in series (interface, plate, and convection), which are determined to be

$$R_{\text{interface}} = \frac{1}{h_c A_c} = \frac{1}{(42{,}000\ \text{W/m}^2\cdot{}^\circ\text{C})(8 \times 10^{-4}\ \text{m}^2)} = 0.030^\circ\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(386\ \text{W/m}\cdot{}^\circ\text{C})(0.01\ \text{m}^2)} = 0.0026^\circ\text{C/W}$$

$$R_{\text{conv}} = \frac{1}{h_o A} = \frac{1}{(25\ \text{W/m}^2\cdot{}^\circ\text{C})(0.01\ \text{m}^2)} = 4.0^\circ\text{C/W}$$

The total thermal resistance is then

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{plate}} + R_{\text{ambient}} = 0.030 + 0.0026 + 4.0 = 4.0326^\circ\text{C/W}$$

Note that the thermal resistance of a copper plate is very small and can be ignored altogether. Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{(70 - 20)^\circ\text{C}}{4.0326^\circ\text{C/W}} = \mathbf{12.4\ W}$$

Therefore, the power transistor should not be operated at power levels greater than 12.4 W if the case temperature is not to exceed 70°C.

The temperature jump at the interface is determined from

$$\Delta T_{\text{interface}} = \dot{Q} R_{\text{interface}} = (12.4\ \text{W})(0.030^\circ\text{C/W}) = \mathbf{0.37^\circ C}$$

which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we will lower the operating temperature of the transistor in this case by less than 0.4°C.

3–3 ■ GENERALIZED THERMAL RESISTANCE NETWORKS

The *thermal resistance* concept or the *electrical analogy* can also be used to solve steady heat transfer problems that involve parallel layers or combined series-parallel arrangements. Although such problems are often two- or even three-dimensional, approximate solutions can be obtained by assuming one-dimensional heat transfer and using the thermal resistance network.

Consider the composite wall shown in Fig. 3–19, which consists of two parallel layers. The thermal resistance network, which consists of two parallel resistances, can be represented as shown in the figure. Noting that the total heat transfer is the sum of the heat transfers through each layer, we have

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{3-29}$$

Utilizing electrical analogy, we get

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}} \tag{3-30}$$

where

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} \tag{3-31}$$

since the resistances are in parallel.

Now consider the combined series-parallel arrangement shown in Fig. 3–20. The total rate of heat transfer through this composite system can again be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \tag{3-32}$$

where

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}} \tag{3-33}$$

and

$$R_1 = \frac{L_1}{k_1 A_1}, \qquad R_2 = \frac{L_2}{k_2 A_2}, \qquad R_3 = \frac{L_3}{k_3 A_3}, \qquad R_{\text{conv}} = \frac{1}{hA_3} \tag{3-34}$$

Once the individual thermal resistances are evaluated, the total resistance and the total rate of heat transfer can easily be determined from the relations above.

The result obtained will be somewhat approximate, since the surfaces of the third layer will probably not be isothermal, and heat transfer between the first two layers is likely to occur.

Two assumptions commonly used in solving complex multidimensional heat transfer problems by treating them as one-dimensional (say, in the

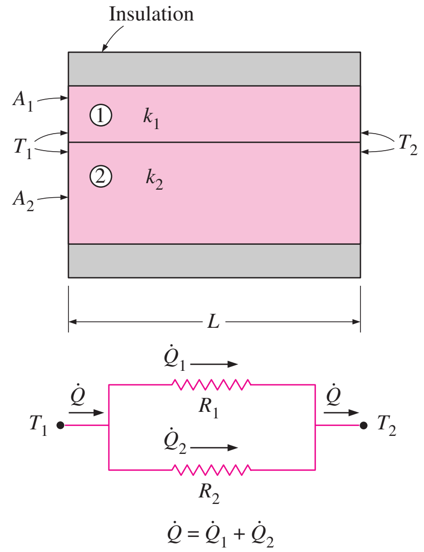


FIGURE 3–19
Thermal resistance network for two parallel layers.

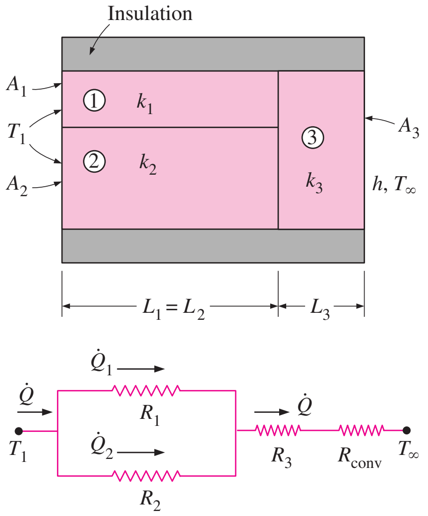


FIGURE 3–20
Thermal resistance network for combined series-parallel arrangement.

x-direction) using the thermal resistance network are (1) any plane wall normal to the x-axis is *isothermal* (i.e., to assume the temperature to vary in the x-direction only) and (2) any plane parallel to the x-axis is *adiabatic* (i.e., to assume heat transfer to occur in the x-direction only). These two assumptions result in different resistance networks, and thus different (but usually close) values for the total thermal resistance and thus heat transfer. The actual result lies between these two values. In geometries in which heat transfer occurs predominantly in one direction, either approach gives satisfactory results.

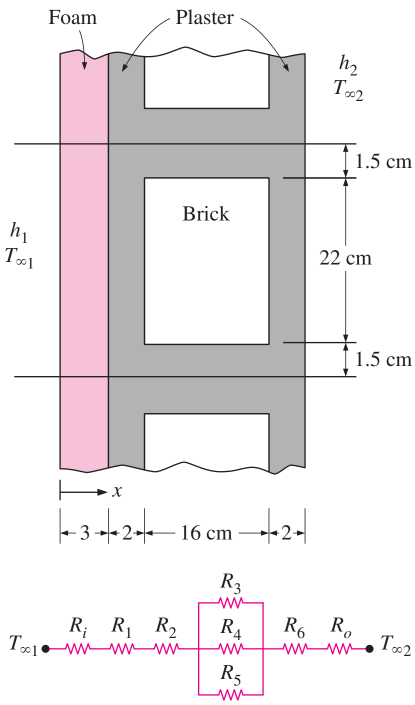


FIGURE 3–21
Schematic for Example 3–6.

EXAMPLE 3–6 Heat Loss through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm × 22-cm cross section horizontal bricks (k = 0.72 W/m · °C) separated by 3-cm-thick plaster layers (k = 0.22 W/m · °C). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam (k = 0.026 W/m · °C) on the inner side of the wall, as shown in Fig. 3–21. The indoor and the outdoor temperatures are 20°C and −10°C, and the convection heat transfer coefficients on the inner and the outer sides are h_1 = 10 W/m² · °C and h_2 = 25 W/m² · °C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

SOLUTION The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the x-direction. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivities are given to be k = 0.72 W/m · °C for bricks, k = 0.22 W/m · °C for plaster layers, and k = 0.026 W/m · °C for the rigid foam.

Analysis There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the x-direction to be *isothermal,* the thermal resistance network for the representative section of the wall becomes as shown in Fig. 3–21. The individual resistances are evaluated as:

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.25 \times 1\ \text{m}^2)} = 0.4^\circ\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03\ \text{m}}{(0.026\ \text{W/m} \cdot {}^\circ\text{C})(0.25 \times 1\ \text{m}^2)} = 4.6^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m} \cdot {}^\circ\text{C})(0.25 \times 1\ \text{m}^2)}$$
$$= 0.36^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16\ \text{m}}{(0.22\ \text{W/m} \cdot {}^\circ\text{C})(0.015 \times 1\ \text{m}^2)}$$
$$= 48.48^\circ\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16\text{ m}}{(0.72\text{ W/m}\cdot{}^\circ\text{C})(0.22\times 1\text{ m}^2)} = 1.01^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2\cdot{}^\circ\text{C})(0.25\times 1\text{ m}^2)} = 0.16^\circ\text{C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03\text{ W/}^\circ\text{C}$$

which gives

$$R_{\text{mid}} = 0.97^\circ\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.4 + 4.6 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.85^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{6.85^\circ\text{C/W}} = 4.38\text{ W} \qquad (\text{per } 0.25\text{ m}^2 \text{ surface area})$$

or 4.38/0.25 = 17.5 W per m^2 area. The total area of the wall is A = 3 m × 5 m = 15 m^2. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (17.5\text{ W/m}^2)(15\text{ m}^2) = \mathbf{263\ W}$$

Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.

Discussion In the above solution, we assumed the temperature at any cross section of the wall normal to the x-direction to be *isothermal.* We could also solve this problem by going to the other extreme and assuming the surfaces parallel to the x-direction to be *adiabatic.* The thermal resistance network in this case will be as shown in Fig. 3–22. By following the approach outlined above, the total thermal resistance in this case is determined to be R_{total} = 6.97°C/W, which is very close to the value 6.85°C/W obtained before. Thus either approach would give roughly the same result in this case. This example demonstrates that either approach can be used in practice to obtain satisfactory results.

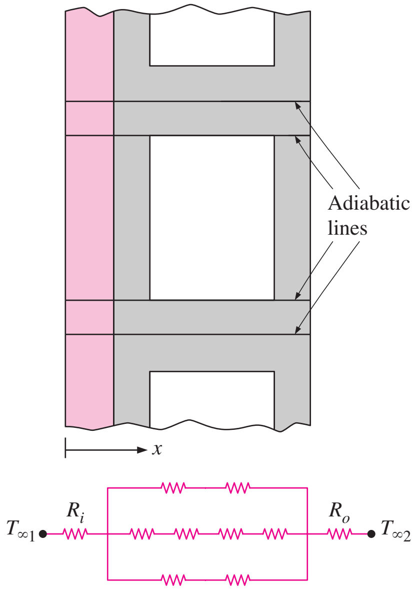


FIGURE 3–22
Alternative thermal resistance network for Example 3–6 for the case of surfaces parallel to the primary direction of heat transfer being adiabatic.

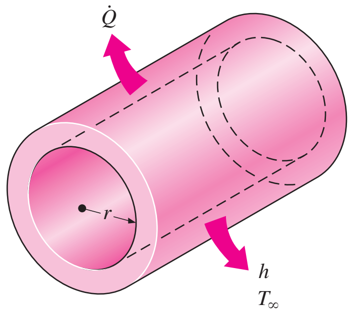


FIGURE 3–23
Heat is lost from a hot water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

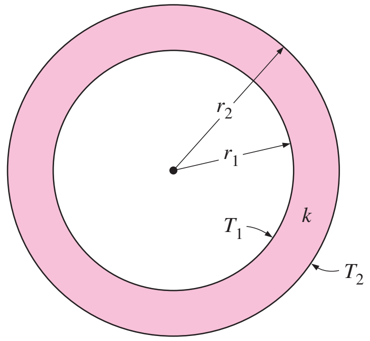


FIGURE 3–24
A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

3–4 ▪ HEAT CONDUCTION IN CYLINDERS AND SPHERES

Consider steady heat conduction through a hot water pipe. Heat is continuously lost to the outdoors through the wall of the pipe, and we intuitively feel that heat transfer through the pipe is in the normal direction to the pipe surface and no significant heat transfer takes place in the pipe in other directions (Fig. 3–23). The wall of the pipe, whose thickness is rather small, separates two fluids at different temperatures, and thus the temperature gradient in the radial direction will be relatively large. Further, if the fluid temperatures inside and outside the pipe remain constant, then heat transfer through the pipe is *steady.* Thus heat transfer through the pipe can be modeled as *steady* and *one-dimensional.* The temperature of the pipe in this case will depend on one direction only (the radial r-direction) and can be expressed as $T = T(r)$. The temperature is independent of the azimuthal angle or the axial distance. This situation is approximated in practice in long cylindrical pipes and spherical containers.

In *steady* operation, there is no change in the temperature of the pipe with time at any point. Therefore, the rate of heat transfer into the pipe must be equal to the rate of heat transfer out of it. In other words, heat transfer through the pipe must be constant, $\dot{Q}_{\text{cond, cyl}} = \text{constant}$.

Consider a long cylindrical layer (such as a circular pipe) of inner radius r_1, outer radius r_2, length L, and average thermal conductivity k (Fig. 3–24). The two surfaces of the cylindrical layer are maintained at constant temperatures T_1 and T_2. There is no heat generation in the layer and the thermal conductivity is constant. For one-dimensional heat conduction through the cylindrical layer, we have $T(r)$. Then Fourier's law of heat conduction for heat transfer through the cylindrical layer can be expressed as

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)} \tag{3-35}$$

where $A = 2\pi rL$ is the heat transfer area at location r. Note that A depends on r, and thus it *varies* in the direction of heat transfer. Separating the variables in the above equation and integrating from $r = r_1$, where $T(r_1) = T_1$, to $r = r_2$, where $T(r_2) = T_2$, gives

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT \tag{3-36}$$

Substituting $A = 2\pi rL$ and performing the integrations give

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)} \tag{3-37}$$

since $\dot{Q}_{\text{cond, cyl}} = \text{constant}$. This equation can be rearranged as

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)} \tag{3-38}$$

where

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times (\text{Length}) \times (\text{Thermal conductivity})} \tag{3-39}$$

is the *thermal resistance* of the cylindrical layer against heat conduction, or simply the **conduction resistance** of the cylinder layer.

We can repeat the analysis above for a *spherical layer* by taking $A = 4\pi r^2$ and performing the integrations in Eq. 3–36. The result can be expressed as

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}} \tag{3-40}$$

where

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})} \tag{3-41}$$

is the *thermal resistance* of the spherical layer against heat conduction, or simply the **conduction resistance** of the spherical layer.

Now consider steady one-dimensional heat flow through a cylindrical or spherical layer that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively, as shown in Fig. 3–25. The thermal resistance network in this case consists of one conduction and two convection resistances in series, just like the one for the plane wall, and the rate of heat transfer under steady conditions can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \tag{3-42}$$

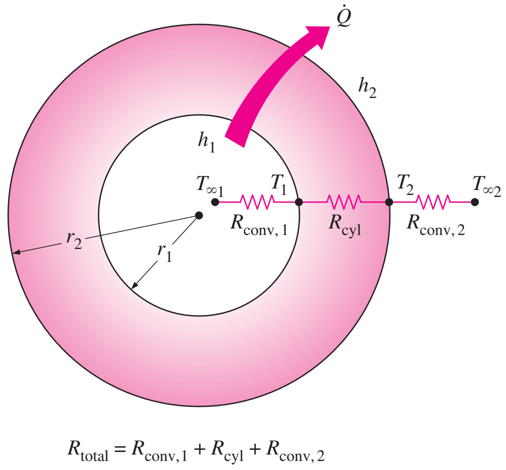


FIGURE 3–25
The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

where

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{cyl}} + R_{\text{conv, 2}}$$
$$= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{(2\pi r_2 L)h_2} \qquad \textbf{(3-43)}$$

for a *cylindrical* layer, and

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{sph}} + R_{\text{conv, 2}}$$
$$= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2} \qquad \textbf{(3-44)}$$

for a spherical layer. Note that A in the convection resistance relation $R_{\text{conv}} = 1/hA$ is the *surface area at which convection occurs*. It is equal to $A = 2\pi rL$ for a cylindrical surface and $A = 4\pi r^2$ for a spherical surface of radius r. Also note that the thermal resistances are in series, and thus the total thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series.

Multilayered Cylinders and Spheres

Steady heat transfer through multilayered cylindrical or spherical shells can be handled just like multilayered plane walls discussed earlier by simply adding an *additional resistance* in series for each *additional layer*. For example, the steady heat transfer rate through the three-layered composite cylinder of length L shown in Fig. 3–26 with convection on both sides can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \qquad \textbf{(3-45)}$$

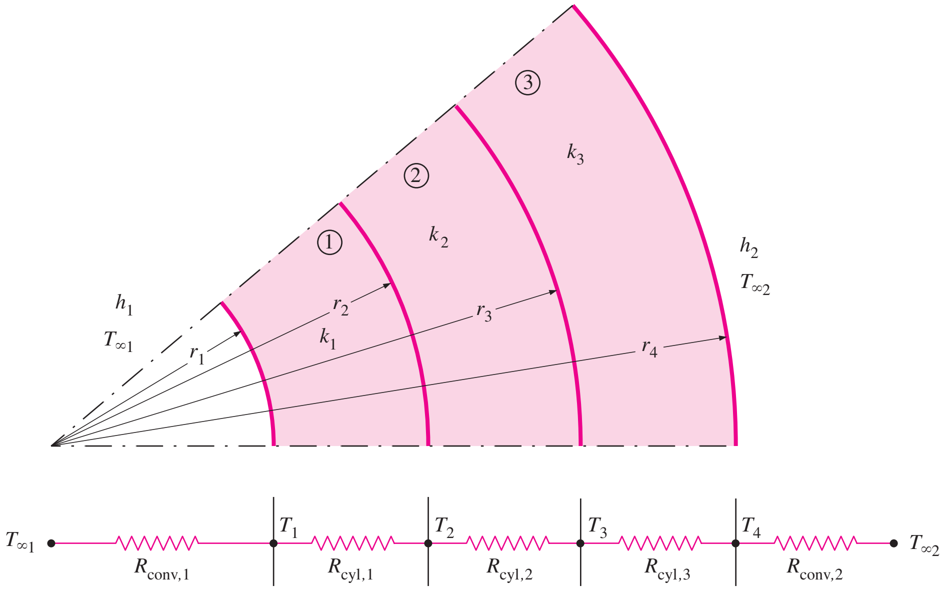


FIGURE 3–26
The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

where R_{total} is the *total thermal resistance,* expressed as

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{cyl, 1}} + R_{\text{cyl, 2}} + R_{\text{cyl, 3}} + R_{\text{conv, 2}}$$
$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4} \quad \textbf{(3-46)}$$

where $A_1 = 2\pi r_1 L$ and $A_4 = 2\pi r_4 L$. Equation 3–46 can also be used for a three-layered spherical shell by replacing the thermal resistances of cylindrical layers by the corresponding spherical ones. Again, note from the thermal resistance network that the resistances are in series, and thus the total thermal resistance is simply the *arithmetic sum* of the individual thermal resistances in the path of heat flow.

Once $\dot{Q}$ is known, we can determine any intermediate temperature T_j by applying the relation $\dot{Q} = (T_i - T_j)/R_{\text{total}, i-j}$ across any layer or layers such that T_i is a *known* temperature at location i and $R_{\text{total}, i-j}$ is the total thermal resistance between locations i and j (Fig. 3–27). For example, once $\dot{Q}$ has been calculated, the interface temperature T_2 between the first and second cylindrical layers can be determined from

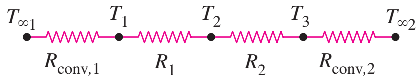
$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv, 1}} + R_{\text{cyl, 1}}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}} \quad \textbf{(3-47)}$$

We could also calculate T_2 from

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{\text{conv, 2}}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}} \quad \textbf{(3-48)}$$

Although both relations will give the same result, we prefer the first one since it involves fewer terms and thus less work.

The thermal resistance concept can also be used for *other geometries,* provided that the proper conduction resistances and the proper surface areas in convection resistances are used.



$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$$
$$= \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$$
$$= \frac{T_1 - T_3}{R_1 + R_2}$$
$$= \frac{T_2 - T_3}{R_2}$$
$$= \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv},2}}$$
$$= \cdots$$

FIGURE 3–27

The ratio $\Delta T/R$ across any layer is equal to $\dot{Q}$, which remains constant in one-dimensional steady conduction.

EXAMPLE 3–7 Heat Transfer to a Spherical Container

A 3-m internal diameter spherical tank made of 2-cm-thick stainless steel (k = 15 W/m · °C) is used to store iced water at $T_{\infty 1}$ = 0°C. The tank is located in a room whose temperature is $T_{\infty 2}$ = 22°C. The walls of the room are also at 22°C. The outer surface of the tank is black and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are h_1 = 80 W/m^2 · °C and h_2 = 10 W/m^2 · °C, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period.

SOLUTION A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

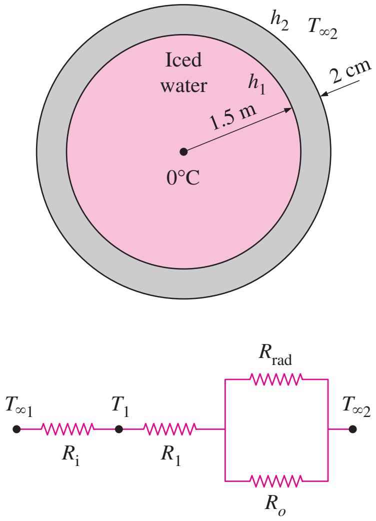


FIGURE 3–28
Schematic for Example 3–7.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be $k = 15$ W/m · °C. The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg. The outer surface of the tank is black and thus its emissivity is $\varepsilon = 1$.

Analysis (*a*) The thermal resistance network for this problem is given in Fig. 3–28. Noting that the inner diameter of the tank is $D_1 = 3$ m and the outer diameter is $D_2 = 3.04$ m, the inner and the outer surface areas of the tank are

$$A_1 = \pi D_1^2 = \pi(3\text{ m})^2 = 28.3\text{ m}^2$$
$$A_2 = \pi D_2^2 = \pi(3.04\text{ m})^2 = 29.0\text{ m}^2$$

Also, the radiation heat transfer coefficient is given by

$$h_{\text{rad}} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

But we do not know the outer surface temperature T_2 of the tank, and thus we cannot calculate h_{rad}. Therefore, we need to assume a T_2 value now and check the accuracy of this assumption later. We will repeat the calculations if necessary using a revised value for T_2.

We note that T_2 must be between 0°C and 22°C, but it must be closer to 0°C, since the heat transfer coefficient inside the tank is much larger. Taking $T_2 = 5°\text{C} = 278$ K, the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{\text{rad}} &= (1)(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(295\text{ K})^2 + (278\text{ K})^2][(295 + 278)\text{ K}] \\ &= 5.34\text{ W/m}^2\cdot\text{K} = 5.34\text{ W/m}^2\cdot°\text{C} \end{aligned}$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A_1} = \frac{1}{(80\text{ W/m}^2\cdot°\text{C})(28.3\text{ m}^2)} = 0.000442°\text{C/W}$$

$$\begin{aligned} R_1 = R_{\text{sphere}} &= \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50)\text{ m}}{4\pi\,(15\text{ W/m}\cdot°\text{C})(1.52\text{ m})(1.50\text{ m})} \\ &= 0.000047°\text{C/W} \end{aligned}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_2} = \frac{1}{(10\text{ W/m}^2\cdot°\text{C})(29.0\text{ m}^2)} = 0.00345°\text{C/W}$$

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_2} = \frac{1}{(5.34\text{ W/m}^2\cdot°\text{C})(29.0\text{ m}^2)} = 0.00646°\text{C/W}$$

The two parallel resistances R_o and R_{rad} can be replaced by an equivalent resistance R_{equiv} determined from

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_o} + \frac{1}{R_{\text{rad}}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7\text{ W/°C}$$

which gives

$$R_{\text{equiv}} = 0.00225°\text{C/W}$$

Now all the resistances are in series, and the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_{\text{equiv}} = 0.000442 + 0.000047 + 0.00225 = 0.00274°\text{C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{\text{total}}} = \frac{(22 - 0)°\text{C}}{0.00274°\text{C/W}} = \mathbf{8029\ W} \qquad (\text{or } \dot{Q} = 8.027 \text{ kJ/s})$$

To check the validity of our original assumption, we now determine the outer surface temperature from

$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{\text{equiv}}} \longrightarrow T_2 = T_{\infty 2} - \dot{Q}R_{\text{equiv}}$$
$$= 22°\text{C} - (8029 \text{ W})(0.00225°\text{C/W}) = 4°\text{C}$$

which is sufficiently close to the 5°C assumed in the determination of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations using 4°C for T_2.

(*b*) The total amount of heat transfer during a 24-h period is

$$Q = \dot{Q}\,\Delta t = (8.029 \text{ kJ/s})(24 \times 3600 \text{ s}) = 673{,}700 \text{ kJ}$$

Noting that it takes 333.7 kJ of energy to melt 1 kg of ice at 0°C, the amount of ice that will melt during a 24-h period is

$$m_{\text{ice}} = \frac{Q}{h_{if}} = \frac{673{,}700 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{2079\ kg}$$

Therefore, about 2 metric tons of ice will melt in the tank every day.

Discussion An easier way to deal with combined convection and radiation at a surface when the surrounding medium and surfaces are at the same temperature is to add the radiation and convection heat transfer coefficients and to treat the result as the convection heat transfer coefficient. That is, to take $h = 10 + 5.34 = 15.34 \text{ W/m}^2 \cdot °\text{C}$ in this case. This way, we can ignore radiation since its contribution is accounted for in the convection heat transfer coefficient. The convection resistance of the outer surface in this case would be

$$R_{\text{combined}} = \frac{1}{h_{\text{combined}} A_2} = \frac{1}{(15.34 \text{ W/m}^2 \cdot °\text{C})(29.0 \text{ m}^2)} = 0.00225°\text{C/W}$$

which is identical to the value obtained for the equivalent resistance for the parallel convection and the radiation resistances.

EXAMPLE 3–8 Heat Loss through an Insulated Steam Pipe

Steam at $T_{\infty 1} = 320°\text{C}$ flows in a cast iron pipe ($k = 80 \text{ W/m} \cdot °\text{C}$) whose inner and outer diameters are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with $k = 0.05 \text{ W/m} \cdot °\text{C}$. Heat is lost to the surroundings at $T_{\infty 2} = 5°\text{C}$ by natural convection and radiation, with a combined heat transfer coefficient of $h_2 = 18$ W/m$^2 \cdot$ °C. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/m$^2 \cdot$ °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

SOLUTION A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 80$ W/m · °C for cast iron and $k = 0.05$ W/m · °C for glass wool insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Fig. 3–29. Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025\text{ m})(1\text{ m}) = 0.157\text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575\text{ m})(1\text{ m}) = 0.361\text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(60\text{ W/m}^2 \cdot °\text{C})(0.157\text{ m}^2)} = 0.106°\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80\text{ W/m} \cdot °\text{C})(1\text{ m})} = 0.0002°\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05\text{ W/m} \cdot °\text{C})(1\text{ m})} = 2.35°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18\text{ W/m}^2 \cdot °\text{C})(0.361\text{ m}^2)} = 0.154°\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61°\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)°\text{C}}{2.61°\text{C/W}} = \mathbf{121\text{ W}} \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length L.

The temperature drops across the pipe and the insulation are determined from Eq. 3–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q}R_{\text{pipe}} = (121\text{ W})(0.0002°\text{C/W}) = \mathbf{0.02°C}$$
$$\Delta T_{\text{insulation}} = \dot{Q}R_{\text{insulation}} = (121\text{ W})(2.35°\text{C/W}) = \mathbf{284°C}$$

That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.

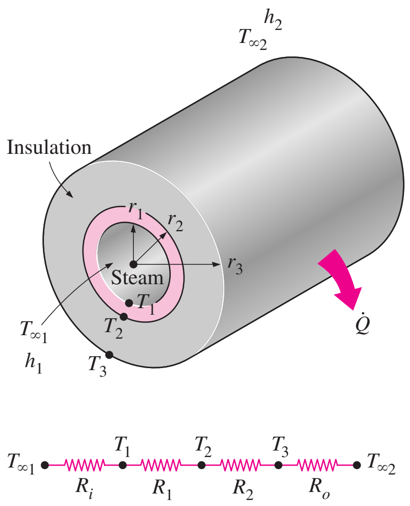


FIGURE 3–29
Schematic for Example 3–8.

Discussion Note that the thermal resistance of the pipe is too small relative to the other resistances and can be neglected without causing any significant error. Also note that the temperature drop across the pipe is practically zero, and thus the pipe can be assumed to be isothermal. The resistance to heat flow in insulated pipes is primarily due to insulation.

3–5 ▪ CRITICAL RADIUS OF INSULATION

We know that adding more insulation to a wall or to the attic always decreases heat transfer. The thicker the insulation, the lower the heat transfer rate. This is expected, since the heat transfer area A is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

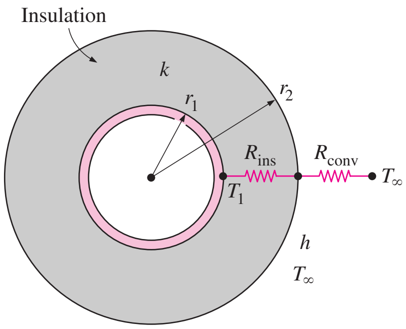


FIGURE 3–30
An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant (Fig. 3–30). The pipe is now insulated with a material whose thermal conductivity is k and outer radius is r_2. Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient h. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as (Fig. 3–31)

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}} \tag{3-49}$$

The variation of $\dot{Q}$ with the outer radius of the insulation r_2 is plotted in Fig. 3–31. The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $d\dot{Q}/dr_2 = 0$ (zero slope). Performing the differentiation and solving for r_2 yields the **critical radius of insulation** for a cylindrical body to be

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)} \tag{3-50}$$

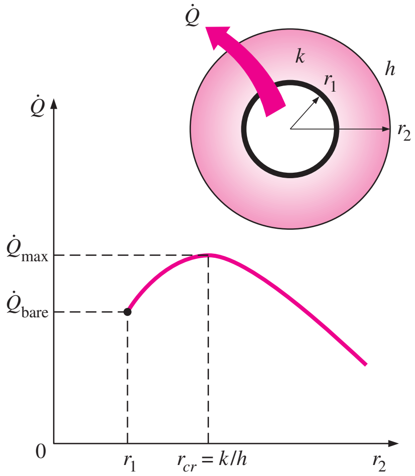


FIGURE 3–31

Note that the critical radius of insulation depends on the thermal conductivity of the insulation k and the external convection heat transfer coefficient h. The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_{cr}$, reaches a maximum when $r_2 = r_{cr}$, and starts to decrease for $r_2 > r_{cr}$. Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when $r_2 < r_{cr}$.

The important question to answer at this point is whether we need to be concerned about the critical radius of insulation when insulating hot water pipes or even hot water tanks. Should we always check and make sure that the outer radius of insulation exceeds the critical radius before we install any insulation? Probably not, as explained here.

The value of the critical radius r_{cr} will be the largest when k is large and h is small. Noting that the lowest value of h encountered in practice is about 5 $W/m^2 \cdot °C$ for the case of natural convection of gases, and that the thermal conductivity of common insulating materials is about 0.05 $W/m^2 \cdot °C$, the largest value of the critical radius we are likely to encounter is

$$r_{cr,\,max} = \frac{k_{max,\,insulation}}{h_{min}} \approx \frac{0.05\ W/m \cdot °C}{5\ W/m^2 \cdot °C} = 0.01\ m = 1\ cm$$

This value would be even smaller when the radiation effects are considered. The critical radius would be much less in forced convection, often less than 1 mm, because of much larger h values associated with forced convection. Therefore, we can insulate hot water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.

The radius of electric wires may be smaller than the critical radius. Therefore, the plastic electrical insulation may actually *enhance* the heat transfer from electric wires and thus keep their steady operating temperatures at lower and thus safer levels.

The discussions above can be repeated for a sphere, and it can be shown in a similar manner that the critical radius of insulation for a spherical shell is

$$r_{cr,\,sphere} = \frac{2k}{h} \tag{3-51}$$

where k is the thermal conductivity of the insulation and h is the convection heat transfer coefficient on the outer surface.

EXAMPLE 3–9 Heat Loss from an Insulated Electric Wire

A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15\ W/m \cdot °C$. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30°C$ with a heat transfer coefficient of $h = 12\ W/m^2 \cdot °C$, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be $k = 0.15$ $W/m \cdot °C$.

Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 3–32. The values of these two resistances are determined to be

$$A_2 = (2\pi r_2)L = 2\pi(0.0035\text{ m})(5\text{ m}) = 0.110\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12\text{ W/m}^2\cdot{}^\circ\text{C})(0.110\text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\text{ W/m}\cdot{}^\circ\text{C})(5\text{ m})} = 0.18^\circ\text{C/W}$$

and therefore

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94^\circ\text{C/W}$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}}$$
$$= 30^\circ\text{C} + (80\text{ W})(0.94^\circ\text{C/W}) = \mathbf{105^\circ C}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3–50 to be

$$r_{\text{cr}} = \frac{k}{h} = \frac{0.15\text{ W/m}\cdot{}^\circ\text{C}}{12\text{ W/m}^2\cdot{}^\circ\text{C}} = 0.0125\text{ m} = 12.5\text{ mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

Discussion It can be shown by repeating the calculations above for a 4-mm-thick plastic cover that the interface temperature drops to 90.6°C when the thickness of the plastic cover is doubled. It can also be shown in a similar manner that the interface reaches a minimum temperature of 83°C when the outer radius of the plastic cover equals the critical radius.

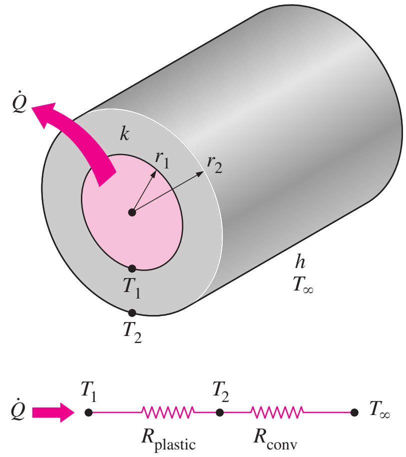


FIGURE 3–32
Schematic for Example 3–9.

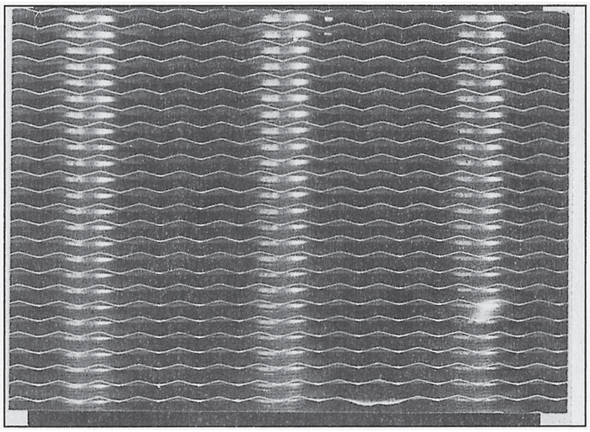


FIGURE 3–33
The thin plate fins of a car radiator greatly increase the rate of heat transfer to the air (photo by Yunus Çengel and James Kleiser).

3–6 ▪ HEAT TRANSFER FROM FINNED SURFACES

The rate of heat transfer from a surface at a temperature T_s to the surrounding medium at T_∞ is given by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where A_s is the heat transfer surface area and h is the convection heat transfer coefficient. When the temperatures T_s and T_∞ are fixed by design considerations, as is often the case, there are *two ways* to increase the rate of heat transfer: to increase the *convection heat transfer coefficient h* or to increase the *surface area A_s*. Increasing h may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate. The alternative is to increase the surface area by attaching to the surface *extended surfaces* called *fins* made of highly conductive materials such as aluminum. Finned surfaces are manufactured by extruding, welding, or wrapping a thin metal sheet on a surface. Fins enhance heat transfer from a surface by exposing a larger surface area to convection and radiation.

Finned surfaces are commonly used in practice to enhance heat transfer, and they often increase the rate of heat transfer from a surface severalfold. The car radiator shown in Fig. 3–33 is an example of a finned surface. The closely packed thin metal sheets attached to the hot water tubes increase the surface area for convection and thus the rate of convection heat transfer from the tubes to the air many times. There are a variety of innovative fin designs available in the market, and they seem to be limited only by imagination (Fig. 3–34).

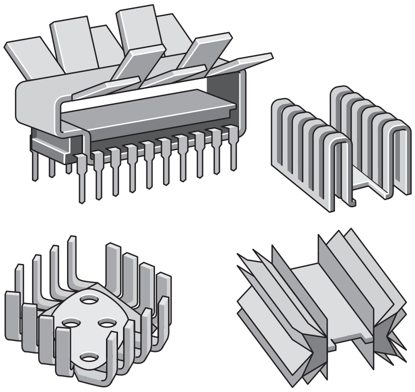


FIGURE 3–34
Some innovative fin designs.

In the analysis of fins, we consider *steady* operation with *no heat generation* in the fin, and we assume the thermal conductivity k of the material to remain constant. We also assume the convection heat transfer coefficient h to be *constant* and *uniform* over the entire surface of the fin for convenience in the analysis. We recognize that the convection heat transfer coefficient h, in general, varies along the fin as well as its circumference, and its value at a point is a strong function of the *fluid motion* at that point. The value of h is usually much lower at the *fin base* than it is at the *fin tip* because the fluid is surrounded by solid surfaces near the base, which seriously disrupt its motion to the point of "suffocating" it, while the fluid near the fin tip has little contact with a solid surface and thus encounters little resistance to flow. Therefore, adding too many fins on a surface may actually decrease the overall heat transfer when the decrease in h offsets any gain resulting from the increase in the surface area.

Fin Equation

Consider a volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and a perimeter of p, as shown in Fig. 3–35. Under steady conditions, the energy balance on this volume element can be expressed as

$$\left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{array}\right) = \left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{array}\right) + \left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{array}\right)$$

or

$$\dot{Q}_{\text{cond}, x} = \dot{Q}_{\text{cond}, x + \Delta x} + \dot{Q}_{\text{conv}}$$

where

$$\dot{Q}_{\text{conv}} = h(p\,\Delta x)(T - T_\infty)$$

Substituting and dividing by Δx, we obtain

$$\frac{\dot{Q}_{\text{cond}, x + \Delta x} - \dot{Q}_{\text{cond}, x}}{\Delta x} + hp(T - T_\infty) = 0 \tag{3-52}$$

Taking the limit as $\Delta x \to 0$ gives

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0 \tag{3-53}$$

From Fourier's law of heat conduction we have

$$\dot{Q}_{\text{cond}} = -kA_c\frac{dT}{dx} \tag{3-54}$$

where A_c is the cross-sectional area of the fin at location x. Substitution of this relation into Eq. 3–53 gives the differential equation governing heat transfer in fins,

$$\frac{d}{dx}\left(kA_c\frac{dT}{dx}\right) - hp(T - T_\infty) = 0 \tag{3-55}$$

In general, the cross-sectional area A_c and the perimeter p of a fin vary with x, which makes this differential equation difficult to solve. In the special case of *constant cross section* and *constant thermal conductivity,* the differential equation 3–55 reduces to

$$\frac{d^2\theta}{dx^2} - a^2\theta = 0 \tag{3-56}$$

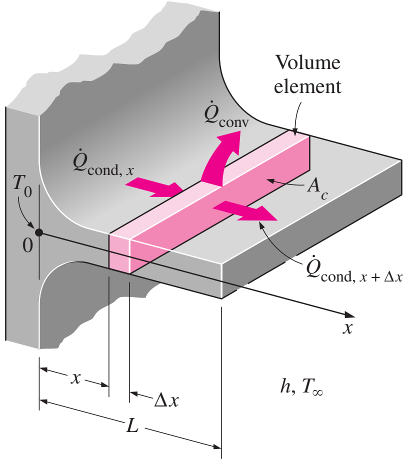


FIGURE 3–35
Volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and perimeter of p.

where

$$a^2 = \frac{hp}{kA_c} \tag{3-57}$$

and $\theta = T - T_\infty$ is the *temperature excess.* At the fin base we have $\theta_b = T_b - T_\infty$.

Equation 3–56 is a linear, homogeneous, second-order differential equation with constant coefficients. A fundamental theory of differential equations states that such an equation has two linearly independent solution functions, and its general solution is the linear combination of those two solution functions. A careful examination of the differential equation reveals that subtracting a constant multiple of the solution function θ from its second derivative yields zero. Thus we conclude that the function θ and its second derivative must be *constant multiples* of each other. The only functions whose derivatives are constant multiples of the functions themselves are the *exponential functions* (or a linear combination of exponential functions such as sine and cosine hyperbolic functions). Therefore, the solution functions of the differential equation above are the exponential functions e^{-ax} or e^{ax} or constant multiples of them. This can be verified by direct substitution. For example, the second derivative of e^{-ax} is a^2e^{-ax}, and its substitution into Eq. 3–56 yields zero. Therefore, the general solution of the differential equation Eq. 3–56 is

$$\theta(x) = C_1e^{ax} + C_2e^{-ax} \tag{3-58}$$

where C_1 and C_2 are arbitrary constants whose values are to be determined from the boundary conditions at the base and at the tip of the fin. Note that we need only two conditions to determine C_1 and C_2 uniquely.

The temperature of the plate to which the fins are attached is normally known in advance. Therefore, at the fin base we have a *specified temperature* boundary condition, expressed as

Boundary condition at fin base: $$\theta(0) = \theta_b = T_b - T_\infty \tag{3-59}$$

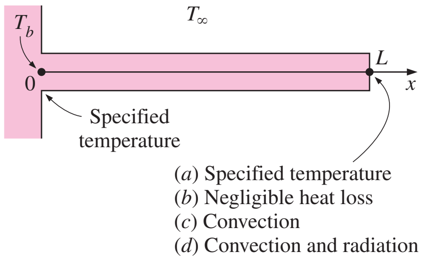


FIGURE 3–36
Boundary conditions at the fin base and the fin tip.

At the fin tip we have several possibilities, including specified temperature, negligible heat loss (idealized as an insulated tip), convection, and combined convection and radiation (Fig. 3–36). Next, we consider each case separately.

1 Infinitely Long Fin ($T_{\text{fin tip}} = T_\infty$)

For a sufficiently long fin of *uniform* cross section (A_c = constant), the temperature of the fin at the fin tip will approach the environment temperature T_∞ and thus θ will approach zero. That is,

Boundary condition at fin tip: $$\theta(L) = T(L) - T_\infty = 0 \quad \text{as} \quad L \to \infty$$

This condition will be satisfied by the function e^{-ax}, but not by the other prospective solution function e^{ax} since it tends to infinity as x gets larger. Therefore, the general solution in this case will consist of a constant multiple of e^{-ax}. The value of the constant multiple is determined from the requirement that at the fin base where $x = 0$ the value of θ will be θ_b. Noting that

$e^{-ax} = e^0 = 1$, the proper value of the constant is θ_b, and the solution function we are looking for is $\theta(x) = \theta_b e^{-ax}$. This function satisfies the differential equation as well as the requirements that the solution reduce to θ_b at the fin base and approach zero at the fin tip for large x. Noting that $\theta = T - T_\infty$ and $a = \sqrt{hp/kA_c}$, the variation of temperature along the fin in this case can be expressed as

Very long fin:
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-ax} = e^{-x\sqrt{hp/kA_c}} \tag{3-60}$$

Note that the temperature along the fin in this case decreases *exponentially* from T_b to T_∞, as shown in Fig. 3–37. The steady rate of *heat transfer* from the entire fin can be determined from Fourier's law of heat conduction

Very long fin:
$$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty) \tag{3-61}$$

where p is the perimeter, A_c is the cross-sectional area of the fin, and x is the distance from the fin base. Alternatively, the rate of heat transfer from the fin could also be determined by considering heat transfer from a differential volume element of the fin and integrating it over the entire surface of the fin. That is,

$$\dot{Q}_{\text{fin}} = \int_{A_{\text{fin}}} h[T(x) - T_\infty]\, dA_{\text{fin}} = \int_{A_{\text{fin}}} h\theta(x)\, dA_{\text{fin}} \tag{3-62}$$

The two approaches described are equivalent and give the same result since, under steady conditions, the heat transfer from the exposed surfaces of the fin is equal to the heat transfer to the fin at the base (Fig. 3–38).

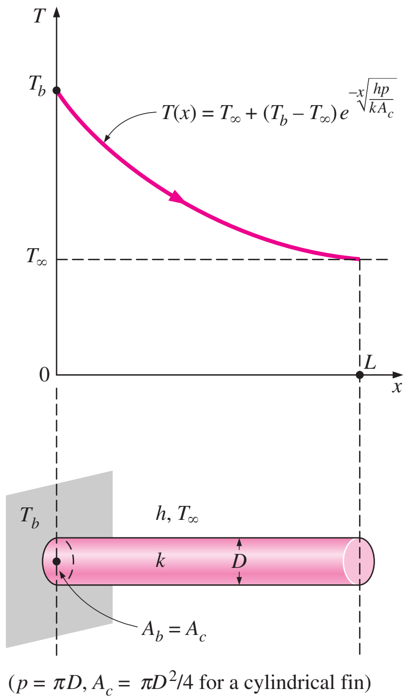


FIGURE 3–37
A long circular fin of uniform cross section and the variation of temperature along it.

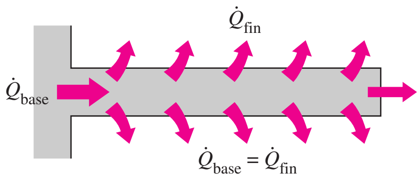


FIGURE 3–38
Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

2 Negligible Heat Loss from the Fin Tip (Insulated fin tip, $\dot{Q}_{\text{fin tip}} = 0$)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic situation is for heat transfer from the fin tip to be negligible since the heat transfer from the fin is proportional to its surface area, and the surface area of the fin tip is usually a negligible fraction of the total fin area. Then the fin tip can be assumed to be insulated, and the condition at the fin tip can be expressed as

Boundary condition at fin tip:
$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0 \tag{3-63}$$

The condition at the fin base remains the same as expressed in Eq. 3–59. The application of these two conditions on the general solution (Eq. 3–58) yields, after some manipulations, this relation for the temperature distribution:

Adiabatic fin tip:
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL} \tag{3-64}$$

The rate of heat transfer from the fin can be determined again from Fourier's law of heat conduction:

Adiabatic fin tip:

$$\dot{Q}_{\text{insulated tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$
$$= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL \qquad \textbf{(3-65)}$$

Note that the heat transfer relations for the very long fin and the fin with negligible heat loss at the tip differ by the factor tanh aL, which approaches 1 as L becomes very large.

3 Convection (or Combined Convection and Radiation) from Fin Tip

The fin tips, in practice, are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection that also includes the effects of radiation. The fin equation can still be solved in this case using the convection at the fin tip as the second boundary condition, but the analysis becomes more involved, and it results in rather lengthy expressions for the temperature distribution and the heat transfer. Yet, in general, the fin tip area is a small fraction of the total fin surface area, and thus the complexities involved can hardly justify the improvement in accuracy.

A practical way of accounting for the heat loss from the fin tip is to replace the *fin length L* in the relation for the *insulated tip* case by a **corrected length** defined as (Fig. 3–39)

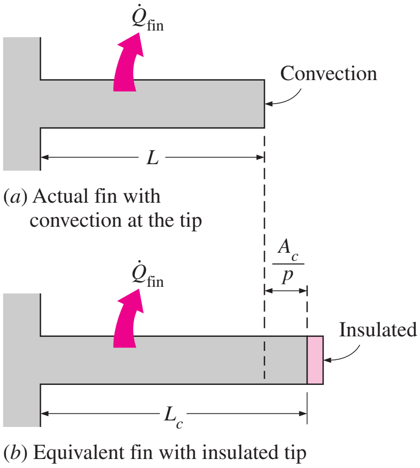


FIGURE 3–39
Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.

Corrected fin length:

$$L_c = L + \frac{A_c}{p} \qquad \textbf{(3-66)}$$

where A_c is the cross-sectional area and p is the perimeter of the fin at the tip. Multiplying the relation above by the perimeter gives $A_{\text{corrected}} = A_{\text{fin (lateral)}} + A_{\text{tip}}$, which indicates that the fin area determined using the corrected length is equivalent to the sum of the lateral fin area plus the fin tip area.

The corrected length approximation gives very good results when the variation of temperature near the fin tip is small (which is the case when $aL \geq 1$) and the heat transfer coefficient at the fin tip is about the same as that at the lateral surface of the fin. Therefore, *fins subjected to convection at their tips can be treated as fins with insulated tips by replacing the actual fin length by the corrected length in Eqs. 3–64 and 3–65.*

Using the proper relations for A_c and p, the corrected lengths for rectangular and cylindrical fins are easily determined to be

$$L_{c,\text{ rectangular fin}} = L + \frac{t}{2} \qquad \text{and} \qquad L_{c,\text{ cylindrical fin}} = L + \frac{D}{4}$$

where t is the thickness of the rectangular fins and D is the diameter of the cylindrical fins.

Fin Efficiency

Consider the surface of a *plane wall* at temperature T_b exposed to a medium at temperature T_∞. Heat is lost from the surface to the surrounding medium by convection with a heat transfer coefficient of h. Disregarding radiation or accounting for its contribution in the convection coefficient h, heat transfer from a surface area A_s is expressed as $\dot{Q} = hA_s(T_s - T_\infty)$.

Now let us consider a fin of constant cross-sectional area $A_c = A_b$ and length L that is attached to the surface with a perfect contact (Fig. 3–40). This time heat will flow from the surface to the fin *by conduction* and from the fin to the surrounding medium *by convection* with the same heat transfer coefficient h. The temperature of the fin will be T_b at the fin base and gradually decrease toward the fin tip. Convection from the fin surface causes the temperature at any cross section to drop somewhat from the midsection toward the outer surfaces. However, the cross-sectional area of the fins is usually very small, and thus the temperature at any cross section can be considered to be uniform. Also, the fin tip can be assumed for convenience and simplicity to be insulated by using the corrected length for the fin instead of the actual length.

In the limiting case of *zero thermal resistance* or *infinite thermal conductivity* ($k \to \infty$), the temperature of the fin will be uniform at the base value of T_b. The heat transfer from the fin will be *maximum* in this case and can be expressed as

$$\dot{Q}_{\text{fin, max}} = hA_{\text{fin}}(T_b - T_\infty) \tag{3-67}$$

In reality, however, the temperature of the fin will drop along the fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference $T(x) - T_\infty$ toward the fin tip, as shown in Fig. 3–41. To account for the effect of this decrease in temperature on heat transfer, we define a **fin efficiency** as

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\begin{array}{c}\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}\end{array}} \tag{3-68}$$

or

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\,\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}\,hA_{\text{fin}}(T_b - T_\infty) \tag{3-69}$$

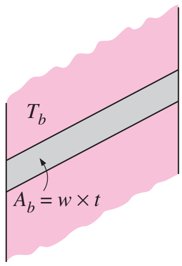
where A_{fin} is the total surface area of the fin. This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of *very long fins* and *fins with insulated tips,* the fin efficiency can be expressed as

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{aL} \tag{3-70}$$

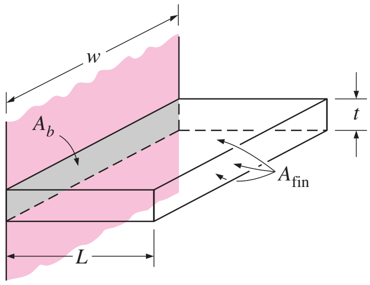
and

$$\eta_{\text{insulated tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{hA_{\text{fin}}(T_b - T_\infty)} = \frac{\tanh aL}{aL} \tag{3-71}$$

since $A_{\text{fin}} = pL$ for fins with constant cross section. Equation 3–71 can also be used for fins subjected to convection provided that the fin length L is replaced by the corrected length L_c.



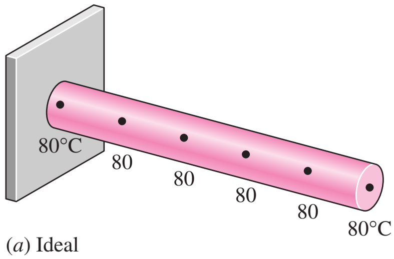
(*a*) Surface without fins



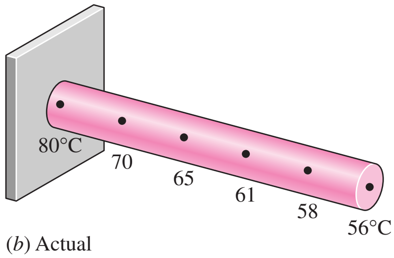
(*b*) Surface with a fin

$A_{\text{fin}} = 2 \times w \times L + w \times t$
$\cong 2 \times w \times L$

FIGURE 3–40
Fins enhance heat transfer from a surface by enhancing surface area.



(*a*) Ideal



(*b*) Actual

FIGURE 3–41
Ideal and actual temperature distribution in a fin.

Fin efficiency relations are developed for fins of various profiles and are plotted in Fig. 3–42 for fins on a *plain surface* and in Fig. 3–43 for *circular fins* of constant thickness. The fin surface area associated with each profile is also given on each figure. For most fins of constant thickness encountered in practice, the fin thickness t is too small relative to the fin length L, and thus the fin tip area is negligible.

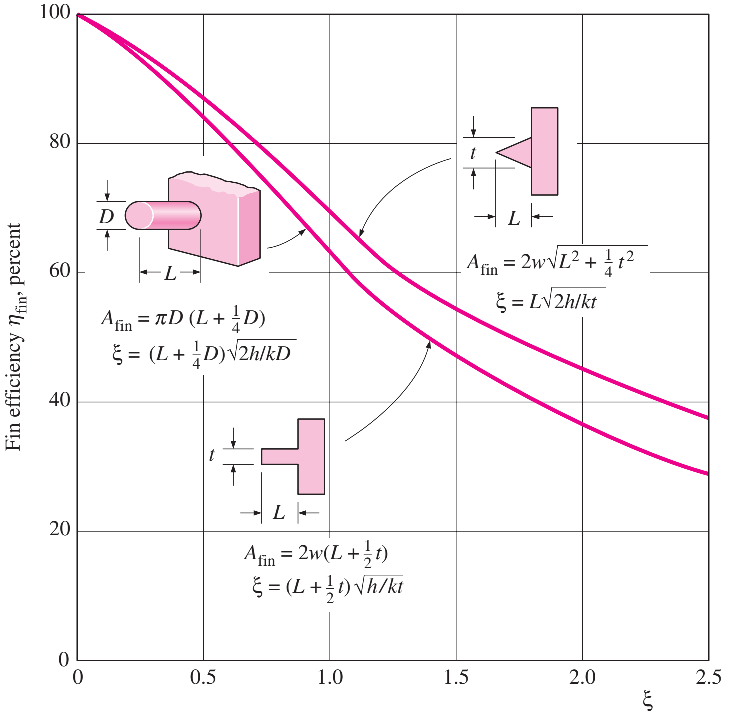


FIGURE 3–42
Efficiency of circular, rectangular, and triangular fins on a plain surface of width w (from Gardner, Ref. 6).

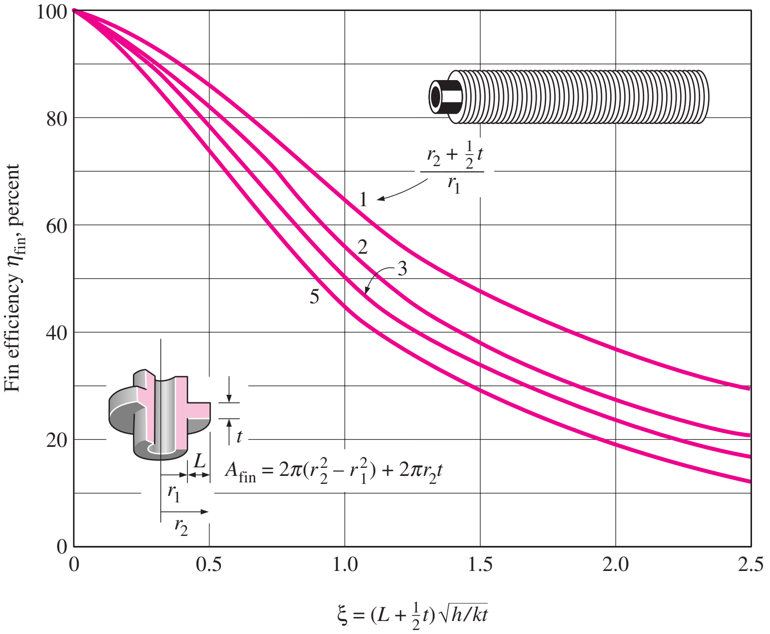


FIGURE 3–43
Efficiency of circular fins of length L and constant thickness t (from Gardner, Ref. 6).

Note that fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles, and thus are more suitable for applications requiring minimum weight such as space applications.

An important consideration in the design of finned surfaces is the selection of the proper *fin length L*. Normally the *longer* the fin, the *larger* the heat transfer area and thus the *higher* the rate of heat transfer from the fin. But also the larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction. Therefore, increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost. Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length. Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically and should be avoided. The efficiency of most fins used in practice is above 90 percent.

Fin Effectiveness

Fins are used to *enhance* heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins. In fact, there is no assurance that adding fins on a surface will *enhance* heat transfer. The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case. The performance of fins expressed in terms of the *fin effectiveness* ε_{fin} is defined as (Fig. 3–44)

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b} \qquad \text{(3-72)}$$

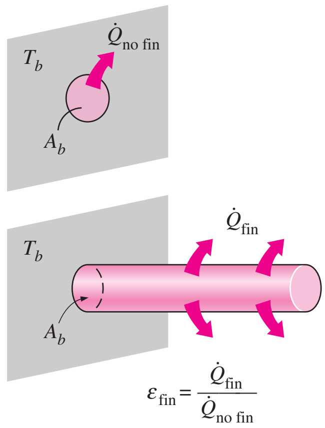


FIGURE 3–44
The effectiveness of a fin.

Here, A_b is the cross-sectional area of the fin at the base and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. An effectiveness of $\varepsilon_{\text{fin}} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all. That is, heat conducted to the fin through the base area A_b is equal to the heat transferred from the same area A_b to the surrounding medium. An effectiveness of $\varepsilon_{\text{fin}} < 1$ indicates that the fin actually acts as *insulation,* slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used. An effectiveness of $\varepsilon_{\text{fin}} > 1$ indicates that fins are *enhancing* heat transfer from the surface, as they should. However, the use of fins cannot be justified unless ε_{fin} is sufficiently larger than 1. Finned surfaces are designed on the basis of *maximizing* effectiveness for a specified cost or *minimizing* cost for a desired effectiveness.

Note that both the fin efficiency and fin effectiveness are related to the performance of the fin, but they are different quantities. However, they are related to each other by

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\eta_{\text{fin}}\, hA_{\text{fin}}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\,\eta_{\text{fin}} \qquad \text{(3-73)}$$

Therefore, the fin effectiveness can be determined easily when the fin efficiency is known, or vice versa.

The rate of heat transfer from a sufficiently *long* fin of *uniform* cross section under steady conditions is given by Eq. 3–61. Substituting this relation into Eq. 3–72, the effectiveness of such a long fin is determined to be

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}} \tag{3-74}$$

since $A_c = A_b$ in this case. We can draw several important conclusions from the fin effectiveness relation above for consideration in the design and selection of the fins:

- The *thermal conductivity k* of the fin material should be as high as possible. Thus it is no coincidence that fins are made from metals, with copper, aluminum, and iron being the most common ones. Perhaps the most widely used fins are made of aluminum because of its low cost and weight and its resistance to corrosion.
- The ratio of the *perimeter* to the *cross-sectional area* of the fin p/A_c should be as high as possible. This criterion is satisfied by *thin* plate fins and *slender* pin fins.
- The use of fins is *most effective* in applications involving a *low convection heat transfer coefficient.* Thus, the use of fins is more easily justified when the medium is a *gas* instead of a liquid and the heat transfer is by *natural convection* instead of by forced convection. Therefore, it is no coincidence that in liquid-to-gas heat exchangers such as the car radiator, fins are placed on the *gas* side.

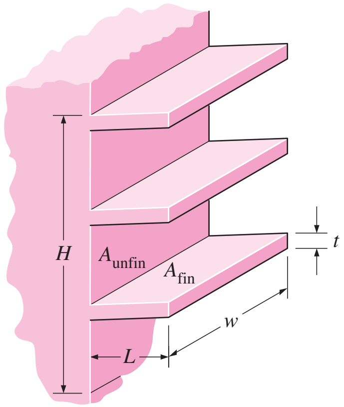
When determining the rate of heat transfer from a finned surface, we must consider the *unfinned portion* of the surface as well as the *fins.* Therefore, the rate of heat transfer for a surface containing *n* fins can be expressed as

$$\begin{aligned} \dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}} \\ &= hA_{\text{unfin}}\,(T_b - T_\infty) + \eta_{\text{fin}}\,hA_{\text{fin}}\,(T_b - T_\infty) \\ &= h(A_{\text{unfin}} + \eta_{\text{fin}}A_{\text{fin}})(T_b - T_\infty) \end{aligned} \tag{3-75}$$

We can also define an **overall effectiveness** for a finned surface as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}}A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}\,(T_b - T_\infty)} \tag{3-76}$$

where $A_{\text{no fin}}$ is the area of the surface when there are no fins, A_{fin} is the total surface area of all the fins on the surface, and A_{unfin} is the area of the unfinned portion of the surface (Fig. 3–45). Note that the overall fin effectiveness depends on the fin density (number of fins per unit length) as well as the effectiveness of the individual fins. The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.



$A_{\text{no fin}} = w \times H$
$A_{\text{unfin}} = w \times H - 3 \times (t \times w)$
$A_{\text{fin}} = 2 \times L \times w + t \times w$ (one fin)
$\approx 2 \times L \times w$

FIGURE 3–45
Various surface areas associated with a rectangular surface with three fins.

Proper Length of a Fin

An important step in the design of a fin is the determination of the appropriate length of the fin once the fin material and the fin cross section are specified. You may be tempted to think that the longer the fin, the larger the surface area and thus the higher the rate of heat transfer. Therefore, for maximum heat transfer, the fin should be infinitely long. However, the temperature drops along the fin exponentially and reaches the environment temperature at some length. The part of the fin beyond this length does not contribute to heat transfer since it is at the temperature of the environment, as shown in Fig. 3–46. Therefore, designing such an "extra long" fin is out of the question since it results in material waste, excessive weight, and increased size and thus increased cost with no benefit in return (in fact, such a long fin will hurt performance since it will suppress fluid motion and thus reduce the convection heat transfer coefficient). Fins that are so long that the temperature approaches the environment temperature cannot be recommended either since the little increase in heat transfer at the tip region cannot justify the large increase in the weight and cost.

To get a sense of the proper length of a fin, we compare heat transfer from a fin of finite length to heat transfer from an infinitely long fin under the same conditions. The ratio of these two heat transfers is

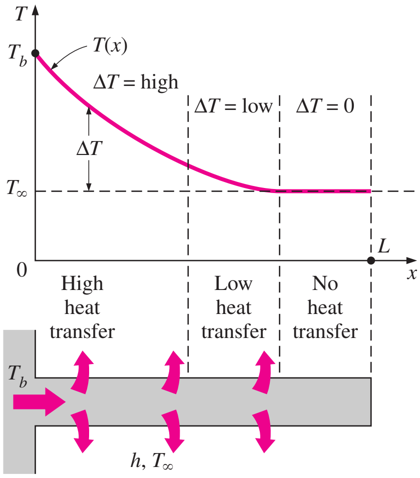


FIGURE 3–46
Because of the gradual temperature drop along the fin, the region near the fin tip makes little or no contribution to heat transfer.

Heat transfer ratio:

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh aL \qquad \textbf{(3-77)}$$

Using a hand calculator, the values of tanh aL are evaluated for some values of aL and the results are given in Table 3–3. We observe from the table that heat transfer from a fin increases with aL almost linearly at first, but the curve reaches a plateau later and reaches a value for the infinitely long fin at about $aL = 5$. Therefore, a fin whose length is $L = \frac{1}{5}a$ can be considered to be an infinitely long fin. We also observe that reducing the fin length by half in that case (from $aL = 5$ to $aL = 2.5$) causes a drop of just 1 percent in heat transfer. We certainly would not hesitate sacrificing 1 percent in heat transfer performance in return for 50 percent reduction in the size and possibly the cost of the fin. In practice, a fin length that corresponds to about $aL = 1$ will transfer 76.2 percent of the heat that can be transferred by an infinitely long fin, and thus it should offer a good compromise between heat transfer performance and the fin size.

A common approximation used in the analysis of fins is to assume the fin temperature varies in one direction only (along the fin length) and the temperature variation along other directions is negligible. Perhaps you are wondering if this one-dimensional approximation is a reasonable one. This is certainly the case for fins made of thin metal sheets such as the fins on a car radiator, but we wouldn't be so sure for fins made of thick materials. Studies have shown that the error involved in one-dimensional fin analysis is negligible (less than about 1 percent) when

$$\frac{h\delta}{k} < 0.2$$

TABLE 3–3
The variation of heat transfer from a fin relative to that from an infinitely long fin

aL	$\dfrac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \tanh aL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

where δ is the characteristic thickness of the fin, which is taken to be the plate thickness t for rectangular fins and the diameter D for cylindrical ones.

Specially designed finned surfaces called *heat sinks,* which are commonly used in the cooling of electronic equipment, involve one-of-a-kind complex geometries, as shown in Table 3–4. The heat transfer performance of heat sinks is usually expressed in terms of their *thermal resistances* R in °C/W, which is defined as

$$\dot{Q}_{\text{fin}} = \frac{T_b - T_\infty}{R} = hA_{\text{fin}}\,\eta_{\text{fin}}\,(T_b - T_\infty) \tag{3-78}$$

A small value of thermal resistance indicates a small temperature drop across the heat sink, and thus a high fin efficiency.

EXAMPLE 3–10 Maximum Power Dissipation of a Transistor

Power transistors that are commonly used in electronic devices consume large amounts of electric power. The failure rate of electronic components increases almost exponentially with operating temperature. As a rule of thumb, the failure rate of electronic components is halved for each 10°C reduction in the junction operating temperature. Therefore, the operating temperature of electronic components is kept below a safe level to minimize the risk of failure.

The sensitive electronic circuitry of a power transistor at the junction is protected by its case, which is a rigid metal enclosure. Heat transfer characteristics of a power transistor are usually specified by the manufacturer in terms of the case-to-ambient thermal resistance, which accounts for both the natural convection and radiation heat transfers.

The case-to-ambient thermal resistance of a power transistor that has a maximum power rating of 10 W is given to be 20°C/W. If the case temperature of the transistor is not to exceed 85°C, determine the power at which this transistor can be operated safely in an environment at 25°C.

SOLUTION The maximum power rating of a transistor whose case temperature is not to exceed 85°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 85°C.

Properties The case-to-ambient thermal resistance is given to be 20°C/W.

Analysis The power transistor and the thermal resistance network associated with it are shown in Fig. 3–47. We notice from the thermal resistance network that there is a single resistance of 20°C/W between the case at T_c = 85°C and the ambient at T_∞ = 25°C, and thus the rate of heat transfer is

$$\dot{Q} = \left(\frac{\Delta T}{R}\right)_{\text{case-ambient}} = \frac{T_c - T_\infty}{R_{\text{case-ambient}}} = \frac{(85 - 25)°\text{C}}{20°\text{C/W}} = \mathbf{3\ W}$$

Therefore, this power transistor should not be operated at power levels above 3 W if its case temperature is not to exceed 85°C.

Discussion This transistor can be used at higher power levels by attaching it to a heat sink (which lowers the thermal resistance by increasing the heat transfer surface area, as discussed in the next example) or by using a fan (which lowers the thermal resistance by increasing the convection heat transfer coefficient).

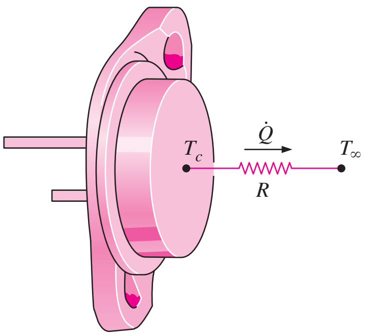
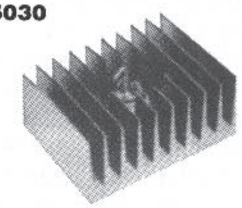
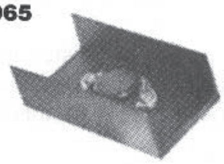
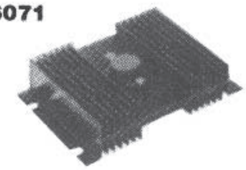
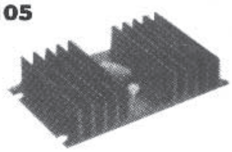
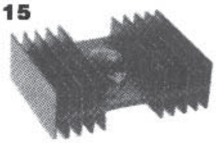
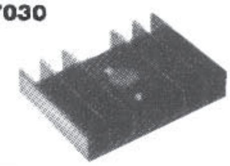


FIGURE 3–47
Schematic for Example 3–10.

TABLE 3–4

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in.) long (courtesy of Vemaline Products, Inc.).

HS 5030 	R = 0.9°C/W (vertical) R = 1.2°C/W (horizontal) Dimensions: 76 mm × 105 mm × 44 mm Surface area: 677 cm^2
HS 6065 	R = 5°C/W Dimensions: 76 mm × 38 mm × 24 mm Surface area: 387 cm^2
HS 6071 	R = 1.4°C/W (vertical) R = 1.8°C/W (horizontal) Dimensions: 76 mm × 92 mm × 26 mm Surface area: 968 cm^2
HS 6105 	R = 1.8°C/W (vertical) R = 2.1°C/W (horizontal) Dimensions: 76 mm × 127 mm × 91 mm Surface area: 677 cm^2
HS 6115 	R = 1.1°C/W (vertical) R = 1.3°C/W (horizontal) Dimensions: 76 mm × 102 mm × 25 mm Surface area: 929 cm^2
HS 7030 	R = 2.9°C/W (vertical) R = 3.1°C/W (horizontal) Dimensions: 76 mm × 97 mm × 19 mm Surface area: 290 cm^2

EXAMPLE 3–11 Selecting a Heat Sink for a Transistor

A 60-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–4. Select a heat sink that will allow the case temperature of the transistor not to exceed 90°C in the ambient air at 30°C.

SOLUTION A commercially available heat sink from Table 3–4 is to be selected to keep the case temperature of a transistor below 90°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The rate of heat transfer from a 60-W transistor at full power is $\dot{Q}$ = 60 W. The thermal resistance between the transistor attached to the heat sink and the ambient air for the specified temperature difference is determined to be

$$\dot{Q} = \frac{\Delta T}{R} \longrightarrow R = \frac{\Delta T}{\dot{Q}} = \frac{(90 - 30)°\text{C}}{60 \text{ W}} = 1.0°\text{C/W}$$

Therefore, the thermal resistance of the heat sink should be below 1.0°C/W. An examination of Table 3–4 reveals that the HS 5030, whose thermal resistance is 0.9°C/W in the vertical position, is the only heat sink that will meet this requirement.

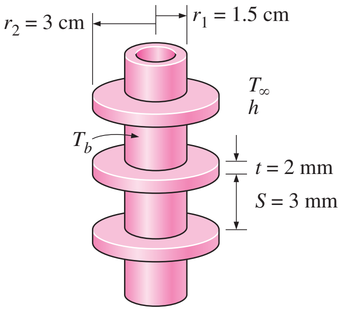


FIGURE 3–48
Schematic for Example 3–12.

EXAMPLE 3–12 Effect of Fins on Heat Transfer from Steam Pipes

Steam in a heating system flows through tubes whose outer diameter is D_1 = 3 cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins (k = 180 W/m · °C) of outer diameter D_2 = 6 cm and constant thickness t = 2 mm are attached to the tube, as shown in Fig. 3–48. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞ = 25°C, with a combined heat transfer coefficient of h = 60 W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

SOLUTION Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be k = 180 W/m · °C.

Analysis In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be

$$\begin{aligned} A_{\text{no fin}} &= \pi D_1 L = \pi(0.03 \text{ m})(1 \text{ m}) = 0.0942 \text{ m}^2 \\ \dot{Q}_{\text{no fin}} &= hA_{\text{no fin}}(T_b - T_\infty) \\ &= (60 \text{ W/m}^2 \cdot °\text{C})(0.0942 \text{ m}^2)(120 - 25)°\text{C} \\ &= 537 \text{ W} \end{aligned}$$

The efficiency of the circular fins attached to a circular tube is plotted in Fig. 3–43. Noting that $L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m in this case, we have

$$\left.\begin{aligned} &\frac{r_2 + \frac{1}{2}t}{r_1} = \frac{(0.03 + \frac{1}{2} \times 0.002)\text{ m}}{0.015\text{ m}} = 2.07 \\ &(L + \tfrac{1}{2}t)\sqrt{\frac{h}{kt}} = (0.015 + \tfrac{1}{2} \times 0.002)\text{ m} \times \sqrt{\frac{60\text{ W/m}^2 \cdot {}^\circ\text{C}}{(180\text{ W/m} \cdot {}^\circ\text{C})(0.002\text{ m})}} = 0.207 \end{aligned}\right\} \eta_{\text{fin}} = 0.95$$

$$\begin{aligned} A_{\text{fin}} &= 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t \\ &= 2\pi[(0.03\text{ m})^2 - (0.015\text{ m})^2] + 2\pi(0.03\text{ m})(0.002\text{ m}) \\ &= 0.00462\text{ m}^2 \end{aligned}$$

$$\begin{aligned} \dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} h A_{\text{fin}}(T_b - T_\infty) \\ &= 0.95(60\text{ W/m}^2 \cdot {}^\circ\text{C})(0.00462\text{ m}^2)(120 - 25)^\circ\text{C} \\ &= 25.0\text{ W} \end{aligned}$$

Heat transfer from the unfinned portion of the tube is

$$\begin{aligned} A_{\text{unfin}} &= \pi D_1 S = \pi(0.03\text{ m})(0.003\text{ m}) = 0.000283\text{ m}^2 \\ \dot{Q}_{\text{unfin}} &= hA_{\text{unfin}}(T_b - T_\infty) \\ &= (60\text{ W/m}^2 \cdot {}^\circ\text{C})(0.000283\text{ m}^2)(120 - 25)^\circ\text{C} \\ &= 1.60\text{ W} \end{aligned}$$

Noting that there are 200 fins and thus 200 interfin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6)\text{ W} = 5320\text{ W}$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = \mathbf{4783\text{ W}} \qquad \text{(per m tube length)}$$

Discussion The overall effectiveness of the finned tube is

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{5320\text{ W}}{537\text{ W}} = 9.9$$

That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of finned surfaces.

3-7 ▪ HEAT TRANSFER IN COMMON CONFIGURATIONS

So far, we have considered heat transfer in *simple* geometries such as large plane walls, long cylinders, and spheres. This is because heat transfer in such geometries can be approximated as *one-dimensional,* and simple analytical solutions can be obtained easily. But many problems encountered in practice are two- or three-dimensional and involve rather complicated geometries for which no simple solutions are available.

An important class of heat transfer problems for which simple solutions are obtained encompasses those involving two surfaces maintained at *constant* temperatures T_1 and T_2. The steady rate of heat transfer between these two surfaces is expressed as

$$Q = Sk(T_1 - T_2) \tag{3-79}$$

where S is the **conduction shape factor,** which has the dimension of *length,* and k is the thermal conductivity of the medium between the surfaces. The conduction shape factor depends on the *geometry* of the system only.

Conduction shape factors have been determined for a number of configurations encountered in practice and are given in Table 3–5 for some common cases. More comprehensive tables are available in the literature. Once the value of the shape factor is known for a specific geometry, the total steady heat transfer rate can be determined from the equation above using the specified two constant temperatures of the two surfaces and the thermal conductivity of the medium between them. Note that conduction shape factors are applicable only when heat transfer between the two surfaces is by *conduction.* Therefore, they cannot be used when the medium between the surfaces is a liquid or gas, which involves natural or forced convection currents.

A comparison of Equations 3-4 and 3-79 reveals that the conduction shape factor S is related to the thermal resistance R by $R = 1/kS$ or $S = 1/kR$. Thus, these two quantities are the inverse of each other when the thermal conductivity of the medium is unity. The use of the conduction shape factors is illustrated with examples 3–13 and 3–14.

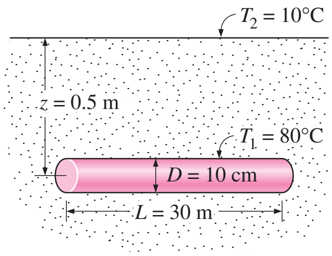


FIGURE 3–49
Schematic for Example 3–13.

EXAMPLE 3–13 Heat Loss from Buried Steam Pipes

A 30-m-long, 10-cm-diameter hot water pipe of a district heating system is buried in the soil 50 cm below the ground surface, as shown in Figure 3–49. The outer surface temperature of the pipe is 80°C. Taking the surface temperature of the earth to be 10°C and the thermal conductivity of the soil at that location to be 0.9 W/m · °C, determine the rate of heat loss from the pipe.

SOLUTION The hot water pipe of a district heating system is buried in the soil. The rate of heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be $k = 0.9$ W/m · °C.

Analysis The shape factor for this configuration is given in Table 3–5 to be

$$S = \frac{2\pi L}{\ln(4z/D)}$$

since $z > 1.5D$, where z is the distance of the pipe from the ground surface, and D is the diameter of the pipe. Substituting,

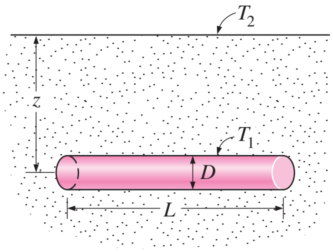
$$S = \frac{2\pi \times (30 \text{ m})}{\ln(4 \times 0.5/0.1)} = 62.9 \text{ m}$$

TABLE 3-5

Conduction shape factors S for several configurations for use in $\dot{Q} = kS(T_1 - T_2)$ to determine the steady rate of heat transfer through a medium of thermal conductivity k between the surfaces at temperatures T_1 and T_2

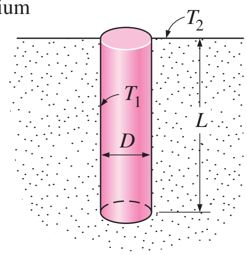
(1) Isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$ and $z > 1.5D$)

$$S = \frac{2\pi L}{\ln(4z/D)}$$



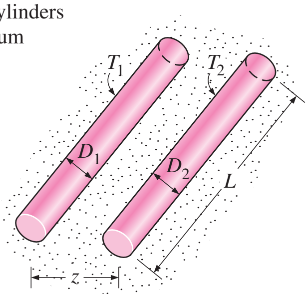
(2) Vertical isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$)

$$S = \frac{2\pi L}{\ln(4L/D)}$$



(3) Two parallel isothermal cylinders placed in an infinite medium ($L >> D_1, D_2, z$)

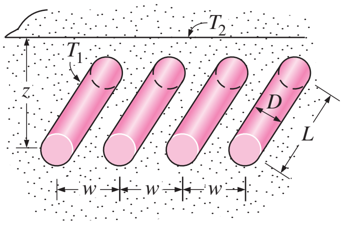
$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$$



(4) A row of equally spaced parallel isothermal cylinders buried in a semi-infinite medium ($L >> D, z$ and $w > 1.5D$)

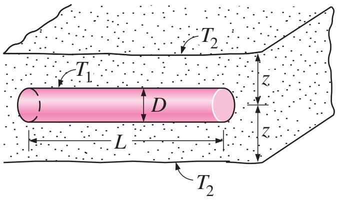
$$S = \frac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\sinh\dfrac{2\pi z}{w}\right)}$$

(per cylinder)



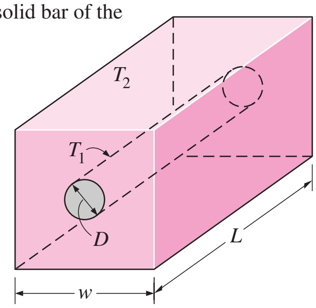
(5) Circular isothermal cylinder of length L in the midplane of an infinite wall ($z > 0.5D$)

$$S = \frac{2\pi L}{\ln(8z/\pi D)}$$



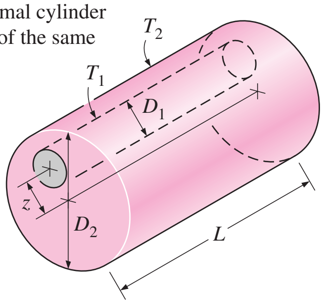
(6) Circular isothermal cylinder of length L at the center of a square solid bar of the same length

$$S = \frac{2\pi L}{\ln(1.08\,w/D)}$$



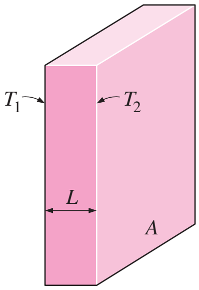
(7) Eccentric circular isothermal cylinder of length L in a cylinder of the same length ($L > D_2$)

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4z^2}{2D_1D_2}\right)}$$



(8) Large plane wall

$$S = \frac{A}{L}$$

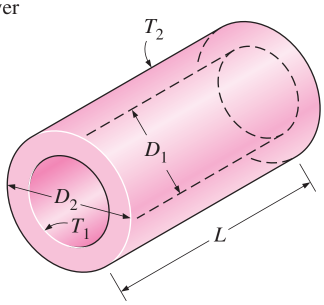


(continued)

TABLE 3-5 (CONCLUDED)

(9) A long cylindrical layer

$$S = \frac{2\pi L}{\ln (D_2/D_1)}$$



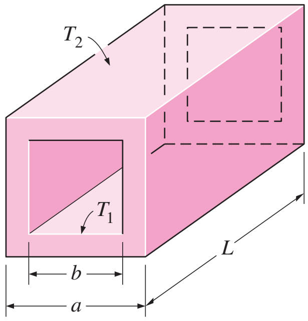
(10) A square flow passage

(*a*) For $a/b > 1.4$,

$$S = \frac{2\pi L}{0.93 \ln (0.948\, a/b)}$$

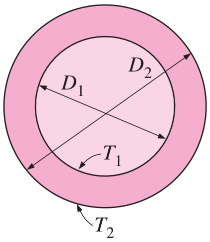
(*b*) For $a/b < 1.41$,

$$S = \frac{2\pi L}{0.785 \ln (a/b)}$$



(11) A spherical layer

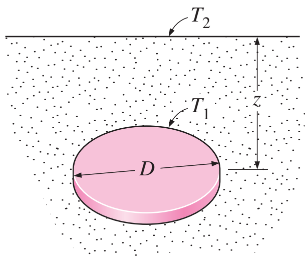
$$S = \frac{2\pi D_1 D_2}{D_2 - D_1}$$



(12) Disk buried parallel to the surface in a semi-infinite medium ($z >> D$)

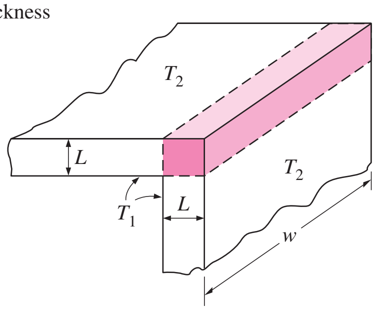
$S = 4\,D$

($S = 2\,D$ when $z = 0$)



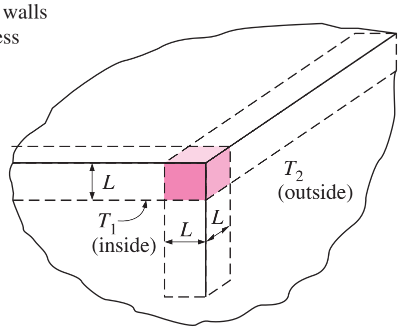
(13) The edge of two adjoining walls of equal thickness

$S = 0.54\,w$



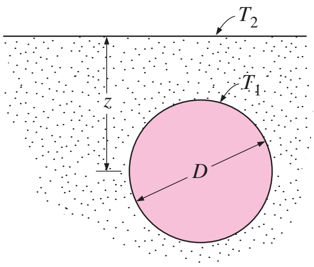
(14) Corner of three walls of equal thickness

$S = 0.15\,L$



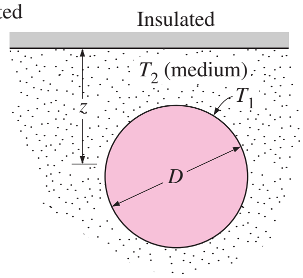
(15) Isothermal sphere buried in a semi-infinite medium

$$S = \frac{2\pi D}{1 - 0.25D/z}$$



(16) Isothermal sphere buried in a semi-infinite medium at T_2 whose surface is insulated

$$S = \frac{2\pi D}{1 + 0.25D/z}$$



Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (62.9 \text{ m})(0.9 \text{ W/m} \cdot {}^\circ\text{C})(80 - 10)^\circ\text{C} = \mathbf{3963 \text{ W}}$$

Discussion Note that this heat is conducted from the pipe surface to the surface of the earth through the soil and then transferred to the atmosphere by convection and radiation.

EXAMPLE 3–14 Heat Transfer between Hot and Cold Water Pipes

A 5-m-long section of hot and cold water pipes run parallel to each other in a thick concrete layer, as shown in Figure 3–50. The diameters of both pipes are 5 cm, and the distance between the centerline of the pipes is 30 cm. The surface temperatures of the hot and cold pipes are 70°C and 15°C, respectively. Taking the thermal conductivity of the concrete to be k = 0.75 W/m · °C, determine the rate of heat transfer between the pipes.

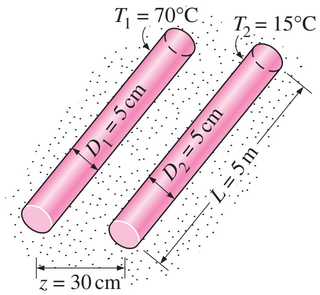


FIGURE 3–50
Schematic for Example 3-14.

SOLUTION Hot and cold water pipes run parallel to each other in a thick concrete layer. The rate of heat transfer between the pipes is to be determined.
Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.
Properties The thermal conductivity of concrete is given to be k = 0.75 W/m · °C.
Analysis The shape factor for this configuration is given in Table 3–5 to be

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$$

where z is the distance between the centerlines of the pipes and L is their length. Substituting,

$$S = \frac{2\pi \times (5 \text{ m})}{\cosh^{-1}\left(\dfrac{4 \times 0.3^2 - 0.05^2 - 0.05^2}{2 \times 0.05 \times 0.05}\right)} = 6.34 \text{ m}$$

Then the steady rate of heat transfer between the pipes becomes

$$\dot{Q} = Sk(T_1 - T_2) = (6.34 \text{ m})(0.75 \text{ W/m} \cdot {}^\circ\text{C})(70 - 15^\circ)\text{C} = \mathbf{262 \text{ W}}$$

Discussion We can reduce this heat loss by placing the hot and cold water pipes further away from each other.

It is well known that insulation reduces heat transfer and saves energy and money. Decisions on the right amount of insulation are based on a heat transfer analysis, followed by an economic analysis to determine the "monetary value" of energy loss. This is illustrated with Example 3–15.

EXAMPLE 3–15 Cost of Heat Loss through Walls in Winter

Consider an electrically heated house whose walls are 9 ft high and have an *R*-value of insulation of 13 (i.e., a thickness-to-thermal conductivity ratio of L/k = 13 h · ft² · °F/Btu). Two of the walls of the house are 40 ft long and the others are 30 ft long. The house is maintained at 75°F at all times, while the temperature of the outdoors varies. Determine the amount of heat lost through the walls of the house on a certain day during which the average temperature of the outdoors is 45°F. Also, determine the cost of this heat loss to the homeowner if the unit cost of electricity is $0.075/kWh. For combined convection and radiation heat transfer coefficients, use the ASHRAE (American Society of Heating, Refrigeration, and Air Conditioning Engineers) recommended values of h_i = 1.46 Btu/h · ft² · °F for the inner surface of the walls and h_o = 4.0 Btu/h · ft² · °F for the outer surface of the walls under 15 mph wind conditions in winter.

SOLUTION An electrically heated house with R-13 insulation is considered. The amount of heat lost through the walls and its cost are to be determined.

Assumptions **1** The indoor and outdoor air temperatures have remained at the given values for the entire day so that heat transfer through the walls is steady. **2** Heat transfer through the walls is one-dimensional since any significant temperature gradients in this case will exist in the direction from the indoors to the outdoors. **3** The radiation effects are accounted for in the heat transfer coefficients.

Analysis This problem involves conduction through the wall and convection at its surfaces and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3–51. The heat transfer area of the walls is

$$A = \text{Circumference} \times \text{Height} = (2 \times 30\text{ ft} + 2 \times 40\text{ ft})(9\text{ ft}) = 1260\text{ ft}^2$$

Then the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv},\,i} = \frac{1}{h_i A} = \frac{1}{(1.46\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(1260\text{ ft}^2)} = 0.00054\text{ h}\cdot{}^\circ\text{F/Btu}$$

$$R_{\text{wall}} = \frac{L}{kA} = \frac{R\text{-value}}{A} = \frac{13\text{ h}\cdot\text{ft}^2\cdot{}^\circ\text{F/Btu}}{1260\text{ ft}^2} = 0.01032\text{ h}\cdot{}^\circ\text{F/Btu}$$

$$R_o = R_{\text{conv},\,o} = \frac{1}{h_c A} = \frac{1}{(4.0\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(1260\text{ ft}^2)} = 0.00020\text{ h}\cdot{}^\circ\text{F/Btu}$$

Noting that all three resistances are in series, the total resistance is

$$R_{\text{total}} = R_i + R_{\text{wall}} + R_o = 0.00054 + 0.01032 + 0.00020 = 0.01106\text{ h}\cdot{}^\circ\text{F/Btu}$$

Then the steady rate of heat transfer through the walls of the house becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(75 - 45)^\circ\text{F}}{0.01106\text{ h}\cdot{}^\circ\text{F/Btu}} = 2712\text{ Btu/h}$$

Finally, the total amount of heat lost through the walls during a 24-h period and its cost to the home owner are

$$Q = \dot{Q}\,\Delta t = (2712\text{ Btu/h})(24\text{-h/day}) = \mathbf{65{,}099\text{ Btu/day} = 19.1\text{ kWh/day}}$$

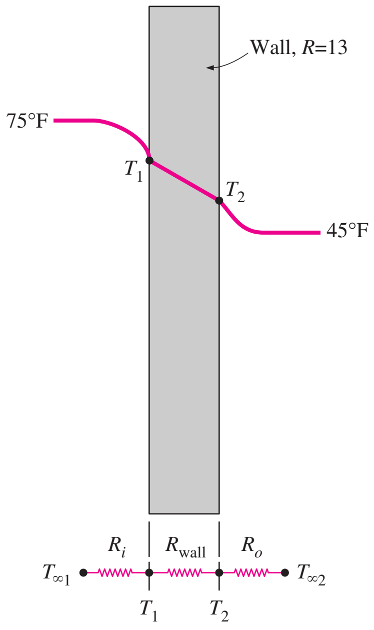


FIGURE 3–51
Schematic for Example 3–15.

since 1 kWh = 3412 Btu, and

$$\text{Heating cost} = (\text{Energy lost})(\text{Cost of energy}) = (19.1 \text{ kWh/day})(\$0.075/\text{kWh}) = \mathbf{\$1.43/day}$$

Discussion The heat losses through the walls of the house that day will cost the home owner $1.43 worth of electricity.

TOPIC OF SPECIAL INTEREST*

Heat Transfer Through Walls and Roofs

Under steady conditions, the rate of heat transfer through any section of a building wall or roof can be determined from

$$\dot{Q} = UA(T_i - T_o) = \frac{A(T_i - T_o)}{R} \qquad \text{(3-80)}$$

where T_i and T_o are the indoor and outdoor air temperatures, A is the heat transfer area, U is the overall heat transfer coefficient (the U-factor), and $R = 1/U$ is the overall unit thermal resistance (the R-value). Walls and roofs of buildings consist of various layers of materials, and the structure and operating conditions of the walls and the roofs may differ significantly from one building to another. Therefore, it is not practical to list the R-values (or U-factors) of different kinds of walls or roofs under different conditions. Instead, the overall R-value is determined from the thermal resistances of the individual components using the thermal resistance network. The overall thermal resistance of a structure can be determined most accurately in a lab by actually assembling the unit and testing it as a whole, but this approach is usually very time consuming and expensive. The analytical approach described here is fast and straightforward, and the results are usually in good agreement with the experimental values.

The unit thermal resistance of a plane layer of thickness L and thermal conductivity k can be determined from $R = L/k$. The thermal conductivity and other properties of common building materials are given in the appendix. The unit thermal resistances of various components used in building structures are listed in Table 3–6 for convenience.

Heat transfer through a wall or roof section is also affected by the convection and radiation heat transfer coefficients at the exposed surfaces. The effects of convection and radiation on the inner and outer surfaces of walls and roofs are usually combined into the *combined convection and radiation heat transfer coefficients* (also called *surface conductances*) h_i and h_o, respectively, whose values are given in Table 3–7 for ordinary surfaces ($\varepsilon = 0.9$) and reflective surfaces ($\varepsilon = 0.2$ or 0.05). Note that surfaces having a low emittance also have a low surface conductance due to the reduction in radiation heat transfer. The values in the table are based on a surface

*This section can be skipped without a loss of continuity.

TABLE 3–7

Combined convection and radiation heat transfer coefficients at window, wall, or roof surfaces (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 22, Table 1).

Position	Direction of Heat Flow	h, W/m² · °C*: Surface Emittance, ε 0.90	0.20	0.05
Still air (both indoors and outdoors)				
Horiz.	Up ↑	9.26	5.17	4.32
Horiz.	Down ↓	6.13	2.10	1.25
45° slope	Up ↑	9.09	5.00	4.15
45° slope	Down ↓	7.50	3.41	2.56
Vertical	Horiz. →	8.29	4.20	3.35
Moving air (any position, any direction)				
Winter condition (winds at 15 mph or 24 km/h)		34.0	—	—
Summer condition (winds at 7.5 mph or 12 km/h)		22.7	—	—

*Multiply by 0.176 to convert to Btu/h · ft² · °F.
Surface resistance can be obtained from $R = 1/h$.

TABLE 3–6

Unit thermal resistance (the *R*-value) of common components used in buildings

Component	*R*-value m² · °C/W	*R*-value ft² · h · °F/Btu
Outside surface (winter)	0.030	0.17
Outside surface (summer)	0.044	0.25
Inside surface, still air	0.12	0.68
Plane air space, vertical, ordinary surfaces ($\varepsilon_{eff} = 0.82$):		
13 mm ($\frac{1}{2}$ in.)	0.16	0.90
20 mm ($\frac{3}{4}$ in.)	0.17	0.94
40 mm (1.5 in.)	0.16	0.90
90 mm (3.5 in.)	0.16	0.91
Insulation, 25 mm (1 in.)		
Glass fiber	0.70	4.00
Mineral fiber batt	0.66	3.73
Urethane rigid foam	0.98	5.56
Stucco, 25 mm (1 in.)	0.037	0.21
Face brick, 100 mm (4 in.)	0.075	0.43
Common brick, 100 mm (4 in.)	0.12	0.79
Steel siding	0.00	0.00
Slag, 13 mm ($\frac{1}{2}$ in.)	0.067	0.38
Wood, 25 mm (1 in.)	0.22	1.25
Wood stud, nominal 2 in. × 4 in. (3.5 in. or 90 mm wide)	0.63	3.58
Wood stud, nominal 2 in. × 6 in. (5.5 in. or 140 mm wide)	0.98	5.56
Clay tile, 100 mm (4 in.)	0.18	1.01
Acoustic tile	0.32	1.79
Asphalt shingle roofing	0.077	0.44
Building paper	0.011	0.06
Concrete block, 100 mm (4 in.):		
Lightweight	0.27	1.51
Heavyweight	0.13	0.71
Plaster or gypsum board, 13 mm ($\frac{1}{2}$ in.)	0.079	0.45
Wood fiberboard, 13 mm ($\frac{1}{2}$ in.)	0.23	1.31
Plywood, 13 mm ($\frac{1}{2}$ in.)	0.11	0.62
Concrete, 200 mm (8 in.):		
Lightweight	1.17	6.67
Heavyweight	0.12	0.67
Cement mortar, 13 mm (1/2 in.)	0.018	0.10
Wood bevel lapped siding, 13 mm × 200 mm (1/2 in. × 8 in.)	0.14	0.81

temperature of 21°C (72°F) and a surface–air temperature difference of 5.5°C (10°F). Also, the equivalent surface temperature of the environment is assumed to be equal to the ambient air temperature. Despite the convenience it offers, this assumption is not quite accurate because of the additional radiation heat loss from the surface to the clear sky. The effect of sky radiation can be accounted for approximately by taking the outside temperature to be the average of the outdoor air and sky temperatures.

The inner surface heat transfer coefficient h_i remains fairly constant throughout the year, but the value of h_o varies considerably because of its dependence on the orientation and wind speed, which can vary from less than 1 km/h in calm weather to over 40 km/h during storms. The commonly used values of h_i and h_o for peak load calculations are

$$h_i = 8.29 \text{ W/m}^2\cdot°\text{C} = 1.46 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F} \qquad \text{(winter and summer)}$$

$$h_o = \begin{cases} 34.0 \text{ W/m}^2\cdot°\text{C} = 6.0 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F} & \text{(winter)} \\ 22.7 \text{ W/m}^2\cdot°\text{C} = 4.0 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F} & \text{(summer)} \end{cases}$$

which correspond to design wind conditions of 24 km/h (15 mph) for winter and 12 km/h (7.5 mph) for summer. The corresponding surface thermal resistances (*R*-values) are determined from $R_i = 1/h_i$ and $R_o = 1/h_o$. The surface conductance values under still air conditions can be used for interior surfaces as well as exterior surfaces in calm weather.

Building components often involve *trapped air spaces* between various layers. Thermal resistances of such air spaces depend on the thickness of the layer, the temperature difference across the layer, the mean air temperature, the emissivity of each surface, the orientation of the air layer, and the direction of heat transfer. The emissivities of surfaces commonly encountered in buildings are given in Table 3–8. The **effective emissivity** of a plane-parallel air space is given by

$$\frac{1}{\varepsilon_{\text{effective}}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \qquad \text{(3-81)}$$

where ε_1 and ε_2 are the emissivities of the surfaces of the air space. Table 3–8 also lists the effective emissivities of air spaces for the cases where (1) the emissivity of one surface of the air space is ε while the emissivity of the other surface is 0.9 (a building material) and (2) the emissivity of both surfaces is ε. Note that the effective emissivity of an air space between building materials is 0.82/0.03 = 27 times that of an air space between surfaces covered with aluminum foil. For specified surface temperatures, radiation heat transfer through an air space is proportional to effective emissivity, and thus the rate of radiation heat transfer in the ordinary surface case is 27 times that of the reflective surface case.

Table 3–9 lists the thermal resistances of 20-mm-, 40-mm-, and 90-mm- (0.75-in., 1.5-in., and 3.5-in.) thick air spaces under various conditions. The thermal resistance values in the table are applicable to air spaces of uniform thickness bounded by plane, smooth, parallel surfaces with no air leakage. Thermal resistances for other temperatures, emissivities, and air spaces can be obtained by interpolation and moderate extrapolation. Note that the presence of a low-emissivity surface reduces radiation heat transfer across an air space and thus significantly increases the thermal resistance. The thermal effectiveness of a low-emissivity surface will decline, however, if the condition of the surface changes as a result of some effects such as condensation, surface oxidation, and dust accumulation.

The *R*-value of a wall or roof structure that involves layers of uniform thickness is determined easily by simply adding up the unit thermal resistances of the layers that are in series. But when a structure involves components such as wood studs and metal connectors, then the thermal resistance network involves parallel connections and possible two-dimensional effects. The overall *R*-value in this case can be determined by assuming (1) parallel heat flow paths through areas of different construction or (2) isothermal planes normal to the direction of heat transfer. The first approach usually overpredicts the overall thermal resistance, whereas the second approach usually underpredicts it. The parallel heat flow path approach is more suitable for wood frame walls and roofs, whereas the isothermal planes approach is more suitable for masonry or metal frame walls.

The thermal contact resistance between different components of building structures ranges between 0.01 and 0.1 m^2 · °C/W, which is negligible in most cases. However, it may be significant for metal building components such as steel framing members.

TABLE 3–8

Emissivities ε of various surfaces and the effective emissivity of air spaces (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 22, Table 3).

		Effective Emissivity of Air Space	
Surface	ε	$\varepsilon_1 = \varepsilon$, $\varepsilon_2 = 0.9$	$\varepsilon_1 = \varepsilon$, $\varepsilon_2 = \varepsilon$
Aluminum foil, bright	0.05*	0.05	0.03
Aluminum sheet	0.12	0.12	0.06
Aluminum-coated paper, polished	0.20	0.20	0.11
Steel, galvanized, bright	0.25	0.24	0.15
Aluminum paint	0.50	0.47	0.35
Building materials: Wood, paper, masonry, nonmetallic paints	0.90	0.82	0.82
Ordinary glass	0.84	0.77	0.72

*Surface emissivity of aluminum foil increases to 0.30 with barely visible condensation, and to 0.70 with clearly visible condensation.

TABLE 3–9

Unit thermal resistances (R-values) of well-sealed plane air spaces (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 22, Table 2)

(*a*) SI units (in $m^2 \cdot °C/W$)

Position of Air Space	Direction of Heat Flow	Mean Temp., °C	Temp. Diff., °C	20-mm Air Space: Effective Emissivity, ε_{eff}				40-mm Air Space: Effective Emissivity, ε_{eff}				90-mm Air Space: Effective Emissivity, ε_{eff}			
				0.03	0.05	0.5	0.82	0.03	0.05	0.5	0.82	0.03	0.05	0.5	0.82
		32.2	5.6	0.41	0.39	0.18	0.13	0.45	0.42	0.19	0.14	0.50	0.47	0.20	0.14
		10.0	16.7	0.30	0.29	0.17	0.14	0.33	0.32	0.18	0.14	0.27	0.35	0.19	0.15
Horizontal	Up ↑	10.0	5.6	0.40	0.39	0.20	0.15	0.44	0.42	0.21	0.16	0.49	0.47	0.23	0.16
		−17.8	11.1	0.32	0.32	0.20	0.16	0.35	0.34	0.22	0.17	0.40	0.38	0.23	0.18
		32.2	5.6	0.52	0.49	0.20	0.14	0.51	0.48	0.20	0.14	0.56	0.52	0.21	0.14
		10.0	16.7	0.35	0.34	0.19	0.14	0.38	0.36	0.20	0.15	0.40	0.38	0.20	0.15
45° slope	Up ↑	10.0	5.6	0.51	0.48	0.23	0.17	0.51	0.48	0.23	0.17	0.55	0.52	0.24	0.17
		−17.8	11.1	0.37	0.36	0.23	0.18	0.40	0.39	0.24	0.18	0.43	0.41	0.24	0.19
		32.2	5.6	0.62	0.57	0.21	0.15	0.70	0.64	0.22	0.15	0.65	0.60	0.22	0.15
		10.0	16.7	0.51	0.49	0.23	0.17	0.45	0.43	0.22	0.16	0.47	0.45	0.22	0.16
Vertical	Horizontal →	10.0	5.6	0.65	0.61	0.25	0.18	0.67	0.62	0.26	0.18	0.64	0.60	0.25	0.18
		−17.8	11.1	0.55	0.53	0.28	0.21	0.49	0.47	0.26	0.20	0.51	0.49	0.27	0.20
		32.2	5.6	0.62	0.58	0.21	0.15	0.89	0.80	0.24	0.16	0.85	0.76	0.24	0.16
		10.0	16.7	0.60	0.57	0.24	0.17	0.63	0.59	0.25	0.18	0.62	0.58	0.25	0.18
45° slope	Down ↓	10.0	5.6	0.67	0.63	0.26	0.18	0.90	0.82	0.28	0.19	0.83	0.77	0.28	0.19
		−17.8	11.1	0.66	0.63	0.30	0.22	0.68	0.64	0.31	0.22	0.67	0.64	0.31	0.22
		32.2	5.6	0.62	0.58	0.21	0.15	1.07	0.94	0.25	0.17	1.77	1.44	0.28	0.18
		10.0	16.7	0.66	0.62	0.25	0.18	1.10	0.99	0.30	0.20	1.69	1.44	0.33	0.21
Horizontal	Down ↓	10.0	5.6	0.68	0.63	0.26	0.18	1.16	1.04	0.30	0.20	1.96	1.63	0.34	0.22
		−17.8	11.1	0.74	0.70	0.32	0.23	1.24	1.13	0.39	0.26	1.92	1.68	0.43	0.29

(*b*) English units (in $h \cdot ft^2 \cdot °F/Btu$)

Position of Air Space	Direction of Heat Flow	Mean Temp., °F	Temp. Diff., °F	0.75-in. Air Space: Effective Emissivity, ε_{eff}				1.5-in. Air Space: Effective Emissivity, ε_{eff}				3.5-in. Air Space: Effective Emissivity, ε_{eff}			
				0.03	0.05	0.5	0.82	0.03	0.05	0.5	0.82	0.03	0.05	0.5	0.82
		90	10	2.34	2.22	1.04	0.75	2.55	2.41	1.08	0.77	2.84	2.66	1.13	0.80
		50	30	1.71	1.66	0.99	0.77	1.87	1.81	1.04	0.80	2.09	2.01	1.10	0.84
Horizontal	Up ↑	50	10	2.30	2.21	1.16	0.87	2.50	2.40	1.21	0.89	2.80	2.66	1.28	0.93
		0	20	1.83	1.79	1.16	0.93	2.01	1.95	1.23	0.97	2.25	2.18	1.32	1.03
		90	10	2.96	2.78	1.15	0.81	2.92	2.73	1.14	0.80	3.18	2.96	1.18	0.82
		50	30	1.99	1.92	1.08	0.82	2.14	2.06	1.12	0.84	2.26	2.17	1.15	0.86
45° slope	Up ↑	50	10	2.90	2.75	1.29	0.94	2.88	2.74	1.29	0.94	3.12	2.95	1.34	0.96
		0	20	2.13	2.07	1.28	1.00	2.30	2.23	1.34	1.04	2.42	2.35	1.38	1.06
		90	10	3.50	3.24	1.22	0.84	3.99	3.66	1.27	0.87	3.69	3.40	1.24	0.85
		50	30	2.91	2.77	1.30	0.94	2.58	2.46	1.23	0.90	2.67	2.55	1.25	0.91
Vertical	Horizontal →	50	10	3.70	3.46	1.43	1.01	3.79	3.55	1.45	1.02	3.63	3.40	1.42	1.01
		0	20	3.14	3.02	1.58	1.18	2.76	2.66	1.48	1.12	2.88	2.78	1.51	1.14
		90	10	3.53	3.27	1.22	0.84	5.07	4.55	1.36	0.91	4.81	4.33	1.34	0.90
		50	30	3.43	3.23	1.39	0.99	3.58	3.36	1.42	1.00	3.51	3.30	1.40	1.00
45° slope	Down ↓	50	10	3.81	3.57	1.45	1.02	5.10	4.66	1.60	1.09	4.74	4.36	1.57	1.08
		0	20	3.75	3.57	1.72	1.26	3.85	3.66	1.74	1.27	3.81	3.63	1.74	1.27
		90	10	3.55	3.29	1.22	0.85	6.09	5.35	1.43	0.94	10.07	8.19	1.57	1.00
		50	30	3.77	3.52	1.44	1.02	6.27	5.63	1.70	1.14	9.60	8.17	1.88	1.22
Horizontal	Down ↓	50	10	3.84	3.59	1.45	1.02	6.61	5.90	1.73	1.15	11.15	9.27	1.93	1.24
		0	20	4.18	3.96	1.81	1.30	7.03	6.43	2.19	1.49	10.90	9.52	2.47	1.62

EXAMPLE 3–16 The *R*-Value of a Wood Frame Wall

Determine the overall unit thermal resistance (the *R*-value) and the overall heat transfer coefficient (the *U*-factor) of a wood frame wall that is built around 38-mm × 90-mm (2 × 4 nominal) wood studs with a center-to-center distance of 400 mm. The 90-mm-wide cavity between the studs is filled with glass fiber insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13-mm wood fiberboard and 13-mm × 200-mm wood bevel lapped siding. The insulated cavity constitutes 75 percent of the heat transmission area while the studs, plates, and sills constitute 21 percent. The headers constitute 4 percent of the area, and they can be treated as studs.

Also, determine the rate of heat loss through the walls of a house whose perimeter is 50 m and wall height is 2.5 m in Las Vegas, Nevada, whose winter design temperature is −2°C. Take the indoor design temperature to be 22°C and assume 20 percent of the wall area is occupied by glazing.

SOLUTION The *R*-value and the *U*-factor of a wood frame wall as well as the rate of heat loss through such a wall in Las Vegas are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The *R*-values of different materials are given in Table 3–6.

Analysis The schematic of the wall as well as the different elements used in its construction are shown here. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factors for the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}}$$

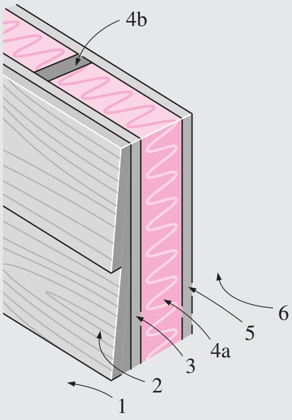
where

$$U_{\text{overall}} = (U \times f_{\text{area}})_{\text{insulation}} + (U \times f_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.75 for the insulation section and 0.25 for the stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 3–6 and calculating others, the total *R*-values for each section can be determined in a systematic manner in the table in this sample.

We conclude that the overall unit thermal resistance of the wall is 2.23 $m^2 \cdot °C/W$, and this value accounts for the effects of the studs and headers. It corresponds to an *R*-value of 2.23 × 5.68 = 12.7 (or nearly *R*-13) in English units. Note that if there were no wood studs and headers in the wall, the overall thermal resistance would be 3.05 $m^2 \cdot °C/W$, which is 37 percent greater than 2.23 $m^2 \cdot °C/W$. Therefore, the wood studs and headers in this case serve as thermal bridges in wood frame walls, and their effect must be considered in the thermal analysis of buildings.

Schematic



Construction	*R*-value, $m^2 \cdot °C/W$ Between Studs	At Studs
1. Outside surface, 24 km/h wind	0.030	0.030
2. Wood bevel lapped siding	0.14	0.14
3. Wood fiberboard sheeting, 13 mm	0.23	0.23
4a. Glass fiber insulation, 90 mm	2.45	—
4b. Wood stud, 38 mm × 90 mm	—	0.63
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12
Total unit thermal resistance of each section, R (in $m^2 \cdot °C/W$)	3.05	1.23
The U-factor of each section, $U = 1/R$, in $W/m^2 \cdot °C$	0.328	0.813
Area fraction of each section, f_{area}	0.75	0.25

Overall U-factor: $U = \Sigma f_{area,\, i}\, U_i = 0.75 \times 0.328 + 0.25 \times 0.813$
$= \mathbf{0.449\ W/m^2 \cdot °C}$

Overall unit thermal resistance: $R = 1/U = \mathbf{2.23\ m^2 \cdot °C/W}$

The perimeter of the building is 50 m and the height of the walls is 2.5 m. Noting that glazing constitutes 20 percent of the walls, the total wall area is

$$A_{wall} = 0.80(\text{Perimeter})(\text{Height}) = 0.80(50\text{ m})(2.5\text{ m}) = 100\text{ m}^2$$

Then the rate of heat loss through the walls under design conditions becomes

$$\begin{aligned}\dot{Q}_{wall} &= (UA)_{wall}\,(T_i - T_o)\\ &= (0.449\text{ W/m}^2 \cdot °\text{C})(100\text{ m}^2)[22 - (-2)°\text{C}]\\ &= \mathbf{1078\ W}\end{aligned}$$

Discussion Note that a 1-kW resistance heater in this house will make up almost all the heat lost through the walls, except through the doors and windows, when the outdoor air temperature drops to −2°C.

EXAMPLE 3–17 The *R*-Value of a Wall with Rigid Foam

The 13-mm-thick wood fiberboard sheathing of the wood stud wall discussed in the previous example is replaced by a 25-mm-thick rigid foam insulation. Determine the percent increase in the *R*-value of the wall as a result.

SOLUTION The overall R-value of the existing wall was determined in Example 3–16 to be 2.23 $m^2 \cdot °C/W$. Noting that the R-values of the fiberboard and the foam insulation are 0.23 $m^2 \cdot °C/W$ and 0.98 $m^2 \cdot °C/W$, respectively, and the added and removed thermal resistances are in series, the overall R-value of the wall after modification becomes

$$\begin{aligned} R_{\text{new}} &= R_{\text{old}} - R_{\text{removed}} + R_{\text{added}} \\ &= 2.23 - 0.23 + 0.98 \\ &= 2.98 \text{ m}^2 \cdot °\text{C/W} \end{aligned}$$

This represents an increase of $(2.98 - 2.23)/2.23 = 0.34$ or **34 percent** in the R-value of the wall. This example demonstrated how to evaluate the new R-value of a structure when some structural members are added or removed.

EXAMPLE 3–18 The *R*-Value of a Masonry Wall

Determine the overall unit thermal resistance (the R-value) and the overall heat transfer coefficient (the U-factor) of a masonry cavity wall that is built around 6-in.-thick concrete blocks made of lightweight aggregate with 3 cores filled with perlite ($R = 4.2$ h · ft² · °F/Btu). The outside is finished with 4-in. face brick with $\frac{1}{2}$-in. cement mortar between the bricks and concrete blocks. The inside finish consists of $\frac{1}{2}$ in. gypsum wallboard separated from the concrete block by $\frac{3}{4}$-in.-thick (1-in. × 3-in. nominal) vertical furring ($R = 4.2$ h · ft² · °F/Btu) whose center-to-center distance is 16 in. Both sides of the $\frac{3}{4}$-in.-thick air space between the concrete block and the gypsum board are coated with reflective aluminum foil ($\varepsilon = 0.05$) so that the effective emissivity of the air space is 0.03. For a mean temperature of 50°F and a temperature difference of 30°F, the R-value of the air space is 2.91 h · ft² · °F/Btu. The reflective air space constitutes 80 percent of the heat transmission area, while the vertical furring constitutes 20 percent.

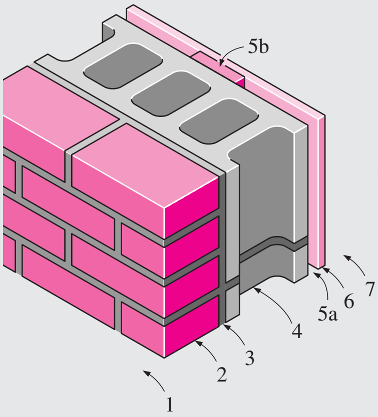
SOLUTION The R-value and the U-factor of a masonry cavity wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3–6.

Analysis The schematic of the wall as well as the different elements used in its construction are shown below. Following the approach described here and using the available R-values from Table 3–6, the overall R-value of the wall is determined in this table.

Schematic



Construction	*R*-value, h · ft² · °F/Btu Between Furring	At Furring
1. Outside surface, 15 mph wind	0.17	0.17
2. Face brick, 4 in.	0.43	0.43
3. Cement mortar, 0.5 in.	0.10	0.10
4. Concrete block, 6 in.	4.20	4.20
5a. Reflective air space, $\frac{3}{4}$ in.	2.91	—
5b. Nominal 1 × 3 vertical furring	—	0.94
6. Gypsum wallboard, 0.5 in.	0.45	0.45
7. Inside surface, still air	0.68	0.68
Total unit thermal resistance of each section, *R*	8.94	6.97
The *U*-factor of each section, $U = 1/R$, in Btu/h · ft² · °F	0.112	0.143
Area fraction of each section, f_{area}	0.80	0.20

Overall *U*-factor: $U = \Sigma f_{\text{area},\,i} U_i = 0.80 \times 0.112 + 0.20 \times 0.143$
$= \mathbf{0.118\ Btu/h \cdot ft^2 \cdot °F}$

Overall unit thermal resistance: $R = 1/U = \mathbf{8.46\ h \cdot ft^2 \cdot °F/Btu}$

Therefore, the overall unit thermal resistance of the wall is 8.46 h · ft² · °F/Btu and the overall *U*-factor is 0.118 Btu/h · ft² · °F. These values account for the effects of the vertical furring.

EXAMPLE 3–19 The *R*-Value of a Pitched Roof

Determine the overall unit thermal resistance (the *R*-value) and the overall heat transfer coefficient (the *U*-factor) of a 45° pitched roof built around nominal 2-in. × 4-in. wood studs with a center-to-center distance of 16 in. The 3.5-in.-wide air space between the studs does not have any reflective surface and thus its effective emissivity is 0.84. For a mean temperature of 90°F and a temperature difference of 30°F, the *R*-value of the air space is 0.86 h · ft² · °F/Btu. The lower part of the roof is finished with $\frac{1}{2}$-in. gypsum wallboard and the upper part with $\frac{5}{8}$-in. plywood, building paper, and asphalt shingle roofing. The air space constitutes 75 percent of the heat transmission area, while the studs and headers constitute 25 percent.

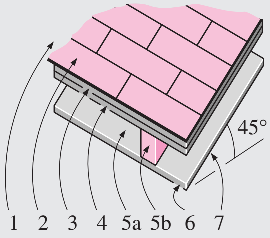
SOLUTION The *R*-value and the *U*-factor of a 45° pitched roof are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the roof is one-dimensional. **3** Thermal properties of the roof and the heat transfer coefficients are constant.

Properties The *R*-values of different materials are given in Table 3–6.

Analysis The schematic of the pitched roof as well as the different elements used in its construction are shown below. Following the approach described above and using the available R-values from Table 3–6, the overall R-value of the roof can be determined in the table here.

Schematic



	Construction	R-value, h · ft² · °F/Btu Between Studs	At Studs
1.	Outside surface, 15 mph wind	0.17	0.17
2.	Asphalt shingle roofing	0.44	0.44
3.	Building paper	0.10	0.10
4.	Plywood deck, $\frac{5}{8}$ in.	0.78	0.78
5a.	Nonreflective air space, 3.5 in.	0.86	—
5b.	Wood stud, 2 in. by 4 in.	—	3.58
6.	Gypsum wallboard, 0.5 in.	0.45	0.45
7.	Inside surface, 45° slope, still air	0.63	0.63
	Total unit thermal resistance of each section, R	3.43	6.15
	The U-factor of each section, $U = 1/R$, in Btu/h · ft² · °F	0.292	0.163
	Area fraction of each section, f_{area}	0.75	0.25

Overall U-factor: $U = \Sigma f_{area,\, i} U_i = 0.75 \times 0.292 + 0.25 \times 0.163$
$= \mathbf{0.260\ Btu/h \cdot ft^2 \cdot °F}$

Overall unit thermal resistance: $R = 1/U = \mathbf{3.85\ h \cdot ft^2 \cdot °F/Btu}$

Therefore, the overall unit thermal resistance of this pitched roof is 3.85 h · ft² · °F/Btu and the overall U-factor is 0.260 Btu/h · ft² · °F. Note that the wood studs offer much larger thermal resistance to heat flow than the air space between the studs.

The construction of wood frame flat ceilings typically involve 2-in. × 6-in. joists on 400-mm (16-in.) or 600-mm (24-in.) centers. The fraction of framing is usually taken to be 0.10 for joists on 400-mm centers and 0.07 for joists on 600-mm centers.

Most buildings have a combination of a ceiling and a roof with an attic space in between, and the determination of the R-value of the roof–attic–ceiling combination depends on whether the attic is vented or not. For adequately ventilated attics, the attic air temperature is practically the same as the outdoor air temperature, and thus heat transfer through the roof is governed by the R-value of the ceiling only. However, heat is also transferred between the roof and the ceiling by radiation, and it needs to be considered (Fig. 3–52). The major function of the roof in this case is to serve as a radiation shield by blocking off solar radiation. Effectively ventilating the attic in summer should not lead one to believe that heat gain to the building through the attic is greatly reduced. This is because most of the heat transfer through the attic is by radiation.

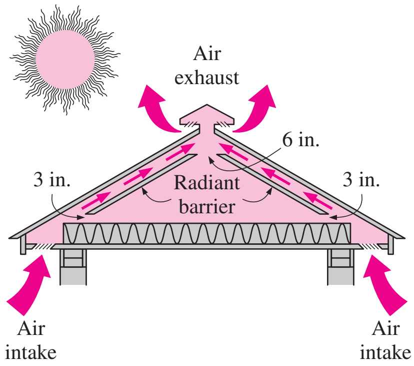


FIGURE 3–52
Ventilation paths for a naturally ventilated attic and the appropriate size of the flow areas around the radiant barrier for proper air circulation (from DOE/CE-0335P, U.S. Dept. of Energy).

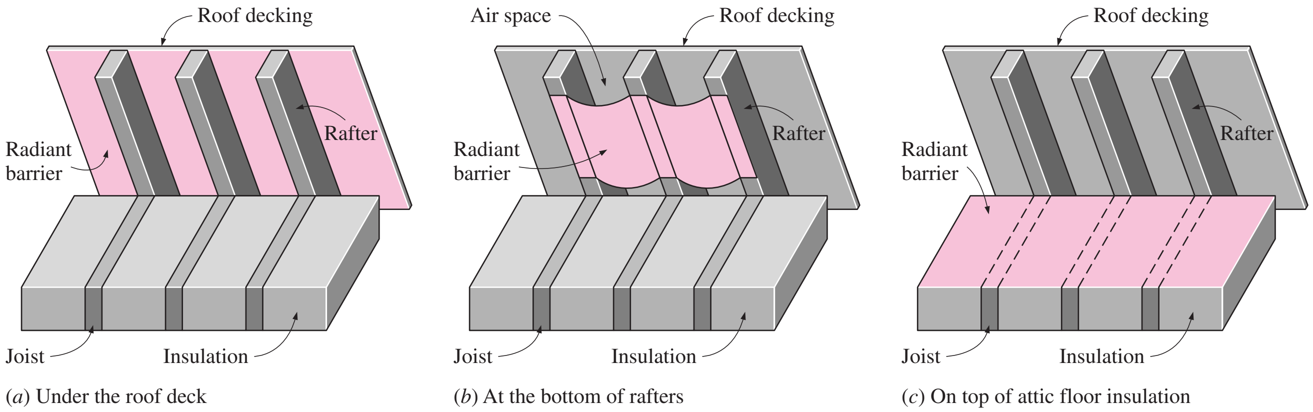


FIGURE 3–53
Three possible locations for an attic radiant barrier (from DOE/CE-0335P, U.S. Dept. of Energy).

Radiation heat transfer between the ceiling and the roof can be minimized by covering at least one side of the attic (the roof or the ceiling side) by a reflective material, called *radiant barrier,* such as aluminum foil or aluminum-coated paper. Tests on houses with *R*-19 attic floor insulation have shown that radiant barriers can reduce summer ceiling heat gains by 16 to 42 percent compared to an attic with the same insulation level and no radiant barrier. Considering that the ceiling heat gain represents about 15 to 25 percent of the total cooling load of a house, radiant barriers will reduce the air conditioning costs by 2 to 10 percent. Radiant barriers also reduce the heat loss in winter through the ceiling, but tests have shown that the percentage reduction in heat losses is less. As a result, the percentage reduction in heating costs will be less than the reduction in the air-conditioning costs. Also, the values given are for new and undusted radiant barrier installations, and percentages will be lower for aged or dusty radiant barriers.

Some possible locations for attic radiant barriers are given in Figure 3–53. In whole house tests on houses with *R*-19 attic floor insulation, radiant barriers have reduced the ceiling heat gain by an average of 35 percent when the radiant barrier is installed on the attic floor, and by 24 percent when it is attached to the bottom of roof rafters. Test cell tests also demonstrated that the best location for radiant barriers is the attic floor, provided that the attic is not used as a storage area and is kept clean.

For unvented attics, any heat transfer must occur through (1) the ceiling, (2) the attic space, and (3) the roof (Fig. 3–54). Therefore, the overall *R*-value of the roof–ceiling combination with an unvented attic depends on the combined effects of the *R*-value of the ceiling and the *R*-value of the roof as well as the thermal resistance of the attic space. The attic space can be treated as an air layer in the analysis. But a more practical way of accounting for its effect is to consider surface resistances on the roof and ceiling surfaces facing each other. In this case, the *R*-values of the ceiling and the roof are first determined separately (by using convection resistances for the still-air case for the attic surfaces). Then it can be shown that the overall *R*-value of the ceiling–roof combination per unit area of the ceiling can be expressed as

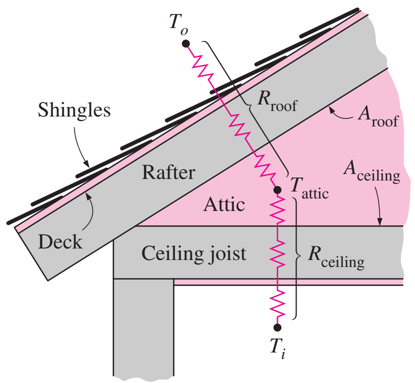


FIGURE 3–54
Thermal resistance network for a pitched roof–attic–ceiling combination for the case of an unvented attic.

$$R = R_{\text{ceiling}} + R_{\text{roof}}\left(\frac{A_{\text{ceiling}}}{A_{\text{roof}}}\right) \tag{3-82}$$

where A_{ceiling} and A_{roof} are the ceiling and roof areas, respectively. The area ratio is equal to 1 for flat roofs and is less than 1 for pitched roofs. For a 45° pitched roof, the area ratio is $A_{\text{ceiling}}/A_{\text{roof}} = 1/\sqrt{2} = 0.707$. Note that the pitched roof has a greater area for heat transfer than the flat ceiling, and the area ratio accounts for the reduction in the unit R-value of the roof when expressed per unit area of the ceiling. Also, the direction of heat flow is up in winter (heat loss through the roof) and down in summer (heat gain through the roof).

The R-value of a structure determined by analysis assumes that the materials used and the quality of workmanship meet the standards. Poor workmanship and substandard materials used during construction may result in R-values that deviate from predicted values. Therefore, some engineers use a safety factor in their designs based on experience in critical applications.

SUMMARY

One-dimensional heat transfer through a simple or composite body exposed to convection from both sides to mediums at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)}$$

where R_{total} is the total thermal resistance between the two mediums. For a plane wall exposed to convection on both sides, the total resistance is expressed as

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

This relation can be extended to plane walls that consist of two or more layers by adding an additional resistance for each additional layer. The elementary thermal resistance relations can be expressed as follows:

Conduction resistance (plane wall): $R_{\text{wall}} = \dfrac{L}{kA}$

Conduction resistance (cylinder): $R_{\text{cyl}} = \dfrac{\ln(r_2/r_1)}{2\pi Lk}$

Conduction resistance (sphere): $R_{\text{sph}} = \dfrac{r_2 - r_1}{4\pi r_1 r_2 k}$

Convection resistance: $R_{\text{conv}} = \dfrac{1}{hA}$

Interface resistance: $R_{\text{interface}} = \dfrac{1}{h_c A} = \dfrac{R_c}{A}$

Radiation resistance: $R_{\text{rad}} = \dfrac{1}{h_{\text{rad}} A}$

where h_c is the thermal contact conductance, R_c is the thermal contact resistance, and the radiation heat transfer coefficient is defined as

$$h_{\text{rad}} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})$$

Once the rate of heat transfer is available, the *temperature drop* across any layer can be determined from

$$\Delta T = \dot{Q}R$$

The thermal resistance concept can also be used to solve steady heat transfer problems involving parallel layers or combined series-parallel arrangements.

Adding insulation to a cylindrical pipe or a spherical shell will increase the rate of heat transfer if the outer radius of the insulation is less than the *critical radius of insulation,* defined as

$$r_{\text{cr, cylinder}} = \frac{k_{\text{ins}}}{h}$$

$$r_{\text{cr, sphere}} = \frac{2k_{\text{ins}}}{h}$$

The effectiveness of an insulation is often given in terms of its *R-value,* the thermal resistance of the material per unit surface area, expressed as

$$R\text{-value} = \frac{L}{k} \qquad \text{(flat insulation)}$$

where L is the thickness and k is the thermal conductivity of the material.

Finned surfaces are commonly used in practice to enhance heat transfer. Fins enhance heat transfer from a surface by exposing a larger surface area to convection. The temperature distribution along the fin for very long fins and for fins with negligible heat transfer at the fin are given by

Very long fin: $$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-x\sqrt{hp/kA_c}}$$

Adiabatic fin tip: $$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL}$$

where $a = \sqrt{hp/kA_c}$, p is the perimeter, and A_c is the cross sectional area of the fin. The rates of heat transfer for both cases are given to be

Very long fin: $$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

Adiabatic fin tip: $$\dot{Q}_{\text{insulated tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL$$

Fins exposed to convection at their tips can be treated as fins with insulated tips by using the corrected length $L_c = L + A_c/p$ instead of the actual fin length.

The temperature of a fin drops along the fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference toward the fin tip. To account for the effect of this decrease in temperature on heat transfer, we define *fin efficiency* as

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

When the fin efficiency is available, the rate of heat transfer from a fin can be determined from

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}\,(T_b - T_\infty)$$

The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case and is expressed in terms of the *fin effectiveness* ε_{fin}, defined as

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$

Here, A_b is the cross-sectional area of the fin at the base and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. The *overall effectiveness* for a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}\,(T_b - T_\infty)}$$

Fin efficiency and fin effectiveness are related to each other by

$$\varepsilon_{\text{fin}} = \frac{A_{\text{fin}}}{A_b}\eta_{\text{fin}}$$

Certain multidimensional heat transfer problems involve two surfaces maintained at constant temperatures T_1 and T_2. The steady rate of heat transfer between these two surfaces is expressed as

$$\dot{Q} = Sk(T_1 - T_2)$$

where S is the *conduction shape factor* that has the dimension of *length* and k is the thermal conductivity of the medium between the surfaces.

REFERENCES AND SUGGESTED READING

1. American Society of Heating, Refrigeration, and Air Conditioning Engineers. *Handbook of Fundamentals.* Atlanta: ASHRAE, 1993.
2. R. V. Andrews. "Solving Conductive Heat Transfer Problems with Electrical-Analogue Shape Factors." *Chemical Engineering Progress* 5 (1955), p. 67.
3. R. Barron. *Cryogenic Systems.* New York: McGraw-Hill, 1967.
4. L. S. Fletcher. "Recent Developments in Contact Conductance Heat Transfer." *Journal of Heat Transfer* 110, no. 4B (1988), pp. 1059–79.
5. E. Fried. "Thermal Conduction Contribution to Heat Transfer at Contacts." *Thermal Conductivity,* vol. 2, ed. R. P. Tye. London: Academic Press, 1969.
6. K. A. Gardner. "Efficiency of Extended Surfaces." *Trans. ASME* 67 (1945), pp. 621–31.
7. F. P. Incropera and D. P. DeWitt. *Introduction to Heat Transfer.* 4th ed. New York: John Wiley & Sons, 2002.
8. D. Q. Kern and A. D. Kraus. *Extended Surface Heat Transfer.* New York: McGraw-Hill, 1972.

9. M. N. Özişik. *Heat Transfer—A Basic Approach.* New York: McGraw-Hill, 1985.

10. G. P. Peterson. "Thermal Contact Resistance in Waste Heat Recovery Systems." *Proceedings of the 18th ASME/ETCE Hydrocarbon Processing Symposium.* Dallas, TX, 1987, pp. 45–51.

11. S. Song, M. M. Yovanovich, and F. O. Goodman. "Thermal Gap Conductance of Conforming Surfaces in Contact." *Journal of Heat Transfer* 115 (1993), p. 533.

12. J. E. Sunderland and K. R. Johnson. "Shape Factors for Heat Conduction through Bodies with Isothermal or Convective Boundary Conditions," *Trans. ASME* 10 (1964), pp. 237–41.

13. N. V. Suryanarayana. *Engineering Heat Transfer.* St. Paul, MN: West Publishing, 1995.

PROBLEMS*

Steady Heat Conduction in Plane Walls

3–1C Consider one-dimensional heat conduction through a cylindrical rod of diameter D and length L. What is the heat transfer area of the rod if (*a*) the lateral surfaces of the rod are insulated and (*b*) the top and bottom surfaces of the rod are insulated?

3–2C Consider heat conduction through a plane wall. Does the energy content of the wall change during steady heat conduction? How about during transient conduction? Explain.

3–3C Consider heat conduction through a wall of thickness L and area A. Under what conditions will the temperature distributions in the wall be a straight line?

3–4C What does the thermal resistance of a medium represent?

3–5C How is the combined heat transfer coefficient defined? What convenience does it offer in heat transfer calculations?

3–6C Can we define the convection resistance per unit surface area as the inverse of the convection heat transfer coefficient?

3–7C Why are the convection and the radiation resistances at a surface in parallel instead of being in series?

3–8C Consider a surface of area A at which the convection and radiation heat transfer coefficients are h_{conv} and h_{rad}, respectively. Explain how you would determine (*a*) the single equivalent heat transfer coefficient, and (*b*) the equivalent thermal resistance. Assume the medium and the surrounding surfaces are at the same temperature.

3–9C How does the thermal resistance network associated with a single-layer plane wall differ from the one associated with a five-layer composite wall?

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

3–10C Consider steady one-dimensional heat transfer through a multilayer medium. If the rate of heat transfer $\dot{Q}$ is known, explain how you would determine the temperature drop across each layer.

3–11C Consider steady one-dimensional heat transfer through a plane wall exposed to convection from both sides to environments at known temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with known heat transfer coefficients h_1 and h_2. Once the rate of heat transfer $\dot{Q}$ has been evaluated, explain how you would determine the temperature of each surface.

3–12C Someone comments that a microwave oven can be viewed as a conventional oven with zero convection resistance at the surface of the food. Is this an accurate statement?

3–13C Consider a window glass consisting of two 4-mm-thick glass sheets pressed tightly against each other. Compare the heat transfer rate through this window with that of one consisting of a single 8-mm-thick glass sheet under identical conditions.

3–14C Consider steady heat transfer through the wall of a room in winter. The convection heat transfer coefficient at the outer surface of the wall is three times that of the inner surface as a result of the winds. On which surface of the wall do you think the temperature will be closer to the surrounding air temperature? Explain.

3–15C The bottom of a pan is made of a 4-mm-thick aluminum layer. In order to increase the rate of heat transfer through the bottom of the pan, someone proposes a design for the bottom that consists of a 3-mm-thick copper layer sandwiched between two 2-mm-thick aluminum layers. Will the new design conduct heat better? Explain. Assume perfect contact between the layers.

3–16C Consider two cold canned drinks, one wrapped in a blanket and the other placed on a table in the same room. Which drink will warm up faster?

3–17 Consider a 4-m-high, 6-m-wide, and 0.3-m-thick brick wall whose thermal conductivity is $k = 0.8$ W/m · °C . On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 14°C and 6°C, respectively. Determine the rate of heat loss through the wall on that day.

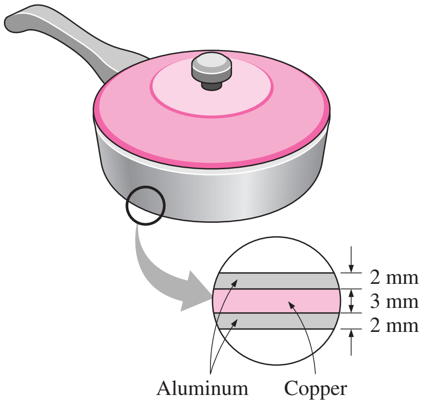


FIGURE P3–15C

3–18 Consider a 1.2-m-high and 2-m-wide glass window whose thickness is 6 mm and thermal conductivity is $k = 0.78$ W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 24°C while the temperature of the outdoors is −5°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 25$ W/m^2 · °C, and disregard any heat transfer by radiation.

3–19 Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass ($k = 0.78$ W/m · °C) separated by a 12-mm-wide stagnant air space ($k = 0.026$ W/m · °C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 24°C while the temperature of the outdoors is −5°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 25$ W/m^2 · °C, and disregard any heat transfer by radiation.
Answers: 114 W, 19.2°C

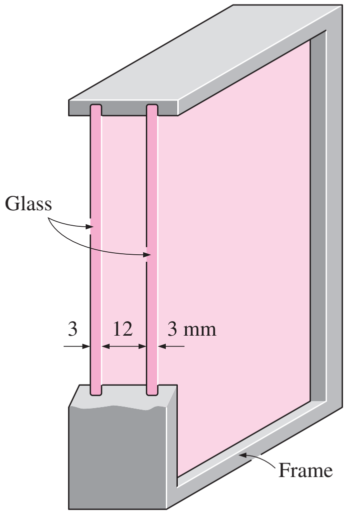


FIGURE P3–19

3–20 Repeat Problem 3–19, assuming the space between the two glass layers is evacuated.

3–21 Reconsider Problem 3–19. Using EES (or other) software, plot the rate of heat transfer through the window as a function of the width of air space in the range of 2 mm to 20 mm, assuming pure conduction through the air. Discuss the results.

3–22E Consider an electrically heated brick house ($k = 0.40$ Btu/h · ft · °F) whose walls are 9 ft high and 1 ft thick. Two of the walls of the house are 40 ft long and the others are 30 ft long. The house is maintained at 70°F at all times while the temperature of the outdoors varies. On a certain day, the temperature of the inner surface of the walls is measured to be at 55°F while the average temperature of the outer surface is observed to remain at 45°F during the day for 10 h and at 35°F at night for 14 h. Determine the amount of heat lost from the house that day. Also determine the cost of that heat loss to the homeowner for an electricity price of $0.09/kWh.

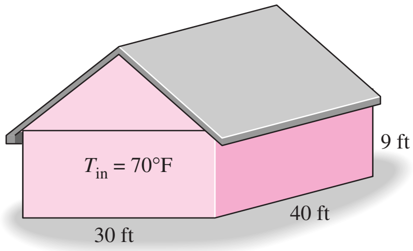


FIGURE P3–22E

3–23 A cylindrical resistor element on a circuit board dissipates 0.15 W of power in an environment at 40°C. The resistor is 1.2 cm long, and has a diameter of 0.3 cm. Assuming heat to be transferred uniformly from all surfaces, determine (*a*) the amount of heat this resistor dissipates during a 24-h period, (*b*) the heat flux on the surface of the resistor, in W/m^2, and (*c*) the surface temperature of the resistor for a combined convection and radiation heat transfer coefficient of 9 W/m^2 · °C.

3–24 Consider a power transistor that dissipates 0.2 W of power in an environment at 30°C. The transistor is 0.4 cm long and has a diameter of 0.5 cm. Assuming heat to be transferred uniformly from all surfaces, determine (*a*) the amount of heat this resistor dissipates during a 24-h period, in kWh; (*b*) the heat flux on the surface of the transistor, in W/m^2; and (*c*) the surface temperature of the resistor for a combined convection and radiation heat transfer coefficient of 18 W/m^2 · °C.

3–25 A 12-cm × 18-cm circuit board houses on its surface 100 closely spaced logic chips, each dissipating 0.07 W in an environment at 40°C. The heat transfer from the back surface of the board is negligible. If the heat transfer coefficient on the

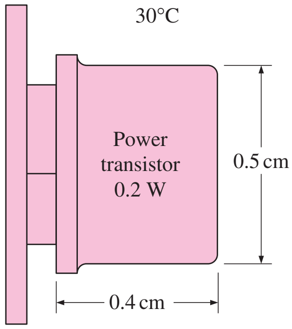


FIGURE P3–24

surface of the board is 10 W/m^2 · °C, determine (*a*) the heat flux on the surface of the circuit board, in W/m^2; (*b*) the surface temperature of the chips; and (*c*) the thermal resistance between the surface of the circuit board and the cooling medium, in °C/W.

3–26 Consider a person standing in a room at 20°C with an exposed surface area of 1.7 m^2. The deep body temperature of the human body is 37°C, and the thermal conductivity of the human tissue near the skin is about 0.3 W/m · °C. The body is losing heat at a rate of 150 W by natural convection and radiation to the surroundings. Taking the body temperature 0.5 cm beneath the skin to be 37°C, determine the skin temperature of the person. *Answer:* 35.5° C

3–27 Water is boiling in a 25-cm-diameter aluminum pan (k = 237 W/m · °C) at 95°C. Heat is transferred steadily to the boiling water in the pan through its 0.5-cm-thick flat bottom at a rate of 800 W. If the inner surface temperature of the bottom of the pan is 108°C, determine (*a*) the boiling heat transfer coefficient on the inner surface of the pan, and (*b*) the outer surface temperature of the bottom of the pan.

3–28E A wall is constructed of two layers of 0.5-in-thick sheetrock (k = 0.10 Btu/h · ft · °F), which is a plasterboard made of two layers of heavy paper separated by a layer of gypsum, placed 5 in. apart. The space between the sheetrocks is filled with fiberglass insulation (k = 0.020 Btu/h · ft · °F). Determine (*a*) the thermal resistance of the wall, and (*b*) its *R*-value of insulation in English units.

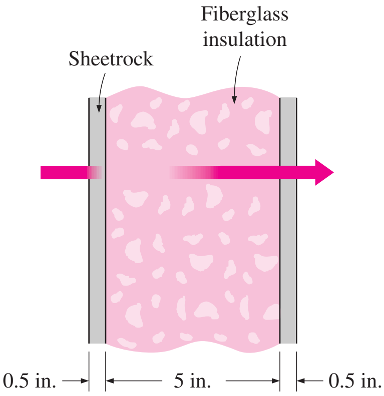


FIGURE P3–28E

3–29 The roof of a house consists of a 3–cm-thick concrete slab (k = 2 W/m · °C) that is 15 m wide and 20 m long. The convection heat transfer coefficients on the inner and outer surfaces of the roof are 5 and 12 W/m^2 · °C, respectively. On a clear winter night, the ambient air is reported to be at 10°C, while the night sky temperature is 100 K. The house and the interior surfaces of the wall are maintained at a constant temperature of 20°C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfers, determine the rate of heat transfer through the roof, and the inner surface temperature of the roof.

If the house is heated by a furnace burning natural gas with an efficiency of 80 percent, and the price of natural gas is \$0.60/therm (1 therm = 105,500 kJ of energy content), determine the money lost through the roof that night during a 14-h period.

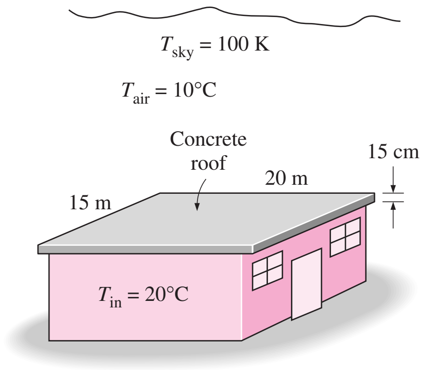


FIGURE P3–29

3–30 A 2-m × 1.5-m section of wall of an industrial furnace burning natural gas is not insulated, and the temperature at the outer surface of this section is measured to be 80°C. The temperature of the furnace room is 30°C, and the combined convection and radiation heat transfer coefficient at the surface of the outer furnace is 10 W/m^2 · °C. It is proposed to insulate this section of the furnace wall with glass wool insulation (k = 0.038 W/m · °C) in order to reduce the heat loss by 90 percent. Assuming the outer surface temperature of the metal section still remains at about 80°C, determine the thickness of the insulation that needs to be used.

The furnace operates continuously and has an efficiency of 78 percent. The price of the natural gas is \$0.55/therm (1 therm = 105,500 kJ of energy content). If the installation of the insulation will cost \$250 for materials and labor, determine how long it will take for the insulation to pay for itself from the energy it saves.

3–31 Repeat Problem. 3–30 for expanded perlite insulation assuming conductivity is k = 0.052 W/m · °C.

3–32 Reconsider Problem 3–30. Using EES (or other) software, investigate the effect of thermal conductivity on the required insulation thickness. Plot the thickness of insulation as a function of the thermal conductivity of the insulation in the range of 0.02 W/m · °C to 0.08 W/m · °C, and discuss the results.

3–33E Consider a house whose walls are 12 ft high and 40 ft long. Two of the walls of the house have no windows, while each of the other two walls has four windows made of 0.25-in.-thick glass ($k = 0.45$ Btu/h · ft · °F), 3 ft × 5 ft in size. The walls are certified to have an R-value of 19 (i.e., an L/k value of 19 h · ft^2 · °F/Btu). Disregarding any direct radiation gain or loss through the windows and taking the heat transfer coefficients at the inner and outer surfaces of the house to be 2 and 4 Btu/h · ft^2 · °F, respectively, determine the ratio of the heat transfer through the walls with and without windows.

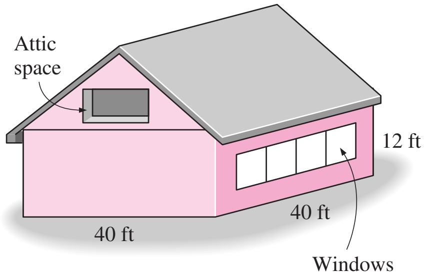


FIGURE P3–33E

3–34 Consider a house that has a 10-m × 20-m base and a 4-m-high wall. All four walls of the house have an R-value of 2.31 m^2 · °C/W. The two 10-m × 4-m walls have no windows. The third wall has five windows made of 0.5-cm-thick glass ($k = 0.78$ W/m · °C), 1.2 m × 1.8 m in size. The fourth wall has the same size and number of windows, but they are double-paned with a 1.5-cm-thick stagnant air space ($k = 0.026$ W/m · °C) enclosed between two 0.5-cm-thick glass layers. The thermostat in the house is set at 22°C and the average temperature outside at that location is 8°C during the seven-month-long heating season. Disregarding any direct radiation gain or loss through the windows and taking the heat transfer coefficients at the inner and outer surfaces of the house to be 7 and 15 W/m^2 · °C, respectively, determine the average rate of heat transfer through each wall.

If the house is electrically heated and the price of electricity is $0.08/kWh, determine the amount of money this household will save per heating season by converting the single-pane windows to double-pane windows.

3–35 The wall of a refrigerator is constructed of fiberglass insulation ($k = 0.035$ W/m · °C) sandwiched between two layers of 1-mm-thick sheet metal ($k = 15.1$ W/m · °C). The refrigerated space is maintained at 3°C, and the average heat transfer coefficients at the inner and outer surfaces of the wall are 4 W/m^2 · °C and 9 W/m^2 · °C, respectively. The kitchen temperature averages 25°C. It is observed that condensation occurs on the outer surfaces of the refrigerator when the temperature of the outer surface drops to 20°C. Determine the minimum thickness of fiberglass insulation that needs to be used in the wall in order to avoid condensation on the outer surfaces.

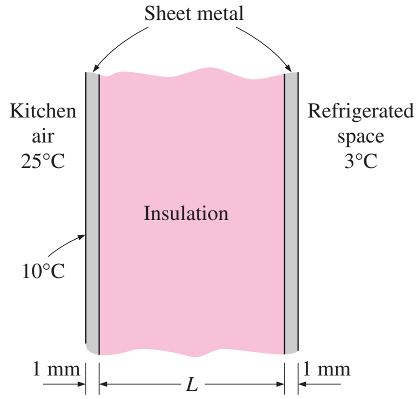


FIGURE P3–35

3–36 Reconsider Problem 3–35. Using EES (or other) software, investigate the effects of the thermal conductivities of the insulation material and the sheet metal on the thickness of the insulation. Let the thermal conductivity vary from 0.02 W/m · °C to 0.08 W/m · °C for insulation and 10 W/m · °C to 400 W/m · °C for sheet metal. Plot the thickness of the insulation as the functions of the thermal conductivities of the insulation and the sheet metal, and discuss the results.

3–37 Heat is to be conducted along a circuit board that has a copper layer on one side. The circuit board is 15 cm long and 15 cm wide, and the thicknesses of the copper and epoxy layers are 0.1 mm and 1.2 mm, respectively. Disregarding heat transfer from side surfaces, determine the percentages of heat conduction along the copper ($k = 386$ W/m · °C) and epoxy ($k = 0.26$ W/m · °C) layers. Also determine the effective thermal conductivity of the board.

Answers: 0.8 percent, 99.2 percent, and 29.9 W/m · °C

3–38E A 0.03-in-thick copper plate ($k = 223$ Btu/h · ft · °F) is sandwiched between two 0.1-in.-thick epoxy boards ($k = 0.15$ Btu/h · ft · °F) that are 7 in. × 9 in. in size. Determine the effective thermal conductivity of the board along its 9-in.-long side. What fraction of the heat conducted along that side is conducted through copper?

Thermal Contact Resistance

3–39C What is thermal contact resistance? How is it related to thermal contact conductance?

3–40C Will the thermal contact resistance be greater for smooth or rough plain surfaces?

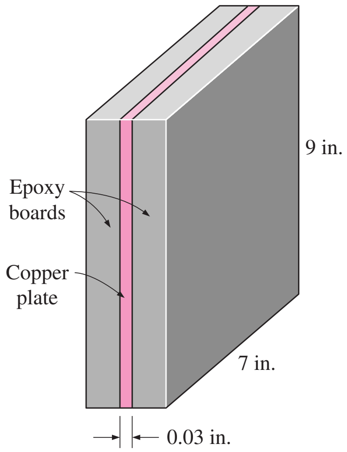


FIGURE P3–38E

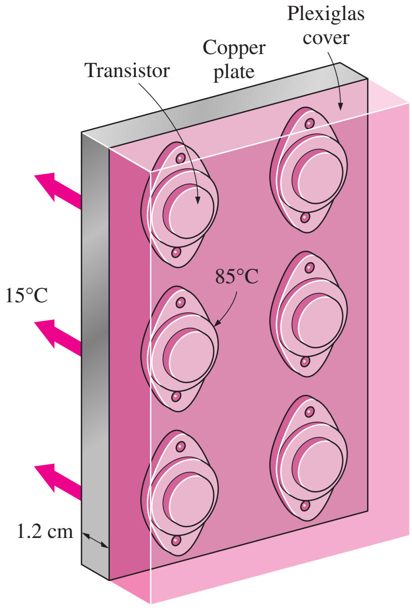


FIGURE P3–46

3–41C A wall consists of two layers of insulation pressed against each other. Do we need to be concerned about the thermal contact resistance at the interface in a heat transfer analysis or can we just ignore it?

3–42C A plate consists of two thin metal layers pressed against each other. Do we need to be concerned about the thermal contact resistance at the interface in a heat transfer analysis or can we just ignore it?

3–43C Consider two surfaces pressed against each other. Now the air at the interface is evacuated. Will the thermal contact resistance at the interface increase or decrease as a result?

3–44C Explain how the thermal contact resistance can be minimized.

3–45 The thermal contact conductance at the interface of two 1-cm-thick copper plates is measured to be 18,000 W/m^2 · °C. Determine the thickness of the copper plate whose thermal resistance is equal to the thermal resistance of the interface between the plates.

3–46 Six identical power transistors with aluminum casing are attached on one side of a 1.2-cm-thick 20-cm × 30-cm copper plate (k = 386 W/m · °C) by screws that exert an average pressure of 10 MPa. The base area of each transistor is 9 cm^2, and each transistor is placed at the center of a 10-cm × 10-cm section of the plate. The interface roughness is estimated to be about 1.4 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 15°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 30 W/m^2 · °C. If the case temperature of the transistor is not to exceed 85°C, determine the maximum power each transistor can dissipate safely, and the temperature jump at the case-plate interface.

3–47 Two 5-cm-diameter, 15–cm-long aluminum bars (k = 176 W/m · °C) with ground surfaces are pressed against each other with a pressure of 20 atm. The bars are enclosed in an insulation sleeve and, thus, heat transfer from the lateral surfaces is negligible. If the top and bottom surfaces of the two-bar system are maintained at temperatures of 150°C and 20°C, respectively, determine (*a*) the rate of heat transfer along the cylinders under steady conditions and (*b*) the temperature drop at the interface. *Answers: (a) 142.4 W, (b) 6.4°C*

3–48 A 1-mm-thick copper plate (k = 386 W/m · °C) is sandwiched between two 5-mm-thick epoxy boards (k = 0.26 W/m · °C) that are 15 cm × 20 cm in size. If the thermal contact conductance on both sides of the copper plate is estimated to be 6000 W/m · °C, determine the error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored.

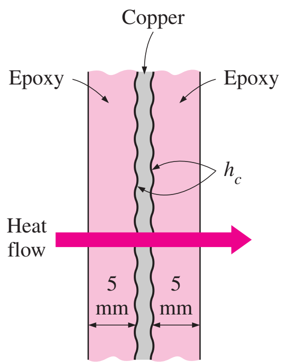


FIGURE P3–48

Generalized Thermal Resistance Networks

3–49C When plotting the thermal resistance network associated with a heat transfer problem, explain when two resistances are in series and when they are in parallel.

3–50C The thermal resistance networks can also be used approximately for multidimensional problems. For what kind of multidimensional problems will the thermal resistance approach give adequate results?

3–51C What are the two approaches used in the development of the thermal resistance network for two-dimensional problems?

3–52 A 4-m-high and 6-m-wide wall consists of a long 18-cm × 30-cm cross section of horizontal bricks (k = 0.72 W/m · °C) separated by 3-cm-thick plaster layers (k = 0.22 W/m · °C). There are also 2-cm-thick plaster layers on each side of the wall, and a 2-cm-thick rigid foam (k = 0.026 W/m · °C) on the inner side of the wall. The indoor and the outdoor temperatures are 22°C and −4°C, and the convection heat transfer coefficients on the inner and the outer sides are h_1 = 10 W/m² · °C and h_2 = 20 W/m² · °C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

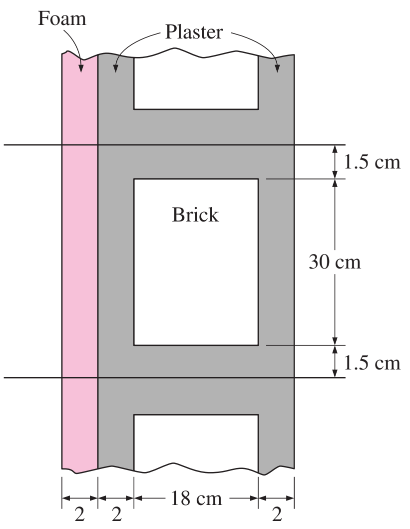


FIGURE P3–52

3–53 Reconsider Problem 3–52. Using EES (or other) software, plot the rate of heat transfer through the wall as a function of the thickness of the rigid foam in the range of 1 cm to 10 cm. Discuss the results.

3–54 A 10-cm-thick wall is to be constructed with 2.5-m-long wood studs (k = 0.11 W/m · °C) that have a cross section of 10 cm × 10 cm. At some point the builder ran out of those studs and started using pairs of 2.5-m-long wood studs that have a cross section of 5 cm × 10 cm nailed to each other instead. The manganese steel nails (k = 50 W/m · °C) are 10 cm long and have a diameter of 0.4 cm. A total of 50 nails are used to connect the two studs, which are mounted to the wall such that the nails cross the wall. The temperature difference between the inner and outer surfaces of the wall is 8°C. Assuming the thermal contact resistance between the two layers to be negligible, determine the rate of heat transfer (*a*) through a solid stud and (*b*) through a stud pair of equal length and width nailed to each other. (*c*) Also determine the effective conductivity of the nailed stud pair.

3–55 A 12-m-long and 5-m-high wall is constructed of two layers of 1-cm-thick sheetrock (k = 0.17 W/m · °C) spaced 12 cm by wood studs (k = 0.11 W/m · °C) whose cross section is 12 cm × 5 cm. The studs are placed vertically 60 cm apart, and the space between them is filled with fiberglass insulation (k = 0.034 W/m · °C). The house is maintained at 20°C and the ambient temperature outside is −5°C. Taking the heat transfer coefficients at the inner and outer surfaces of the house to be 8.3 and 34 W/m² · °C, respectively, determine (*a*) the thermal resistance of the wall considering a representative section of it and (*b*) the rate of heat transfer through the wall.

3–56E A 10-in.-thick, 30-ft-long, and 10-ft-high wall is to be constructed using 9-in.-long solid bricks (k = 0.40 Btu/h · ft · °F) of cross section 7 in. × 7 in., or identical size bricks with nine square air holes (k = 0.015 Btu/h · ft · °F) that are 9 in. long and have a cross section of 1.5 in. × 1.5 in. There is a 0.5-in.-thick plaster layer (k = 0.10 Btu/h · ft · °F) between two adjacent bricks on all four sides and on both sides of the wall. The house is maintained at 80°F and the ambient temperature outside is 30°F. Taking the heat transfer coefficients at the inner and outer surfaces of the wall to be 1.5 and 4 Btu/h · ft² · °F, respectively, determine the rate of heat transfer through the wall constructed of (*a*) solid bricks and (*b*) bricks with air holes.

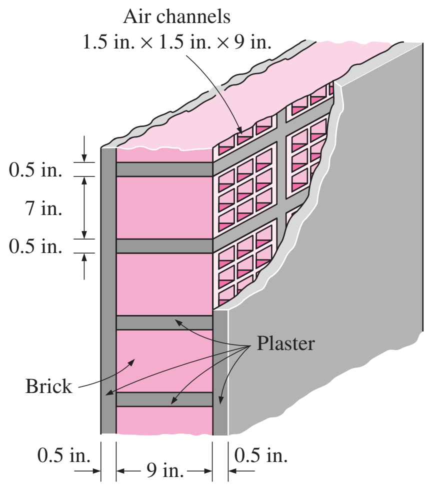


FIGURE P3–56E

3–57 Consider a 5-m-high, 8-m-long, and 0.22-m-thick wall whose representative cross section is as given in the figure. The thermal conductivities of various materials used, in W/m · °C, are $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, and $k_E = 35$. The left and right surfaces of the wall are maintained at uniform temperatures of 300°C and 100°C, respectively. Assuming heat transfer through the wall to be one-dimensional, determine (*a*) the rate of heat transfer through the wall; (*b*) the temperature at the point where the sections *B*, *D*, and *E* meet; and (*c*) the temperature drop across the section *F*. Disregard any contact resistances at the interfaces.

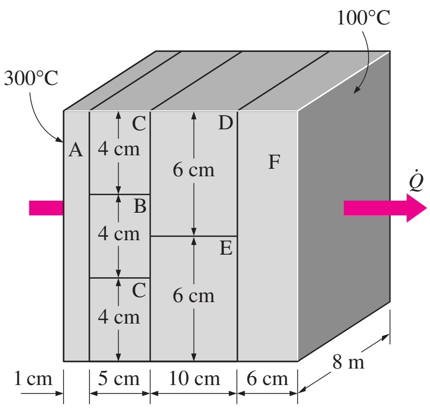


FIGURE P3–57

3–58 Repeat Problem 3–57 assuming that the thermal contact resistance at the interfaces D-F and E-F is 0.00012 m^2 · °C/W.

3–59 Clothing made of several thin layers of fabric with trapped air in between, often called ski clothing, is commonly used in cold climates because it is light, fashionable, and a very effective thermal insulator. So it is no surprise that such clothing has largely replaced thick and heavy old-fashioned coats.

Consider a jacket made of five layers of 0.1-mm-thick synthetic fabric (k = 0.13 W/m · °C) with 1.5-mm-thick air space (k = 0.026 W/m · °C) between the layers. Assuming the inner surface temperature of the jacket to be 28°C and the surface area to be 1.1 m^2, determine the rate of heat loss through the jacket when the temperature of the outdoors is −5°C and the heat transfer coefficient at the outer surface is 25 W/m^2 · °C.

FIGURE P3–59

What would your response be if the jacket is made of a single layer of 0.5-mm-thick synthetic fabric? What should be the thickness of a wool fabric (k = 0.035 W/m · °C) if the person is to achieve the same level of thermal comfort wearing a thick wool coat instead of a five-layer ski jacket?

3–60 Repeat Problem 3–59 assuming the layers of the jacket are made of cotton fabric (k = 0.06 W/m · °C).

3–61 A 5-m-wide, 4-m-high, and 40-m-long kiln used to cure concrete pipes is made of 20-cm-thick concrete walls and ceiling (k = 0.9 W/m · °C). The kiln is maintained at 40°C by injecting hot steam into it. The two ends of the kiln, 4 m × 5 m in size, are made of a 3-mm-thick sheet metal covered with 2-cm-thick Styrofoam (k = 0.033 W/m · °C). The convection heat transfer coefficients on the inner and the outer surfaces of the kiln are 3000 W/m^2 · °C and 25 W/m^2 · °C, respectively. Disregarding any heat loss through the floor, determine the rate of heat loss from the kiln when the ambient air is at −4°C.

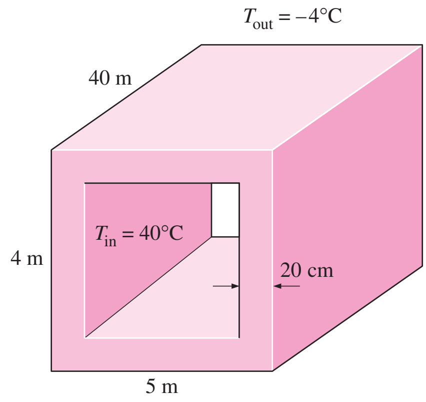


FIGURE P3–61

3–62 Reconsider Problem 3–61. Using EES (or other) software, investigate the effects of the thickness of the wall and the convection heat transfer coefficient on the outer surface of the rate of heat loss from the kiln. Let the thickness vary from 10 cm to 30 cm and the convection heat transfer coefficient from 5 W/m^2 · °C to 50 W/m^2 · °C. Plot the rate of heat transfer as functions of wall thickness and the convection heat transfer coefficient, and discuss the results.

3–63E Consider a 6-in. × 8-in. epoxy glass laminate (k = 0.10 Btu/h · ft · °F) whose thickness is 0.05 in. In order to reduce the thermal resistance across its thickness, cylindrical copper fillings (k = 223 Btu/h · ft · °F) of 0.02 in. diameter are to be planted throughout the board, with a center-to-center distance of 0.06 in. Determine the new value of the thermal resistance of the epoxy board for heat conduction across its thickness as a result of this modification.

Answer: 0.00064 h · °F/Btu

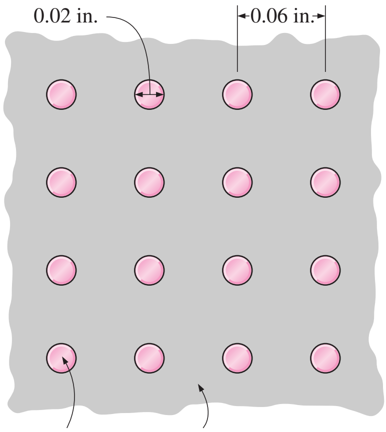


FIGURE P3–63E

Heat Conduction in Cylinders and Spheres

3–64C What is an infinitely long cylinder? When is it proper to treat an actual cylinder as being infinitely long, and when is it not?

3–65C Consider a short cylinder whose top and bottom surfaces are insulated. The cylinder is initially at a uniform temperature T_i and is subjected to convection from its side surface to a medium at temperature T_∞, with a heat transfer coefficient of h. Is the heat transfer in this short cylinder one- or two-dimensional? Explain.

3–66C Can the thermal resistance concept be used for a solid cylinder or sphere in steady operation? Explain.

3–67 A 5-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel (k = 15 W/m · °C) is used to store iced water at 0°C. The tank is located in a room whose temperature is 30°C. The walls of the room are also at 30°C. The outer surface of the tank is black (emissivity ε = 1), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are 80 W/m^2 · °C and 10 W/m^2 · °C, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water at atmospheric pressure is h_{if} = 333.7 kJ/kg.

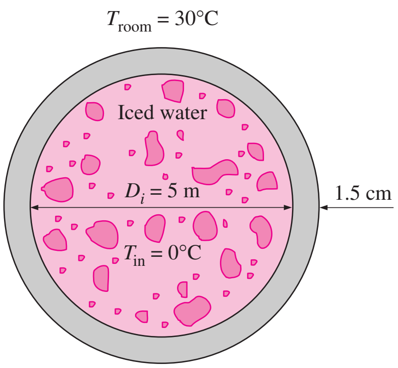


FIGURE P3–67

3–68 Steam at 320°C flows in a stainless steel pipe (k = 15 W/m · °C) whose inner and outer diameters are 5 cm and 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation (k = 0.038 W/m · °C). Heat is lost to the surroundings at 5°C by natural convection and radiation, with a combined natural convection and radiation heat transfer coefficient of 15 W/m^2 · °C. Taking the heat transfer coefficient inside the pipe to be 80 W/m^2 · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

3–69 Reconsider Problem 3–68. Using EES (or other) software, investigate the effect of the thickness of the insulation on the rate of heat loss from the steam and the temperature drop across the insulation layer. Let the insulation thickness vary from 1 cm to 10 cm. Plot the rate of heat loss and the temperature drop as a function of insulation thickness, and discuss the results.

3–70 A 50-m-long section of a steam pipe whose outer diameter is 10 cm passes through an open space at 15°C. The average temperature of the outer surface of the pipe is measured to be 150°C. If the combined heat transfer coefficient on the outer surface of the pipe is 20 W/m^2 · °C, determine (*a*) the rate of heat loss from the steam pipe, (*b*) the annual cost of this energy lost if steam is generated in a natural gas furnace that has an efficiency of 75 percent and the price of natural gas is \$0.52/therm (1 therm = 105,500 kJ), and (*c*) the thickness of fiberglass insulation (k = 0.035 W/m · °C) needed in order to save 90 percent of the heat lost. Assume the pipe temperature to remain constant at 150°C.

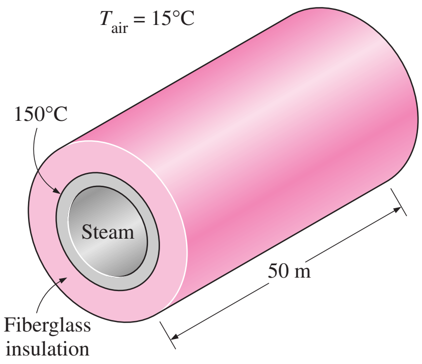


FIGURE P3–70

3–71 Consider a 2-m-high electric hot water heater that has a diameter of 40 cm and maintains the hot water at 55°C. The tank is located in a small room whose average temperature is

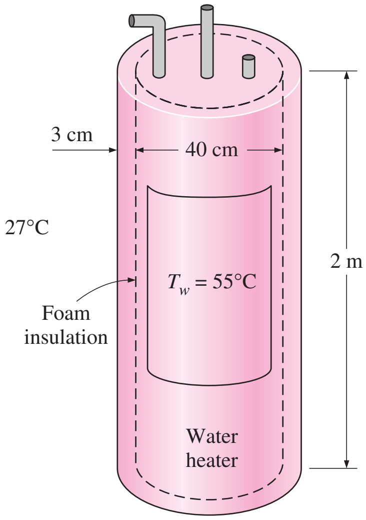


FIGURE P3–71

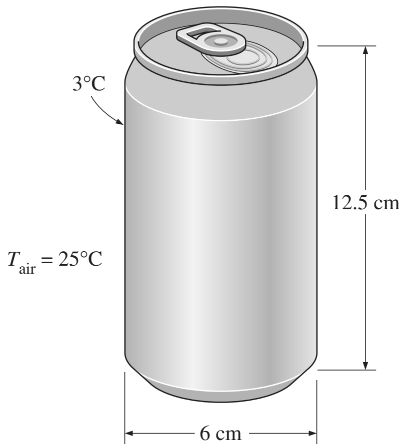


FIGURE P3–73

27°C, and the heat transfer coefficients on the inner and outer surfaces of the heater are 50 and 12 W/m^2 · °C, respectively. The tank is placed in another 46-cm-diameter sheet metal tank of negligible thickness, and the space between the two tanks is filled with foam insulation (k = 0.03 W/m · °C). The thermal resistances of the water tank and the outer thin sheet metal shell are very small and can be neglected. The price of electricity is \$0.08/kWh, and the home owner pays \$280 a year for water heating. Determine the fraction of the hot water energy cost of this household that is due to the heat loss from the tank.

Hot water tank insulation kits consisting of 3-cm-thick fiberglass insulation (k = 0.035 W/m · °C) large enough to wrap the entire tank are available in the market for about \$30. If such an insulation is installed on this water tank by the home owner himself, how long will it take for this additional insulation to pay for itself? *Answers:* 17.5 percent, 1.5 years

3–72 Reconsider Problem 3–71. Using EES (or other) software, plot the fraction of energy cost of hot water due to the heat loss from the tank as a function of the hot water temperature in the range of 40°C to 90°C. Discuss the results.

3–73 Consider a cold aluminum canned drink that is initially at a uniform temperature of 3°C. The can is 12.5 cm high and has a diameter of 6 cm. If the combined convection/radiation heat transfer coefficient between the can and the surrounding air at 25°C is 10 W/m^2 · °C, determine how long it will take for the average temperature of the drink to rise to 10°C.

In an effort to slow down the warming of the cold drink, a person puts the can in a perfectly fitting 1-cm-thick cylindrical rubber insulation (k = 0.13 W/m · °C). Now how long will it take for the average temperature of the drink to rise to 10°C? Assume the top of the can is not covered.

3–74 Repeat Problem 3–73, assuming a thermal contact resistance of 0.00008 m^2 · °C/W between the can and the insulation.

3–75E Steam at 450°F is flowing through a steel pipe (k = 8.7 Btu/h · ft · °F) whose inner and outer diameters are 3.5 in. and 4.0 in., respectively, in an environment at 55°F. The pipe is insulated with 2-in.-thick fiberglass insulation (k = 0.020 Btu/h · ft · °F). If the heat transfer coefficients on the inside and the outside of the pipe are 30 and 5 Btu/h · ft^2 · °F, respectively, determine the rate of heat loss from the steam per foot length of the pipe. What is the error involved in neglecting the thermal resistance of the steel pipe in calculations?

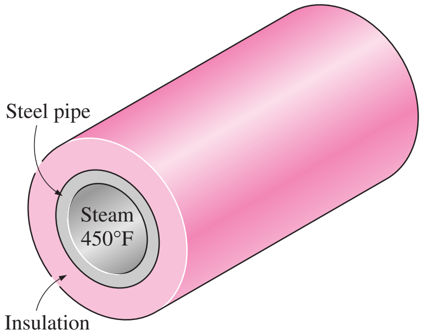


FIGURE P3–75E

3–76 Hot water at an average temperature of 90°C is flowing through a 15-m section of a cast iron pipe (k = 52 W/m · °C) whose inner and outer diameters are 4 cm and 4.6 cm, respectively. The outer surface of the pipe, whose emissivity is 0.7, is exposed to the cold air at 10°C in the basement, with a heat transfer coefficient of 15 W/m^2 · °C. The heat transfer coefficient at the inner surface of the pipe is 120 W/m^2 · °C. Taking the walls of the basement to be at 10°C also, determine the rate of heat loss from the hot water. Also, determine the average velocity of the water in the pipe if the temperature of the water drops by 3°C as it passes through the basement.

3–77 Repeat Problem 3–76 for a pipe made of copper (k = 386 W/m · °C) instead of cast iron.

3–78E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper pipes (k = 223 Btu/h · ft · °F) of inner diameter 0.4 in. and outer diameter 0.6 in. at an average temperature of 70°F. The heat of vaporization of water at 100°F is 1037 Btu/lbm. The heat transfer coefficients are 1500 Btu/h · ft^2 · °F on the steam side and 35 Btu/h · ft^2 · °F on the water side. Determine the length of the tube required to condense steam at a rate of 120 lbm/h. *Answer:* 1148 ft

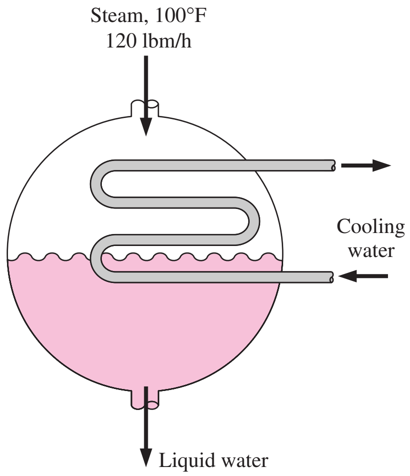


FIGURE P3–78E

3–79E Repeat Problem 3–78E, assuming that a 0.01-in.-thick layer of mineral deposit (k = 0.5 Btu/h · ft · °F) has formed on the inner surface of the pipe.

3–80 Reconsider Problem 3–78E. Using EES (or other) software, investigate the effects of the thermal conductivity of the pipe material and the outer diameter of the pipe on the length of the tube required. Let the thermal conductivity vary from 10 Btu/h · ft · °F to 400 Btu/h · ft · °F and the outer diameter from 0.5 in. to 1.0 in. Plot the length of the tube as functions of pipe conductivity and the outer pipe diameter, and discuss the results.

3–81 The boiling temperature of nitrogen at atmospheric pressure at sea level (1 atm pressure) is −196°C. Therefore, nitrogen is commonly used in low-temperature scientific studies since the temperature of liquid nitrogen in a tank open to the atmosphere will remain constant at −196°C until it is depleted. Any heat transfer to the tank will result in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg and a density of 810 kg/m^3 at 1 atm.

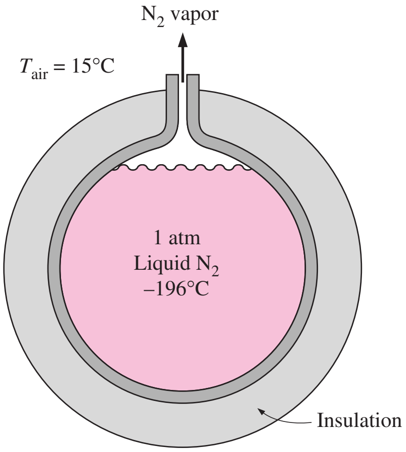


FIGURE P3–81

Consider a 3-m-diameter spherical tank that is initially filled with liquid nitrogen at 1 atm and −196°C. The tank is exposed to ambient air at 15°C, with a combined convection and radiation heat transfer coefficient of 35 W/m^2 · °C. The temperature of the thin-shelled spherical tank is observed to be almost the same as the temperature of the nitrogen inside. Determine the rate of evaporation of the liquid nitrogen in the tank as a result of the heat transfer from the ambient air if the tank is (*a*) not insulated, (*b*) insulated with 5-cm-thick fiberglass insulation (k = 0.035 W/m · °C), and (*c*) insulated with 2-cm-thick superinsulation which has an effective thermal conductivity of 0.00005 W/m · °C.

3–82 Repeat Problem 3–81 for liquid oxygen, which has a boiling temperature of −183°C, a heat of vaporization of 213 kJ/kg, and a density of 1140 kg/m^3 at 1 atm pressure.

Critical Radius of Insulation

3–83C What is the critical radius of insulation? How is it defined for a cylindrical layer?

3–84C A pipe is insulated such that the outer radius of the insulation is less than the critical radius. Now the insulation is taken off. Will the rate of heat transfer from the pipe increase or decrease for the same pipe surface temperature?

3–85C A pipe is insulated to reduce the heat loss from it. However, measurements indicate that the rate of heat loss has increased instead of decreasing. Can the measurements be right?

3–86C Consider a pipe at a constant temperature whose radius is greater than the critical radius of insulation. Someone claims that the rate of heat loss from the pipe has increased when some insulation is added to the pipe. Is this claim valid?

3–87C Consider an insulated pipe exposed to the atmosphere. Will the critical radius of insulation be greater on calm days or on windy days? Why?

3–88 A 2-mm-diameter and 10-m-long electric wire is tightly wrapped with a 1-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m · °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30°C$ with a heat transfer coefficient of $h = 24$ W/m^2 · °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine if doubling the thickness of the plastic cover will increase or decrease this interface temperature.

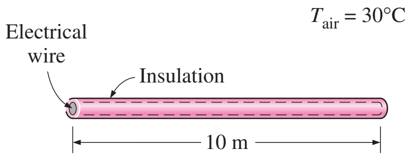


FIGURE P3–88

3–89E A 0.083-in.-diameter electrical wire at 115°F is covered by 0.02-in.-thick plastic insulation ($k = 0.075$ Btu/h · ft · °F). The wire is exposed to a medium at 50°F, with a combined convection and radiation heat transfer coefficient of 2.5 Btu/h · ft^2 · °F. Determine if the plastic insulation on the wire will increase or decrease heat transfer from the wire.
Answer: It helps

3–90E Repeat Problem 3–89E, assuming a thermal contact resistance of 0.001 h · ft^2 · °F/Btu at the interface of the wire and the insulation.

3–91 A 5-mm-diameter spherical ball at 50°C is covered by a 1-mm-thick plastic insulation ($k = 0.13$ W/m · °C). The ball is exposed to a medium at 15°C, with a combined convection and radiation heat transfer coefficient of 20 W/m^2 · °C. Determine if the plastic insulation on the ball will help or hurt heat transfer from the ball.

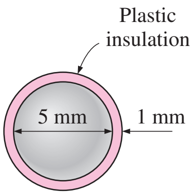


FIGURE P3–91

3–92 Reconsider Problem 3–91. Using EES (or other) software, plot the rate of heat transfer from the ball as a function of the plastic insulation thickness in the range of 0.5 mm to 20 mm. Discuss the results.

Heat Transfer from Finned Surfaces

3–93C What is the reason for the widespread use of fins on surfaces?

3–94C What is the difference between the fin effectiveness and the fin efficiency?

3–95C The fins attached to a surface are determined to have an effectiveness of 0.9. Do you think the rate of heat transfer from the surface has increased or decreased as a result of the addition of these fins?

3–96C Explain how the fins enhance heat transfer from a surface. Also, explain how the addition of fins may actually decrease heat transfer from a surface.

3–97C How does the overall effectiveness of a finned surface differ from the effectiveness of a single fin?

3–98C Hot water is to be cooled as it flows through the tubes exposed to atmospheric air. Fins are to be attached in order to enhance heat transfer. Would you recommend attaching the fins inside or outside the tubes? Why?

3–99C Hot air is to be cooled as it is forced to flow through the tubes exposed to atmospheric air. Fins are to be added in order to enhance heat transfer. Would you recommend attaching the fins inside or outside the tubes? Why? When would you recommend attaching fins both inside and outside the tubes?

3–100C Consider two finned surfaces that are identical except that the fins on the first surface are formed by casting or extrusion, whereas they are attached to the second surface afterwards by welding or tight fitting. For which case do you think the fins will provide greater enhancement in heat transfer? Explain.

3–101C The heat transfer surface area of a fin is equal to the sum of all surfaces of the fin exposed to the surrounding medium, including the surface area of the fin tip. Under what conditions can we neglect heat transfer from the fin tip?

3–102C Does the (*a*) efficiency and (*b*) effectiveness of a fin increase or decrease as the fin length is increased?

3–103C Two pin fins are identical, except that the diameter of one of them is twice the diameter of the other. For which fin will the (*a*) fin effectiveness and (*b*) fin efficiency be higher? Explain.

3–104C Two plate fins of constant rectangular cross section are identical, except that the thickness of one of them is twice the thickness of the other. For which fin will the (*a*) fin effectiveness and (*b*) fin efficiency be higher? Explain.

3–105C Two finned surfaces are identical, except that the convection heat transfer coefficient of one of them is twice that of the other. For which finned surface will the (*a*) fin effectiveness and (*b*) fin efficiency be higher? Explain.

3–106 Obtain a relation for the fin efficiency for a fin of constant cross-sectional area A_c, perimeter p, length L, and thermal conductivity k exposed to convection to a medium at T_∞ with a heat transfer coefficient h. Assume the fins are sufficiently long so that the temperature of the fin at the tip is nearly T_∞. Take the temperature of the fin at the base to be T_b and neglect heat

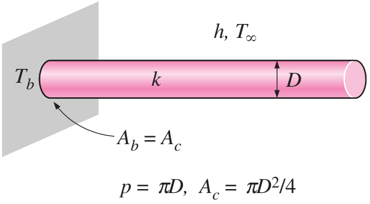


FIGURE P3–106

transfer from the fin tips. Simplify the relation for (*a*) a circular fin of diameter D and (*b*) rectangular fins of thickness t.

3–107 The case-to-ambient thermal resistance of a power transistor that has a maximum power rating of 15 W is given to be 25°C/W. If the case temperature of the transistor is not to exceed 80°C, determine the power at which this transistor can be operated safely in an environment at 40°C.

3–108 A 40-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–4. Select a heat sink that will allow the case temperature of the transistor not to exceed 90° in the ambient air at 20°.

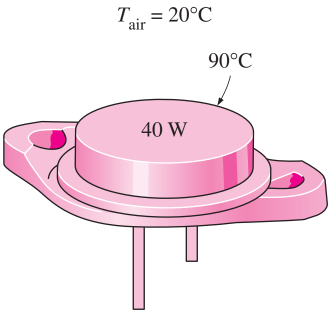


FIGURE P3–108

3–109 A 30-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 3–4. Select a heat sink that will allow the case temperature of the transistor not to exceed 80°C in the ambient air at 35°C.

3–110 Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°C. Circular aluminum alloy 2024-T6 fins (k = 186 W/m · °C) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞ = 25°C, with a heat transfer coefficient of 40 W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins. *Answer:* 2639 W

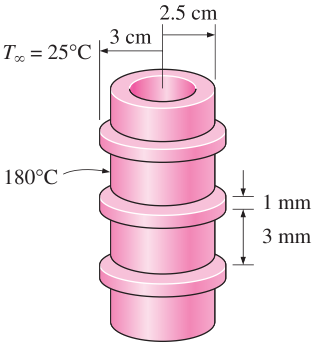


FIGURE P3–110

3–111E Consider a stainless steel spoon (k = 8.7 Btu/h · ft · °F) partially immersed in boiling water at 200°F in a kitchen at 75°F. The handle of the spoon has a cross section of 0.08 in. × 0.5 in., and extends 7 in. in the air from the free surface of the water. If the heat transfer coefficient at the exposed surfaces of the spoon handle is 3 Btu/h · ft^2 · °F, determine the temperature difference across the exposed surface of the spoon handle. State your assumptions. *Answer:* 124.6°F

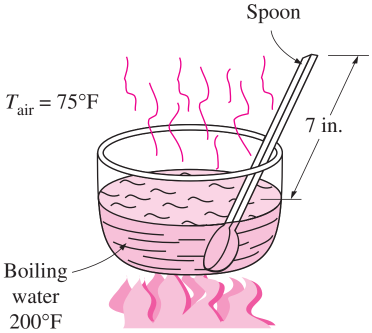


FIGURE P3–111E

3–112E Repeat Problem 3–111 for a silver spoon (k = 247 Btu/h · ft · °F).

3–113E Reconsider Problem 3–111E. Using EES (or other) software, investigate the effects of the thermal conductivity of the spoon material and the length of its extension in the air on the temperature difference across the exposed surface of the spoon handle. Let the thermal conductivity vary from 5 Btu/h · ft · °F to 225 Btu/h · ft · °F and the length from 5 in. to 12 in. Plot the temperature difference as the functions of thermal conductivity and length, and discuss the results.

3–114 A 0.3-cm-thick, 12-cm-high, and 18-cm-long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.04 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 20 W/m · °C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to a medium at 40°C, with a heat transfer coefficient of 50 W/m^2 · °C. (*a*) Determine the temperatures on the two sides of the circuit board. (*b*) Now a 0.2-cm-thick, 12-cm-high, and

18-cm-long aluminum plate (k = 237 W/m · °C) with 864 2-cm-long aluminum pin fins of diameter 0.25 cm is attached to the back side of the circuit board with a 0.02-cm-thick epoxy adhesive (k = 1.8 W/m · °C). Determine the new temperatures on the two sides of the circuit board.

3–115 Repeat Problem 3–114 using a copper plate with copper fins (k = 386 W/m · °C) instead of aluminum ones.

3–116 A hot surface at 100°C is to be cooled by attaching 3-cm-long, 0.25-cm-diameter aluminum pin fins (k = 237 W/m · °C) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m^2 · °C. Determine the rate of heat transfer from the surface for a 1-m × 1-m section of the plate. Also determine the overall effectiveness of the fins.

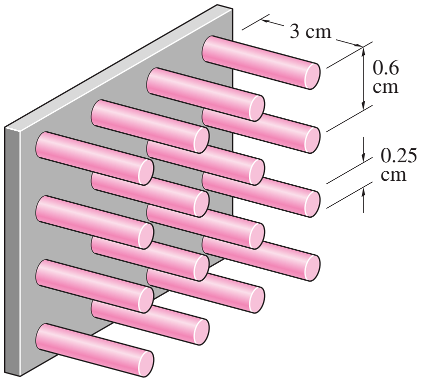


FIGURE P3–116

3–117 Repeat Problem 3–116 using copper fins (k = 386 W/m · °C) instead of aluminum ones.

3–118 Reconsider Problem 3–116. Using EES (or other) software, investigate the effect of the center-to-center distance of the fins on the rate of heat transfer from the surface and the overall effectiveness of the fins. Let the center-to-center distance vary from 0.4 cm to 2.0 cm. Plot the rate of heat transfer and the overall effectiveness as a function of the center-to-center distance, and discuss the results.

3–119 Two 3-m-long and 0.4-cm-thick cast iron (k = 52 W/m · °C) steam pipes of outer diameter 10 cm are connected to each other through two 1-cm-thick flanges of outer diameter 20 cm. The steam flows inside the pipe at an average temperature of 200°C with a heat transfer coefficient of 180 W/m^2 · °C. The outer surface of the pipe is exposed to an ambient at 12°C, with a heat transfer coefficient of 25 W/m^2 · °C. (*a*) Disregarding the flanges, determine the average outer surface temperature of the pipe. (*b*) Using this temperature for the base of the flange and treating the flanges as the fins, determine the fin efficiency and the rate of heat transfer from the flanges. (*c*) What length of pipe is the flange section equivalent to for heat transfer purposes?

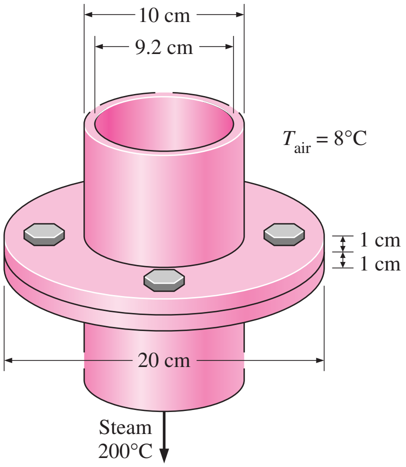


FIGURE P3–119

Heat Transfer in Common Configurations

3-120C What is a conduction shape factor? How is it related to the thermal resistance?

3-121C What is the value of conduction shape factors in engineering?

3-122 A 20-m-long and 8-cm-diameter hot water pipe of a district heating system is buried in the soil 80 cm below the ground surface. The outer surface temperature of the pipe is 60°C. Taking the surface temperature of the earth to be 5°C and the thermal conductivity of the soil at that location to be 0.9 W/m · °C, determine the rate of heat loss from the pipe.

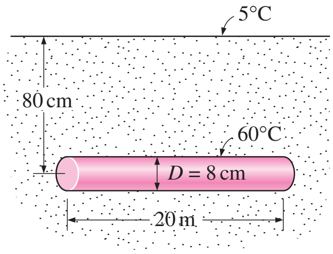


FIGURE P3–122

3–123 Reconsider Problem 3–122. Using EES (or other) software, plot the rate of heat loss from the pipe as a function of the burial depth in the range of 20 cm to 2.0 m. Discuss the results.

3-124 Hot and cold water pipes 8 m long run parallel to each other in a thick concrete layer. The diameters of both pipes are 5 cm, and the distance between the centerlines of the pipes is 40 cm. The surface temperatures of the hot and cold pipes are 60°C and 15°C, respectively. Taking the thermal conductivity of the concrete to be k = 0.75 W/m · °C, determine the rate of heat transfer between the pipes. *Answer:* 306 W

3–125 Reconsider Problem 3–124. Using EES (or other) software, plot the rate of heat transfer between the pipes as a function of the distance between the centerlines of the pipes in the range of 10 cm to 1.0 m. Discuss the results.

3-126E A row of 3-ft-long and 1-in.-diameter used uranium fuel rods that are still radioactive are buried in the ground parallel to each other with a center-to-center distance of 8 in. at a depth 15 ft from the ground surface at a location where the thermal conductivity of the soil is 0.6 Btu/h · ft · °F. If the surface temperature of the rods and the ground are 350°F and 60°F, respectively, determine the rate of heat transfer from the fuel rods to the atmosphere through the soil.

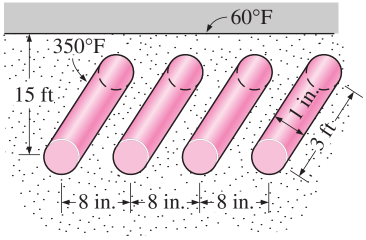


FIGURE P3–126

3-127 Hot water at an average temperature of 60°C and an average velocity of 0.6 m/s is flowing through a 5-m section of a thin-walled hot water pipe that has an outer diameter of 2.5 cm. The pipe passes through the center of a 14-cm-thick wall filled with fiberglass insulation (k = 0.035 W/m · °C). If the surfaces of the wall are at 18°C, determine (a) the rate of heat transfer from the pipe to the air in the rooms and (b) the temperature drop of the hot water as it flows through this 5-m-long section of the wall. *Answers:* 23.5 W, 0.02°C

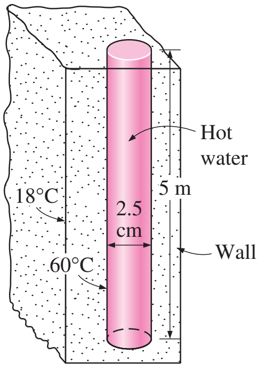


FIGURE P3–127

3-128 Hot water at an average temperature of 80°C and an average velocity of 1.5 m/s is flowing through a 25-m section of a pipe that has an outer diameter of 5 cm. The pipe extends 2 m in the ambient air above the ground, dips into the ground

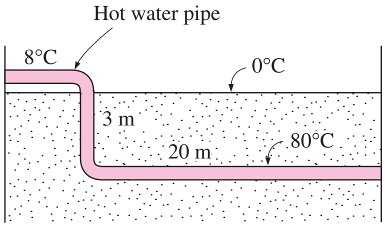


FIGURE P3–128

(k = 1.5 W/m · °C) vertically for 3 m, and continues horizontally at this depth for 20 m more before it enters the next building. The first section of the pipe is exposed to the ambient air at 8°C, with a heat transfer coefficient of 22 W/m^2 · °C. If the surface of the ground is covered with snow at 0°C, determine (a) the total rate of heat loss from the hot water and (b) the temperature drop of the hot water as it flows through this 25-m-long section of the pipe.

3-129 Consider a house with a flat roof whose outer dimensions are 12 m × 12 m. The outer walls of the house are 6 m high. The walls and the roof of the house are made of 20-cm-thick concrete (k = 0.75 W/m · °C). The temperatures of the inner and outer surfaces of the house are 15°C and 3°C, respectively. Accounting for the effects of the edges of adjoining surfaces, determine the rate of heat loss from the house through its walls and the roof. What is the error involved in ignoring the effects of the edges and corners and treating the roof as a 12 m × 12 m surface and the walls as 6 m × 12 m surfaces for simplicity?

3-130 Consider a 10-m-long thick-walled concrete duct (k = 0.75 W/m · °C) of square cross-section. The outer dimensions of the duct are 20 cm × 20 cm, and the thickness of the duct wall is 2 cm. If the inner and outer surfaces of the duct are at 100°C and 15°C, respectively, determine the rate of heat transfer through the walls of the duct. *Answer:* 22.9 kW

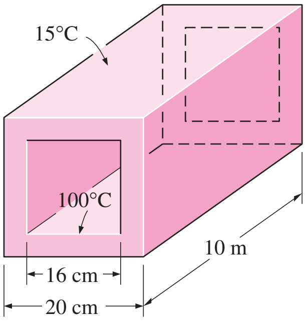


FIGURE P3–130

3-131 A 3-m-diameter spherical tank containing some radioactive material is buried in the ground (k = 1.4 W/m · °C). The distance between the top surface of the tank and the ground surface is 4 m. If the surface temperatures of the tank and the ground are 140°C and 15°C, respectively, determine the rate of heat transfer from the tank.

3–132 Reconsider Problem 3–131. Using EES (or other) software, plot the rate of heat transfer from the tank as a function of the tank diameter in the range of 0.5 m to 5.0 m. Discuss the results.

3-133 Hot water at an average temperature of 85°C passes through a row of eight parallel pipes that are 4 m long and have an outer diameter of 3 cm, located vertically in the middle of a concrete wall (k = 0.75 W/m · °C) that is 4 m high, 8 m long, and 15 cm thick. If the surfaces of the concrete walls are exposed to a medium at 32°C, with a heat transfer coefficient of 12 W/m^2 · °C, determine the rate of heat loss from the hot water and the surface temperature of the wall.

Special Topics: Heat Transfer through the Walls and Roofs

3–134C What is the R-value of a wall? How does it differ from the unit thermal resistance of the wall? How is it related to the U-factor?

3–135C What is effective emissivity for a plane-parallel air space? How is it determined? How is radiation heat transfer through the air space determined when the effective emissivity is known?

3–136C The unit thermal resistances (R-values) of both 40-mm and 90-mm vertical air spaces are given in Table 3–9 to be 0.22 m^2 · °C/W, which implies that more than doubling the thickness of air space in a wall has no effect on heat transfer through the wall. Do you think this is a typing error? Explain.

3–137C What is a radiant barrier? What kind of materials are suitable for use as radiant barriers? Is it worthwhile to use radiant barriers in the attics of homes?

3–138C Consider a house whose attic space is ventilated effectively so that the air temperature in the attic is the same as the ambient air temperature at all times. Will the roof still have any effect on heat transfer through the ceiling? Explain.

3–139 Determine the summer R-value and the U-factor of a wood frame wall that is built around 38-mm × 140-mm wood studs with a center-to-center distance of 400 mm. The 140-mm-wide cavity between the studs is filled with mineral fiber batt insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13-mm wood fiberboard and 13-mm × 200-mm wood bevel lapped siding. The insulated cavity constitutes 80 percent of the heat transmission area, while the studs, headers, plates, and sills constitute 20 percent. *Answers:* 3.213 m^2 · °C/W, 0.311 W/m^2 · °C

3–140 The 13-mm-thick wood fiberboard sheathing of the wood stud wall in Problem 3–139 is replaced by a 25-mm-thick rigid foam insulation. Determine the percent increase in the R-value of the wall as a result.

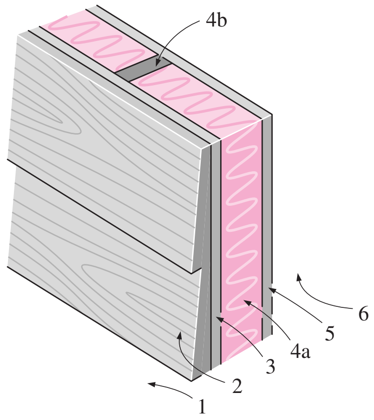


FIGURE P3–139

3–141E Determine the winter R-value and the U-factor of a masonry cavity wall that is built around 4-in.-thick concrete blocks made of lightweight aggregate. The outside is finished with 4-in. face brick with $\frac{1}{2}$-in. cement mortar between the bricks and concrete blocks. The inside finish consists of $\frac{1}{2}$-in. gypsum wallboard separated from the concrete block by $\frac{3}{4}$-in.-thick (1-in. by 3-in. nominal) vertical furring whose center-to-center distance is 16 in. Neither side of the $\frac{3}{4}$-in.-thick air space between the concrete block and the gypsum board is coated with any reflective film. When determining the R-value of the air space, the temperature difference across it can be taken to be 30°F with a mean air temperature of 50°F. The air space constitutes 80 percent of the heat transmission area, while the vertical furring and similar structures constitute 20 percent.

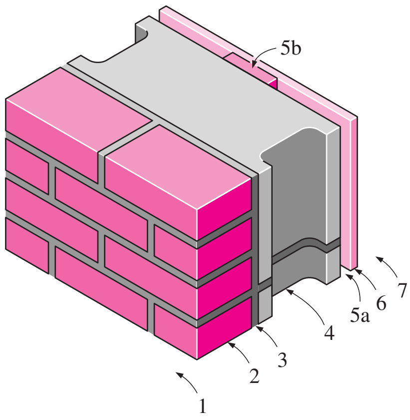


FIGURE P3–141E

3–142 Consider a flat ceiling that is built around 38-mm × 90-mm wood studs with a center-to-center distance of 400 mm. The lower part of the ceiling is finished with 13-mm gypsum wallboard, while the upper part consists of a wood subfloor (R = 0.166 m^2 · °C/W), a 13-mm plywood, a layer of felt (R = 0.011 m^2 · °C/W), and linoleum (R = 0.009 m^2 · °C/W). Both

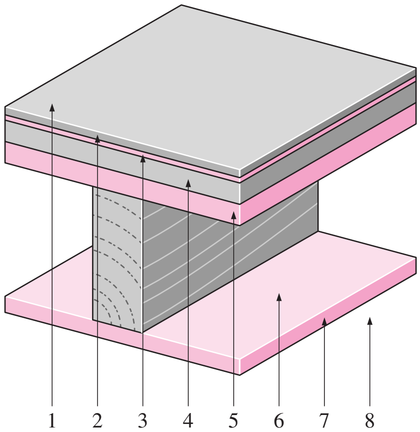


FIGURE P3–142

sides of the ceiling are exposed to still air. The air space constitutes 82 percent of the heat transmission area, while the studs and headers constitute 18 percent. Determine the winter R-value and the U-factor of the ceiling assuming the 90-mm-wide air space between the studs (*a*) does not have any reflective surface, (*b*) has a reflective surface with $\varepsilon = 0.05$ on one side, and (*c*) has reflective surfaces with $\varepsilon = 0.05$ on both sides. Assume a mean temperature of 10°C and a temperature difference of 5.6°C for the air space.

3–143 Determine the winter R-value and the U-factor of a masonry cavity wall that consists of 100-mm common bricks, a 90-mm air space, 100-mm concrete blocks made of lightweight aggregate, 20-mm air space, and 13-mm gypsum wallboard separated from the concrete block by 20-mm-thick (1-in. × 3-in. nominal) vertical furring whose center-to-center distance is 400 mm. Neither side of the two air spaces is coated with any reflective films. When determining the R-value of the air spaces, the temperature difference across them can be taken to be 16.7°C with a mean air temperature of 10°C. The air space constitutes 84 percent of the heat transmission area, while the vertical furring and similar structures constitute 16 percent. *Answers:* 1.02 m^2 · °C/W, 0.978 W/m^2 · °C

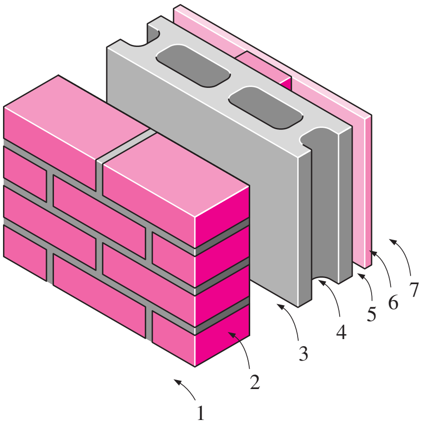


FIGURE P3–143

3–144 Repeat Problem 3–143 assuming one side of both air spaces is coated with a reflective film of $\varepsilon = 0.05$.

3–145 Determine the winter R-value and the U-factor of a masonry wall that consists of the following layers: 100-mm face bricks, 100-mm common bricks, 25-mm urethane rigid foam insulation, and 13-mm gypsum wallboard.
Answers: 1.404 m^2 · °C/W, 0.712 W/m^2 · °C

3–146 The overall heat transfer coefficient (the U-value) of a wall under winter design conditions is $U = 1.55$ W/m^2 · °C. Determine the U-value of the wall under summer design conditions.

3–147 The overall heat transfer coefficient (the U-value) of a wall under winter design conditions is $U = 2.25$ W/m^2 · °C. Now a layer of 100-mm face brick is added to the outside, leaving a 20-mm air space between the wall and the bricks. Determine the new U-value of the wall. Also, determine the rate of heat transfer through a 3-m-high, 7-m-long section of the wall after modification when the indoor and outdoor temperatures are 22°C and −5°C, respectively.

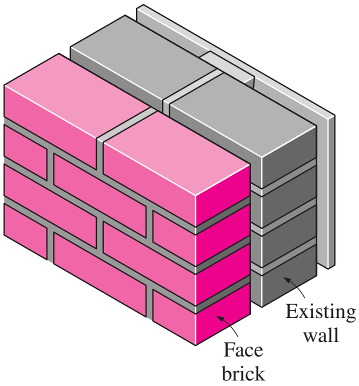


FIGURE P3–147

3–148 Determine the summer and winter R-values, in m^2 · °C/W, of a masonry wall that consists of 100-mm face bricks, 13-mm of cement mortar, 100-mm lightweight concrete block, 40-mm air space, and 20-mm plasterboard.
Answers: 0.809 and 0.795 m^2 · °C/W

3–149E The overall heat transfer coefficient of a wall is determined to be $U = 0.09$ Btu/h · ft^2 · °F under the conditions of still air inside and winds of 7.5 mph outside. What will the U-factor be when the wind velocity outside is doubled?
Answer: 0.0907 Btu/h · ft^2 · °F

3–150 Two homes are identical, except that the walls of one house consist of 200-mm lightweight concrete blocks, 20-mm air space, and 20-mm plasterboard, while the walls of the other house involve the standard R-2.4 m^2 · °C/W frame wall construction. Which house do you think is more energy efficient?

3–151 Determine the R-value of a ceiling that consists of a layer of 19-mm acoustical tiles whose top surface is covered with a highly reflective aluminum foil for winter conditions. Assume still air below and above the tiles.

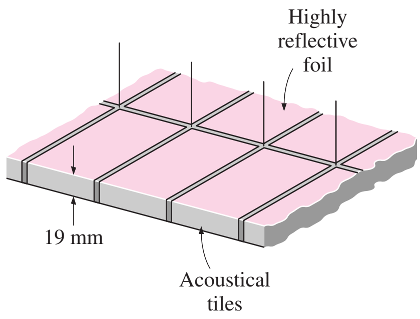


FIGURE P3–151

Review Problems

3–152E Steam is produced in the copper tubes (k = 223 Btu/h · ft · °F) of a heat exchanger at a temperature of 250°F by another fluid condensing on the outside surfaces of the tubes at 350°F. The inner and outer diameters of the tube are 1 in. and 1.3 in., respectively. When the heat exchanger was new, the rate of heat transfer per foot length of the tube was 2×10^4 Btu/h. Determine the rate of heat transfer per foot length of the tube when a 0.01-in.-thick layer of limestone (k = 1.7 Btu/h · ft · °F) has formed on the inner surface of the tube after extended use.

3–153E Repeat Problem 3–152E, assuming that a 0.01-in.-thick limestone layer has formed on both the inner and outer surfaces of the tube.

3–154 A 1.2-m-diameter and 6-m-long cylindrical propane tank is initially filled with liquid propane whose density is 581 kg/m^3. The tank is exposed to the ambient air at 30°C, with a heat transfer coefficient of 25 W/m^2 · °C. Now a crack develops at the top of the tank and the pressure inside drops to 1 atm while the temperature drops to −42°C, which is the boiling temperature of propane at 1 atm. The heat of vaporization of propane at 1 atm is 425 kJ/kg. The propane is slowly vaporized as a result of the heat transfer from the ambient air into the tank, and the propane vapor escapes the tank at −42°C through the crack. Assuming the propane tank to be at about the same temperature as the propane inside at all times, determine how long it will take for the propane tank to empty if the tank is (*a*) not insulated and (*b*) insulated with 7.5-cm-thick glass wool insulation (k = 0.038 W/m · °C).

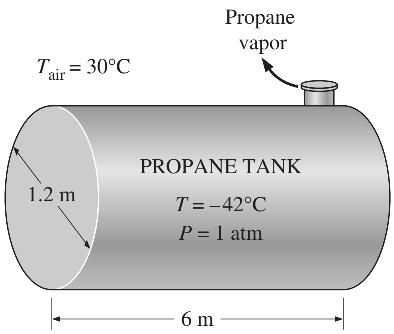


FIGURE P3–154

3–155 Hot water is flowing at an average velocity of 1.5 m/s through a cast iron pipe (k = 52 W/m · °C) whose inner and outer diameters are 3 cm and 3.5 cm, respectively. The pipe passes through a 15-m-long section of a basement whose temperature is 15°C. If the temperature of the water drops from 70°C to 67°C as it passes through the basement and the heat transfer coefficient on the inner surface of the pipe is 400 W/m^2 · °C, determine the combined convection and radiation heat transfer coefficient at the outer surface of the pipe.
Answer: 272.5 W/m^2 · °C

3–156 Newly formed concrete pipes are usually cured first overnight by steam in a curing kiln maintained at a temperature of 45°C before the pipes are cured for several days outside. The heat and moisture to the kiln is provided by steam flowing in a pipe whose outer diameter is 12 cm. During a plant inspection, it was noticed that the pipe passes through a 10-m section that is completely exposed to the ambient air before it reaches the kiln. The temperature measurements indicate that the average temperature of the outer surface of the steam pipe is 82°C when the ambient temperature is 8°C. The combined convection and radiation heat transfer coefficient at the outer surface of the pipe is estimated to be 25 W/m^2 · °C. Determine the amount of heat lost from the steam during a 10-h curing process that night.

Steam is supplied by a gas-fired steam generator that has an efficiency of 80 percent, and the plant pays $0.60/therm of natural gas (1 therm = 105,500 kJ). If the pipe is insulated and 90 percent of the heat loss is saved as a result, determine the amount of money this facility will save a year as a result of insulating the steam pipes. Assume that the concrete pipes are cured 110 nights a year. State your assumptions.

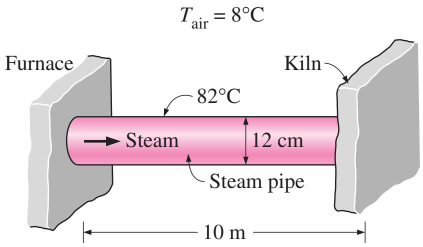


FIGURE P3–156

3–157 Consider an 18-cm × 18-cm multilayer circuit board dissipating 27 W of heat. The board consists of four layers of 0.2-mm-thick copper (k = 386 W/m · °C) and three layers of

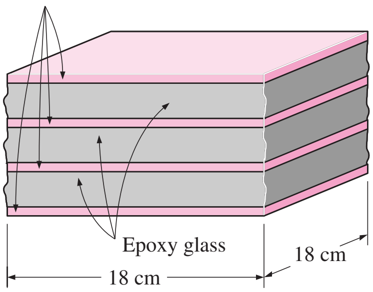


FIGURE P3–157

1.5-mm-thick epoxy glass (k = 0.26 W/m · °C) sandwiched together, as shown in the figure. The circuit board is attached to a heat sink from both ends, and the temperature of the board at those ends is 35°C. Heat is considered to be uniformly generated in the epoxy layers of the board at a rate of 0.5 W per 1-cm × 18-cm epoxy laminate strip (or 1.5 W per 1-cm × 18-cm strip of the board). Considering only a portion of the board because of symmetry, determine the magnitude and location of the maximum temperature that occurs in the board. Assume heat transfer from the top and bottom faces of the board to be negligible.

3–158 The plumbing system of a house involves a 0.5-m section of a plastic pipe (k = 0.16 W/m · °C) of inner diameter 2 cm and outer diameter 2.4 cm exposed to the ambient air. During a cold and windy night, the ambient air temperature remains at about −5°C for a period of 14 h. The combined convection and radiation heat transfer coefficient on the outer surface of the pipe is estimated to be 40 W/m² · °C, and the heat of fusion of water is 333.7 kJ/kg. Assuming the pipe to contain stationary water initially at 0°C, determine if the water in that section of the pipe will completely freeze that night.

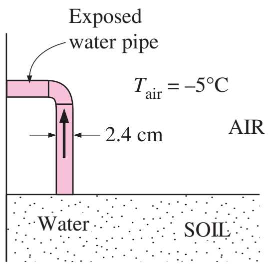


FIGURE P3–158

3–159 Repeat Problem 3–158 for the case of a heat transfer coefficient of 10 W/m² · °C on the outer surface as a result of putting a fence around the pipe that blocks the wind.

3–160E The surface temperature of a 3-in.-diameter baked potato is observed to drop from 300°F to 200°F in 5 minutes in an environment at 70°F. Determine the average heat transfer coefficient between the potato and its surroundings. Using this heat transfer coefficient and the same surface temperature, determine how long it will take for the potato to experience the same temperature drop if it is wrapped completely in a 0.12-in.-thick towel (k = 0.035 Btu/h · ft · °F). You may use the properties of water for potato.

3–161E Repeat Problem 3–160E assuming there is a 0.02-in.-thick air space (k = 0.015 Btu/h · ft · °F) between the potato and the towel.

3–162 An ice chest whose outer dimensions are 30 cm × 40 cm × 50 cm is made of 3-cm-thick Styrofoam (k = 0.033 W/m · °C). Initially, the chest is filled with 45 kg of ice at 0°C, and the inner surface temperature of the ice chest can be taken to be 0°C at all times. The heat of fusion of ice at 0°C is 333.7 kJ/kg, and the heat transfer coefficient between the outer surface of the ice chest and surrounding air at 35°C is 18 W/m² · °C. Disregarding any heat transfer from the 40-cm × 50-cm base of the ice chest, determine how long it will take for the ice in the chest to melt completely.

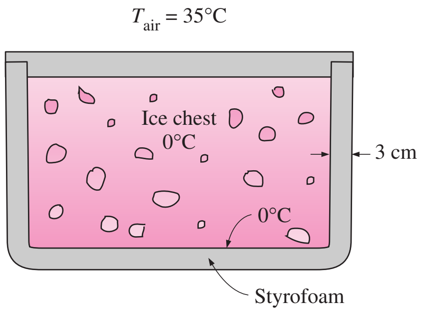


FIGURE P3–162

3–163 A 4-m-high and 6-m-long wall is constructed of two large 2-cm-thick steel plates (k = 15 W/m · °C) separated by 1-cm-thick and 20-cm-wide steel bars placed 99 cm apart. The

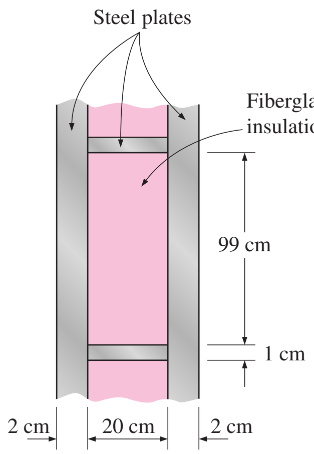


FIGURE P3–163

remaining space between the steel plates is filled with fiberglass insulation (k = 0.035 W/m · °C). If the temperature difference between the inner and the outer surfaces of the walls is 22°C, determine the rate of heat transfer through the wall. Can we ignore the steel bars between the plates in heat transfer analysis since they occupy only 1 percent of the heat transfer surface area?

3–164 A 0.2-cm-thick, 10-cm-high, and 15-cm-long circuit board houses electronic components on one side that dissipate a total of 15 W of heat uniformly. The board is impregnated with conducting metal fillings and has an effective thermal conductivity of 12 W/m · °C. All the heat generated in the components is conducted across the circuit board and is dissipated from the back side of the board to a medium at 37°C, with a heat transfer coefficient of 45 W/m^2 · °C. (*a*) Determine the surface temperatures on the two sides of the circuit board. (*b*) Now a 0.1-cm-thick, 10-cm-high, and 15-cm-long aluminum plate (k = 237 W/m · °C) with 20 0.2-cm-thick, 2-cm-long, and 15-cm-wide aluminum fins of rectangular profile are attached to the back side of the circuit board with a 0.03–cm-thick epoxy adhesive (k = 1.8 W/m · °C). Determine the new temperatures on the two sides of the circuit board.

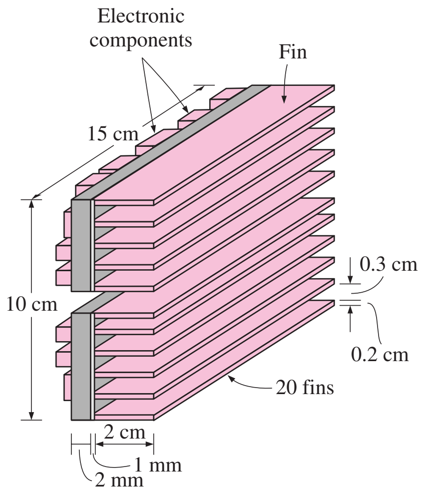


FIGURE P3–164

3–165 Repeat Problem 3–164 using a copper plate with copper fins (k = 386 W/m · °C) instead of aluminum ones.

3-166 A row of 10 parallel pipes that are 5 m long and have an outer diameter of 6 cm are used to transport steam at 150°C through the concrete floor (k = 0.75 W/m · °C) of a 10-m × 5-m room that is maintained at 25°C. The combined convection and radiation heat transfer coefficient at the floor is 12 W/m^2 · °C. If the surface temperature of the concrete floor is not to exceed 40°C, determine how deep the steam pipes should be buried below the surface of the concrete floor.

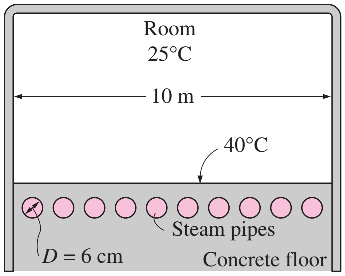


FIGURE P3–166

3–167 Consider two identical people each generating 60 W of metabolic heat steadily while doing sedentary work, and dissipating it by convection and perspiration. The first person is wearing clothes made of 1-mm-thick leather (k = 0.159 W/m · °C) that covers half of the body while the second one is wearing clothes made of 1-mm-thick synthetic fabric (k = 0.13 W/m · °C) that covers the body completely. The ambient air is at 30°C, the heat transfer coefficient at the outer surface is 15 W/m^2 · °C, and the inner surface temperature of the clothes can be taken to be 32°C. Treating the body of each person as a 25-cm-diameter 1.7-m-long cylinder, determine the fractions of heat lost from each person by perspiration.

3–168 A 6-m-wide 2.8-m-high wall is constructed of one layer of common brick (k = 0.72 W/m · °C) of thickness 20 cm, one inside layer of light-weight plaster (k = 0.36 W/m · °C) of thickness 1 cm, and one outside layer of cement based covering (k = 1.40 W/m · °C) of thickness 2 cm. The inner surface of the wall is maintained at 23°C while the outer surface is exposed to outdoors at 8°C with a combined convection and radiation heat transfer coefficient of 17 W/m^2 · °C. Determine the rate of heat transfer through the wall and temperature drops across the plaster, brick, covering, and surface-ambient air.

3–169 Reconsider Problem 3–168. It is desired to insulate the wall in order to decrease the heat loss by 85 percent. For the same inner surface temperature, determine the thickness of insulation and the outer surface temperature if the wall is insulated with (*a*) polyurethane foam (k = 0.025 W/m · °C) and (*b*) glass fiber (k = 0.036 W/m · °C).

3–170 Cold conditioned air at 12°C is flowing inside a 1.5-cm-thick square aluminum (k = 237 W/m · °C) duct of inner cross section 22 cm × 22 cm at a mass flow rate of 0.8 kg/s. The duct is exposed to air at 33°C with a combined convection-radiation heat transfer coefficient of 8 W/m^2 · °C. The convection heat transfer coefficient at the inner surface is 75 W/m^2 · °C. If the air temperature in the duct should not increase by more than 1°C determine the maximum length of the duct.

3–171 When analyzing heat transfer through windows, it is important to consider the frame as well as the glass area. Consider a 2-m-wide 1.5-m-high wood-framed window with

85 percent of the area covered by 3-mm-thick single-pane glass (k = 0.7 W/m · °C). The frame is 5 cm thick, and is made of pine wood (k = 0.12 W/m · °C). The heat transfer coefficient is 7 W/m^2 · °C inside and 13 W/m^2 · °C outside. The room is maintained at 24°C, and the temperature outdoors is 40°C. Determine the percent error involved in heat transfer when the window is assumed to consist of glass only.

3–172 Steam at 235°C is flowing inside a steel pipe (k = 61 W/m · °C) whose inner and outer diameters are 10 cm and 12 cm, respectively, in an environment at 20°C. The heat transfer coefficients inside and outside the pipe are 105 W/m^2 · °C and 14 W/m^2 · °C, respectively. Determine (*a*) the thickness of the insulation (k = 0.038 W/m · °C) needed to reduce the heat loss by 95 percent and (*b*) the thickness of the insulation needed to reduce the exposed surface temperature of insulated pipe to 40°C for safety reasons.

3–173 When the transportation of natural gas in a pipeline is not feasible for economic or other reasons, it is first liquefied at about −160°C, and then transported in specially insulated tanks placed in marine ships. Consider a 6-m-diameter spherical tank that is filled with liquefied natural gas (LNG) at −160°C. The tank is exposed to ambient air at 18°C with a heat transfer coefficient of 22 W/m^2 · °C. The tank is thin-shelled and its temperature can be taken to be the same as the LNG temperature. The tank is insulated with 5-cm-thick super insulation that has an effective thermal conductivity of 0.00008 W/m · °C. Taking the density and the specific heat of LNG to be 425 kg/m^3 and 3.475 kJ/kg · °C, respectively, estimate how long it will take for the LNG temperature to rise to −150°C.

3–174 A 15-cm × 20-cm hot surface at 85°C is to be cooled by attaching 4-cm-long aluminum (k = 237 W/m · °C) fins of 2-mm × 2-mm square cross section. The temperature of surrounding medium is 25°C and the heat transfer coefficient on the surfaces can be taken to be 20 W/m^2 · °C. If it is desired to triple the rate of heat transfer from the bare hot surface, determine the number of fins that needs to be attached.

3–175 Reconsider Problem 3–174. Using EES (or other) software, plot the number of fins as a function of the increase in the heat loss by fins relative to no fin case (i.e., overall effectiveness of the fins) in the range of 1.5 to 5. Discuss the results. Is it realistic to assume the heat transfer coefficient to remain constant?

3–176 A 1.4-m-diameter spherical steel tank filled with iced water at 0°C is buried underground at a location where the thermal conductivity of the soil is k = 0.55 W/m · °C. The distance between the tank center and the ground surface is 2.4 m. For ground surface temperature of 18°C, determine the rate of heat transfer to the iced water in the tank. What would your answer be if the soil temperature were 18°C and the ground surface were insulated?

3–177 A 0.6-m-diameter 1.9-m-long cylindrical tank containing liquefied natural gas (LNG) at −160°C is placed at the center of a 1.9-m-long 1.4-m × 1.4-m square solid bar made of an insulating material with k = 0.0006 W/m · °C. If the outer surface temperature of the bar is 20°C, determine the rate of heat transfer to the tank. Also, determine the LNG temperature after one month. Take the density and the specific heat of LNG to be 425 kg/m^3 and 3.475 kJ/kg · °C, respectively.

Design and Essay Problems

3–178 The temperature in deep space is close to absolute zero, which presents thermal challenges for the astronauts who do space walks. Propose a design for the clothing of the astronauts that will be most suitable for the thermal environment in space. Defend the selections in your design.

3–179 In the design of electronic components, it is very desirable to attach the electronic circuitry to a substrate material that is a very good thermal conductor but also a very effective electrical insulator. If the high cost is not a major concern, what material would you propose for the substrate?

3–180 Using cylindrical samples of the same material, devise an experiment to determine the thermal contact resistance. Cylindrical samples are available at any length, and the thermal conductivity of the material is known.

3–181 Find out about the wall construction of the cabins of large commercial airplanes, the range of ambient conditions under which they operate, typical heat transfer coefficients on the inner and outer surfaces of the wall, and the heat generation rates inside. Determine the size of the heating and air-conditioning system that will be able to maintain the cabin at 20°C at all times for an airplane capable of carrying 400 people.

3–182 Repeat Problem 3–181 for a submarine with a crew of 60 people.

3–183 A house with 200-m^2 floor space is to be heated with geothermal water flowing through pipes laid in the ground under the floor. The walls of the house are 4 m high, and there are 10 single-paned windows in the house that are 1.2 m wide and 1.8 m high. The house has R-19 (in h · ft^2 · °F/Btu) insulation in the walls and R-30 on the ceiling. The floor temperature is not to exceed 40°C. Hot geothermal water is available at 90°C, and the inner and outer diameter of the pipes to be used are 2.4 cm and 3.0 cm. Design such a heating system for this house in your area.

3–184 Using a timer (or watch) and a thermometer, conduct this experiment to determine the rate of heat gain of your refrigerator. First, make sure that the door of the refrigerator is not opened for at least a few hours to make sure that steady operating conditions are established. Start the timer when the refrigerator stops running and measure the time Δt_1 it stays off before it kicks in. Then measure the time Δt_2 it stays on. Noting that the heat removed during Δt_2 is equal to the heat gain of the refrigerator during $\Delta t_1 + \Delta t_2$ and using the power consumed by the refrigerator when it is running, determine the average rate of heat gain for your refrigerator, in watts. Take the COP (coefficient of performance) of your refrigerator to be 1.3 if it is not available.

Now, clean the condenser coils of the refrigerator and remove any obstacles on the way of airflow through the coils By replacing these measurements, determine the improvement in the COP of the refrigerator.