CHAPTER 6

FUNDAMENTALS OF CONVECTION

So far, we have considered *conduction,* which is the mechanism of heat transfer through a solid or a quiescent fluid. We now consider *convection,* which is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion.

Convection is classified as *natural* (or *free*) and *forced convection,* depending on how the fluid motion is initiated. In forced convection, the fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, which manifests itself as the rise of warmer fluid and the fall of the cooler fluid. Convection is also classified as *external* and *internal,* depending on whether the fluid is forced to flow over a surface or in a channel.

We start this chapter with a general physical description of the convection mechanism. We then discuss the *velocity* and *thermal boundary layers,* and *laminar and turbulent flows.* We continue with the discussion of the dimensionless *Reynolds, Prandtl,* and *Nusselt numbers,* and their physical significance. Next we derive the *convection equations* of on the basis of mass, momentum, and energy conservation, and obtain solutions for *flow over a flat plate.* We then nondimensionalize the convection equations, and obtain functional forms of friction and convection coefficients. Finally, we present analogies between momentum and heat transfer.

CONTENTS

6–1 ▪ PHYSICAL MECHANISM OF CONVECTION

We mentioned earlier that there are three basic mechanisms of heat transfer: conduction, convection, and radiation. Conduction and convection are similar in that both mechanisms require the presence of a material medium. But they are different in that convection requires the presence of fluid motion.

Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid (Fig. 6–1).

Convection heat transfer is complicated by the fact that it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer, since it brings hotter and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction. In fact, the higher the fluid velocity, the higher the rate of heat transfer.

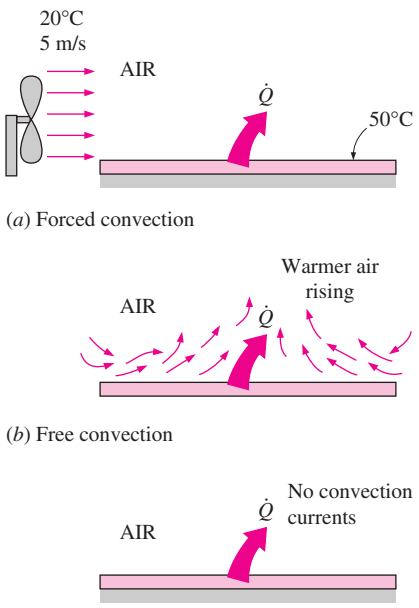


FIGURE 6–1
Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

To clarify this point further, consider steady heat transfer through a fluid contained between two parallel plates maintained at different temperatures, as shown in Figure 6–2. The temperatures of the fluid and the plate will be the same at the points of contact because of the continuity of temperature. Assuming no fluid motion, the energy of the hotter fluid molecules near the hot plate will be transferred to the adjacent cooler fluid molecules. This energy will then be transferred to the next layer of the cooler fluid molecules. This energy will then be transferred to the next layer of the cooler fluid, and so on, until it is finally transferred to the other plate. This is what happens during conduction through a fluid. Now let us use a syringe to draw some fluid near the hot plate and inject it near the cold plate repeatedly. You can imagine that this will speed up the heat transfer process considerably, since some energy is carried to the other side as a result of fluid motion.

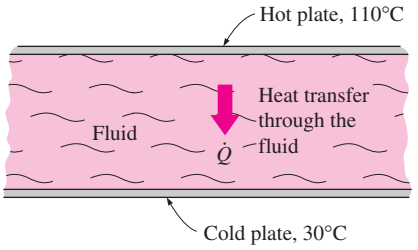


FIGURE 6–2
Heat transfer through a fluid sandwiched between two parallel plates.

Consider the cooling of a hot iron block with a fan blowing air over its top surface, as shown in Figure 6–3. We know that heat will be transferred from the hot block to the surrounding cooler air, and the block will eventually cool. We also know that the block will cool faster if the fan is switched to a higher speed. Replacing air by water will enhance the convection heat transfer even more.

Experience shows that convection heat transfer strongly depends on the fluid properties *dynamic viscosity* μ, *thermal conductivity k, density* ρ, and *specific heat* C_p, as well as the *fluid velocity* $\mathcal{V}$. It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being streamlined or turbulent). Thus, we expect the convection heat transfer relations to be rather complex because of the dependence of convection on so many variables. This is not surprising, since convection is the most complex mechanism of heat transfer.

Despite the complexity of convection, the rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by **Newton's law of cooling** as

$$\dot{q}_{\text{conv}} = h(T_s - T_\infty) \qquad (\text{W/m}^2) \tag{6-1}$$

or

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad (\text{W}) \tag{6-2}$$

where

h = convection heat transfer coefficient, W/m$^2 \cdot$ °C
A_s = heat transfer surface area, m^2
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C

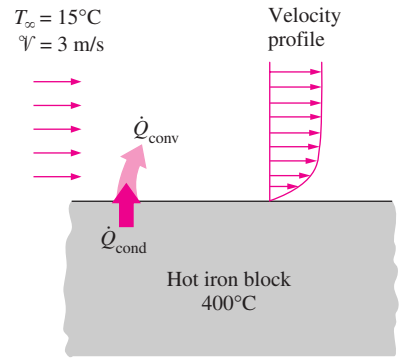


FIGURE 6–3
The cooling of a hot block by forced convection.

Judging from its units, the **convection heat transfer coefficient** h can be defined as *the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.*

You should not be deceived by the simple appearance of this relation, because the convection heat transfer coefficient h depends on the several of the mentioned variables, and thus is difficult to determine.

When a fluid is forced to flow over a solid surface that is nonporous (i.e., impermeable to the fluid), it is observed that the fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, the fluid layer in direct contact with a solid surface "sticks" to the surface and there is no slip. In fluid flow, this phenomenon is known as the **no-slip condition,** and it is due to the viscosity of the fluid (Fig. 6–4).

The no-slip condition is responsible for the development of the velocity profile for flow. Because of the friction between the fluid layers, the layer that sticks to the wall slows the adjacent fluid layer, which slows the next layer, and so on. A consequence of the no-slip condition is that all velocity profiles must have zero values at the points of contact between a fluid and a solid. The only exception to the no-slip condition occurs in extremely rarified gases.

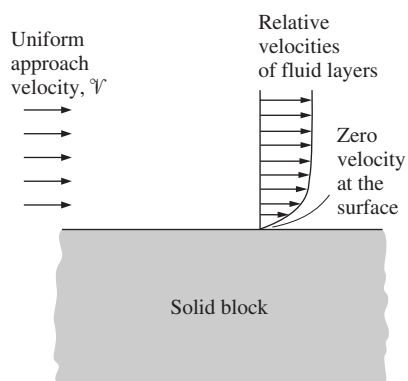


FIGURE 6–4
A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

A similar phenomenon occurs for the temperature. When two bodies at different temperatures are brought into contact, heat transfer occurs until both bodies assume the same temperature at the point of contact. Therefore, a fluid and a solid surface will have the same temperature at the point of contact. This is known as **no-temperature-jump condition.**

An implication of the no-slip and the no-temperature jump conditions is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction,* since the fluid layer is motionless, and can be expressed as

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k_{\text{fluid}} \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (\text{W/m}^2) \tag{6-3}$$

where T represents the temperature distribution in the fluid and $(\partial T/\partial y)_{y=0}$ is the *temperature gradient* at the surface. This heat is then *convected away* from the surface as a result of fluid motion. Note that convection heat transfer from a solid surface to a fluid is merely the conduction heat transfer from the solid surface to the fluid layer adjacent to the surface. Therefore, we can equate Eqs. 6-1 and 6-3 for the heat flux to obtain

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C}) \tag{6-4}$$

for the determination of the *convection heat transfer coefficient* when the temperature distribution within the fluid is known.

The convection heat transfer coefficient, in general, varies along the flow (or x-) direction. The *average* or *mean* convection heat transfer coefficient for a surface in such cases is determined by properly averaging the *local* convection heat transfer coefficients over the entire surface.

Nusselt Number

In convection studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables. It is also common practice to nondimensionalize the heat transfer coefficient h with the Nusselt number, defined as

$$\text{Nu} = \frac{hL_c}{k} \tag{6-5}$$

where k is the thermal conductivity of the fluid and L_c is the *characteristic length.* The Nusselt number is named after Wilhelm Nusselt, who made significant contributions to convective heat transfer in the first half of the twentieth century, and it is viewed as the *dimensionless convection heat transfer coefficient.*

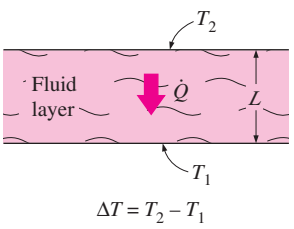


FIGURE 6–5
Heat transfer through a fluid layer of thickness L and temperature difference ΔT.

To understand the physical significance of the Nusselt number, consider a fluid layer of thickness L and temperature difference $\Delta T = T_2 - T_1$, as shown in Fig. 6–5. Heat transfer through the fluid layer will be by *convection* when the fluid involves some motion and by *conduction* when the fluid layer is motionless. Heat flux (the rate of heat transfer per unit time per unit surface area) in either case will be

$$\dot{q}_{\text{conv}} = h\Delta T \tag{6-6}$$

and

$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L} \tag{6-7}$$

Taking their ratio gives

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu} \tag{6-8}$$

which is the Nusselt number. Therefore, the Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. The larger the Nusselt number, the more effective the convection. A Nusselt number of Nu = 1 for a fluid layer represents heat transfer across the layer by pure conduction.

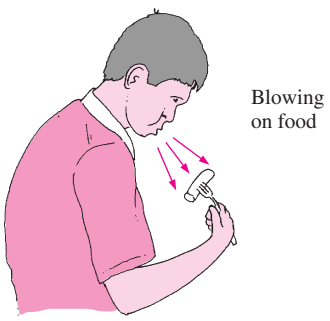


FIGURE 6–6
We resort to forced convection whenever we need to increase the rate of heat transfer.

We use forced convection in daily life more often than you might think (Fig. 6–6). We resort to forced convection whenever we want to increase the

rate of heat transfer from a hot object. For example, we turn on the fan on hot summer days to help our body cool more effectively. The higher the fan speed, the better we feel. We *stir* our soup and *blow* on a hot slice of pizza to make them cool faster. The air on *windy* winter days feels much colder than it actually is. The simplest solution to heating problems in electronics packaging is to use a large enough fan.

6–2 ▪ CLASSIFICATION OF FLUID FLOWS

Convection heat transfer is closely tied with fluid mechanics, which is the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries. There are a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify the fluid flow problems, and below we present some general categories.

Viscous versus Inviscid Flow

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is called the **viscosity,** which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids, and by the molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the effects of viscosity are significant are called **viscous flows.** The effects of viscosity are very small in some flows, and neglecting those effects greatly simplifies the analysis without much loss in accuracy. Such idealized flows of zero-viscosity fluids are called frictionless or **inviscid flows.**

Internal versus External Flow

A fluid flow is classified as being internal and external, depending on whether the fluid is forced to flow in a confined channel or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow.** The flow in a pipe or duct is **internal flow** if the fluid is completely bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and air flow over an exposed pipe during a windy day is external flow (Fig. 6–7). The flow of liquids in a pipe is called *open-channel flow* if the pipe is partially filled with the liquid and there is a free surface. The flow of water in rivers and irrigation ditches are examples of such flows.

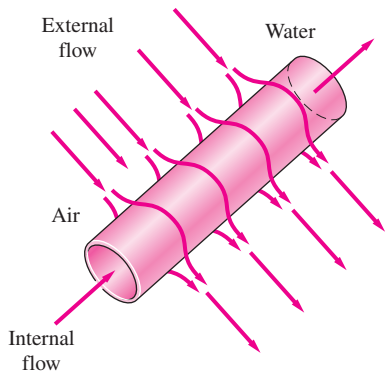


FIGURE 6–7
Internal flow of water in a pipe and the external flow of air over the same pipe.

Compressible versus Incompressible Flow

A fluid flow is classified as being *compressible* or *incompressible,* depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually classified as *incompressible substances.* A pressure of 210 atm, for example, will cause the density of liquid water at 1 atm to change by just 1 percent. Gases, on the other hand, are highly compressible. A

pressure change of just 0.01 atm, for example, will cause a change of 1 percent in the density of atmospheric air. However, gas flows can be treated as incompressible if the density changes are under about 5 percent, which is usually the case when the flow velocity is less than 30 percent of the velocity of sound in that gas (i.e., the Mach number of flow is less than 0.3). The velocity of sound in air at room temperature is 346 m/s. Therefore, the compressibility effects of air can be neglected at speeds under 100 m/s. Note that the flow of a gas is not necessarily a compressible flow.

Laminar versus Turbulent Flow

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth streamlines is called **laminar.** The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities characterized by velocity fluctuations is called **turbulent.** The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the heat transfer rates and the required power for pumping.

Natural (or Unforced) versus Forced Flow

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In **forced flow,** a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In **natural flows,** any fluid motion is due to a natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid. This thermosiphoning effect is commonly used to replace pumps in solar water heating systems by placing the water tank sufficiently above the solar collectors (Fig. 6–8).

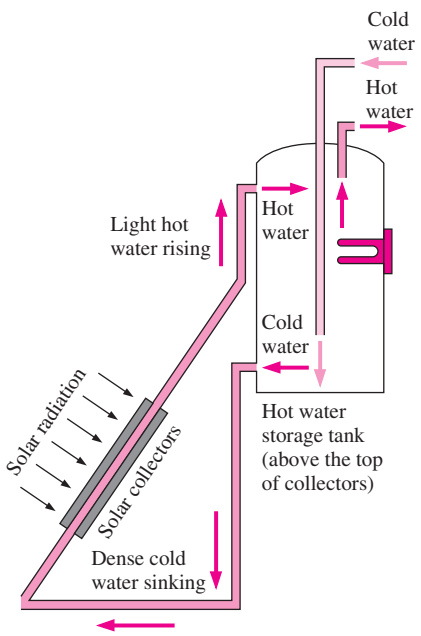


FIGURE 6–8
Natural circulation of water in a solar water heater by thermosiphoning.

Steady versus Unsteady (Transient) Flow

The terms *steady* and *uniform* are used frequently in engineering, and thus it is important to have a clear understanding of their meanings. The term **steady** implies *no change with time.* The opposite of steady is **unsteady,** or **transient.** The term *uniform,* however, implies *no change with location* over a specified region.

Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as *steady-flow devices.* During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant.

One-, Two-, and Three-Dimensional Flows

A flow field is best characterized by the velocity distribution, and thus a flow is said to be one-, two-, or three-dimensional if the flow velocity $\mathcal{V}$ varies in one, two, or three primary dimensions, respectively. A typical fluid flow involves a three-dimensional geometry and the velocity may vary in all three dimensions rendering the flow three-dimensional [$\mathcal{V}(x, y, z)$ in rectangular or $\mathcal{V}(r, \theta, z)$ in cylindrical coordinates]. However, the variation of velocity in

certain direction can be small relative to the variation in other directions, and can be ignored with negligible error. In such cases, the flow can be modeled conveniently as being one- or two-dimensional, which is easier to analyze.

When the entrance effects are disregarded, fluid flow in a circular pipe is *one-dimensional* since the velocity varies in the radial r direction but not in the angular θ- or axial z-directions (Fig. 6–9). That is, the velocity profile is the same at any axial z-location, and it is symmetric about the axis of the pipe. Note that even in this simplest flow, the velocity cannot be uniform across the cross section of the pipe because of the no-slip condition. However, for convenience in calculations, the velocity can be assumed to be constant and thus *uniform* at a cross section. Fluid flow in a pipe usually approximated as *one-dimensional uniform flow.*

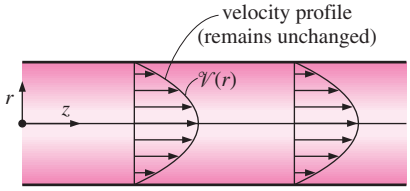


FIGURE 6–9
One-dimensional flow in a circular pipe.

6–3 ▪ VELOCITY BOUNDARY LAYER

Consider the parallel flow of a fluid over a *flat plate,* as shown in Fig. 6–10. Surfaces that are slightly contoured such as turbine blades can also be approximated as flat plates with reasonable accuracy. The x-coordinate is measured along the plate surface from the *leading edge* of the plate in the direction of the flow, and y is measured from the surface in the normal direction. The fluid approaches the plate in the x-direction with a uniform upstream velocity of $\mathcal{V}$, which is practically identical to the free-stream velocity u_∞ over the plate away from the surface (this would not be the case for cross flow over blunt bodies such as a cylinder).

For the sake of discussion, we can consider the fluid to consist of adjacent layers piled on top of each other. The velocity of the particles in the first fluid layer adjacent to the plate becomes zero because of the no-slip condition. This motionless layer slows down the particles of the neighboring fluid layer as a result of friction between the particles of these two adjoining fluid layers at different velocities. This fluid layer then slows down the molecules of the next layer, and so on. Thus, the presence of the plate is felt up to some normal distance δ from the plate beyond which the free-stream velocity u_∞ remains essentially unchanged. As a result, the x-component of the fluid velocity, u, will vary from 0 at $y = 0$ to nearly u_∞ at $y = \delta$ (Fig. 6–11).

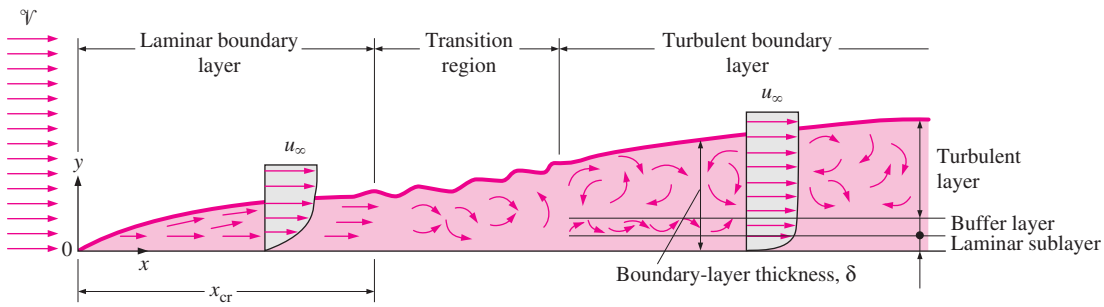


FIGURE 6–10
The development of the boundary layer for flow over a flat plate, and the different flow regimes.

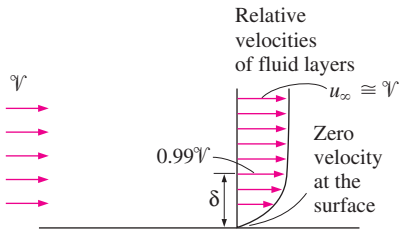


FIGURE 6–11
The development of a boundary layer on a surface is due to the no-slip condition.

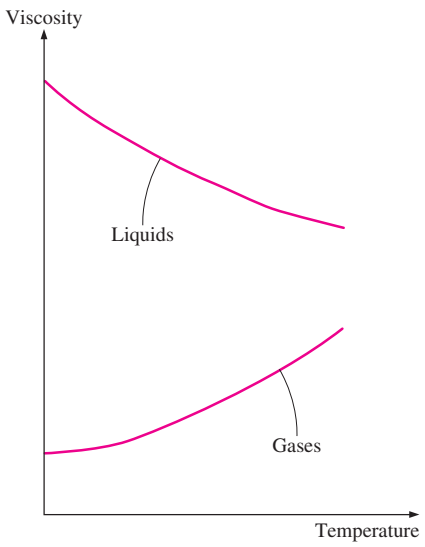


FIGURE 6–12
The viscosity of liquids decreases and the viscosity of gases increases with temperature.

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the **velocity boundary layer.** The *boundary layer thickness,* δ, is typically defined as the distance y from the surface at which $u = 0.99u_\infty$.

The hypothetical line of $u = 0.99u_\infty$ divides the flow over a plate into two regions: the **boundary layer region,** in which the viscous effects and the velocity changes are significant, and the **inviscid flow region,** in which the frictional effects are negligible and the velocity remains essentially constant.

Surface Shear Stress

Consider the flow of a fluid over the surface of a plate. The fluid layer in contact with the surface will try to drag the plate along via friction, exerting a *friction force* on it. Likewise, a faster fluid layer will try to drag the adjacent slower layer and exert a friction force because of the friction between the two layers. Friction force per unit area is called **shear stress,** and is denoted by τ. Experimental studies indicate that the shear stress for most fluids is proportional to the *velocity gradient,* and the shear stress at the wall surface is as

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad (\text{N/m}^2) \tag{6-9}$$

where the constant of proportionality μ is called the **dynamic viscosity** of the fluid, whose unit is kg/m · s (or equivalently, N · s/m², or Pa · s, or poise = 0.1 Pa · s).

The fluids that that obey the linear relationship above are called **Newtonian fluids,** after Sir Isaac Newton who expressed it first in 1687. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids. In this text we will consider Newtonian fluids only.

In fluid flow and heat transfer studies, the ratio of dynamic viscosity to density appears frequently. For convenience, this ratio is given the name **kinematic viscosity** ν and is expressed as $\nu = \mu/\rho$. Two common units of kinematic viscosity are m²/s and *stoke* (1 stoke = 1 cm²/s = 0.0001 m²/s).

The viscosity of a fluid is a measure of its *resistance to flow,* and it is a strong function of temperature. The viscosities of liquids *decrease* with temperature, whereas the viscosities of gases *increase* with temperature (Fig. 6–12). The viscosities of some fluids at 20°C are listed in Table 6–1. Note that the viscosities of different fluids differ by several orders of magnitude.

The determination of the surface shear stress τ_s from Eq. 6-9 is not practical since it requires a knowledge of the flow velocity profile. A more practical approach in external flow is to relate τ_s to the upstream velocity $\mathcal{V}$ as

$$\tau_s = C_f \frac{\rho \mathcal{V}^2}{2} \qquad (\text{N/m}^2) \tag{6-10}$$

where C_f is the dimensionless **friction coefficient,** whose value in most cases is determined experimentally, and ρ is the density of the fluid. Note that the friction coefficient, in general, will vary with location along the surface. Once the average friction coefficient over a given surface is available, the friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2} \qquad \text{(N)} \tag{6-11}$$

where A_s is the surface area.

The friction coefficient is an important parameter in heat transfer studies since it is directly related to the heat transfer coefficient and the power requirements of the pump or fan.

6–4 ▪ THERMAL BOUNDARY LAYER

We have seen that a velocity boundary layer develops when a fluid flows over a surface as a result of the fluid layer adjacent to the surface assuming the surface velocity (i.e., zero velocity relative to the surface). Also, we defined the velocity boundary layer as the region in which the fluid velocity varies from zero to $0.99u_\infty$. Likewise, a *thermal boundary layer* develops when a fluid at a specified temperature flows over a surface that is at a different temperature, as shown in Fig. 6–13.

Consider the flow of a fluid at a uniform temperature of T_∞ over an isothermal flat plate at temperature T_s. The fluid particles in the layer adjacent to the surface will reach thermal equilibrium with the plate and assume the surface temperature T_s. These fluid particles will then exchange energy with the particles in the adjoining-fluid layer, and so on. As a result, a temperature profile will develop in the flow field that ranges from T_s at the surface to T_∞ sufficiently far from the surface. The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the **thermal boundary layer.** The *thickness* of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference* $T - T_s$ *equals* $0.99(T_\infty - T_s)$. Note that for the special case of $T_s = 0$, we have $T = 0.99T_\infty$ at the outer edge of the thermal boundary layer, which is analogous to $u = 0.99u_\infty$ for the velocity boundary layer.

The thickness of the thermal boundary layer increases in the flow direction, since the effects of heat transfer are felt at greater distances from the surface further down stream.

The convection heat transfer rate anywhere along the surface is directly related to the temperature gradient at that location. Therefore, the shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it. In flow over a heated (or cooled) surface, both velocity and thermal boundary layers will develop simultaneously. Noting that the fluid velocity will have a strong influence on the temperature profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convection heat transfer.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number,** defined as

$$\text{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \tag{6-12}$$

TABLE 6–1

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	12.1
20°C	1.49
40°C	0.27
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.0003
100°C (vapor)	0.000013
Blood, 37°C	0.0004
Gasoline	0.00029
Ammonia	0.00022
Air	0.000018
Hydrogen, 0°C	0.000009

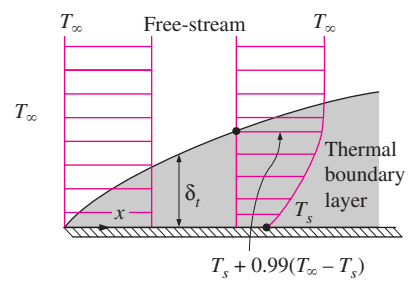


FIGURE 6–13

Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface).

TABLE 6–2

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

It is named after Ludwig Prandtl, who introduced the concept of boundary layer in 1904 and made significant contributions to boundary layer theory. The Prandtl numbers of fluids range from less than 0.01 for liquid metals to more than 100,000 for heavy oils (Table 6–2). Note that the Prandtl number is in the order of 10 for water.

The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate. Heat diffuses very quickly in liquid metals ($Pr \ll 1$) and very slowly in oils ($Pr \gg 1$) relative to momentum. Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

6–5 ▪ LAMINAR AND TURBULENT FLOWS

If you have been around smokers, you probably noticed that the cigarette smoke rises in a smooth plume for the first few centimeters and then starts fluctuating randomly in all directions as it continues its journey toward the lungs of others (Fig. 6–14). Likewise, a careful inspection of flow in a pipe reveals that the fluid flow is streamlined at low velocities but turns chaotic as the velocity is increased above a critical value, as shown in Figure 6–15. The flow regime in the first case is said to be **laminar,** characterized by *smooth streamlines* and *highly-ordered motion,* and **turbulent** in the second case, where it is characterized by *velocity fluctuations* and *highly-disordered motion.* The **transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.

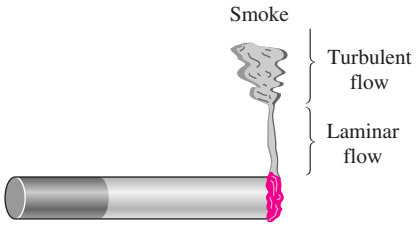


FIGURE 6–14
Laminar and turbulent flow regimes of cigarette smoke.

We can verify the existence of these laminar, transition, and turbulent flow regimes by injecting some dye streak into the flow in a glass tube, as the British scientist Osborn Reynolds (1842–1912) did over a century ago. We will observe that the dye streak will form a *straight and smooth line* at low velocities when the flow is laminar (we may see some blurring because of molecular diffusion), will have *bursts of fluctuations* in the transition regime, and will *zigzag rapidly and randomly* when the flow becomes fully turbulent. These zigzags and the dispersion of the dye are indicative of the fluctuations in the main flow and the rapid mixing of fluid particles from adjacent layers.

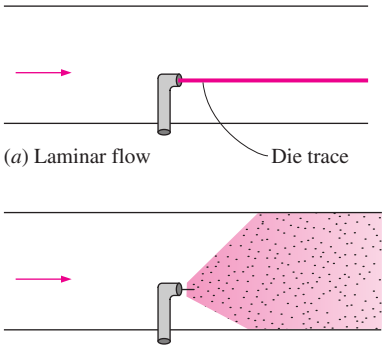


FIGURE 6–15
The behavior of colored fluid injected into the flow in laminar and turbulent flows in a tube.

Typical velocity profiles in laminar and turbulent flow are also given in Figure 6–10. Note that the velocity profile is approximately parabolic in laminar flow and becomes flatter in turbulent flow, with a sharp drop near the surface. The turbulent boundary layer can be considered to consist of three layers. The very thin layer next to the wall where the viscous effects are dominant is the **laminar sublayer.** The velocity profile in this layer is nearly linear, and the flow is streamlined. Next to the laminar sublayer is the **buffer layer,** in which the turbulent effects are significant but not dominant of the diffusion effects, and next to it is the **turbulent layer,** in which the turbulent effects dominate.

The *intense mixing* of the fluid in turbulent flow as a result of rapid fluctuations enhances heat and momentum transfer between fluid particles, which increases the friction force on the surface and the convection heat transfer rate. It also causes the boundary layer to enlarge. Both the friction and heat transfer coefficients reach maximum values when the flow becomes *fully turbulent.* So it will come as no surprise that a special effort is made in the design

of heat transfer coefficients associated with turbulent flow. The enhancement in heat transfer in turbulent flow does not come for free, however. It may be necessary to use a larger pump to overcome the larger friction forces accompanying the higher heat transfer rate.

Reynolds Number

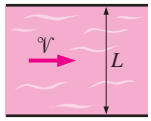
The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, free-stream velocity, surface temperature,* and *type of fluid,* among other things. After exhaustive experiments in the 1880s, Osborn Reynolds discovered that the flow regime depends mainly on the ratio of the *inertia forces* to *viscous forces* in the fluid. This ratio is called the **Reynolds number,** which is a *dimensionless* quantity, and is expressed for external flow as (Fig. 6–16)

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V} L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu} \tag{6-13}$$

where $\mathcal{V}$ is the upstream velocity (equivalent to the free-stream velocity u_∞ for a flat plate), L_c is the characteristic length of the geometry, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid. For a flat plate, the characteristic length is the distance x from the leading edge. Note that kinematic viscosity has the unit m^2/s, which is identical to the unit of thermal diffusivity, and can be viewed as *viscous diffusivity* or *diffusivity for momentum.*

At *large* Reynolds numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. At *small* Reynolds numbers, however, the viscous forces are large enough to overcome the inertia forces and to keep the fluid "in line." Thus the flow is *turbulent* in the first case and *laminar* in the second.

The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number.** The value of the critical Reynolds number is different for different geometries. For flow over a flat plate, the generally accepted value of the critical Reynolds number is $\mathrm{Re}_{cr} = \mathcal{V} x_{cr}/\nu = u_\infty x_{cr}/\nu = 5 \times 10^5$, where x_{cr} is the distance from the leading edge of the plate at which transition from laminar to turbulent flow occurs. The value of Re_{cr} may change substantially, however, depending on the level of turbulence in the free stream.

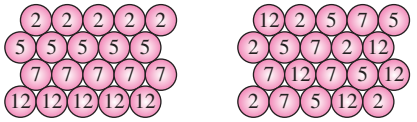


$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\rho \mathcal{V}^2/L}{\mu \mathcal{V}/L^2} = \frac{\rho \mathcal{V} L}{\mu} = \frac{\mathcal{V} L}{\nu}$$

FIGURE 6–16
The Reynolds number can be viewed as the ratio of the inertia forces to viscous forces acting on a fluid volume element.

6–6 ▪ HEAT AND MOMENTUM TRANSFER IN TURBULENT FLOW

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress and heat transfer. Turbulent flow is characterized by random and rapid fluctuations of groups of fluid particles, called *eddies,* throughout the boundary layer. These fluctuations provide an additional mechanism for momentum and heat transfer. In laminar flow, fluid particles flow in an orderly manner along streamlines, and both momentum and heat are transferred across streamlines by molecular diffusion. In turbulent flow, the transverse motion of eddies transport momentum and heat to other regions of flow before they mix with the rest of the fluid and lose their identity, greatly enhancing momentum and heat



(*a*) Before turbulence (*b*) After turbulence

FIGURE 6–17
The intense mixing in turbulent flow brings fluid particles at different temperatures into close contact, and thus enhances heat transfer.

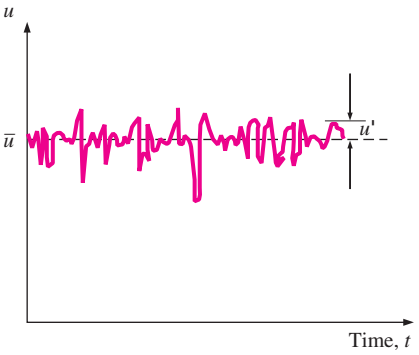


FIGURE 6–18
Fluctuations of the velocity component u with time at a specified location in turbulent flow.

transfer. As a result, turbulent flow is associated with much higher values of friction and heat transfer coefficients (Fig. 6–17).

Even when the mean flow is steady, the eddying motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow). Figure 6–18 shows the variation of the instantaneous velocity component u with time at a specified location, as can be measured with a hot-wire anemometer probe or other sensitive device. We observe that the instantaneous values of the velocity fluctuate about a mean value, which suggests that the velocity can be expressed as the sum of a *mean value* $\bar{u}$ and a *fluctuating component* u',

$$u = \bar{u} + u' \tag{6-14}$$

This is also the case for other properties such as the velocity component v in the y direction, and thus $v = \bar{v} + v'$, $P = \bar{P} + P'$, and $T = \bar{T} + T'$. The mean value of a property at some location is determined by averaging it over a time interval that is sufficiently large so that the net effect of fluctuations is zero. Therefore, the time average of fluctuating components is zero, e.g., $\overline{u'} = 0$. The magnitude of u' is usually just a few percent of $\bar{u}$, but the high frequencies of eddies (in the order of a thousand per second) makes them very effective for the transport of momentum and thermal energy. In *steady* turbulent flow, the mean values of properties (indicated by an overbar) are independent of time.

Consider the upward eddy motion of a fluid during flow over a surface. The mass flow rate of fluid per unit area normal to flow is $\rho v'$. Noting that $h = C_pT$ represents the energy of the fluid and T' is the eddy temperature relative to the mean value, the rate of thermal energy transport by turbulent eddies is $\dot{q}_t = \rho C_p \overline{v'T'}$. By a similar argument on momentum transfer, the turbulent shear stress can be shown to be $\tau_t = -\rho\overline{u'v'}$. Note that $\overline{u'v'} \neq 0$ even though $\overline{u'} = 0$ and $\overline{v'} = 0$, and experimental results show that $\overline{u'v'}$ is a negative quantity. Terms such as $-\rho\overline{u'v'}$ are called **Reynolds stresses.**

The random eddy motion of groups of particles resembles the random motion of molecules in a gas—colliding with each other after traveling a certain distance and exchanging momentum and heat in the process. Therefore, momentum and heat transport by eddies in turbulent boundary layers is analogous to the molecular momentum and heat diffusion. Then turbulent wall shear stress and turbulent heat transfer can be expressed in an analogous manner as

$$\tau_t = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y} \quad \text{and} \quad \dot{q}_t = \rho C_p \overline{v'T'} = -k_t \frac{\partial \bar{T}}{\partial y} \tag{6-15}$$

where μ_t is called the **turbulent viscosity,** which accounts for momentum transport by turbulent eddies, and k_t is called the **turbulent thermal conductivity,** which accounts for thermal energy transport by turbulent eddies. Then the total shear stress and total heat flux can be expressed conveniently as

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial \bar{u}}{\partial y} = \rho(\nu + \varepsilon_M)\frac{\partial \bar{u}}{\partial y} \tag{6-16}$$

and

$$\dot{q}_{\text{total}} = -(k + k_t)\frac{\partial \bar{T}}{\partial y} = -\rho C_p(\alpha + \varepsilon_H)\frac{\partial \bar{T}}{\partial y} \tag{6-17}$$

where $\varepsilon_M = \mu_t/\rho$ is the **eddy diffusivity of momentum** and $\varepsilon_H = k_t/\rho C_p$ is the **eddy diffusivity of heat.**

Eddy motion and thus eddy diffusivities are much larger than their molecular counterparts in the core region of a turbulent boundary layer. The eddy motion loses its intensity close to the wall, and diminishes at the wall because of the no-slip condition. Therefore, the velocity and temperature profiles are nearly uniform in the core region of a turbulent boundary layer, but very steep in the thin layer adjacent to the wall, resulting in large velocity and temperature gradients at the wall surface. So it is no surprise that the wall shear stress and wall heat flux are much larger in turbulent flow than they are in laminar flow (Fig. 6–19).

Note that molecular diffusivities ν and α (as well as μ and k) are fluid properties, and their values can be found listed in fluid handbooks. Eddy diffusivities ε_M and ε_H (as well as μ_t and k_t), however are *not* fluid properties and their values depend on flow conditions. Eddy diffusivities ε_M and ε_H decrease towards the wall, becoming zero at the wall.

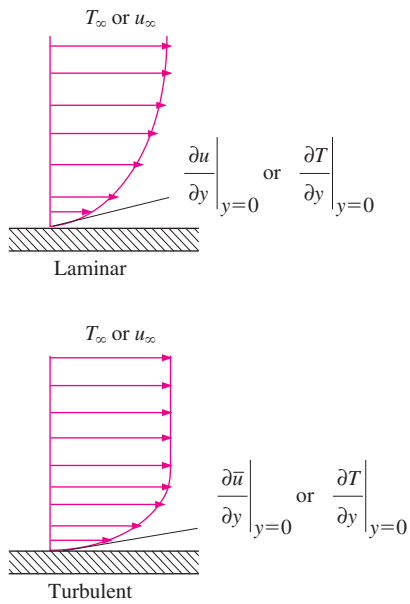


FIGURE 6–19
The velocity and temperature gradients at the wall, and thus the wall shear stress and heat transfer rate, are much larger for turbulent flow than they are for laminar flow (T is shown relative to T_s).

6–7 ▪ DERIVATION OF DIFFERENTIAL CONVECTION EQUATIONS*

In this section we derive the governing equations of fluid flow in the boundary layers. To keep the analysis at a manageable level, we assume the flow to be steady and two-dimensional, and the fluid to be Newtonian with constant properties (density, viscosity, thermal conductivity, etc.).

Consider the parallel flow of a fluid over a surface. We take the flow direction along the surface to be x and the direction normal to the surface to be y, and we choose a differential volume element of length dx, height dy, and unit depth in the z-direction (normal to the paper) for analysis (Fig. 6–20). The fluid flows over the surface with a uniform free-stream velocity u_∞, but the velocity within boundary layer is two-dimensional: the x-component of the velocity is u, and the y-component is v. Note that $u = u(x, y)$ and $v = v(x, y)$ in steady two-dimensional flow.

Next we apply three fundamental laws to this fluid element: Conservation of mass, conservation of momentum, and conservation of energy to obtain the continuity, momentum, and energy equations for laminar flow in boundary layers.

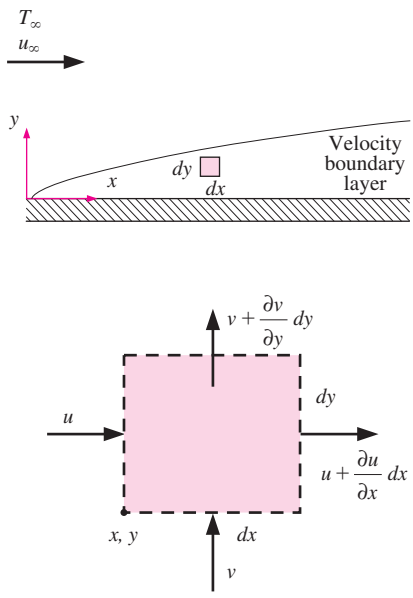


FIGURE 6–20
Differential control volume used in the derivation of mass balance in velocity boundary layer in two-dimensional flow over a surface.

Conservation of Mass Equation

The conservation of mass principle is simply a statement that mass cannot be created or destroyed, and all the mass must be accounted for during an analysis. In steady flow, the amount of mass within the control volume remains constant, and thus the conservation of mass can be expressed as

$$\begin{pmatrix}\text{Rate of mass flow}\\ \text{into the control volume}\end{pmatrix} = \begin{pmatrix}\text{Rate of mass flow}\\ \text{out of the control volume}\end{pmatrix} \qquad \textbf{(6-18)}$$

*This and the upcoming sections of this chapter deal with theoretical aspects of convection, and can be skipped and be used as a reference if desired without a loss in continuity.

Noting that mass flow rate is equal to the product of density, mean velocity, and cross-sectional area normal to flow, the rate at which fluid enters the control volume from the left surface is $\rho u(dy \cdot 1)$. The rate at which the fluid leaves the control volume from the right surface can be expressed as

$$\rho\left(u + \frac{\partial u}{\partial x}dx\right)(dy \cdot 1) \tag{6-19}$$

Repeating this for the y direction and substituting the results into Eq. 6-18, we obtain

$$\rho u(dy \cdot 1) + \rho v(dx \cdot 1) = \rho\left(u + \frac{\partial u}{\partial x}dx\right)(dy \cdot 1) + \rho\left(v + \frac{\partial v}{\partial y}dy\right)(dx \cdot 1) \tag{6-20}$$

Simplifying and dividing by $dx \cdot dy \cdot 1$ gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6-21}$$

This is the *conservation of mass* relation, also known as the **continuity equation,** or **mass balance** for steady two-dimensional flow of a fluid with constant density.

Conservation of Momentum Equations

The differential forms of the equations of motion in the velocity boundary layer are obtained by applying Newton's second law of motion to a differential control volume element in the boundary layer. Newton's second law is an expression for the conservation of momentum, and can be stated as *the net force acting on the control volume is equal to the mass times the acceleration of the fluid element within the control volume, which is also equal to the net rate of momentum outflow from the control volume.*

The forces acting on the control volume consist of *body forces* that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and are proportional to the volume of the body, and *surface forces* that act on the control surface (such as the pressure forces due to hydrostatic pressure and shear stresses due to viscous effects) and are proportional to the surface area. The surface forces appear as the control volume is isolated from its surroundings for analysis, and the effect of the detached body is replaced by a force at that location. Note that pressure represents the compressive force applied on the fluid element by the surrounding fluid, and is always directed to the surface.

We express Newton's second law of motion for the control volume as

$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a specified direction}\end{pmatrix} = \begin{pmatrix}\text{Net force (body and surface)}\\ \text{acting in that direction}\end{pmatrix} \tag{6-22}$$

or

$$\delta m \cdot a_x = F_{\text{surface},\,x} + F_{\text{body},\,x} \tag{6-23}$$

where the mass of the fluid element within the control volume is

$$\delta m = \rho(dx \cdot dy \cdot 1) \tag{6-24}$$

Noting that flow is steady and two-dimensional and thus $u = u(x, y)$, the total differential of u is

$$du = \frac{\partial u}{\partial x}\,dx + \frac{\partial u}{\partial y}\,dy \tag{6-25}$$

Then the acceleration of the fluid element in the x direction becomes

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \tag{6-26}$$

You may be tempted to think that acceleration is zero in steady flow since acceleration is the rate of change of velocity with time, and in steady flow there is no change with time. Well, a garden hose nozzle will tell us that this understanding is not correct. Even in steady flow and thus constant mass flow rate, water will accelerate through the nozzle (Fig. 6–21). *Steady* simply means no change with time at a specified location (and thus $\partial u/\partial t = 0$), but the value of a quantity may change from one location to another (and thus $\partial u/\partial x$ and $\partial u/\partial y$ may be different from zero). In the case of a nozzle, the velocity of water remains constant at a specified point, but it changes from inlet to the exit (water accelerates along the nozzle, which is the reason for attaching a nozzle to the garden hose in the first place).

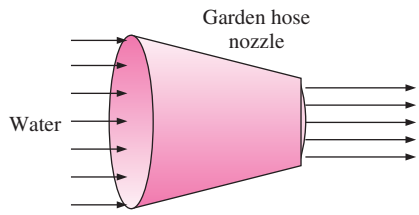


FIGURE 6–21
During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space.

The forces acting on a surface are due to pressure and viscous effects. In two-dimensional flow, the *viscous stress* at any point on an imaginary surface within the fluid can be resolved into two perpendicular components: one normal to the surface called *normal stress* (which should not be confused with pressure) and another along the surface called *shear stress.* The normal stress is related to the velocity gradients $\partial u/\partial x$ and $\partial v/\partial y$, that are much smaller than $\partial u/\partial y$, to which shear stress is related. Neglecting the normal stresses for simplicity, the surface forces acting on the control volume in the x-direction will be as shown in Fig. 6–22. Then the net surface force acting in the x-direction becomes

$$\begin{aligned} F_{\text{surface},\,x} &= \left(\frac{\partial \tau}{\partial y}\,dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}\,dx\right)(dy \cdot 1) = \left(\frac{\partial \tau}{\partial y} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1) \\ &= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1) \end{aligned} \tag{6-27}$$

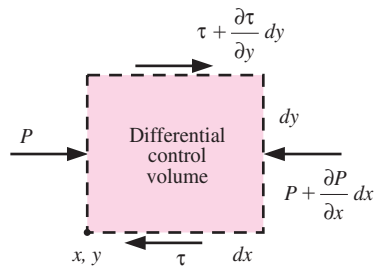


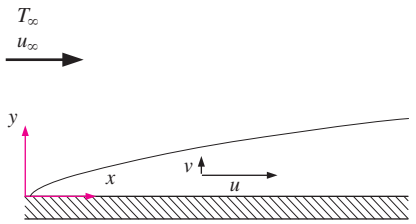
FIGURE 6–22
Differential control volume used in the derivation of x-momentum equation in velocity boundary layer in two-dimensional flow over a surface.

since $\tau = \mu(\partial u/\partial y)$. Substituting Eqs. 6-21, 6-23, and 6-24 into Eq. 6-20 and dividing by $dx \cdot dy \cdot 1$ gives

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \tag{6-28}$$

This is the relation for the **conservation of momentum** in the x-direction, and is known as the ***x*-momentum equation.** Note that we would obtain the same result if we used momentum flow rates for the left-hand side of this equation instead of mass times acceleration. If there is a body force acting in the x-direction, it can be added to the right side of the equation provided that it is expressed per unit volume of the fluid.

In a boundary layer, the velocity component in the flow direction is much larger than that in the normal direction, and thus $u \gg v$, and $\partial v/\partial x$ and $\partial v/\partial y$ are



1) Velocity components:
$v \ll u$
2) Velocity grandients:
$\frac{\partial v}{\partial x} \approx 0, \frac{\partial v}{\partial y} \approx 0$
$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$
3) Temperature gradients:
$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

FIGURE 6–23
Boundary layer approximations.

negligible. Also, u varies greatly with y in the normal direction from zero at the wall surface to nearly the free-stream value across the relatively thin boundary layer, while the variation of u with x along the flow is typically small. Therefore, $\partial u/\partial y \gg \partial u/\partial x$. Similarly, if the fluid and the wall are at different temperatures and the fluid is heated or cooled during flow, heat conduction will occur primarily in the direction normal to the surface, and thus $\partial T/\partial y \gg \partial T/\partial x$. That is, the velocity and temperature gradients normal to the surface are much greater than those along the surface. These simplifications are known as the **boundary layer approximations.** These approximations greatly simplify the analysis usually with little loss in accuracy, and make it possible to obtain analytical solutions for certain types of flow problems (Fig. 6–23).

When gravity effects and other body forces are negligible and the boundary layer approximations are valid, applying Newton's second law of motion on the volume element in the y-direction gives the *y-momentum equation* to be

$$\frac{\partial P}{\partial y} = 0 \qquad \textbf{(6-29)}$$

That is, *the variation of pressure in the direction normal to the surface is negligible,* and thus $P = P(x)$ and $\partial P/\partial x = dP/dx$. Then it follows that for a given x, the pressure in the boundary layer is equal to the pressure in the free stream, and the pressure determined by a separate analysis of fluid flow in the free stream (which is typically easier because of the absence of viscous effects) can readily be used in the boundary layer analysis.

The velocity components in the free stream region of a flat plate are $u = u_\infty$ = constant and $v = 0$. Substituting these into the x-momentum equations (Eq. 6-28) gives $\partial P/\partial x = 0$. Therefore, for flow over a flat plate, the pressure remains constant over the entire plate (both inside and outside the boundary layer).

Conservation of Energy Equation

The energy balance for any system undergoing any process is expressed as $E_{in} - E_{out} = \Delta E_{system}$, which states that the change in the energy content of a system during a process is equal to the difference between the energy input and the energy output. During a *steady-flow process,* the total energy content of a control volume remains constant (and thus $\Delta E_{system} = 0$), and the amount of energy entering a control volume in all forms must be equal to the amount of energy leaving it. Then the rate form of the general energy equation reduces for a steady-flow process to $\dot{E}_{in} - \dot{E}_{out} = 0$.

Noting that energy can be transferred by heat, work, and mass only, the energy balance for a steady-flow control volume can be written explicitly as

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by work}} + (\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} = 0 \qquad \textbf{(6-30)}$$

The total energy of a flowing fluid stream per unit mass is $e_{stream} = h + \text{ke} + \text{pe}$ where h is the enthalpy (which is the sum of internal energy and flow energy), $\text{pe} = gz$ is the potential energy, and $\text{ke} = \mathcal{V}^2/2 = (u^2 + v^2)/2$ is the kinetic energy of the fluid per unit mass. The kinetic and potential energies are usually very small relative to enthalpy, and therefore it is common practice to neglect them (besides, it can be shown that if kinetic energy is included in the analysis below, all the terms due to this inclusion cancel each other). We

assume the density ρ, specific heat C_p, viscosity μ, and the thermal conductivity k of the fluid to be constant. Then the energy of the fluid per unit mass can be expressed as $e_{\text{stream}} = h = C_pT$.

Energy is a scalar quantity, and thus energy interactions in all directions can be combined in one equation. Noting that mass flow rate of the fluid entering the control volume from the left is $\rho u(dy \cdot 1)$, the rate of energy transfer to the control volume by mass in the x-direction is, from Fig. 6–24,

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by mass},\,x} = (\dot{m}e_{\text{stream}})_x - \left[(\dot{m}e_{\text{stream}})_x + \frac{\partial(\dot{m}e_{\text{stream}})_x}{\partial x}dx\right]$$
$$= -\frac{\partial[\rho u(dy \cdot 1)C_pT]}{\partial x}dx = -\rho C_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy \quad \textbf{(6-31)}$$

Repeating this for the y-direction and adding the results, the net rate of energy transfer to the control volume by mass is determined to be

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by mass}} = -\rho C_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy - \rho C_p\left(v\frac{\partial T}{\partial y} + T\frac{\partial v}{\partial y}\right)dx\,dy$$
$$= -\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dx\,dy \quad \textbf{(6-32)}$$

since $\partial u/\partial x + \partial v/\partial y = 0$ from the continuity equation.

The net rate of heat conduction to the volume element in the x-direction is

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by heat},\,x} = \dot{Q}_x - \left(\dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x}dx\right)$$
$$= -\frac{\partial}{\partial x}\left(-k(dy \cdot 1)\frac{\partial T}{\partial x}\right)dx = k\frac{\partial^2 T}{\partial x^2}dx\,dy \quad \textbf{(6-33)}$$

Repeating this for the y-direction and adding the results, the net rate of energy transfer to the control volume by heat conduction becomes

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by heat}} = k\frac{\partial^2 T}{\partial x^2}dx\,dy + k\frac{\partial^2 T}{\partial y^2}dx\,dy = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)dx\,dy \quad \textbf{(6-34)}$$

Another mechanism of energy transfer to and from the fluid in the control volume is the work done by the body and surface forces. The work done by a body force is determined by multiplying this force by the velocity in the direction of the force and the volume of the fluid element, and this work needs to be considered only in the presence of significant gravitational, electric, or magnetic effects. The surface forces consist of the forces due to fluid pressure and the viscous shear stresses. The work done by pressure (the flow work) is already accounted for in the analysis above by using enthalpy for the microscopic energy of the fluid instead of internal energy. The shear stresses that result from viscous effects are usually very small, and can be neglected in many cases. This is especially the case for applications that involve low or moderate velocities.

Then the energy equation for the steady two-dimensional flow of a fluid with constant properties and negligible shear stresses is obtained by substituting Eqs. 6-32 and 6-34 into 6-30 to be

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad \textbf{(6-35)}$$

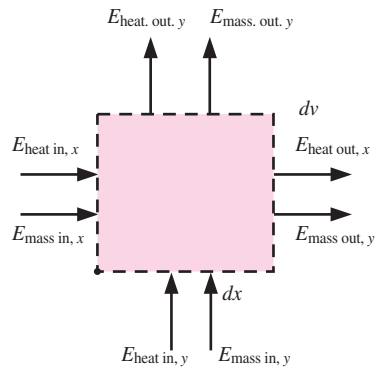


FIGURE 6–24
The energy transfers by heat and mass flow associated with a differential control volume in the thermal boundary layer in steady two-dimensional flow.

which states that *the net energy convected by the fluid out of the control volume is equal to the net energy transferred into the control volume by heat conduction.*

When the viscous shear stresses are not negligible, their effect is accounted for by expressing the energy equation as

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi \tag{6-36}$$

where the *viscous dissipation function* Φ is obtained after a lengthy analysis (see an advanced book such as the one by *Schlichting* (Ref. 9) for details) to be

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \tag{6-37}$$

Viscous dissipation may play a dominant role in high-speed flows, especially when the viscosity of the fluid is high (like the flow of oil in journal bearings). This manifests itself as a significant rise in fluid temperature due to the conversion of the kinetic energy of the fluid to thermal energy. Viscous dissipation is also significant for high-speed flights of aircraft.

For the special case of a stationary fluid, $u = v = 0$ and the energy equation reduces, as expected, to the steady two-dimensional heat conduction equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{6-38}$$

EXAMPLE 6–1 Temperature Rise of Oil in a Journal Bearing

The flow of oil in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. Such flows are known as Couette flow.

Consider two large isothermal plates separated by 2-mm-thick oil film. The upper plates moves at a constant velocity of 12 m/s, while the lower plate is stationary. Both plates are maintained at 20°C. (*a*) Obtain relations for the velocity and temperature distributions in the oil. (*b*) Determine the maximum temperature in the oil and the heat flux from the oil to each plate (Fig. 6–25).

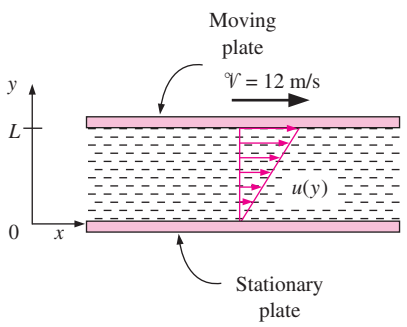


FIGURE 6–25
Schematic for Example 6–1.

SOLUTION Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the total heat transfer rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in the *z* direction.

Properties The properties of oil at 20°C are (Table A-10):

$$k = 0.145\ \text{W/m}\cdot\text{K} \quad \text{and} \quad \mu = 0.800\ \text{kg/m}\cdot\text{s} = 0.800\ \text{N}\cdot\text{s/m}^2$$

Analysis (*a*) We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation (Eq. 6-21) reduces to

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\partial u}{\partial x} = 0 \quad \rightarrow u = u(y)$$

Therefore, the x-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the x-momentum equation (Eq. 6-28) reduces to

$$x\text{-momentum: } \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \qquad \rightarrow \qquad \frac{d^2u}{dy^2} = 0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y) = C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y) = \frac{y}{L}\mathcal{V}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

$$\text{Energy:} \qquad 0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad \rightarrow \qquad k\frac{d^2T}{dy^2} = -\mu\left(\frac{\mathcal{V}}{L}\right)^2$$

since $\partial u/\partial y = \mathcal{V}/L$. Dividing both sides by k and integrating twice give

$$T(y) = -\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2 + C_3 y + C_4$$

Applying the boundary conditions $T(0) = T_0$ and $T(L) = T_0$ gives the temperature distribution to be

$$T(y) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

(*b*) The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy} = \frac{\mu\mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for y,

$$\frac{dT}{dy} = \frac{\mu\mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right) = 0 \qquad \rightarrow \qquad y = \frac{L}{2}$$

Therefore, maximum temperature will occur at mid plane, which is not surprising since both plates are maintained at the same temperature. The maximum temperature is the value of temperature at $y = L/2$,

$$T_{\max} = T\left(\frac{L}{2}\right) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{L/2}{L} - \frac{(L/2)^2}{L^2}\right) = T_0 + \frac{\mu\mathcal{V}^2}{8k}$$

$$= 20 + \frac{(0.8\ \text{N}\cdot\text{s/m}^2)(12\text{m/s})^2}{8(0.145\ \text{W/m}\cdot{}^\circ\text{C})}\left(\frac{1\text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{119°C}$$

Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{\mu\mathcal{V}^2}{2kL}(1-0) = -\frac{\mu\mathcal{V}^2}{2L}$$

$$= -\frac{(0.8\ \text{N}\cdot\text{s/m}^2)(12\ \text{m/s})^2}{2(0.002\ \text{m})}\left(\frac{1W}{1N\cdot m/s}\right) = \mathbf{-28{,}800\ W/m^2}$$

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{\mu\mathcal{V}^2}{2kL}(1-2) = \frac{\mu\mathcal{V}^2}{2L} = -\dot{q}_0 = \mathbf{28{,}800\ W/m^2}$$

Therefore, heat fluxes at the two plates are equal in magnitude but opposite in sign.

Discussion A temperature rise of 99°C confirms our suspicion that viscous dissipation is very significant. Also, the heat flux is equivalent to the rate of mechanical energy dissipation. Therefore, mechanical energy is being converted to thermal energy at a rate of 57.2 kW/m^2 of plate area to overcome friction in the oil. Finally, calculations are done using oil properties at 20°C, but the oil temperature turned out to be much higher. Therefore, knowing the strong dependence of viscosity on temperature, calculations should be repeated using properties at the average temperature of 70°C to improve accuracy.

6–8 ▪ SOLUTIONS OF CONVECTION EQUATIONS FOR A FLAT PLATE

Consider laminar flow of a fluid over a *flat plate,* as shown in Fig. 6–19. Surfaces that are slightly contoured such as turbine blades can also be approximated as flat plates with reasonable accuracy. The x-coordinate is measured along the plate surface from the leading edge of the plate in the direction of the flow, and y is measured from the surface in the normal direction. The fluid approaches the plate in the x-direction with a uniform upstream velocity, which is equivalent to the free stream velocity u_∞.

When viscous dissipation is negligible, the continuity, momentum, and energy equations (Eqs. 6-21, 6-28, and 6-35) reduce for steady, incompressible, laminar flow of a fluid with constant properties over a flat plate to

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \textbf{(6-39)}$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad \textbf{(6-40)}$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \textbf{(6-41)}$$

with the boundary conditions (Fig. 6–26)

$$\begin{aligned} &\text{At } x = 0: && u(0, y) = u_\infty, && T(0, y) = T_\infty \\ &\text{At } y = 0: && u(x, 0) = 0, && v(x, 0) = 0,\ T(x, 0) = T_s \\ &\text{As } y \to \infty: && u(x, \infty) = u_\infty, && T(x, \infty) = T_\infty \end{aligned} \qquad \textbf{(6-42)}$$

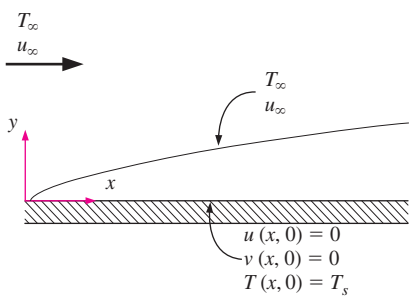


FIGURE 6–26
Boundary conditions for flow over a flat plate.

When fluid properties are assumed to be constant and thus independent of temperature, the first two equations can be solved separately for the velocity components u and v. Once the velocity distribution is available, we can determine

the friction coefficient and the boundary layer thickness using their definitions. Also, knowing u and v, the temperature becomes the only unknown in the last equation, and it can be solved for temperature distribution.

The continuity and momentum equations were first solved in 1908 by the German engineer H. Blasius, a student of L. Prandtl. This was done by transforming the two partial differential equations into a single ordinary differential equation by introducing a new independent variable, called the **similarity variable.** The finding of such a variable, assuming it exists, is more of an art than science, and it requires to have a good insight of the problem.

Noticing that the general shape of the velocity profile remains the same along the plate, Blasius reasoned that the nondimensional velocity profile u/u_∞ should remain unchanged when plotted against the nondimensional distance y/δ, where δ is the thickness of the local velocity boundary layer at a given x. That is, although both δ and u at a given y vary with x, the velocity u at a fixed y/δ remains constant. Blasius was also aware from the work of Stokes that δ is proportional to $\sqrt{\nu x/u_\infty}$, and thus he defined a *dimensionless similarity variable* as

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}} \tag{6-43}$$

and thus $u/u_\infty = \text{function}(\eta)$. He then introduced a *stream function* $\psi(x, y)$ as

$$u = \frac{\partial \psi}{\partial y} \qquad \text{and} \qquad v = -\frac{\partial \psi}{\partial x} \tag{6-44}$$

so that the continuity equation (Eq. 6-39) is automatically satisfied and thus eliminated (this can be verified easily by direct substitution). He then defined a function $f(\eta)$ as the dependent variable as

$$f(\eta) = \frac{\psi}{u_\infty\sqrt{\nu x/u_\infty}} \tag{6-45}$$

Then the velocity components become

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = u_\infty\sqrt{\frac{\nu x}{u_\infty}}\frac{df}{d\eta}\sqrt{\frac{u_\infty}{\nu x}} = u_\infty\frac{df}{d\eta} \tag{6-46}$$

$$v = -\frac{\partial \psi}{\partial x} = -u_\infty\sqrt{\frac{\nu x}{u_\infty}}\frac{\partial f}{\partial x} - \frac{u_\infty}{2}\sqrt{\frac{\nu}{u_\infty x}}\,f = \frac{1}{2}\sqrt{\frac{u_\infty \nu}{x}}\left(\eta\frac{df}{d\eta} - f\right) \tag{6-47}$$

By differentiating these u and v relations, the derivatives of the velocity components can be shown to be

$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x}\eta\frac{d^2f}{d\eta^2}, \qquad \frac{\partial u}{\partial y} = u_\infty\sqrt{\frac{u_\infty}{\nu x}}\frac{d^2f}{d\eta^2}, \qquad \frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x}\frac{d^3f}{d\eta^3} \tag{6-48}$$

Substituting these relations into the momentum equation and simplifying, we obtain

$$2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0 \tag{6-49}$$

which is a third-order nonlinear differential equation. Therefore, the system of two partial differential equations is transformed into a single ordinary

differential equation by the use of a similarity variable. Using the definitions of f and η, the boundary conditions in terms of the similarity variables can be expressed as

$$f(0) = 0, \qquad \left.\frac{df}{d\eta}\right|_{\eta=0} = 0, \qquad \text{and} \qquad \left.\frac{df}{d\eta}\right|_{\eta=\infty} = 1 \tag{6-50}$$

The transformed equation with its associated boundary conditions cannot be solved analytically, and thus an alternative solution method is necessary. The problem was first solved by Blasius in 1908 using a power series expansion approach, and this original solution is known as the *Blasius solution.* The problem is later solved more accurately using different numerical approaches, and results from such a solution are given in Table 6–3. The nondimensional velocity profile can be obtained by plotting u/u_∞ against η. The results obtained by this simplified analysis are in excellent agreement with experimental results.

Recall that we defined the boundary layer thickness as the distance from the surface for which $u/u_\infty = 0.99$. We observe from Table 6–3 that the value of η corresponding to $u/u_\infty = 0.992$ is $\eta = 5.0$. Substituting $\eta = 5.0$ and $y = \delta$ into the definition of the similarity variable (Eq. 6-43) gives $5.0 = \delta\sqrt{u_\infty/\nu x}$. Then the velocity boundary layer thickness becomes

$$\delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5.0x}{\sqrt{\mathrm{Re}_x}} \tag{6-51}$$

since $\mathrm{Re}_x = u_\infty x/\nu$, where x is the distance from the leading edge of the plate. Note that the boundary layer thickness increases with increasing kinematic viscosity ν and with increasing distance from the leading edge x, but it decreases with increasing free-stream velocity u_∞. Therefore, a large free-stream velocity will suppress the boundary layer and cause it to be thinner.

The shear stress on the wall can be determined from its definition and the $\partial u/\partial y$ relation in Eq. 6-48:

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} \left.\frac{d^2 f}{d\eta^2}\right|_{\eta=0} \tag{6-52}$$

Substituting the value of the second derivative of f at $\eta = 0$ from Table 6–3 gives

$$\tau_w = 0.332 u_\infty \sqrt{\frac{\rho\mu u_\infty}{x}} = \frac{0.332\rho u_\infty^2}{\sqrt{\mathrm{Re}_x}} \tag{6-53}$$

Then the local skin friction coefficient becomes

$$C_{f,x} = \frac{\tau_w}{\rho \mathcal{V}^2/2} = \frac{\tau_w}{\rho u_\infty^2/2} = 0.664\ \mathrm{Re}_x^{-1/2} \tag{6-54}$$

Note that unlike the boundary layer thickness, wall shear stress and the skin friction coefficient decrease along the plate as $x^{-1/2}$.

TABLE 6–3

Similarity function f and its derivatives for laminar boundary layer along a flat plate.

η	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
∞	∞	1	0

The Energy Equation

Knowing the velocity profile, we are now ready to solve the energy equation for temperature distribution for the case of constant wall temperature T_s. First we introduce the dimensionless temperature θ as

$$\theta(x, y) = \frac{T(x, y) - T_s}{T_\infty - T_s} \tag{6-55}$$

Noting that both T_s and T_∞ are constant, substitution into the energy equation gives

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \alpha\frac{\partial^2\theta}{\partial y^2} \tag{6-56}$$

Temperature profiles for flow over an isothermal flat plate are similar, just like the velocity profiles, and thus we expect a similarity solution for temperature to exist. Further, the thickness of the thermal boundary layer is proportional to $\sqrt{\nu x/u_\infty}$, just like the thickness of the velocity boundary layer, and thus the similarity variable is also η, and $\theta = \theta(\eta)$. Using the chain rule and substituting the u and v expressions into the energy equation gives

$$u_\infty\frac{df}{d\eta}\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial x} + \frac{1}{2}\sqrt{\frac{u_\infty\nu}{x}}\left(\eta\frac{df}{d\eta} - f\right)\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial y} = \alpha\frac{d^2\theta}{d\eta^2}\left(\frac{\partial\eta}{\partial y}\right)^2 \tag{6-57}$$

Simplifying and noting that $\Pr = \nu/\alpha$ give

$$2\frac{d^2\theta}{d\eta^2} + \Pr f\frac{d\theta}{d\eta} = 0 \tag{6-58}$$

with the boundary conditions $\theta(0) = 0$ and $\theta(\infty) = 1$. Obtaining an equation for θ as a function of η alone confirms that the temperature profiles are similar, and thus a similarity solution exists. Again a closed-form solution cannot be obtained for this boundary value problem, and it must be solved numerically.

It is interesting to note that for $\Pr = 1$, this equation reduces to Eq. 6-49 when θ is replaced by $df/d\eta$, which is equivalent to u/u_∞ (see Eq. 6-46). The boundary conditions for θ and $df/d\eta$ are also identical. Thus we conclude that the velocity and thermal boundary layers coincide, and the nondimensional velocity and temperature profiles (u/u_∞ and θ) are identical for steady, incompressible, laminar flow of a fluid with constant properties and $\Pr = 1$ over an isothermal flat plate (Fig. 6–27). The value of the temperature gradient at the surface ($y = 0$ or $\eta = 0$) in this case is, from Table 6–3, $d\theta/d\eta = d^2f/d\eta^2 = 0.332$.

Equation 6-58 is solved for numerous values of Prandtl numbers. For $\Pr > 0.6$, the nondimensional temperature gradient at the surface is found to be proportional to $\Pr^{1/3}$, and is expressed as

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = 0.332\,\Pr^{1/3} \tag{6-59}$$

The temperature gradient at the surface is

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{\partial\theta}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{d\theta}{d\eta}\right|_{\eta=0}\left.\frac{\partial\eta}{\partial y}\right|_{y=0} = 0.332\,\Pr^{1/3}(T_\infty - T_s)\sqrt{\frac{u_\infty}{\nu x}} \tag{6-60}$$

Then the local convection coefficient and Nusselt number become

$$h_x = \frac{q_s}{T_s - T_\infty} = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = 0.332\,\Pr^{1/3}k\sqrt{\frac{u_\infty}{\nu x}} \tag{6-61}$$

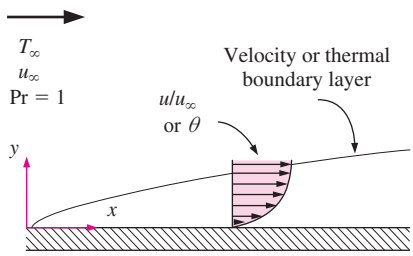


FIGURE 6–27
When $\Pr = 1$, the velocity and thermal boundary layers coincide, and the nondimensional velocity and temperature profiles are identical for steady, incompressible, laminar flow over a flat plate.

and

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Pr}^{1/3}\mathrm{Re}_x^{1/2} \qquad \mathrm{Pr} > 0.6 \tag{6-62}$$

The Nu_x values obtained from this relation agree well with measured values.

Solving Eq. 6-58 numerically for the temperature profile for different Prandtl numbers, and using the definition of the thermal boundary layer, it is determined that $\delta/\delta_t \cong \mathrm{Pr}^{1/3}$. Then the thermal boundary layer thickness becomes

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{5.0x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}} \tag{6-63}$$

Note that these relations are valid only for laminar flow over an isothermal flat plate. Also, the effect of variable properties can be accounted for by evaluating all such properties at the film temperature defined as $T_f = (T_s + T_\infty)/2$.

The Blasius solution gives important insights, but its value is largely historical because of the limitations it involves. Nowadays both laminar and turbulent flows over surfaces are routinely analyzed using numerical methods.

6–9 ▪ NONDIMENSIONALIZED CONVECTION EQUATIONS AND SIMILARITY

When viscous dissipation is negligible, the continuity, momentum, and energy equations for steady, incompressible, laminar flow of a fluid with constant properties are given by Eqs. 6-21, 6-28, and 6-35.

These equations and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by relevant and meaningful constant quantities: all lengths by a characteristic length L (which is the length for a plate), all velocities by a reference velocity $\mathcal{V}$ (which is the free stream velocity for a plate), pressure by $\rho\mathcal{V}^2$ (which is twice the free stream dynamic pressure for a plate), and temperature by a suitable temperature difference (which is $T_\infty - T_s$ for a plate). We get

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{\mathcal{V}}, \quad v^* = \frac{v}{\mathcal{V}}, \quad P^* = \frac{P}{\rho\mathcal{V}^2}, \quad \text{and} \quad T^* = \frac{T - T_s}{T_\infty - T_s}$$

where the asterisks are used to denote nondimensional variables. Introducing these variables into Eqs. 6-21, 6-28, and 6-35 and simplifying give

Continuity:
$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{6-64}$$

Momentum:
$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{dP^*}{dx^*} \tag{6-65}$$

Energy:
$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L\,\mathrm{Pr}}\frac{\partial^2 T^*}{\partial y^{*2}} \tag{6-66}$$

with the boundary conditions

$$\begin{aligned} &u^*(0, y^*) = 1, \quad u^*(x^*, 0) = 0, \quad u^*(x^*, \infty) = 1, \quad v^*(x^*, 0) = 0, \\ &T^*(0, y^*) = 1, \quad T^*(x^*, 0) = 0, \quad T^*(x^*, \infty) = 1 \end{aligned} \tag{6-67}$$

where $\mathrm{Re}_L = \mathcal{V}L/\nu$ is the dimensionless Reynolds number and $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number. For a given type of geometry, the solutions of problems with the same Re and Nu numbers are similar, and thus Re and Nu numbers serve as *similarity parameters.* Two physical phenomena are *similar* if they have the same dimensionless forms of governing differential equations and boundary conditions (Fig. 6–28).

A major advantage of nondimensionalizing is the significant reduction in the number of parameters. The original problem involves 6 parameters (L, $\mathcal{V}$, T_∞, T_s, ν, α), but the nondimensionalized problem involves just 2 parameters (Re_L and Pr). For a given geometry, problems that have the same values for the similarity parameters have identical solutions. For example, determining the convection heat transfer coefficient for flow over a given surface will require numerical solutions or experimental investigations for several fluids, with several sets of velocities, surface lengths, wall temperatures, and free stream temperatures. The same information can be obtained with far fewer investigations by grouping data into the dimensionless Re and Pr numbers. Another advantage of similarity parameters is that they enable us to group the results of a large number of experiments and to report them conveniently in terms of such parameters (Fig. 6–29).

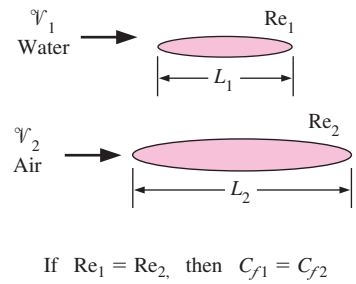


FIGURE 6–28
Two geometrically similar bodies have the same value of friction coefficient at the same Reynolds number.

6–10 ▪ FUNCTIONAL FORMS OF FRICTION AND CONVECTION COEFFICIENTS

The three nondimensionalized boundary layer equations (Eqs. 6-64, 6-65, and 6-66) involve three unknown functions u^*, v^*, and T^*, two independent variables x^* and y^*, and two parameters Re_L and Pr. The pressure $P^*(x^*)$ depends on the geometry involved (it is constant for a flat plate), and it has the same value inside and outside the boundary layer at a specified x^*. Therefore, it can be determined separately from the free stream conditions, and dP^*/dx^* in Eq. 6-65 can be treated as a known function of x^*. Note that the boundary conditions do not introduce any new parameters.

For a given geometry, the solution for u^* can be expressed as

$$u^* = f_1(x^*, y^*, \mathrm{Re}_L) \tag{6-68}$$

Then the shear stress at the surface becomes

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{\mu \mathcal{V}}{L} \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} = \frac{\mu \mathcal{V}}{L} f_2(x^*, \mathrm{Re}_L) \tag{6-69}$$

Substituting into its definition gives the local friction coefficient,

$$C_{f,x} = \frac{\tau_s}{\rho \mathcal{V}^2/2} = \frac{\mu \mathcal{V}/L}{\rho \mathcal{V}^2/2} f_2(x^*, \mathrm{Re}_L) = \frac{2}{\mathrm{Re}_L} f_2(x^*, \mathrm{Re}_l) = f_3(x^*, \mathrm{Re}_L) \tag{6-70}$$

Thus we conclude that the friction coefficient for a given geometry can be expressed in terms of the Reynolds number Re and the dimensionless space variable x^* alone (instead of being expressed in terms of x, L, $\mathcal{V}$, ρ, and μ). This is a very significant finding, and shows the value of nondimensionalized equations.

Similarly, the solution of Eq. 6-66 for the dimensionless temperature T^* for a given geometry can be expressed as

Parameters before nondimensionalizing

L, $\mathcal{V}$, T_∞, T_s, ν, α

Parameters after nondimensionalizing:

Re, Pr

FIGURE 6–29
The number of parameters is reduced greatly by nondimensionalizing the convection equations.

$$T^* = g_1(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr}) \tag{6-71}$$

Using the definition of T^*, the convection heat transfer coefficient becomes

$$h = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = \frac{-k(T_\infty - T_s)}{L(T_s - T_\infty)} \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} = \frac{k}{L} \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \tag{6-72}$$

Substituting this into the Nusselt number relation gives [or alternately, we can rearrange the relation above in dimensionless form as $hL/k = (\partial T^*/\partial y^*)|_{y^*=0}$ and define the dimensionless group hL/k as the Nusselt number]

$$\mathrm{Nu}_x = \frac{hL}{k} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} = g_2(x^*, \mathrm{Re}_L, \mathrm{Pr}) \tag{6-73}$$

Note that the Nusselt number is equivalent to the *dimensionless temperature gradient at the surface,* and thus it is properly referred to as the dimensionless heat transfer coefficient (Fig. 6–30). Also, the Nusselt number for a given geometry can be expressed in terms of the Reynolds number Re, the Prandtl number Pr, and the space variable x^*, and such a relation can be used for different fluids flowing at different velocities over similar geometries of different lengths.

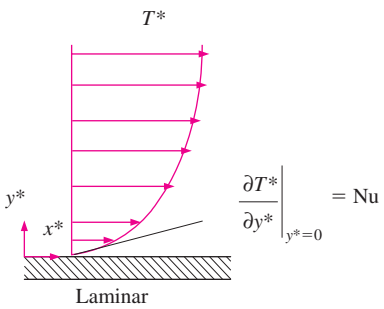


FIGURE 6–30
The Nusselt number is equivalent to the dimensionless temperature gradient at the surface.

The average friction and heat transfer coefficients are determined by integrating $C_{f,x}$ and Nu_x over the surface of the given body with respect to x^* from 0 to 1. Integration will remove the dependence on x^*, and the average friction coefficient and Nusselt number can be expressed as

$$C_f = f_4(\mathrm{Re}_L) \qquad \text{and} \qquad \mathrm{Nu} = g_3(\mathrm{Re}_L, \mathrm{Pr}) \tag{6-74}$$

These relations are extremely valuable as they state that for a given geometry, the friction coefficient can be expressed as a function of Reynolds number alone, and the Nusselt number as a function of Reynolds and Prandtl numbers alone (Fig. 6–31). Therefore, experimentalists can study a problem with a minimum number of experiments, and report their friction and heat transfer coefficient measurements conveniently in terms of Reynolds and Prandtl numbers. For example, a friction coefficient relation obtained with air for a given surface can also be used for water at the same Reynolds number. But it should be kept in mind that the validity of these relations is limited by the limitations on the boundary layer equations used in the analysis.

Local Nusselt number:

Nu_x = function (x^*, Re_L, Pr)

Average Nusselt number:

Nu = function (Re_L, Pr)

A common form of Nusselt number:

$Nu = C\,\mathrm{Re}_L^m \mathrm{Pr}^n$

FIGURE 6–31
For a given geometry, the average Nusselt number is a function of Reynolds and Prandtl numbers.

The experimental data for heat transfer is often represented with reasonable accuracy by a simple power-law relation of the form

$$\mathrm{Nu} = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n \tag{6-75}$$

where m and n are constant exponents (usually between 0 and 1), and the value of the constant C depends on geometry. Sometimes more complex relations are used for better accuracy.

6–11 ▪ ANALOGIES BETWEEN MOMENTUM AND HEAT TRANSFER

In forced convection analysis, we are primarily interested in the determination of the quantities C_f (to calculate shear stress at the wall) and Nu (to calculate heat transfer rates). Therefore, it is very desirable to have a relation between

C_f and Nu so that we can calculate one when the other is available. Such relations are developed on the basis of the similarity between momentum and heat transfers in boundary layers, and are known as *Reynolds analogy* and *Chilton–Colburn analogy.*

Reconsider the nondimensionalized momentum and energy equations for steady, incompressible, laminar flow of a fluid with constant properties and negligible viscous dissipation (Eqs. 6-65 and 6-66). When Pr = 1 (which is approximately the case for gases) and $\partial P^*/\partial x^* = 0$ (which is the case when, $u = u_\infty = \mathcal{V} = constant$ in the free stream, as in flow over a flat plate), these equations simplify to

Momentum:
$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{6-76}$$

Energy:
$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L}\frac{\partial^2 T^*}{\partial y^{*2}} \tag{6-77}$$

which are exactly of the same form for the dimensionless velocity u^* and temperature T^*. The boundary conditions for u^* and T^* are also identical. Therefore, the functions u^* and T^* must be identical, and thus the first derivatives of u^* and T^* at the surface must be equal to each other,

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \tag{6-78}$$

Then from Eqs. 6-69, 6-70, and 6-73 we have

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x \qquad (\mathrm{Pr} = 1) \tag{6-79}$$

which is known as the **Reynolds analogy** (Fig. 6–32). This is an important analogy since it allows us to determine the heat transfer coefficient for fluids with Pr ≈ 1 from a knowledge of friction coefficient which is easier to measure. Reynolds analogy is also expressed alternately as

$$\frac{C_{f,x}}{2} = \mathrm{St}_x \qquad (\mathrm{Pr} = 1) \tag{6-80}$$

where

$$\mathrm{St} = \frac{h}{\rho C_p \mathcal{V}} = \frac{\mathrm{Nu}}{\mathrm{Re}_L \mathrm{Pr}} \tag{6-81}$$

is the **Stanton number,** which is also a dimensionless heat transfer coefficient.

Reynolds analogy is of limited use because of the restrictions Pr = 1 and $\partial P^*/\partial x^* = 0$ on it, and it is desirable to have an analogy that is applicable over a wide range of Pr. This is done by adding a Prandtl number correction. The friction coefficient and Nusselt number for a flat plate are determined in Section 6-8 to be

$$C_{f,x} = 0.664\,\mathrm{Re}_x^{-1/2} \qquad \text{and} \qquad \mathrm{Nu}_x = 0.332\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2} \tag{6-82}$$

Taking their ratio and rearranging give the desired relation, known as the **modified Reynolds analogy** or **Chilton–Colburn analogy,**

Profiles: $u^* = T$

Gradients: $\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$

Analogy: $C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x$

FIGURE 6–32
When Pr = 1 and $\partial P^*/\partial x^* \approx 0$, the nondimensional velocity and temperature profiles become identical, and Nu is related to C_f by Reynolds analogy.

$$C_{f,x}\frac{Re_L}{2} = \mathrm{Nu}_x\,\mathrm{Pr}^{-1/3} \qquad \text{or} \qquad \frac{C_{f,x}}{2} = \frac{h_x}{\rho C_p \mathcal{V}}\mathrm{Pr}^{2/3} \equiv j_H \tag{6-83}$$

for $0.6 < \mathrm{Pr} < 60$. Here j_H is called the *Colburn j-factor.* Although this relation is developed using relations for laminar flow over a flat plate (for which $\partial P^*/\partial x^* = 0$), experimental studies show that it is also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients. For laminar flow, however, the analogy is not applicable unless $\partial P^*/\partial x^* \approx 0$. Therefore, it does not apply to laminar flow in a pipe. Analogies between C_f and Nu that are more accurate are also developed, but they are more complex and beyond the scope of this book. The analogies given above can be used for both local and average quantities.

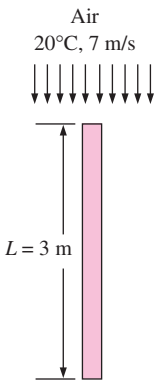


FIGURE 6–33
Schematic for Example 6–2.

EXAMPLE 6–2 Finding Convection Coefficient from Drag Measurement

A 2-m × 3-m flat plate is suspended in a room, and is subjected to air flow parallel to its surfaces along its 3-m-long side. The free stream temperature and velocity of air are 20°C and 7 m/s. The total drag force acting on the plate is measured to be 0.86 N. Determine the average convection heat transfer coefficient for the plate (Fig. 6–33).

SOLUTION A flat plate is subjected to air flow, and the drag force acting on it is measured. The average convection coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 20°C and 1 atm are (Table A-15):

$$\rho = 1.204\ \mathrm{kg/m^3}, \quad C_p = 1.007\ \mathrm{kJ/kg\cdot K}, \quad \mathrm{Pr} = 0.7309$$

Analysis The flow is along the 3-m side of the plate, and thus the characteristic length is $L = 3$ m. Both sides of the plate are exposed to air flow, and thus the total surface area is

$$A_s = 2WL = 2(2\ \mathrm{m})(3\ \mathrm{m}) = 12\ \mathrm{m^2}$$

For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from Eq. 6-11,

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2}$$

Solving for C_f and substituting,

$$C_f = \frac{F_f}{\rho A_s \mathcal{V}^2/2} = \frac{0.86\ \mathrm{N}}{(1.204\ \mathrm{kg/m^3})(12\ \mathrm{m^2})(7\ \mathrm{m/s})^2/2}\left(\frac{1\mathrm{kg\cdot m/s^2}}{1\ \mathrm{N}}\right) = 0.00243$$

Then the average heat transfer coefficient can be determined from the modified Reynolds analogy (Eq. 6-83) to be

$$h = \frac{C_f}{2}\frac{\rho \mathcal{V} C_p}{\mathrm{Pr}^{2/3}} = \frac{0.00243}{2}\frac{(1.204\ \mathrm{kg/m^3})(7\ \mathrm{m/s})(1007\ \mathrm{J/kg\cdot °C})}{0.7309^{2/3}} = 12.7\ \mathrm{W/m^2\cdot °C}$$

Discussion This example shows the great utility of momentum-heat transfer analogies in that the convection heat transfer coefficient can be determined from a knowledge of friction coefficient, which is easier to determine.

SUMMARY

Convection heat transfer is expressed by *Newton's law of cooling* as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where h is the convection heat transfer coefficient, T_s is the surface temperature, and T_∞ is the free-stream temperature. The convection coefficient is also expressed as

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty}$$

The *Nusselt number,* which is the dimensionless heat transfer coefficient, is defined as

$$\text{Nu} = \frac{hL_c}{k}$$

where k is the thermal conductivity of the fluid and L_c is the characteristic length.

The highly ordered fluid motion characterized by smooth streamlines is called *laminar.* The highly disordered fluid motion that typically occurs at high velocities is characterized by velocity fluctuations is called *turbulent.* The random and rapid fluctuations of groups of fluid particles, called *eddies,* provide an additional mechanism for momentum and heat transfer.

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the *velocity boundary layer.* The *boundary layer thickness,* δ, is defined as the distance from the surface at which $u = 0.99u_\infty$. The hypothetical line of $u = 0.99u_\infty$ divides the flow over a plate into the *boundary layer region* in which the viscous effects and the velocity changes are significant, and the *inviscid flow region,* in which the frictional effects are negligible.

The friction force per unit area is called *shear stress,* and the shear stress at the wall surface is expressed as

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad \text{or} \qquad \tau_s = C_f \frac{\rho \mathcal{V}^2}{2}$$

where μ is the dynamic viscosity, $\mathcal{V}$ is the upstream velocity, and C_f is the dimensionless *friction coefficient.* The property $\nu = \mu/\rho$ is the *kinematic viscosity.* The friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2}$$

The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the *thermal boundary layer.* The *thickness* of the thermal boundary layer δ_t at any location along the surface is the distance from the surface at which the temperature difference $T - T_s$ equals $0.99(T_\infty - T_s)$. The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless *Prandtl number,* defined as

$$\text{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

For external flow, the dimensionless *Reynolds number* is expressed as

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{\mathcal{V} L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu}$$

For a flat plate, the characteristic length is the distance x from the leading edge. The Reynolds number at which the flow becomes turbulent is called the *critical Reynolds number.* For flow over a flat plate, its value is taken to be $\text{Re}_{\text{cr}} = \mathcal{V} x_{\text{cr}}/\nu = 5 \times 10^5$.

The continuity, momentum, and energy equations for steady two-dimensional incompressible flow with constant properties are determined from mass, momentum, and energy balances to be

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

Energy: $$\rho C_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi$$

where the *viscous dissipation function* Φ is

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$

Using the boundary layer approximations and a similarity variable, these equations can be solved for parallel steady incompressible flow over a flat plate, with the following results:

Velocity boundary layer thickness: $$\delta = \frac{5.0}{\sqrt{\mathcal{V}/\nu x}} = \frac{5.0x}{\sqrt{\text{Re}_x}}$$

Local friction coefficient: $$C_{f,x} = \frac{\tau_w}{\rho \mathcal{V}^2/2} = 0.664\,\text{Re}_x^{-1/2}$$

Local Nusselt number: $$\text{Nu}_x = \frac{h_x x}{k} = 0.332\,\text{Pr}^{1/3}\,\text{Re}_x^{1/2}$$

Thermal boundary layer thickness: $$\delta_t = \frac{\delta}{\text{Pr}^{1/3}} = \frac{5.0x}{\text{Pr}^{1/3}\sqrt{\text{Re}_x}}$$

The average friction coefficient and Nusselt number are expressed in functional form as

$$C_f = f_4(\mathrm{Re}_L) \qquad \text{and} \qquad \mathrm{Nu} = g_3(\mathrm{Re}_L, \mathrm{Pr})$$

The Nusselt number can be expressed by a simple power-law relation of the form

$$\mathrm{Nu} = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

where m and n are constant exponents, and the value of the constant C depends on geometry. The *Reynolds analogy* relates the convection coefficient to the friction coefficient for fluids with $\mathrm{Pr} \approx 1$, and is expressed as

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x \qquad \text{or} \qquad \frac{C_{f,x}}{2} = \mathrm{St}_x$$

where

$$\mathrm{St} = \frac{h}{\rho C_p \mathcal{V}} = \frac{\mathrm{Nu}}{\mathrm{Re}_L\,\mathrm{Pr}}$$

is the *Stanton number.* The analogy is extended to other Prandtl numbers by the *modified Reynolds analogy* or *Chilton–Colburn analogy,* expressed as

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x\mathrm{Pr}^{-1/3}$$

or

$$\frac{C_{f,x}}{2} = \frac{h_x}{\rho C_p \mathcal{V}}\mathrm{Pr}^{2/3} \equiv j_H \quad (0.6 < \mathrm{Pr} < 60)$$

These analogies are also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients.

REFERENCES AND SUGGESTED READING

1. H. Blasius. "The Boundary Layers in Fluids with Little Friction (in German)." *Z. Math. Phys.,* 56, 1 (1908); pp. 1–37; English translation in National Advisory Committee for Aeronautics Technical Memo No. 1256, February 1950.
2. R. W. Fox and A. T. McDonald. *Introduction to Fluid Mechanics.* 5th. ed. New York, Wiley, 1999.
3. J. P. Holman. *Heat Transfer.* 9th ed. New York: McGraw-Hill, 2002.
4. F. P. Incropera and D. P. DeWitt. *Introduction to Heat Transfer.* 4th ed. New York: John Wiley & Sons, 2002.
5. W. M. Kays and M. E. Crawford. *Convective Heat and Mass Transfer.* 3rd ed. New York: McGraw-Hill, 1993.
6. F. Kreith and M. S. Bohn. *Principles of Heat Transfer.* 6th ed. Pacific Grove, CA: Brooks/Cole, 2001.
7. M. N. Özisik. *Heat Transfer—A Basic Approach.* New York: McGraw-Hill, 1985.
8. O. Reynolds. "On the Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and the Law of Resistance in Parallel Channels." *Philosophical Transactions of the Royal Society of London* 174 (1883), pp. 935–82.
9. H. Schlichting. *Boundary Layer Theory.* 7th ed. New York: McGraw-Hill, 1979.
10. G. G. Stokes. "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums." *Cambridge Philosophical Transactions,* IX, 8, 1851.
11. N. V. Suryanarayana. *Engineering Heat Transfer.* St. Paul, MN: West, 1995.
12. F. M. White. *Heat and Mass Transfer.* Reading, MA: Addison-Wesley, 1988.

PROBLEMS*

Mechanism and Types of Convection

6–1C What is forced convection? How does it differ from natural convection? Is convection caused by winds forced or natural convection?

6–2C What is external forced convection? How does it differ from internal forced convection? Can a heat transfer system involve both internal and external convection at the same time? Give an example.

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

6–3C In which mode of heat transfer is the convection heat transfer coefficient usually higher, natural convection or forced convection? Why?

6–4C Consider a hot baked potato. Will the potato cool faster or slower when we blow the warm air coming from our lungs on it instead of letting it cool naturally in the cooler air in the room? Explain.

6–5C What is the physical significance of the Nusselt number? How is it defined?

6–6C When is heat transfer through a fluid conduction and when is it convection? For what case is the rate of heat transfer higher? How does the convection heat transfer coefficient differ from the thermal conductivity of a fluid?

6–7C Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?

6–8 During air cooling of potatoes, the heat transfer coefficient for combined convection, radiation, and evaporation is determined experimentally to be as shown:

Air Velocity, m/s	Heat Transfer Coefficient, W/m^2 · °C
0.66	14.0
1.00	19.1
1.36	20.2
1.73	24.4

Consider a 10-cm-diameter potato initially at 20°C with a thermal conductivity of 0.49 W/m · °C. Potatoes are cooled by refrigerated air at 5°C at a velocity of 1 m/s. Determine the initial rate of heat transfer from a potato, and the initial value of the temperature gradient in the potato at the surface.

Answers: 9.0 W, −585°C/m

6–9 An average man has a body surface area of 1.8 m^2 and a skin temperature of 33°C. The convection heat transfer coefficient for a clothed person walking in still air is expressed as $h = 8.6\mathcal{V}^{0.53}$ for $0.5 < \mathcal{V} < 2$ m/s, where $\mathcal{V}$ is the walking velocity in m/s. Assuming the average surface temperature of the clothed person to be 30°C, determine the rate of heat loss from an average man walking in still air at 10°C by convection at a walking velocity of (*a*) 0.5 m/s, (*b*) 1.0 m/s, (*c*) 1.5 m/s, and (*d*) 2.0 m/s.

6–10 The convection heat transfer coefficient for a clothed person standing in moving air is expressed as $h = 14.8\mathcal{V}^{0.69}$ for $0.15 < \mathcal{V} < 1.5$ m/s, where $\mathcal{V}$ is the air velocity. For a person with a body surface area of 1.7 m^2 and an average surface temperature of 29°C, determine the rate of heat loss from the person in windy air at 10°C by convection for air velocities of (*a*) 0.5 m/s, (*b*) 1.0 m/s, and (*c*) 1.5 m/s.

6–11 During air cooling of oranges, grapefruit, and tangelos, the heat transfer coefficient for combined convection, radiation, and evaporation for air velocities of $0.11 < \mathcal{V} < 0.33$ m/s is determined experimentally and is expressed as $h = 5.05\, k_{air}\mathrm{Re}^{1/3}/D$, where the diameter D is the characteristic length. Oranges are cooled by refrigerated air at 5°C and 1 atm at a velocity of 0.5 m/s. Determine (*a*) the initial rate of heat transfer from a 7-cm-diameter orange initially at 15°C with a thermal conductivity of 0.50 W/m · °C, (*b*) the value of the initial temperature gradient inside the orange at the surface, and (*c*) the value of the Nusselt number.

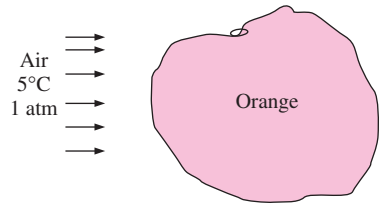


FIGURE P6–11

Velocity and Thermal Boundary Layers

6–12C What is viscosity? What causes viscosity in liquids and in gases? Is dynamic viscosity typically higher for a liquid or for a gas?

6–13C What is Newtonian fluid? Is water a Newtonian fluid?

6–14C What is the no-slip condition? What causes it?

6–15C Consider two identical small glass balls dropped into two identical containers, one filled with water and the other with oil. Which ball will reach the bottom of the container first? Why?

6–16C How does the dynamic viscosity of (*a*) liquids and (*b*) gases vary with temperature?

6–17C What fluid property is responsible for the development of the velocity boundary layer? For what kind of fluids will there be no velocity boundary layer on a flat plate?

6–18C What is the physical significance of the Prandtl number? Does the value of the Prandtl number depend on the type of flow or the flow geometry? Does the Prandtl number of air change with pressure? Does it change with temperature?

6–19C Will a thermal boundary layer develop in flow over a surface even if both the fluid and the surface are at the same temperature?

Laminar and Turbulent Flows

6–20C How does turbulent flow differ from laminar flow? For which flow is the heat transfer coefficient higher?

6–21C What is the physical significance of the Reynolds number? How is it defined for external flow over a plate of length L?

6–22C What does the friction coefficient represent in flow over a flat plate? How is it related to the drag force acting on the plate?

6–23C What is the physical mechanism that causes the friction factor to be higher in turbulent flow?

6–24C What is turbulent viscosity? What is it caused by?

6–25C What is turbulent thermal conductivity? What is it caused by?

Convection Equations and Similarity Solutions

6–26C Under what conditions can a curved surface be treated as a flat plate in fluid flow and convection analysis?

6–27C Express continuity equation for steady two-dimensional flow with constant properties, and explain what each term represents.

6–28C Is the acceleration of a fluid particle necessarily zero in steady flow? Explain.

6–29C For steady two-dimensional flow, what are the boundary layer approximations?

6–30C For what types of fluids and flows is the viscous dissipation term in the energy equation likely to be significant?

6–31C For steady two-dimensional flow over an isothermal flat plate in the x-direction, express the boundary conditions for the velocity components u and v, and the temperature T at the plate surface and at the edge of the boundary layer.

6–32C What is a similarity variable, and what is it used for? For what kinds of functions can we expect a similarity solution for a set of partial differential equations to exist?

6–33C Consider steady, laminar, two-dimensional flow over an isothermal plate. Does the thickness of the velocity boundary layer increase or decrease with (*a*) distance from the leading edge, (*b*) free-stream velocity, and (*c*) kinematic viscosity?

6–34C Consider steady, laminar, two-dimensional flow over an isothermal plate. Does the wall shear stress increase, decrease, or remain constant with distance from the leading edge?

6–35C What are the advantages of nondimensionalizing the convection equations?

6–36C Consider steady, laminar, two-dimensional, incompressible flow with constant properties and a Prandtl number of unity. For a given geometry, is it correct to say that both the average friction and heat transfer coefficients depend on the Reynolds number only?

6–37 Oil flow in a journal bearing can be treated as parallel flow between two large isothermal plates with one plate moving at a constant velocity of 12 m/s and the other stationary. Consider such a flow with a uniform spacing of 0.7 mm between the plates. The temperatures of the upper and lower plates are 40°C and 15°C, respectively. By simplifying and solving the continuity, momentum, and energy equations, determine (*a*) the velocity and temperature distributions in the oil, (*b*) the maximum temperature and where it occurs, and (*c*) the heat flux from the oil to each plate.

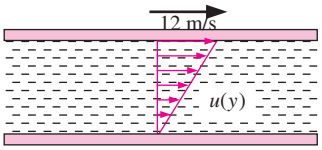


FIGURE P6–37

6–38 Repeat Problem 6–37 for a spacing of 0.4 mm.

6–39 A 6-cm-diameter shaft rotates at 3000 rpm in a 20-cm-long bearing with a uniform clearance of 0.2 mm. At steady operating conditions, both the bearing and the shaft in the vicinity of the oil gap are at 50°C, and the viscosity and thermal conductivity of lubricating oil are 0.05 N · s/m^2 and 0.17 W/m · K. By simplifying and solving the continuity, momentum, and energy equations, determine (*a*) the maximum temperature of oil, (*b*) the rates of heat transfer to the bearing and the shaft, and (*c*) the mechanical power wasted by the viscous dissipation in the oil. *Answers:* (*a*) 53.3°C, (*b*) 419 W, (*c*) 838 W

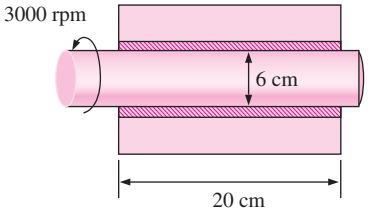


FIGURE P6-39

6–40 Repeat Problem 6–39 by assuming the shaft to have reached peak temperature and thus heat transfer to the shaft to be negligible, and the bearing surface still to be maintained at 50°C.

6–41 Reconsider Problem 6–39. Using EES (or other) software, investigate the effect of shaft velocity on the mechanical power wasted by viscous dissipation. Let the shaft rotation vary from 0 rpm to 5000 rpm. Plot the power wasted versus the shaft rpm, and discuss the results.

6–42 Consider a 5-cm-diameter shaft rotating at 2500 rpm in a 10-cm-long bearing with a clearance of 0.5 mm. Determine the power required to rotate the shaft if the fluid in the gap is (*a*) air, (*b*) water, and (*c*) oil at 40°C and 1 atm.

6–43 Consider the flow of fluid between two large parallel isothermal plates separated by a distance L. The upper plate is moving at a constant velocity of $\mathcal{V}$ and maintained at temperature T_0 while the lower plate is stationary and insulated. By simplifying and solving the continuity, momentum, and energy equations, obtain relations for the maximum temperature of fluid, the location where it occurs, and heat flux at the upper plate.

6–44 Reconsider Problem 6–43. Using the results of this problem, obtain a relation for the volumetric heat generation rate $\dot{g}$, in W/m^3. Then express the convection problem as an equivalent

conduction problem in the oil layer. Verify your model by solving the conduction problem and obtaining a relation for the maximum temperature, which should be identical to the one obtained in the convection analysis.

6–45 A 5-cm-diameter shaft rotates at 4500 rpm in a 15-cm-long, 8-cm-outer-diameter cast iron bearing (k = 70 W/m · K) with a uniform clearance of 0.6 mm filled with lubricating oil (μ = 0.03 N · s/m^2 and k = 0.14 W/m · K). The bearing is cooled externally by a liquid, and its outer surface is maintained at 40°C. Disregarding heat conduction through the shaft and assuming one-dimensional heat transfer, determine (*a*) the rate of heat transfer to the coolant, (*b*) the surface temperature of the shaft, and (*c*) the mechanical power wasted by the viscous dissipation in oil.

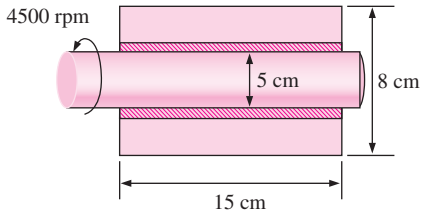


FIGURE P6–45

6–46 Repeat Problem 6–45 for a clearance of 1 mm.

Momentum and Heat Transfer Analogies

6–47C How is Reynolds analogy expressed? What is the value of it? What are its limitations?

6–48C How is the modified Reynolds analogy expressed? What is the value of it? What are its limitations?

6–49 A 4-m × 4-m flat plate maintained at a constant temperature of 80°C is subjected to parallel flow of air at 1 atm, 20°C, and 10 m/s. The total drag force acting on the upper surface of the plate is measured to be 2.4 N. Using momentum-heat transfer analogy, determine the average convection heat transfer coefficient, and the rate of heat transfer between the upper surface of the plate and the air.

6–50 A metallic airfoil of elliptical cross section has a mass of 50 kg, surface area of 12 m^2, and a specific heat of 0.50 kJ/kg · °C). The airfoil is subjected to air flow at 1 atm, 25°C, and 8 m/s along its 3-m-long side. The average temperature of the airfoil is observed to drop from 160°C to 150°C within 2 min of cooling. Assuming the surface temperature of the airfoil to be equal to its average temperature and using momentum-heat transfer analogy, determine the average friction coefficient of the airfoil surface. *Answer:* 0.000227

6–51 Repeat Problem 6–50 for an air-flow velocity of 12 m/s.

6–52 The electrically heated 0.6-m-high and 1.8-m-long windshield of a car is subjected to parallel winds at 1 atm, 0°C, and 80 km/h. The electric power consumption is observed to be 50 W when the exposed surface temperature of the windshield is 4°C. Disregarding radiation and heat transfer from the inner surface and using the momentum-heat transfer analogy, determine drag force the wind exerts on the windshield.

6–53 Consider an airplane cruising at an altitude of 10 km where standard atmospheric conditions are −50°C and 26.5 kPa at a speed of 800 km/h. Each wing of the airplane can be modeled as a 25-m × 3-m flat plate, and the friction coefficient of the wings is 0.0016. Using the momentum-heat transfer analogy, determine the heat transfer coefficient for the wings at cruising conditions. *Answer:* 89.6 W/m^2 · °C

Design and Essay Problems

6–54 Design an experiment to measure the viscosity of liquids using a vertical funnel with a cylindrical reservoir of height h and a narrow flow section of diameter D and length L. Making appropriate assumptions, obtain a relation for viscosity in terms of easily measurable quantities such as density and volume flow rate.

6–55 A facility is equipped with a wind tunnel, and can measure the friction coefficient for flat surfaces and airfoils. Design an experiment to determine the mean heat transfer coefficient for a surface using friction coefficient data.

CHAPTER 7

EXTERNAL FORCED CONVECTION

In Chapter 6 we considered the general and theoretical aspects of forced convection, with emphasis on differential formulation and analytical solutions. In this chapter we consider the practical aspects of forced convection to or from flat or curved surfaces subjected to *external flow*, characterized by the freely growing boundary layers surrounded by a free flow region that involves no velocity and temperature gradients.

We start this chapter with an overview of external flow, with emphasis on friction and pressure drag, flow separation, and the evaluation of average drag and convection coefficients. We continue with *parallel flow over flat plates.* In Chapter 6, we solved the boundary layer equations for steady, laminar, parallel flow over a flat plate, and obtained relations for the local friction coefficient and the Nusselt number. Using these relations as the starting point, we determine the average friction coefficient and Nusselt number. We then extend the analysis to turbulent flow over flat plates with and without an unheated starting length.

Next we consider *cross flow over cylinders and spheres*, and present graphs and empirical correlations for the drag coefficients and the Nusselt numbers, and discuss their significance. Finally, we consider *cross flow over tube banks* in aligned and staggered configurations, and present correlations for the pressure drop and the average Nusselt number for both configurations.

CONTENTS

7–1 ▪ DRAG AND HEAT TRANSFER IN EXTERNAL FLOW

Fluid flow over solid bodies frequently occurs in practice, and it is responsible for numerous physical phenomena such as the *drag force* acting on the automobiles, power lines, trees, and underwater pipelines; the *lift* developed by airplane wings; *upward draft* of rain, snow, hail, and dust particles in high winds; and the *cooling* of metal or plastic sheets, steam and hot water pipes, and extruded wires. Therefore, developing a good understanding of external flow and external forced convection is important in the mechanical and thermal design of many engineering systems such as aircraft, automobiles, buildings, electronic components, and turbine blades.

The flow fields and geometries for most external flow problems are too complicated to be solved analytically, and thus we have to rely on correlations based on experimental data. The availability of high-speed computers has made it possible to conduct series of "numerical experimentations" quickly by solving the governing equations numerically, and to resort to the expensive and time-consuming testing and experimentation only in the final stages of design. In this chapter we will mostly rely on relations developed experimentally.

The velocity of the fluid relative to an immersed solid body sufficiently far from the body (outside the boundary layer) is called the **free-stream velocity**, and is denoted by u_∞. It is usually taken to be equal to the **upstream velocity** $\mathcal{V}$ also called the **approach velocity**, which is the velocity of the approaching fluid far ahead of the body. This idealization is nearly exact for very thin bodies, such as a flat plate parallel to flow, but approximate for blunt bodies such as a large cylinder. The fluid velocity ranges from zero at the surface (the no-slip condition) to the free-stream value away from the surface, and the subscript "infinity" serves as a reminder that this is the value at a distance where the presence of the body is not felt. The upstream velocity, in general, may vary with location and time (e.g., the wind blowing past a building). But in the design and analysis, the upstream velocity is usually assumed to be *uniform* and *steady* for convenience, and this is what we will do in this chapter.

Friction and Pressure Drag

You may have seen high winds knocking down trees, power lines, and even trailers, and have felt the strong "push" the wind exerts on your body. You experience the same feeling when you extend your arm out of the window of a moving car. The force a flowing fluid exerts on a body in the flow direction is called **drag** (Fig. 7–1)

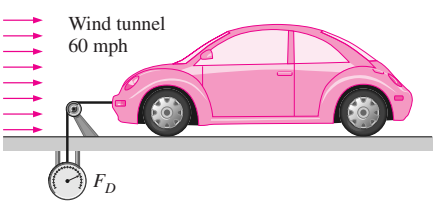


FIGURE 7–1
Schematic for measuring the drag force acting on a car in a wind tunnel.

A stationary fluid exerts only normal pressure forces on the surface of a body immersed in it. A moving fluid, however, also exerts tangential shear forces on the surface because of the no-slip condition caused by viscous effects. Both of these forces, in general, have components in the direction of flow, and thus the drag force is due to the combined effects of pressure and wall shear forces in the flow direction. The components of the pressure and wall shear forces in the normal direction to flow tend to move the body in that direction, and their sum is called **lift**.

In general, both the skin friction (wall shear) and pressure contribute to the drag and the lift. In the special case of a thin flat plate aligned parallel to the flow direction, the drag force depends on the wall shear only and is

independent of pressure. When the flat plate is placed normal to the flow direction, however, the drag force depends on the pressure only and is independent of the wall shear since the shear stress in this case acts in the direction normal to flow (Fig. 7–2). For slender bodies such as wings, the shear force acts nearly parallel to the flow direction. The drag force for such slender bodies is mostly due to shear forces (the skin friction).

The drag force F_D depends on the density ρ of the fluid, the upstream velocity $\mathcal{V}$, and the size, shape, and orientation of the body, among other things. The drag characteristics of a body is represented by the dimensionless **drag coefficient** C_D defined as

Drag coefficient:

$$C_D = \frac{F_D}{\frac{1}{2}\rho \mathcal{V}^2 A} \tag{7-1}$$

where A is the *frontal area* (the area projected on a plane normal to the direction of flow) for blunt bodies—bodies that tends to block the flow. The frontal area of a cylinder of diameter D and length L, for example, is $A = LD$. For parallel flow over flat plates or thin airfoils, A is the surface area. The drag coefficient is primarily a function of the shape of the body, but it may also depend on the Reynolds number and the surface roughness.

The drag force is the net force exerted by a fluid on a body in the direction of flow due to the combined effects of wall shear and pressure forces. The part of drag that is due directly to wall shear stress τ_w is called the **skin friction drag** (or just *friction drag*) since it is caused by frictional effects, and the part that is due directly to pressure P is called the **pressure drag** (also called the *form drag* because of its strong dependence on the form or shape of the body). When the friction and pressure drag coefficients are available, the total drag coefficient is determined by simply adding them,

$$C_D = C_{D,\,\text{friction}} + C_{D,\,\text{pressure}} \tag{7-2}$$

The *friction drag* is the component of the wall shear force in the direction of flow, and thus it depends on the orientation of the body as well as the magnitude of the wall shear stress τ_w. The friction drag is *zero* for a surface normal to flow, and *maximum* for a surface parallel to flow since the friction drag in this case equals the total shear force on the surface. Therefore, for parallel flow over a flat plate, the drag coefficient is equal to the *friction drag coefficient,* or simply the *friction coefficient* (Fig. 7–3). That is,

Flat plate:

$$C_D = C_{D,\,\text{friction}} = C_f \tag{7-3}$$

Once the average friction coefficient C_f is available, the drag (or friction) force over the surface can be determined from Eq. 7-1. In this case A is the surface area of the plate exposed to fluid flow. When both sides of a thin plate are subjected to flow, A becomes the total area of the top and bottom surfaces. Note that the friction coefficient, in general, will vary with location along the surface.

Friction drag is a strong function of viscosity, and an "idealized" fluid with zero viscosity would produce zero friction drag since the wall shear stress would be zero (Fig. 7–4). The pressure drag would also be zero in this case during steady flow regardless of the shape of the body since there will be no pressure losses. For flow in the horizontal direction, for example, the pressure along a horizontal line will be constant (just like stationary fluids) since the

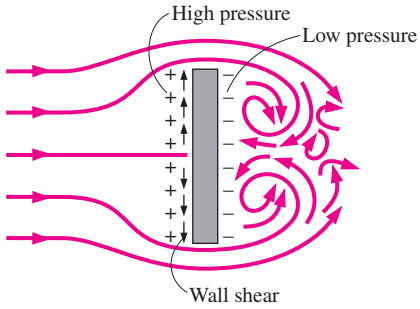


FIGURE 7–2
Drag force acting on a flat plate normal to flow depends on the pressure only and is independent of the wall shear, which acts normal to flow.

$C_{D,\,\text{pressure}} = 0$

$C_D = C_{D,\,\text{friction}} = C_f$

$F_{D,\,\text{pressure}} = 0$

$F_D = F_{D,\,\text{friction}} = F_f = C_f A \dfrac{\rho \mathcal{V}^2}{2}$

FIGURE 7–3
For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

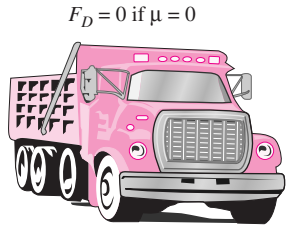


FIGURE 7–4
For the flow of an "idealized" fluid with zero viscosity past a body, both the friction drag and pressure drag are zero regardless of the shape of the body.

upstream velocity is constant, and thus there will be no net pressure force acting on the body in the horizontal direction. Therefore, the total drag is zero for the case of ideal inviscid fluid flow.

At low Reynolds numbers, most drag is due to friction drag. This is especially the case for highly streamlined bodies such as airfoils. The friction drag is also proportional to the surface area. Therefore, bodies with a larger surface area will experience a larger friction drag. Large commercial airplanes, for example, reduce their total surface area and thus drag by retracting their wing extensions when they reach the cruising altitudes to save fuel. The friction drag coefficient is independent of *surface roughness* in laminar flow, but is a strong function of surface roughness in turbulent flow due to surface roughness elements protruding further into the highly viscous laminar sublayer.

The pressure drag is proportional to the *difference* between the pressures acting on the front and back of the immersed body, and the frontal area. Therefore, the pressure drag is usually dominant for blunt bodies, negligible for streamlined bodies such as airfoils, and zero for thin flat plates parallel to the flow.

When a fluid is forced to flow over a curved surface at sufficiently high velocities, it will detach itself from the surface of the body. The low-pressure region behind the body where recirculating and back flows occur is called the *separation* region. The larger the separation area is, the larger the pressure drag will be. The effects of flow separation are felt far downstream in the form of reduced velocity (relative to the upstream velocity). The region of flow trailing the body where the effect of the body on velocity is felt is called the *wake* (Fig. 7–5). The separated region comes to an end when the two flow streams reattach, but the wake keeps growing behind the body until the fluid in the wake region regains its velocity. The viscous effects are the most significant in the boundary layer, the separated region, and the wake. The flow outside these regions can be considered to be inviscid.

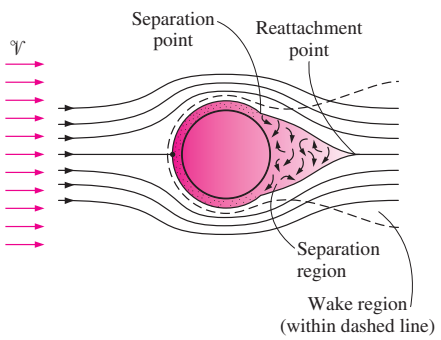


FIGURE 7–5
Separation and reattachment during flow over a cylinder, and the wake region.

Heat Transfer

The phenomena that affect drag force also affect heat transfer, and this effect appears in the Nusselt number. By nondimensionalizing the boundary layer equations, it was shown in Chapter 6 that the local and average Nusselt numbers have the functional form

$$\mathrm{Nu}_x = f_1(x^*, \mathrm{Re}_x, \mathrm{Pr}) \qquad \text{and} \qquad \mathrm{Nu} = f_2(\mathrm{Re}_L, \mathrm{Pr}) \tag{7-4a, b}$$

The experimental data for heat transfer is often represented conveniently with reasonable accuracy by a simple power-law relation of the form

$$\mathrm{Nu} = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n \tag{7-5}$$

where m and n are constant exponents, and the value of the constant C depends on geometry and flow.

The fluid temperature in the thermal boundary layer varies from T_s at the surface to about T_∞ at the outer edge of the boundary. The fluid properties also vary with temperature, and thus with position across the boundary layer. In order to account for the variation of the properties with temperature, the fluid properties are usually evaluated at the so-called **film temperature**, defined as

$$T_f = \frac{T_s + T_\infty}{2} \tag{7-6}$$

which is the *arithmetic average* of the surface and the free-stream temperatures. The fluid properties are then assumed to remain constant at those values during the entire flow. An alternative way of accounting for the variation of properties with temperature is to evaluate all properties at the free stream temperature and to multiply the Nusselt number relation in Eq. 7-5 by $(\mathrm{Pr}_\infty/\mathrm{Pr}_s)^r$ or $(\mu_\infty/\mu_s)^r$.

The local drag and convection coefficients vary along the surface as a result of the changes in the velocity boundary layers in the flow direction. We are usually interested in the drag force and the heat transfer rate for the *entire* surface, which can be determined using the *average* friction and convection coefficient. Therefore, we present correlations for both local (identified with the subscript x) and average friction and convection coefficients. When relations for local friction and convection coefficients are available, the *average* friction and convection coefficients for the entire surface can be determined by integration from

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx \tag{7-7}$$

and

$$h = \frac{1}{L}\int_0^L h_x\,dx \tag{7-8}$$

When the average drag and convection coefficients are available, the drag force can be determined from Eq. 7-1 and the rate of heat transfer to or from an isothermal surface can be determined from

$$\dot{Q} = hA_s(T_s - T_\infty) \tag{7-9}$$

where A_s is the surface area.

7–2 ▪ PARALLEL FLOW OVER FLAT PLATES

Consider the parallel flow of a fluid over a flat plate of length L in the flow direction, as shown in Figure 7–6. The x-coordinate is measured along the plate surface from the leading edge in the direction of the flow. The fluid approaches the plate in the x-direction with uniform upstream velocity $\mathcal{V}$ and temperature T_∞. The flow in the velocity boundary layer starts out as laminar, but if the plate is sufficiently long, the flow will become turbulent at a distance x_{cr} from the leading edge where the Reynolds number reaches its critical value for transition.

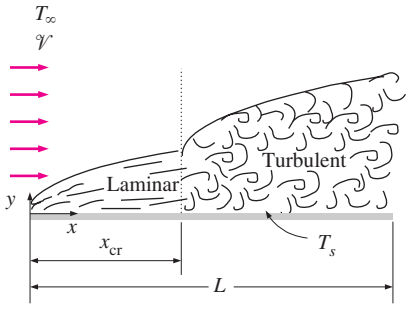


FIGURE 7–6
Laminar and turbulent regions of the boundary layer during flow over a flat plate.

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, upstream velocity, surface temperature,* and the *type of fluid,* among other things, and is best characterized by the Reynolds number. The Reynolds number at a distance x from the leading edge of a flat plate is expressed as

$$\mathrm{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\nu} \tag{7-10}$$

Note that the value of the Reynolds number varies for a flat plate along the flow, reaching $\mathrm{Re}_L = \mathcal{V}L/\nu$ at the end of the plate.

For flow over a flat plate, transition from laminar to turbulent is usually taken to occur at the *critical Reynolds number* of

$$\mathrm{Re}_{\mathrm{cr}} = \frac{\rho \mathcal{V} x_{\mathrm{cr}}}{\mu} = 5 \times 10^5 \tag{7-11}$$

The value of the critical Reynolds number for a flat plate may vary from 10^5 to 3×10^6, depending on the surface roughness and the turbulence level of the free stream.

Friction Coefficient

Based on analysis, the boundary layer thickness and the local friction coefficient at location x for laminar flow over a flat plate were determined in Chapter 6 to be

$$\textit{Laminar:} \quad \delta_{v,x} = \frac{5x}{\mathrm{Re}_x^{1/2}} \quad \text{and} \quad C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}, \quad \mathrm{Re}_x < 5 \times 10^5 \tag{7-12a, b}$$

The corresponding relations for turbulent flow are

$$\textit{Turbulent:} \quad \delta_{v,x} = \frac{0.382x}{\mathrm{Re}_x^{1/5}} \quad \text{and} \quad C_{f,x} = \frac{0.0592}{\mathrm{Re}_x^{1/5}}, \quad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \tag{7-13a, b}$$

where x is the distance from the leading edge of the plate and $\mathrm{Re}_x = \mathcal{V}x/\nu$ is the Reynolds number at location x. Note that $C_{f,x}$. is proportional to $\mathrm{Re}_x^{-1/2}$ and thus to $x^{-1/2}$ for laminar flow. Therefore, $C_{f,x}$ is supposedly *infinite* at the leading edge (x = 0) and decreases by a factor of $x^{-1/2}$ in the flow direction. The local friction coefficients are higher in turbulent flow than they are in laminar flow because of the intense mixing that occurs in the turbulent boundary layer. Note that $C_{f,x}$ reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-1/5}$ in the flow direction.

The *average* friction coefficient over the entire plate is detennined by substituting the relations above into Eq. 7-7 and performing the integrations (Fig.7–7). We get

$$\begin{aligned}
C_f &= \frac{1}{L}\int_0^L C_{f,x}\,dx \\
&= \frac{1}{L}\int_0^L \frac{0.664}{\mathrm{Re}_x^{1/2}}\,dx \\
&= \frac{0.664}{L}\int_0^L \left(\frac{\mathcal{V}x}{\nu}\right)^{-1/2} dx \\
&= \frac{0.664}{L}\left(\frac{\mathcal{V}}{\nu}\right)^{-1/2} \left.\frac{x^{1/2}}{\frac{1}{2}}\right|_0^L \\
&= \frac{2\times 0.664}{L}\left(\frac{\mathcal{V}L}{\nu}\right)^{-1/2} \\
&= \frac{1.328}{\mathrm{Re}_L^{1/2}}
\end{aligned}$$

FIGURE 7–7
The average friction coefficient over a surface is determined by integrating the local friction coefficient over the entire surface.

$$\textit{Laminar:} \quad C_f = \frac{1.328}{\mathrm{Re}_L^{1/2}} \quad \mathrm{Re}_L < 5 \times 10^5 \tag{7-14}$$

$$\textit{Turbulent:} \quad C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \quad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \tag{7-15}$$

The first relation gives the average friction coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average friction coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region (that is, $x_{\mathrm{cr}} \ll L$ where the length of the plate x_{cr} over which the flow is laminar can be determined from $\mathrm{Re}_{\mathrm{cr}} = 5 \times 10^5 = \mathcal{V}x_{\mathrm{cr}}/\nu$).

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the *average* friction coefficient over the entire plate is determined by performing the integration in Eq. 7-7 over two parts: the laminar region $0 \le x \le x_{cr}$ and the turbulent region $x_{cr} < x \le L$ as

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,\,x\ \text{laminar}}\,dx + \int_{x_{cr}}^{L} C_{f,\,x,\ \text{turbulent}}\,dx\right) \tag{7-16}$$

Note that we included the transition region with the turbulent region. Again taking the critical Reynolds number to be $\text{Re}_{cr} = 5 \times 10^5$ and performing the integrations of Eq. 7-16 after substituting the indicated expressions, the *average* friction coefficient over the entire plate is determined to be

$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L} \qquad 5 \times 10^5 \le \text{Re}_L \le 10^7 \tag{7-17}$$

The constants in this relation will be different for different critical Reynolds numbers. Also, the surfaces are assumed to be *smooth*, and the free stream to be *turbulent free*. For laminar flow, the friction coefficient depends on only the Reynolds number, and the surface roughness has no effect. For turbulent flow, however, surface roughness causes the friction coefficient to increase severalfold, to the point that in fully turbulent regime the friction coefficient is a function of surface roughness alone, and independent of the Reynolds number (Fig. 7–8).

A curve fit of experimental data for the average friction coefficient in this regime is given by Schlichting as

Rough surface, turbulent:
$$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5} \tag{7-18}$$

were ε is the surface roughness, and L is the length of the plate in the flow direction. In the absence of a better relation, the relation above can be used for turbulent flow on rough surfaces for $\text{Re} > 10^6$, especially when $\varepsilon/L > 10^{-4}$.

Relative roughness, ε/L	Friction coefficient C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

*Smooth surface for Re = 10^7. Others calculated from Eq. 7-18.

FIGURE 7–8
For turbulent flow, surface roughness may cause the friction coefficient to increase severalfold.

Heat Transfer Coefficient

The local Nusselt number at a location x for laminar flow over a flat plate was determined in Chapter 6 by solving the differential energy equation to be

Laminar:
$$\text{Nu}_x = \frac{h_x x}{k} = 0.332\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3} \qquad \text{Pr} > 0.60 \tag{7-19}$$

The corresponding relation for turbulent flow is

Turbulent:
$$\text{Nu}_x = \frac{h_x x}{k} = 0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_x \le 10^7 \end{array} \tag{7-20}$$

Note that h_x is proportional to $\text{Re}_x^{0.5}$ and thus to $x^{-0.5}$ for laminar flow. Therefore, h_x is *infinite* at the leading edge ($x = 0$) and decreases by a factor of $x^{-0.5}$ in the flow direction. The variation of the boundary layer thickness δ and the friction and heat transfer coefficients along an isothermal flat plate are shown in Figure 7–9. The local friction and heat transfer coefficients are higher in

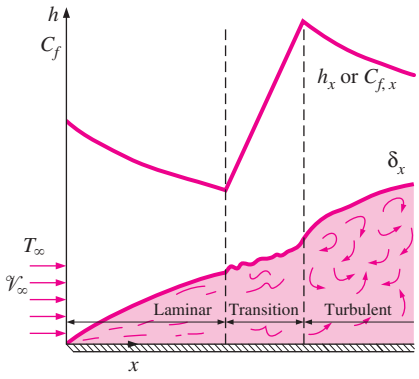


FIGURE 7–9
The variation of the local friction and heat transfer coefficients for flow over a flat plate.

turbulent flow than they are in laminar flow. Also, h_x reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-0.2}$ in the flow direction, as shown in the figure.

The *average* Nusselt number over the entire plate is determined by substituting the relations above into Eq. 7-8 and performing the integrations. We get

Laminar: $$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re}_L < 5 \times 10^5 \tag{7-21}$$

Turbulent: $$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array} \tag{7-22}$$

The first relation gives the average heat transfer coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average heat transfer coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region.

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the *average* heat transfer coefficient over the entire plate is determined by performing the integration in Eq. 7-8 over two parts as

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\,\text{laminar}}\,dx + \int_{x_{cr}}^{L} h_{x,\,\text{trubulent}}\,dx\right) \tag{7-23}$$

Again taking the critical Reynolds number to be $\mathrm{Re}_{cr} = 5 \times 10^5$ and performing the integrations in Eq. 7-23 after substituting the indicated expressions, the *average* Nusselt number over the *entire* plate is determined to be (Fig. 7–10)

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array} \tag{7-24}$$

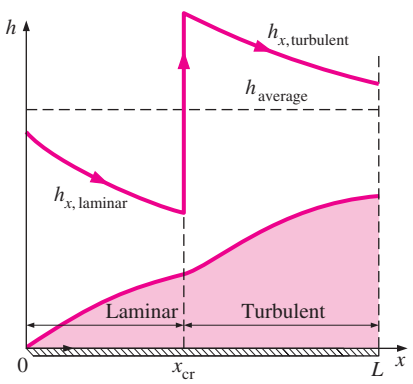


FIGURE 7–10
Graphical representation of the average heat transfer coefficient for a flat plate with combined laminar and turbulent flow.

The constants in this relation will be different for different critical Reynolds numbers.

Liquid metals such as mercury have high thermal conductivities, and are commonly used in applications that require high heat transfer rates. However, they have very small Prandtl numbers, and thus the thermal boundary layer develops much faster than the velocity boundary layer. Then we can assume the velocity in the thermal boundary layer to be constant at the free stream value and solve the energy equation. It gives

$$\mathrm{Nu}_x = 0.565(\mathrm{Re}_x\,\mathrm{Pr})^{1/2} \qquad \mathrm{Pr} < 0.05 \tag{7-25}$$

It is desirable to have a single correlation that applies to *all fluids*, including liquid metals. By curve-fitting existing data, Churchill and Ozoe (Ref. 3) proposed the following relation which is applicable for *all Prandtl numbers* and is claimed to be accurate to ±1%,

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \tag{7-26}$$

These relations have been obtained for the case of *isothermal* surfaces but could also be used approximately for the case of nonisothermal surfaces by assuming the surface temperature to be constant at some average value.

Also, the surfaces are assumed to be *smooth*, and the free stream to be *turbulent free*. The effect of variable properties can be accounted for by evaluating all properties at the film temperature.

Flat Plate with Unheated Starting Length

So far we have limited our consideration to situations for which the entire plate is heated from the leading edge. But many practical applications involve surfaces with an unheated starting section of length ξ, shown in Figure 7–11, and thus there is no heat transfer for $0 < x < \xi$. In such cases, the velocity boundary layer starts to develop at the leading edge ($x = 0$), but the thermal boundary layer starts to develop where heating starts ($x = \xi$).

Consider a flat plate whose heated section is maintained at a constant temperature ($T = T_s$ constant for $x > \xi$). Using integral solution methods (see Kays and Crawford, 1994), the local Nusselt numbers for both laminar and turbulent flows are determined to be

Laminar:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}} \tag{7-27}$$

Turbulent:
$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}} \tag{7-28}$$

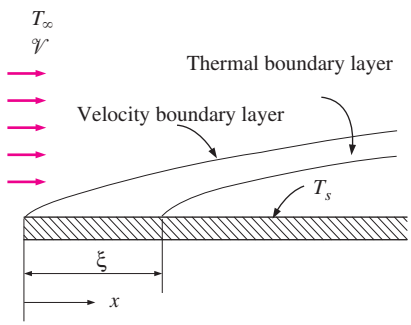


FIGURE 7–11
Flow over a flat plate with an unheated starting length.

for $x > \xi$. Note that for $\xi = 0$, these Nu_x relations reduce to $\mathrm{Nu}_{x\,(\text{for } \xi=0)}$, which is the Nusselt number relation for a flat plate without an unheated starting length. Therefore, the terms in brackets in the denominator serve as correction factors for plates with unheated starting lengths.

The determination of the average Nusselt number for the heated section of a plate requires the integration of the local Nusselt number relations above, which cannot be done analytically. Therefore, integrations must be done numerically. The results of numerical integrations have been correlated for the average convection coefficients [Thomas, (1977) Ref. 11] as

Laminar:
$$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L} \tag{7-29}$$

Turbulent:
$$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)}\,h_{x=L} \tag{7-30}$$

The first relation gives the average convection coefficient for the entire heated section of the plate when the flow is laminar over the entire plate. Note that for $\xi = 0$ it reduces to $h_L = 2h_{x=L}$, as expected. The second relation gives the average convection coefficient for the case of turbulent flow over the entire plate or when the laminar flow region is small relative to the turbulent region.

Uniform Heat Flux

When a flat plate is subjected to *uniform heat flux* instead of uniform temperature, the local Nusselt number is given by

Laminar:
$$\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \tag{7-31}$$

Turbulent:
$$\mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \tag{7-32}$$

These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case. When the plate involves an unheated starting length, the relations developed for the uniform surface temperature case can still be used provided that Eqs. 7-31 and 7-32 are used for $\mathrm{Nu}_{x(\text{for } \xi = 0)}$ in Eqs. 7-27 and 7-28, respectively.

When heat flux $\dot{q}_s$ is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance x are determined from

$$\dot{Q} = \dot{q}_s A_s \tag{7-33}$$

and

$$\dot{q}_s = h_x[T_s(x) - T_\infty] \qquad \rightarrow \qquad T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x} \tag{7-34}$$

where A_s is the heat transfer surface area.

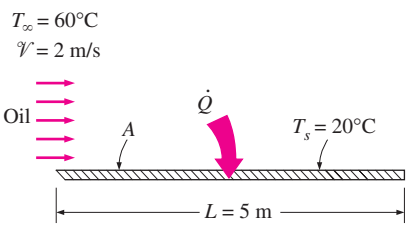


FIGURE 7–12
Schematic for Example 7-1.

EXAMPLE 7–1 Flow of Hot Oil over a Flat Plate

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s (Fig. 7–12). Determine the total drag force and the rate of heat transfer per unit width of the entire plate.

SOLUTION Engine oil flows over a flat plate. The total drag force and the rate of heat transfer per unit width of the plate are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5 \times 10^5$.

Properties The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40°C$ are (Table A–14).

$$\rho = 876 \text{ kg/m}^3 \qquad \mathrm{Pr} = 2870$$
$$k = 0.144 \text{ W/m} \cdot °\text{C} \qquad \nu = 242 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis Noting that $L = 5$ m, the Reynolds number at the end of the plate is

$$\mathrm{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(2 \text{ m/s})(5 \text{ m})}{0.242 \times 10^{-5} \text{ m}^2/\text{s}} = 4.13 \times 10^4$$

which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is

$$C_f = 1.328\,\mathrm{Re}_L^{-0.5} = 1.328 \times (4.13 \times 10^3)^{-0.5} = 0.0207$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the plate per unit width becomes

$$F_D = C_f A_s \frac{\rho \mathcal{V}^2}{2} = 0.0207 \times (5 \times 1 \text{ m}^2)\frac{(876 \text{ kg/m}^3)(2 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$
$$= \mathbf{181\ N}$$

The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

This force per unit width corresponds to the weight of a mass of about 18 kg. Therefore, a person who applies an equal and opposite force to the plate to keep

it from moving will feel like he or she is using as much force as is necessary to hold a 18-kg mass from dropping.

Similarly, the Nusselt number is determined using the laminar flow relations for a flat plate,

$$\text{Nu} = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918$$

Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.144\ \text{W/m}\cdot{}^\circ\text{C}}{5\ \text{m}}(1918) = 55.2\ \text{W/m}^2\cdot{}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (55.2\ \text{W/m}^2\cdot{}^\circ\text{C})(5 \times 1\ \text{m}^2)(60 - 20)^\circ\text{C} = \mathbf{11{,}040\ W}$$

Discussion Note that heat transfer is always from the higher-temperature medium to the lower-temperature one. In this case, it is from the oil to the plate. The heat transfer rate is per m width of the plate. The heat transfer for the entire plate can be obtained by multiplying the value obtained by the actual width of the plate.

EXAMPLE 7–2 Cooling of a Hot Block by Forced Air at High Elevation

The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5 m × 6 m flat plate whose temperature is 140°C (Fig. 7-13). Determine the rate of heat transfer from the plate if the air flows parallel to the (*a*) 6-m-long side and (*b*) the 1.5-m side.

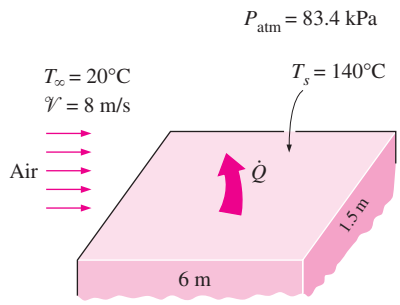


FIGURE 7–13
Schematic for Example 7-2.

SOLUTION The top surface of a hot block is to be cooled by forced air. The rate of heat transfer is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas.

Properties The properties k, μ, C_p, and Pr of ideal gases are independent of pressure, while the properties ν and α are inversely proportional to density and thus pressure. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (140 + 20)/2 = 80°\text{C}$ and 1 atm pressure are (Table A–15)

$$k = 0.02953\ \text{W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 0.7154$$
$$\nu_{@\ 1\ \text{atm}} = 2.097 \times 10^{-5}\ \text{m}^2/\text{s}$$

The atmospheric pressure in Denver is $P = (83.4\ \text{kPa})/(101.325\ \text{kPa/atm}) = 0.823$ atm. Then the kinematic viscosity of air in Denver becomes

$$\nu = \nu_{@\ 1\ \text{atm}}/P = (2.097 \times 10^{-5}\ \text{m}^2/\text{s})/0.823 = 2.548 \times 10^{-5}\ \text{m}^2/\text{s}$$

Analysis (*a*) When air flow is parallel to the long side, we have $L = 6$ m, and the Reynolds number at the end of the plate becomes

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8\ \text{m/s})(6\ \text{m})}{2.548 \times 10^{-5}\ \text{m}^2/\text{s}} = 1.884 \times 10^6$$

which is greater than the critical Reynolds number. Thus, we have combined laminar and turbulent flow, and the average Nusselt number for the entire plate is determined to be

$$\begin{aligned}\text{Nu} = \frac{hL}{k} &= (0.037\,\text{Re}_L^{0.8} - 871)\text{Pr}^{1/3}\\ &= [0.037(1.884 \times 10^6)^{0.8} - 871]0.7154^{1/3}\\ &= 2687\end{aligned}$$

Then

$$h = \frac{k}{L}\text{Nu} = \frac{0.02953\ \text{W/m}\cdot{}^\circ\text{C}}{6\ \text{m}}(2687) = 13.2\ \text{W/m}^2\cdot{}^\circ\text{C}$$

$$A_s = wL = (1.5\ \text{m})(6\ \text{m}) = 9\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (13.2\ \text{W/m}^2\cdot{}^\circ\text{C})(9\ \text{m}^2)(140 - 20)^\circ\text{C} = \mathbf{1.43 \times 10^4\ W}$$

Note that if we disregarded the laminar region and assumed turbulent flow over the entire plate, we would get Nu = 3466 from Eq. 7–22, which is 29 percent higher than the value calculated above.

(*b*) When air flow is along the short side, we have $L = 1.5$ m, and the Reynolds number at the end of the plate becomes

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8\ \text{m/s})(1.5\ \text{m})}{2.548 \times 10^{-5}\ \text{m}^2/\text{s}} = 4.71 \times 10^5$$

which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate, and the average Nusselt number is

$$\text{Nu} = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664 \times (4.71 \times 10^5)^{0.5} \times 0.7154^{1/3} = 408$$

Then

$$h = \frac{k}{L}\text{Nu} = \frac{0.02953\ \text{W/m}\cdot{}^\circ\text{C}}{1.5\ \text{m}}(408) = 8.03\ \text{W/m}^2\cdot{}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (8.03\ \text{W/m}^2\cdot{}^\circ\text{C})(9\ \text{m}^2)(140 - 20)^\circ\text{C} = \mathbf{8670\ W}$$

which is considerably less than the heat transfer rate determined in case (*a*).

Discussion Note that the *direction* of fluid flow can have a significant effect on convection heat transfer to or from a surface (Fig. 7-14). In this case, we can increase the heat transfer rate by 65 percent by simply blowing the air along the long side of the rectangular plate instead of the short side.

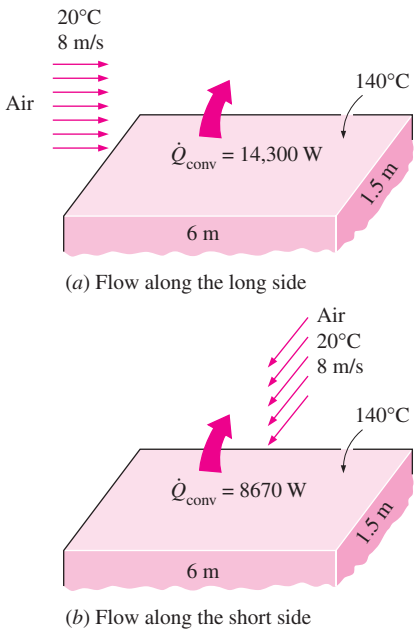


FIGURE 7–14
The direction of fluid flow can have a significant effect on convection heat transfer.

EXAMPLE 7–3 Cooling of Plastic Sheets by Forced Air

The forming section of a plastics plant puts out a continuous sheet of plastic that is 4 ft wide and 0.04 in. thick at a velocity of 30 ft/min. The temperature of the plastic sheet is 200°F when it is exposed to the surrounding air, and a 2-ft-long section of the plastic sheet is subjected to air flow at 80°F at a velocity of 10 ft/s on both sides along its surfaces normal to the direction of motion

of the sheet, as shown in Figure 7–15. Determine (*a*) the rate of heat transfer from the plastic sheet to air by forced convection and radiation and (*b*) the temperature of the plastic sheet at the end of the cooling section. Take the density, specific heat, and emissivity of the plastic sheet to be $\rho = 75$ lbm/ft^3, $C_p = 0.4$ Btu/lbm · °F, and $\varepsilon = 0.9$.

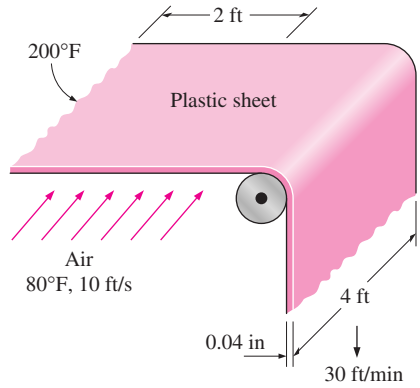


FIGURE 7–15
Schematic for Example 7–3.

SOLUTION Plastic sheets are cooled as they leave the forming section of a plastics plant. The rate of heat loss from the plastic sheet by convection and radiation and the exit temperature of the plastic sheet are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$. **3** Air is an ideal gas. **4** The local atmospheric pressure is 1 atm. **5** The surrounding surfaces are at the temperature of the room air.

Properties The properties of the plastic sheet are given in the problem statement. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (200 + 80)/2 = 140°\text{F}$ and 1 atm pressure are (Table A–15E)

$$k = 0.01623 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F} \qquad \text{Pr} = 0.7202$$
$$\nu = 0.7344 \text{ ft}^2/\text{h} = 0.204 \times 10^{-3} \text{ ft}^2/\text{s}$$

Analysis (*a*) We expect the temperature of the plastic sheet to drop somewhat as it flows through the 2-ft-long cooling section, but at this point we do not know the magnitude of that drop. Therefore, we assume the plastic sheet to be isothermal at 200°F to get started. We will repeat the calculations if necessary to account for the temperature drop of the plastic sheet.

Noting that $L = 4$ ft, the Reynolds number at the end of the air flow across the plastic sheet is

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(10 \text{ ft/s})(4 \text{ ft})}{0.204 \times 10^{-3} \text{ ft}^2/\text{s}} = 1.961 \times 10^5$$

which is less than the critical Reynolds number. Thus, we have *laminar flow* over the entire sheet, and the Nusselt number is determined from the laminar flow relations for a flat plate to be

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} = 0.664 \times (1.961 \times 10^5)^{0.5} \times (0.7202)^{1/3} = 263.6$$

Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.01623 \text{ Btu/h} \cdot \text{ft} \cdot °\text{F}}{4 \text{ ft}}(263.6) = 1.07 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F}$$
$$A_s = (2 \text{ ft})(4 \text{ ft})(2 \text{ sides}) = 16 \text{ ft}^2$$

and

$$\begin{aligned}\dot{Q}_{\text{conv}} &= hA_s(T_s - T_\infty) \\ &= (1.07 \text{ Btu/h} \cdot \text{ft}^2 \cdot °\text{F})(16 \text{ ft}^2)(200 - 80)°\text{F} \\ &= 2054 \text{ Btu/h}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) \\ &= (0.9)(0.1714 \times 10^{-8} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4)(16 \text{ ft}^2)[(660 \text{ R})^4 - (540 \text{ R})^4] \\ &= 2584 \text{ Btu/h}\end{aligned}$$

Therefore, the rate of cooling of the plastic sheet by combined convection and radiation is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 2054 + 2584 = \mathbf{4638\ Btu/h}$$

(*b*) To find the temperature of the plastic sheet at the end of the cooling section, we need to know the mass of the plastic rolling out per unit time (or the mass flow rate), which is determined from

$$\dot{m} = \rho A_c \mathcal{V}_{\text{plastic}} = (75\ \text{lbm/ft}^3)\left(\frac{4 \times 0.04}{12}\ \text{ft}^3\right)\left(\frac{30}{60}\ \text{ft/s}\right) = 0.5\ \text{lbm/s}$$

Then, an energy balance on the cooled section of the plastic sheet yields

$$\dot{Q} = \dot{m}\, C_p(T_2 - T_1) \quad \rightarrow \quad T_2 = T_1 + \frac{\dot{Q}}{\dot{m} C_p}$$

Noting that $\dot{Q}$ is a negative quantity (heat loss) for the plastic sheet and substituting, the temperature of the plastic sheet as it leaves the cooling section is determined to be

$$T_2 = 200°\text{F} + \frac{-4638\ \text{Btu/h}}{(0.5\ \text{lbm/s})(0.4\ \text{Btu/lbm} \cdot °\text{F})}\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = \mathbf{193.6°F}$$

Discussion The average temperature of the plastic sheet drops by about 6.4°F as it passes through the cooling section. The calculations now can be repeated by taking the average temperature of the plastic sheet to be 196.8°F instead of 200°F for better accuracy, but the change in the results will be insignificant because of the small change in temperature.

7–3 ▪ FLOW ACROSS CYLINDERS AND SPHERES

Flow across cylinders and spheres is frequently encountered in practice. For example, the tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes, and both flows must be considered in the analysis of the heat exchanger. Also, many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D*. Thus, the Reynolds number is defined as $\text{Re} = \mathcal{V}D/\nu$ where $\mathcal{V}$ is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $\text{Re}_{\text{cr}} \approx 2 \times 10^5$. That is, the boundary layer remains laminar for about $\text{Re} \lesssim 2 \times 10^5$ and becomes turbulent for $\text{Re} \gtrsim 2 \times 10^5$.

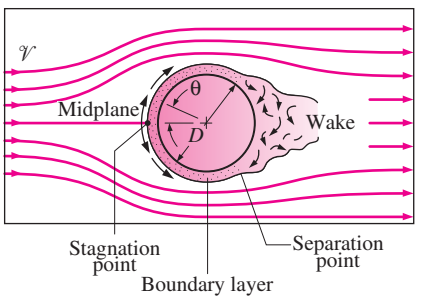


FIGURE 7–16
Typical flow patterns in cross flow over a cylinder.

Cross flow over a cylinder exhibits complex flow patterns, as shown in Figure 7–16. The fluid approaching the cylinder branches out and encircles the cylinder, forming a boundary layer that wraps around the cylinder. The fluid particles on the midplane strike the cylinder at the stagnation point, bringing the fluid to a complete stop and thus raising the pressure at that point. The pressure decreases in the flow direction while the fluid velocity increases.

At very low upstream velocities ($\text{Re} \lesssim 1$), the fluid completely wraps around the cylinder and the two arms of the fluid meet on the rear side of the cylinder

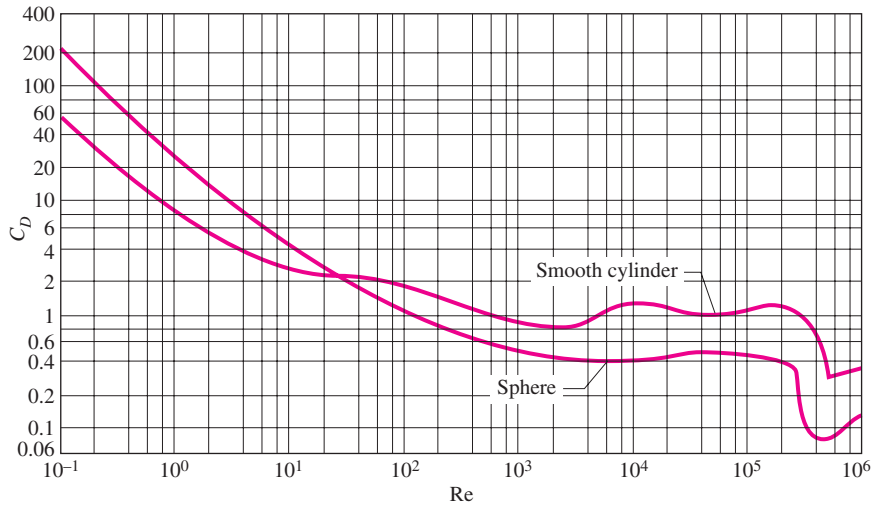


FIGURE 7–17
Average drag coefficient for cross flow over a smooth circular cylinder and a smooth sphere (from Schlichting, Ref. 10).

in an orderly manner. Thus, the fluid follows the curvature of the cylinder. At higher velocities, the fluid still hugs the cylinder on the frontal side, but it is too fast to remain attached to the surface as it approaches the top of the cylinder. As a result, the boundary layer detaches from the surface, forming a separation region behind the cylinder. Flow in the wake region is characterized by random vortex formation and pressures much lower than the stagnation point pressure.

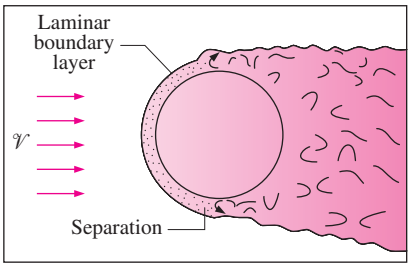
The nature of the flow across a cylinder or sphere strongly affects the total drag coefficient C_D. Both the *friction drag* and the *pressure drag* can be significant. The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow. The drag force is primarily due to friction drag at low Reynolds numbers (Re $<$ 10) and to pressure drag at high Reynolds numbers (Re $>$ 5000). Both effects are significant at intermediate Reynolds numbers.

The average drag coefficients C_D for cross flow over a smooth single circular cylinder and a sphere are given in Figure 7–17. The curves exhibit different behaviors in different ranges of Reynolds numbers:

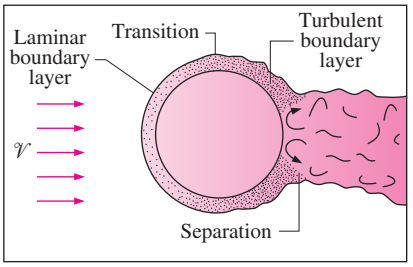
- For Re $\lesssim$ 1, we have creeping flow, and the drag coefficient decreases with increasing Reynolds number. For a sphere, it is $C_D = 24/\text{Re}$. There is no flow separation in this regime.
- At about Re $=$ 10, separation starts occurring on the rear of the body with vortex shedding starting at about Re $\approx$ 90. The region of separation increases with increasing Reynolds number up to about Re $= 10^3$. At this point, the drag is mostly (about 95 percent) due to pressure drag. The drag coefficient continues to decrease with increasing Reynolds number in this range of $10 < \text{Re} < 10^3$. (A decrease in the drag coefficient does not necessarily indicate a decrease in drag. The drag force is proportional to the square of the velocity, and the increase in velocity at higher Reynolds numbers usually more than offsets the decrease in the drag coefficient.)

- In the moderate range of $10^3 < \text{Re} < 10^5$, the drag coefficient remains relatively constant. This behavior is characteristic of blunt bodies. The flow in the boundary layer is laminar in this range, but the flow in the separated region past the cylinder or sphere is highly turbulent with a wide turbulent wake.
- There is a sudden drop in the drag coefficient somewhere in the range of $10^5 < \text{Re} < 10^6$ (usually, at about 2×10^5). This large reduction in C_D is due to the flow in the boundary layer becoming *turbulent,* which moves the separation point further on the rear of the body, reducing the size of the wake and thus the magnitude of the pressure drag. This is in contrast to streamlined bodies, which experience an increase in the drag coefficient (mostly due to friction drag) when the boundary layer becomes turbulent.

Flow separation occurs at about $\theta \approx 80°$ (measured from the stagnation point) when the boundary layer is *laminar* and at about $\theta \approx 140°$ when it is *turbulent* (Fig. 7–18). The delay of separation in turbulent flow is caused by the rapid fluctuations of the fluid in the transverse direction, which enables the turbulent boundary layer to travel further along the surface before separation occurs, resulting in a narrower wake and a smaller pressure drag. In the range of Reynolds numbers where the flow changes from laminar to turbulent, even the drag force F_D decreases as the velocity (and thus Reynolds number) increases. This results in a sudden decrease in drag of a flying body and instabilities in flight.



(*a*) Laminar flow ($\text{Re} < 2 \times 10^5$)



(*b*) Turbulence occurs ($\text{Re} > 2 \times 10^5$)

FIGURE 7–18
Turbulence delays flow separation.

Effect of Surface Roughness

We mentioned earlier that *surface roughness,* in general, increases the drag coefficient in turbulent flow. This is especially the case for streamlined bodies. For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may actually *decrease* the drag coefficient, as shown in Figure 7–19 for a sphere. This is done by tripping the flow into turbulence at a lower Reynolds number, and thus causing the fluid to close in behind the body, narrowing the wake and reducing pressure drag considerably. This results in a much smaller drag coefficient and thus drag force for a rough-surfaced cylinder or sphere in a certain range of Reynolds number compared to a smooth one of identical size at the same velocity. At $\text{Re} = 10^5$, for example, $C_D = 0.1$ for a rough sphere with $\varepsilon/D = 0.0015$, whereas $C_D = 0.5$ for a smooth one. Therefore, the drag coefficient in this case is reduced by a factor of 5 by simply roughening the surface. Note, however, that at $\text{Re} = 10^6$, $C_D = 0.4$ for the rough sphere while $C_D = 0.1$ for the smooth one. Obviously, roughening the sphere in this case will increase the drag by a factor of 4 (Fig. 7–20).

The discussion above shows that roughening the surface can be used to great advantage in reducing drag, but it can also backfire on us if we are not careful—specifically, if we do not operate in the right range of Reynolds number. With this consideration, golf balls are intentionally roughened to induce *turbulence* at a lower Reynolds number to take advantage of the sharp *drop* in the drag coefficient at the onset of turbulence in the boundary layer (the typical velocity range of golf balls is 15 to 150 m/s, and the Reynolds number is less than 4×10^5). The critical Reynolds number of dimpled golf balls is

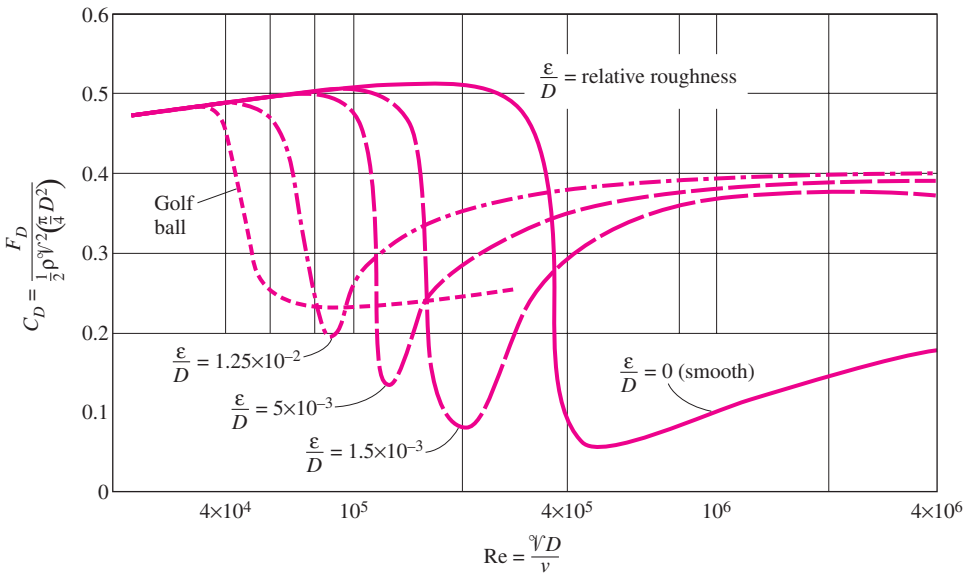


FIGURE 7–19
The effect of surface roughness on the drag coefficient of a sphere (from Blevins, Ref. 1).

about 4×10^4. The occurrence of turbulent flow at this Reynolds number reduces the drag coefficient of a golf ball by half, as shown in Figure 7–19. For a given hit, this means a longer distance for the ball. Experienced golfers also give the ball a spin during the hit, which helps the rough ball develop a lift and thus travel higher and further. A similar argument can be given for a tennis ball. For a table tennis ball, however, the distances are very short, and the balls never reach the speeds in the turbulent range. Therefore, the surfaces of table tennis balls are made smooth.

Once the drag coefficient is available, the drag force acting on a body in cross flow can be determined from Eq. 7-1 where A is the *frontal area* ($A = LD$ for a cylinder of length L and $A = \pi D^2/4$ for a sphere). It should be kept in mind that the free-stream turbulence and disturbances by other bodies in flow (such as flow over tube bundles) may affect the drag coefficients significantly.

	C_D	
Re	Smooth surface	Rough surface, $\varepsilon/D = 0.0015$
10^5	0.5	0.1
10^6	0.1	0.4

FIGURE 7–20
Surface roughness may increase or decrease the drag coefficient of a spherical object, depending on the value of the Reynolds number.

EXAMPLE 7–4 Drag Force Acting on a Pipe in a River

A 2.2-cm-outer-diameter pipe is to cross a river at a 30-m-wide section while being completely immersed in water (Fig. 7–21). The average flow velocity of water is 4 m/s and the water temperature is 15°C. Determine the drag force exerted on the pipe by the river.

SOLUTION A pipe is crossing a river. The drag force that acts on the pipe is to be determined.

Assumptions **1** The outer surface of the pipe is smooth so that Figure 7–17 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The direction of water flow is normal to the pipe. **4** Turbulence in river flow is not considered.

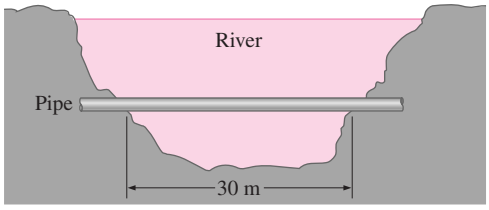


FIGURE 7–21
Schematic for Example 7–4.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m³ and $\mu = 1.138 \times 10^{-3}$ kg/m · s (Table A-9).

Analysis Noting that D = 0.022 m, the Reynolds number for flow over the pipe is

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{\rho\mathcal{V}D}{\mu} = \frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})(0.022\ \text{m})}{1.138 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} = 7.73 \times 10^4$$

The drag coefficient corresponding to this value is, from Figure 7-17, $C_D = 1.0$. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the pipe becomes

$$F_D = C_D A \frac{\rho\mathcal{V}^2}{2} = 1.0(30 \times 0.022\ \text{m}^2)\frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2}\right)$$
$$= \mathbf{5275\ N}$$

Discussion Note that this force is equivalent to the weight of a mass over 500 kg. Therefore, the drag force the river exerts on the pipe is equivalent to hanging a total of over 500 kg in mass on the pipe supported at its ends 30 m apart. The necessary precautions should be taken if the pipe cannot support this force.

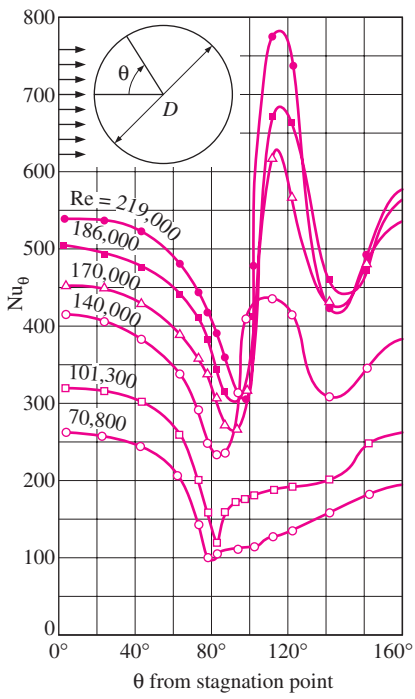


FIGURE 7–22
Variation of the local heat transfer coefficient along the circumference of a circular cylinder in cross flow of air (from Giedt, Ref. 5).

Heat Transfer Coefficient

Flows across cylinders and spheres, in general, involve *flow separation,* which is difficult to handle analytically. Therefore, such flows must be studied experimentally or numerically. Indeed, flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.

The complicated flow pattern across a cylinder greatly influences heat transfer. The variation of the local Nusselt number Nu_θ around the periphery of a cylinder subjected to cross flow of air is given in Figure 7–22. Note that, for all cases, the value of Nu_θ starts out relatively high at the stagnation point ($\theta = 0°$) but decreases with increasing θ as a result of the thickening of the laminar boundary layer. On the two curves at the bottom corresponding to Re = 70,800 and 101,300, Nu_θ reaches a minimum at $\theta \approx 80°$, which is the separation point in laminar flow. Then Nu_θ increases with increasing θ as a result of the intense mixing in the separated flow region (the wake). The curves

at the top corresponding to Re = 140,000 to 219,000 differ from the first two curves in that they have *two* minima for Nu_θ. The sharp increase in Nu_θ at about $\theta \approx 90°$ is due to the transition from laminar to turbulent flow. The later decrease in Nu_θ is again due to the thickening of the boundary layer. Nu_θ reaches its second minimum at about $\theta \approx 140°$, which is the flow separation point in turbulent flow, and increases with θ as a result of the intense mixing in the turbulent wake region.

The discussions above on the local heat transfer coefficients are insightful; however, they are of little value in heat transfer calculations since the calculation of heat transfer requires the *average* heat transfer coefficient over the entire surface. Of the several such relations available in the literature for the average Nusselt number for cross flow over a cylinder, we present the one proposed by Churchill and Bernstein:

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} \tag{7-35}$$

This relation is quite comprehensive in that it correlates available data well for Re Pr > 0.2. The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$, which is the average of the free-stream and surface temperatures.

For flow over a *sphere,* Whitaker recommends the following comprehensive correlation:

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \tag{7-36}$$

which is valid for $3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s. Although the two relations above are considered to be quite accurate, the results obtained from them can be off by as much as 30 percent.

The average Nusselt number for flow across cylinders can be expressed compactly as

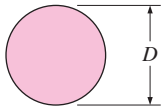
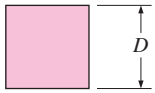
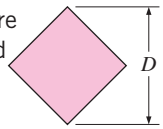
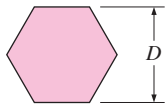
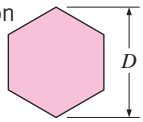
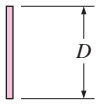
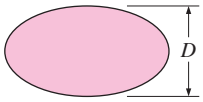
$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n \tag{7-37}$$

where $n = \frac{1}{3}$ and the experimentally determined constants C and m are given in Table 7–1 for circular as well as various noncircular cylinders. The characteristic length D for use in the calculation of the Reynolds and the Nusselt numbers for different geometries is as indicated on the figure. All fluid properties are evaluated at the film temperature.

The relations for cylinders above are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. Also, they are applicable to *smooth* surfaces. *Surface roughness* and the *free-stream turbulence* may affect the drag and heat transfer coefficients significantly. Eq. 7-37 provides a simpler alternative to Eq. 7-35 for flow over cylinders. However, Eq. 7-35 is more accurate, and thus should be preferred in calculations whenever possible.

TABLE 7-1

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\ Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\ Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\ Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\ Pr^{1/3}$

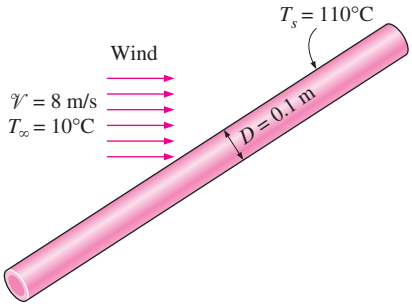


FIGURE 7–23
Schematic for Example 7–5.

EXAMPLE 7–5 **Heat Loss from a Steam Pipe in Windy Air**

A long 10-cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds (Fig. 7–23). Determine the rate of heat loss from the pipe per unit of its length

when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.

SOLUTION A steam pipe is exposed to windy air. The rate of heat loss from the steam is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas.

Properties The properties of air at the average film temperature of $T_f = (T_s + T_\infty)/2 = (110 + 10)/2 = 60°C$ and 1 atm pressure are (Table A-15)

$$k = 0.02808 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7202$$
$$\nu = 1.896 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The Reynolds number is

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{(8 \text{ m/s})(0.1 \text{ m})}{1.896 \times 10^{-5} \text{ m}^2/\text{s}} = 4.219 \times 10^4$$

The Nusselt number can be determined from

$$\text{Nu} = \frac{hD}{k} = 0.3 + \frac{0.62 \text{ Re}^{1/2} \text{ Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$
$$= 0.3 + \frac{0.62(4.219 \times 10^4)^{1/2}(0.7202)^{1/3}}{[1 + (0.4/0.7202)^{2/3}]^{1/4}}\left[1 + \left(\frac{4.219 \times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5}$$
$$= 124$$

and

$$h = \frac{k}{D}\text{Nu} = \frac{0.02808 \text{ W/m} \cdot °\text{C}}{0.1 \text{ m}}(124) = 34.8 \text{ W/m}^2 \cdot °\text{C}$$

Then the rate of heat transfer from the pipe per unit of its length becomes

$$A_s = pL = \pi DL = \pi(0.1 \text{ m})(1 \text{ m}) = 0.314 \text{ m}^2$$
$$\dot{Q} = hA_s(T_s - T_\infty) = (34.8 \text{ W/m}^2 \cdot \text{C})(0.314 \text{ m}^2)(110 - 10)°\text{C} = \mathbf{1093 \text{ W}}$$

The rate of heat loss from the entire pipe can be obtained by multiplying the value above by the length of the pipe in m.

Discussion The simpler Nusselt number relation in Table 7–1 in this case would give Nu = 128, which is 3 percent higher than the value obtained above using Eq. 7-35.

EXAMPLE 7–6 **Cooling of a Steel Ball by Forced Air**

A 25-cm-diameter stainless steel ball (ρ = 8055 kg/m³, C_p = 480 J/kg · °C) is removed from the oven at a uniform temperature of 300°C (Fig. 7–24). The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200°C. Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.

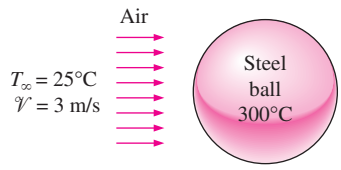


FIGURE 7–24
Schematic for Example 7–6.

SOLUTION A hot stainless steel ball is cooled by forced air. The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas. **4** The outer surface temperature of the ball is uniform at all times. **5** The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of (300 + 200)/2 = 250°C in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

Properties The dynamic viscosity of air at the average surface temperature is $\mu_s = \mu_{@\,250°C} = 2.76 \times 10^{-5}$ kg/m · s. The properties of air at the free-stream temperature of 25°C and 1 atm are (Table A-15)

$$k = 0.02551 \text{ W/m} \cdot °\text{C} \qquad \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu = 1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr} = 0.7296$$

Analysis The Reynolds number is determined from

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{(3 \text{ m/s})(0.25 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 4.802 \times 10^4$$

The Nusselt number is

$$\begin{aligned}\text{Nu} = \frac{hD}{k} &= 2 + [0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}]\,\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \\ &= 2 + [0.4(4.802 \times 10^4)^{1/2} + 0.06(4.802 \times 10^4)^{2/3}](0.7296)^{0.4} \\ &\quad \times \left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{1/4} \\ &= 135\end{aligned}$$

Then the average convection heat transfer coefficient becomes

$$h = \frac{k}{D}\text{Nu} = \frac{0.02551 \text{ W/m} \cdot °\text{C}}{0.25 \text{ m}}(135) = \mathbf{13.8\ W/m^2 \cdot °C}$$

In order to estimate the time of cooling of the ball from 300°C to 200°C, we determine the *average* rate of heat transfer from Newton's law of cooling by using the *average* surface temperature. That is,

$$A_s = \pi D^2 = \pi(0.25 \text{ m})^2 = 0.1963 \text{ m}^2$$
$$\dot{Q}_{\text{ave}} = hA_s(T_{s,\,\text{ave}} - T_\infty) = (13.8 \text{ W/m}^2 \cdot °\text{C})(0.1963 \text{ m}^2)(250 - 25)°\text{C} = 610 \text{ W}$$

Next we determine the *total* heat transferred from the ball, which is simply the change in the energy of the ball as it cools from 300°C to 200°C:

$$m = \rho V = \rho\tfrac{1}{6}\pi D^3 = (8055 \text{ kg/m}^3)\,\tfrac{1}{6}\pi(0.25 \text{ m})^3 = 65.9 \text{ kg}$$
$$Q_{\text{total}} = mC_p(T_2 - T_1) = (65.9 \text{ kg})(480 \text{ J/kg} \cdot °\text{C})(300 - 200)°\text{C} = 3{,}163{,}000 \text{ J}$$

In this calculation, we assumed that the entire ball is at 200°C, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$\Delta t \approx \frac{Q}{\dot{Q}_{\text{ave}}} = \frac{3{,}163{,}000 \text{ J}}{610 \text{ J/s}} = 5185 \text{ s} = \mathbf{1 \text{ h } 26 \text{ min}}$$

Discussion The time of cooling could also be determined more accurately using the transient temperature charts or relations introduced in Chapter 4. But the simplifying assumptions we made above can be justified if all we need is a ballpark value. It will be naive to expect the time of cooling to be exactly 1 h 26 min, but, using our engineering judgment, it is realistic to expect the time of cooling to be somewhere between one and two hours.

7–4 ▪ FLOW ACROSS TUBE BANKS

Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such as the condensers and evaporators of power plants, refrigerators, and air conditioners. In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.

In a heat exchanger that involves a tube bank, the tubes are usually placed in a *shell* (and thus the *name shell-and-tube heat exchanger*), especially when the fluid is a liquid, and the fluid flows through the space between the tubes and the shell. There are numerous types of shell-and-tube heat exchangers, some of which are considered in Chap. 13. In this section we will consider the general aspects of flow over a tube bank, and try to develop a better and more intuitive understanding of the performance of heat exchangers involving a tube bank.

Flow *through* the tubes can be analyzed by considering flow through a single tube, and multiplying the results by the number of tubes. This is not the case for flow *over* the tubes, however, since the tubes affect the flow pattern and turbulence level downstream, and thus heat transfer to or from them, as shown in Figure 7–25. Therefore, when analyzing heat transfer from a tube bank in cross flow, we must consider all the tubes in the bundle at once.

The tubes in a tube bank are usually arranged either *in-line* or *staggered* in the direction of flow, as shown in Figure 7–26. The outer tube diameter D is taken as the characteristic length. The arrangement of the tubes in the tube bank is characterized by the *transverse pitch* S_T, *longitudinal pitch* S_L, and the *diagonal pitch* S_D between tube centers. The diagonal pitch is determined from

$$S_D = \sqrt{S_L^2 + (S_T/2)^2} \tag{7-38}$$

As the fluid enters the tube bank, the flow area decreases from $A_1 = S_T L$ to $A_T = (S_T - \text{D})L$ between the tubes, and thus flow velocity increases. In staggered arrangement, the velocity may increase further in the diagonal region if the tube rows are very close to each other. In tube banks, the flow characteristics are dominated by the maximum velocitiy $\mathcal{V}_{\max}$ that occurs within the tube bank rather than the approach velocity $\mathcal{V}$. Therefore, the Reynolds number is defined on the basis of maximum velocity as

$$\text{Re}_D = \frac{\rho \mathcal{V}_{\max} D}{\mu} = \frac{\mathcal{V}_{\max} D}{\nu} \tag{7-39}$$

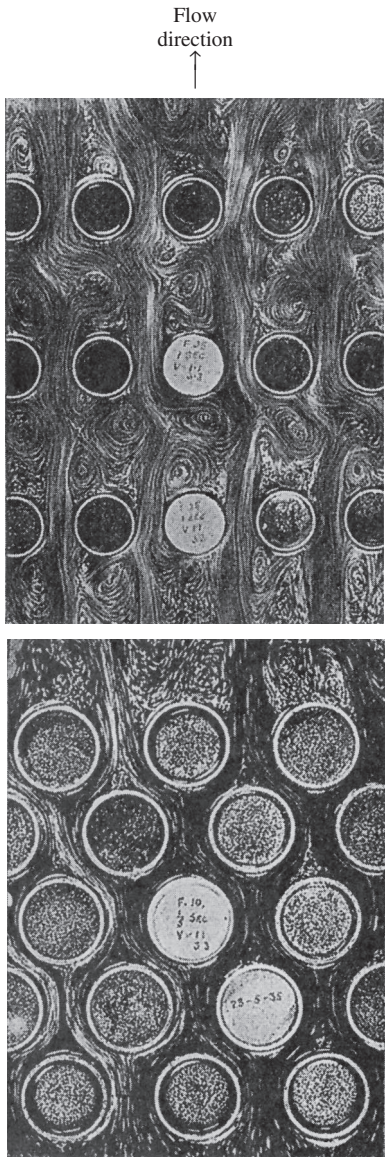


FIGURE 7-25
Flow patterns for staggered and in-line tube banks (photos by R. D. Willis, Ref 12).

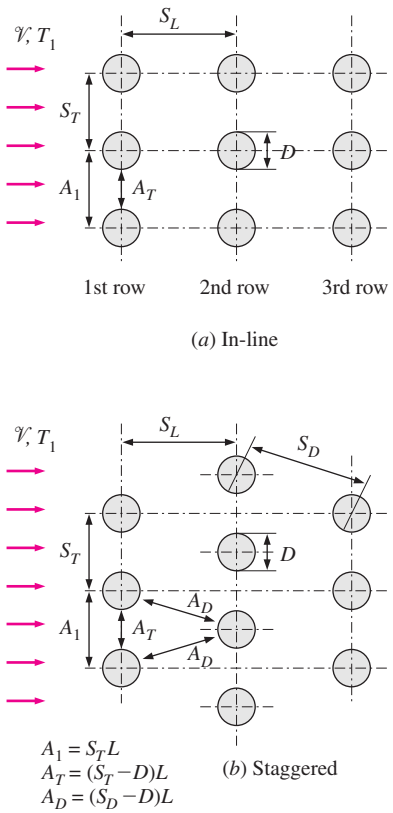


FIGURE 7–26
Arrangement of the tubes in in-line and staggered tube banks (A_1, A_T, and A_D are flow areas at indicated locations, and L is the length of the tubes).

The maximum velocity is determined from the conservation of mass requirement for steady incompressible flow. For *in-line* arrangement, the maximum velocity occurs at the minimum flow area between the tubes, and the conservation of mass can be expressed as (see Fig. 7-26*a*) $\rho \mathcal{V} A_1 = \rho \mathcal{V}_{max} A_T$ or $\mathcal{V} S_T = \mathcal{V}_{max}(S_T - D)$. Then the maximum velocity becomes

$$\mathcal{V}_{max} = \frac{S_T}{S_T - D}\mathcal{V} \tag{7-40}$$

In *staggered* arrangement, the fluid approaching through area A_1 in Figure 7–26*b* passes through area A_T and then through area $2A_D$ as it wraps around the pipe in the next row. If $2A_D > A_T$, maximum velocity will still occur at A_T between the tubes, and thus the $\mathcal{V}_{max}$ relation Eq. 7-40 can also be used for staggered tube banks. But if $2A_D < A_T$ [or, if $2(S_D - D) < (S_T - D)$], maximum velocity will occur at the diagonal cross sections, and the maximum velocity in this case becomes

Staggered and $S_D < (S_T + D)/2$:
$$\mathcal{V}_{max} = \frac{S_T}{2(S_D - D)}\mathcal{V} \tag{7-41}$$

since $\rho \mathcal{V} A_1 = \rho \mathcal{V}_{max}(2A_D)$ or $\mathcal{V} S_T = 2\mathcal{V}_{max}(S_D - D)$.

The nature of flow around a tube in the first row resembles flow over a single tube discussed in section 7–3, especially when the tubes are not too close to each other. Therefore, each tube in a tube bank that consists of a single transverse row can be treated as a single tube in cross-flow. The nature of flow around a tube in the second and subsequent rows is very different, however, because of wakes formed and the turbulence caused by the tubes upstream. The level of turbulence, and thus the heat transfer coefficient, increases with row number because of the combined effects of upstream rows. But there is no significant change in turbulence level after the first few rows, and thus the heat transfer coefficient remains constant.

Flow through tube banks is studied experimentally since it is too complex to be treated analytically. We are primarily interested in the average heat transfer coefficient for the entire tube bank, which depends on the number of tube rows along the flow as well as the arrangement and the size of the tubes.

Several correlations, all based on experimental data, have been proposed for the average Nusselt number for cross flow over tube banks. More recently, Zukauskas has proposed correlations whose general form is

$$\mathrm{Nu}_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25} \tag{7-42}$$

where the values of the constants C, m, and n depend on value Reynolds number. Such correlations are given in Table 7–2 explicitly for $0.7 < \mathrm{Pr} < 500$ and $0 < \mathrm{Re}_D < 2 \times 10^6$. The uncertainty in the values of Nusselt number obtained from these relations is ± 15 percent. Note that all properties except Pr_s are to be evaluated at the arithmetic mean temperature of the fluid determined from

$$T_m = \frac{T_i + T_e}{2} \tag{7-43}$$

where T_i and T_e are the fluid temperatures at the inlet and the exit of the tube bank, respectively.

TABLE 7–2

Nusselt number correlations for cross flow over tube banks for N > 16 and 0.7 < Pr < 500 (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\ Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\ Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.27\ Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.033\ Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\ Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\ Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.35(S_T/S_L)^{0.2}\ Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.031(S_T/S_L)^{0.2}\ Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

The average Nusselt number relations in Table 7–2 are for tube banks with 16 or more rows. Those relations can also be used for tube banks with N_L provided that they are modified as

$$Nu_{D,\,N_L} = F Nu_D \tag{7-44}$$

where F is a *correction factor* F whose values are given in Table 7–3. For $Re_D > 1000$, the correction factor is independent of Reynolds number.

Once the Nusselt number and thus the average heat transfer coefficient for the entire tube bank is known, the heat transfer rate can be determined from Newton's law of cooling using a suitable temperature difference ΔT. The first thought that comes to mind is to use $\Delta T = T_s - T_m = T_s - (T_i + T_e)/2$. But this will, in general, over predict the heat transfer rate. We will show in the next chapter that the proper temperature difference for internal flow (flow over tube banks is still internal flow through the shell) is the *logarithmic mean temperature difference* $\Delta T_{\ln}$ defined as

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \tag{7-45}$$

We will also show that the exit temperature of the fluid T_e can be determined from

TABLE 7–3

Correction factor F to be used in $Nu_{D,\,N_L} = F Nu_D$ for $N_L < 16$ and $Re_D > 1000$ (from Zukauskas, Ref 15, 1987).

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right) \tag{7-46}$$

where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid. Here N is the total number of tubes in the bank, N_T is the number of tubes in a transverse plane, L is the length of the tubes, and $\mathcal{V}$ is the velocity of the fluid just before entering the tube bank. Then the heat transfer rate can be determined from

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}C_p(T_e - T_i) \tag{7-47}$$

The second relation is usually more convenient to use since it does not require the calculation of $\Delta T_{\ln}$.

Pressure Drop

Another quantity of interest associated with tube banks is the *pressure drop* ΔP, which is the difference between the pressures at the inlet and the exit of the tube bank. It is a measure of the resistance the tubes offer to flow over them, and is expressed as

$$\Delta P = N_L f\chi\frac{\rho\mathcal{V}_{\max}^2}{2} \tag{7-48}$$

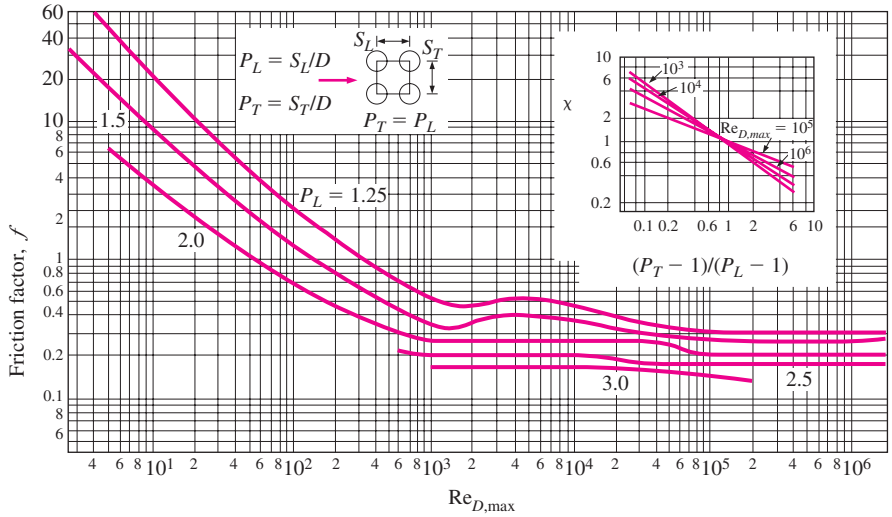
where f is the friction factor and χ is the correction factor, both plotted in Figures 7–27*a* and 7–27*b* against the Reynolds number based on the maximum velocity $\mathcal{V}_{\max}$. The friction factor in Figure 7–27*a* is for a *square* in-line tube bank ($S_T = S_L$), and the correction factor given in the insert is used to account for the effects of deviation of rectangular in-line arrangements from square arrangement. Similarly, the friction factor in Figure 7–27*b* is for an *equilateral* staggered tube bank ($S_T = S_D$), and the correction factor is to account for the effects of deviation from equilateral arrangement. Note that $\chi = 1$ for both square and equilateral triangle arrangements. Also, pressure drop occurs in the flow direction, and thus we used N_L (the number of rows) in the ΔP relation.

The power required to move a fluid through a tube bank is proportional to the pressure drop, and when the pressure drop is available, the pumping power required can be determined from

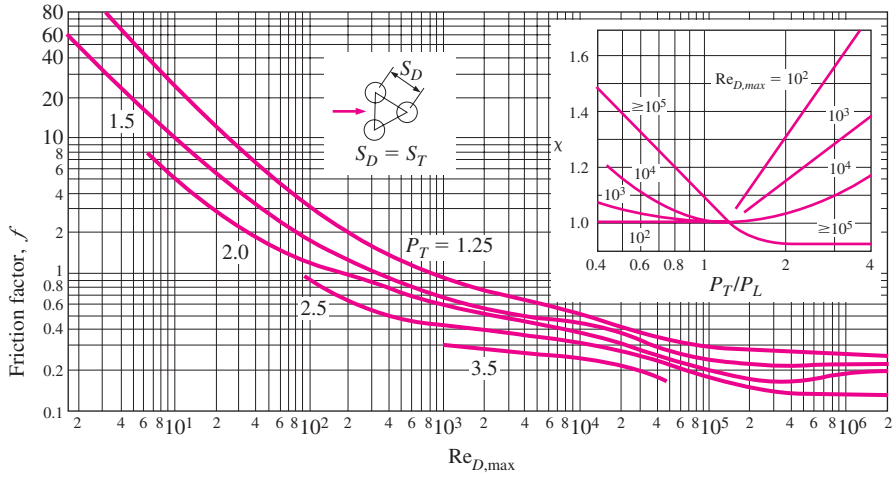
$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho} \tag{7-49}$$

where $\dot{V} = \mathcal{V}(N_T S_T L)$ is the volume flow rate and $\dot{m} = \rho\dot{V} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid through the tube bank. Note that the power required to keep a fluid flowing through the tube bank (and thus the operating cost) is proportional to the pressure drop. Therefore, the benefits of enhancing heat transfer in a tube bank via rearrangement should be weighed against the cost of additional power requirements.

In this section we limited our consideration to tube banks with base surfaces (no fins). Tube banks with finned surfaces are also commonly used in practice, especially when the fluid is a gas, and heat transfer and pressure drop correlations can be found in the literature for tube banks with pin fins, plate fins, strip fins, etc.



(*a*) In-line arrangement



(*b*) Staggered arrangement

FIGURE 7–27
Friction factor f and correction factor χ for tube banks (from Zukauskas, Ref. 16, 1985).

EXAMPLE 7–7 Preheating Air by Geothermal Water in a Tube Bank

In an industrial facility, air is to be preheated before entering a furnace by geothermal water at 120°C flowing through the tubes of a tube bank located in a duct. Air enters the duct at 20°C and 1 atm with a mean velocity of 4.5 m/s, and flows over the tubes in normal direction. The outer diameter of the tubes is 1.5 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of $S_L = S_T = 5$ cm. There are 6 rows in the flow direction with 10 tubes in each row, as shown in Figure 7–28. Determine the rate of heat transfer per unit length of the tubes, and the pressure drop across the tube bank.

SOLUTION Air is heated by geothermal water in a tube bank. The rate of heat transfer to air and the pressure drop of air are to be determined.

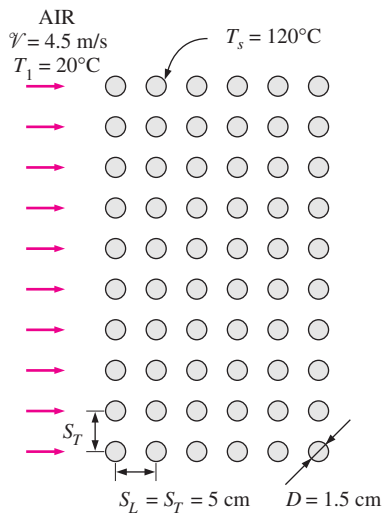


FIGURE 7–28
Schematic for Example 7–7.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of geothermal water.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 60°C (will be checked later) and 1 atm are Table A–15):

$$k = 0.02808 \text{ W/m} \cdot \text{K}, \qquad \rho = 1.06 \text{ kg/m}^3$$
$$C_p = 1.007 \text{ kJ/kg} \cdot \text{K}, \qquad \text{Pr} = 0.7202$$
$$\mu = 2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr}_s = \text{Pr}_{@Ts} = 0.7073$$

Also, the density of air at the inlet temperature of 20°C (for use in the mass flow rate calculation at the inlet) is $\rho_1 = 1.204$ kg/m^3

Analysis It is given that $D = 0.015$ m, $S_L = S_T = 0.05$ m, and $\mathcal{V} = 4.5$ m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$\mathcal{V}_{\max} = \frac{S_T}{S_T - D}\mathcal{V} = \frac{0.05}{0.05 - 0.015}(4.5 \text{ m/s}) = 6.43 \text{ m/s}$$
$$\text{Re}_D = \frac{\rho \mathcal{V}_{\max} D}{\mu} = \frac{(1.06 \text{ kg/m}^3)(6.43 \text{ m/s})(0.015 \text{ m})}{2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 5091$$

The average Nusselt number is determined using the proper relation from Table 7–2 to be

$$\text{Nu}_D = 0.27\, \text{Re}_D^{0.63}\, \text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$$
$$= 0.27(5091)^{0.63}(0.7202)^{0.36}(0.7202/0.7073)^{0.25} = 52.2$$

This Nusselt number is applicable to tube banks with $N_L > 16$. In our case, the number of rows is $N_L = 6$, and the corresponding correction factor from Table 7–3 is $F = 0.945$. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = F\text{Nu}_D = (0.945)(52.2) = 49.3$$
$$h = \frac{\text{Nu}_{D,N_L} k}{D} = \frac{49.3(0.02808 \text{ W/m} \cdot {}^\circ\text{C})}{0.015 \text{ m}} = 92.2 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

The total number of tubes is $N = N_L \times N_T = 6 \times 10 = 60$. For a unit tube length ($L = 1$ m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 60\pi(0.015 \text{ m})(1 \text{ m}) = 2.827 \text{ m}^2$$
$$\dot{m} = \dot{m}_1 = \rho_1 \mathcal{V}(N_T S_T L)$$
$$= (1.204 \text{ kg/m}^3)(4.5 \text{ m/s})(10)(0.05 \text{ m})(1 \text{ m}) = 2.709 \text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m} C_p}\right)$$
$$= 120 - (120 - 20)\exp\left(-\frac{(2.827 \text{ m}^2)(92.2 \text{ W/m}^2 \cdot {}^\circ\text{C})}{(2.709 \text{ kg/s})(1007 \text{ J/kg} \cdot {}^\circ\text{C})}\right) = 29.11^\circ\text{C}$$

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{(120 - 29.11) - (120 - 20)}{\ln[(120 - 29.11)/(120 - 20)]} = 95.4°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (92.2\ \text{W/m}^2 \cdot °\text{C})(2.827\ \text{m}^2)(95.4°\text{C}) = \mathbf{2.49 \times 10^4\ W}$$

The rate of heat transfer can also be determined in a simpler way from

$$\dot{Q} = hA_s\Delta T_{\text{in}} = \dot{m}C_p(T_e - T_i)$$
$$= (2.709\ \text{kg/s})(1007\ \text{J/kg} \cdot °\text{C})(29.11 - 20)°\text{C} = 2.49 \times 10^4\ \text{W}$$

For this square in-line tube bank, the friction coefficient corresponding to $\text{Re}_D = 5088$ and $S_L/D = 5/1.5 = 3.33$ is, from Fig. 7–27*a*, $f = 0.16$. Also, $\chi = 1$ for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f \chi \frac{\rho V_{\max}^2}{2}$$
$$= 6(0.16)(1)\frac{(1.06\ \text{kg/m}^3)(6.43\ \text{m/s})^3}{2}\left(\frac{1\text{N}}{1\ \text{kg} \cdot \text{m/s}^2}\right) = \mathbf{21\ Pa}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2 = (20 + 110.9)/2 = 65.4°\text{C}$, which is fairly close to the assumed value of 60°C. Therefore, there is no need to repeat calculations by reevaluating the properties at 65.4°C (it can be shown that doing so would change the results by less than 1 percent, which is much less than the uncertainty in the equations and the charts used).

TOPIC OF SPECIAL INTEREST

Reducing Heat Transfer through Surfaces: Thermal Insulation

Thermal insulations are materials or combinations of materials that are used primarily to provide resistance to heat flow (Fig. 7–29). You are probably familiar with several kinds of insulation available in the market. Most insulations are heterogeneous materials made of low thermal conductivity materials, and they involve air pockets. This is not surprising since air has one of the lowest thermal conductivities and is readily available. The *Styrofoam* commonly used as a packaging material for TVs, VCRs, computers, and just about anything because of its light weight is also an excellent insulator.

Temperature difference is the driving force for heat flow, and the greater the temperature difference, the larger the rate of heat transfer. We can slow down the heat flow between two mediums at different temperatures by putting "barriers" on the path of heat flow. Thermal insulations serve as such barriers, and they play a major role in the design and manufacture of all energy-efficient devices or systems, and they are usually the cornerstone of energy conservation projects. A 1991 Drexel University study of the energy-intensive U.S. industries revealed that insulation saves the U.S.

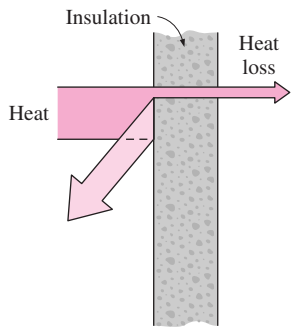


FIGURE 7–29
Thermal insulation retards heat transfer by acting as a barrier in the path of heat flow.

*This section can be skipped without a loss in continuity.

industry nearly 2 billion barrels of oil per year, valued at $60 billion a year in energy costs, and more can be saved by practicing better insulation techniques and retrofitting the older industrial facilities.

Heat is generated in furnaces or heaters by burning a fuel such as coal, oil, or natural gas or by passing electric current through a *resistance heater.* Electricity is rarely used for heating purposes since its unit cost is much higher. The heat generated is absorbed by the medium in the furnace and its surfaces, causing a temperature rise above the ambient temperature. This temperature difference drives heat transfer from the hot medium to the ambient, and insulation reduces the amount of heat loss and thus saves fuel and money. Therefore, insulation *pays for itself* from the energy it saves. Insulating properly requires a one-time capital investment, but its effects are dramatic and long term. The payback period of insulation is often less than one year. That is, the money insulation saves during the first year is usually greater than its initial material and installation costs. On a broader perspective, insulation also helps the environment and fights air pollution and the greenhouse effect by reducing the amount of fuel burned and thus the amount of CO_2 and other gases released into the atmosphere (Fig. 7–30).

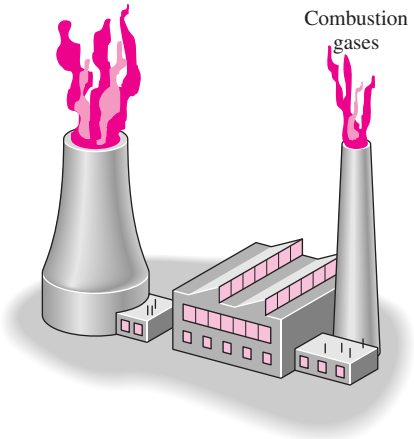


FIGURE 7–30
Insulation also helps the environment by reducing the amount of fuel burned and the air pollutants released.

Saving energy with insulation is not limited to hot surfaces. We can also save energy and money by insulating *cold surfaces* (surfaces whose temperature is below the ambient temperature) such as chilled water lines, cryogenic storage tanks, refrigerated trucks, and air-conditioning ducts. The source of "coldness" is *refrigeration,* which requires energy input, usually electricity. In this case, heat is transferred from the surroundings to the cold surfaces, and the refrigeration unit must now work harder and longer to make up for this heat gain and thus it must consume more electrical energy. A cold canned drink can be kept cold much longer by wrapping it in a blanket. A refrigerator with well-insulated walls will consume much less electricity than a similar refrigerator with little or no insulation. Insulating a house will result in reduced cooling load, and thus reduced electricity consumption for air-conditioning.

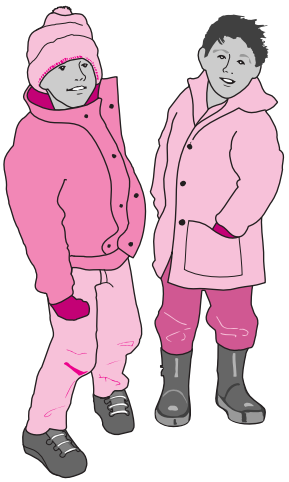


FIGURE 7–31
In cold weather, we minimize heat loss from our bodies by putting on thick layers of insulation (coats or furs).

Whether we realize it or not, we have an *intuitive* understanding and appreciation of thermal insulation. As babies we feel much better in our blankies, and as children we know we should wear a sweater or coat when going outside in cold weather (Fig. 7–31). When getting out of a pool after swimming on a windy day, we quickly wrap in a towel to stop shivering. Similarly, early man used animal furs to keep warm and built shelters using mud bricks and wood. Cork was used as a roof covering for centuries. The need for effective thermal insulation became evident with the development of mechanical refrigeration later in the nineteenth century, and a great deal of work was done at universities and government and private laboratories in the 1910s and 1920s to identify and characterize thermal insulation.

Thermal insulation in the form of *mud, clay, straw, rags,* and *wood strips* was first used in the eighteenth century on steam engines to keep workmen from being burned by hot surfaces. As a result, boiler room temperatures dropped and it was noticed that fuel consumption was also reduced. The realization of improved engine efficiency and energy savings prompted the search for materials with improved thermal efficiency. One of the first such materials was *mineral wool* insulation, which, like many materials, was

discovered by accident. About 1840, an iron producer in Wales aimed a stream of high-pressure steam at the slag flowing from a blast furnace, and manufactured mineral wool was born. In the early 1860s, this slag wool was a by-product of manufacturing cannons for the Civil War and quickly found its way into many industrial uses. By 1880, builders began installing mineral wool in houses, with one of the most notable applications being General Grant's house. The insulation of this house was described in an article: "it keeps the house cool in summer and warm in winter; it prevents the spread of fire; and it deadens the sound between floors" [Edmunds (1989), Ref. 4]. An article published in 1887 in *Scientific American* detailing the benefits of insulating the entire house gave a major boost to the use of insulation in residential buildings.

The energy crisis of the 1970s had a tremendous impact on the public awareness of energy and limited energy reserves and brought an emphasis on *energy conservation.* We have also seen the development of new and more effective insulation materials since then, and a considerable increase in the use of insulation. Thermal insulation is used in more places than you may be aware of. The walls of your house are probably filled with some kind of insulation, and the roof is likely to have a thick layer of insulation. The "thickness" of the walls of your refrigerator is due to the insulation layer sandwiched between two layers of sheet metal (Fig. 7–32). The walls of your range are also insulated to conserve energy, and your hot water tank contains less water than you think because of the 2- to 4-cm-thick insulation in the walls of the tank. Also, your hot water pipe may look much thicker than the cold water pipe because of insulation.

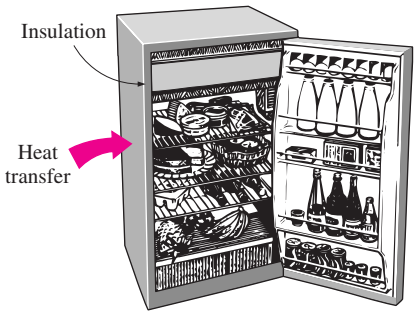


FIGURE 7–32
The insulation layers in the walls of a refrigerator reduce the amount of heat flow into the refrigerator and thus the running time of the refrigerator, saving electricity.

Reasons for Insulating

If you examine the engine compartment of your car, you will notice that the firewall between the engine and the passenger compartment as well as the inner surface of the hood are insulated. The reason for insulating the hood is not to conserve the waste heat from the engine but to protect people from burning themselves by touching the hood surface, which will be too hot if not insulated. As this example shows, the use of insulation is not limited to energy conservation. Various reasons for using insulation can be summarized as follows:

- **Energy Conservation** Conserving energy by reducing the rate of heat flow is the primary reason for insulating surfaces. Insulation materials that will perform satisfactorily in the temperature range of −268°C to 1000°C (−450°F to 1800°F) are widely available.
- **Personnel Protection and Comfort** A surface that is too hot poses a danger to people who are working in that area of accidentally touching the hot surface and burning themselves (Fig. 7–33). To prevent this danger and to comply with the OSHA (Occupational Safety and Health Administration) standards, the temperatures of hot surfaces should be reduced to below 60°C (140°F) by insulating them. Also, the excessive heat coming off the hot surfaces creates an unpleasant environment in which to work, which adversely affects the performance or productivity of the workers, especially in summer months.

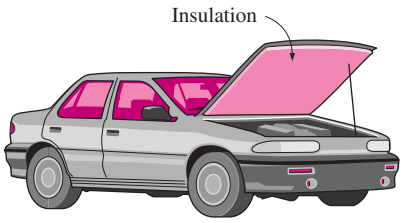


FIGURE 7–33
The hood of the engine compartment of a car is insulated to reduce its temperature and to protect people from burning themselves.

- **Maintaining Process Temperature** Some processes in the chemical industry are temperature-sensitive, and it may become necessary to insulate the process tanks and flow sections heavily to maintain the same temperature throughout.
- **Reducing Temperature Variation and Fluctuations** The temperature in an enclosure may vary greatly between the midsection and the edges if the enclosure is not insulated. For example, the temperature near the walls of a poorly insulated house is much lower than the temperature at the midsections. Also, the temperature in an uninsulated enclosure will follow the temperature changes in the environment closely and fluctuate. Insulation minimizes temperature nonuniformity in an enclosure and slows down fluctuations.
- **Condensation and Corrosion Prevention** Water vapor in the air condenses on surfaces whose temperature is below the dew point, and the outer surfaces of the tanks or pipes that contain a cold fluid frequently fall below the dew-point temperature unless they have adequate insulation. The liquid water on exposed surfaces of the metal tanks or pipes may promote corrosion as well as algae growth.
- **Fire Protection** Damage during a fire may be minimized by keeping valuable combustibles in a safety box that is well insulated. Insulation may lower the rate of heat flow to such levels that the temperature in the box never rises to unsafe levels during fire.
- **Freezing Protection** Prolonged exposure to subfreezing temperatures may cause water in pipes or storage vessels to freeze and burst as a result of heat transfer from the water to the cold ambient. The bursting of pipes as a result of freezing can cause considerable damage. Adequate insulation will slow down the heat loss from the water and prevent freezing during limited exposure to subfreezing temperatures. For example, covering vegetables during a cold night will protect them from freezing, and burying water pipes in the ground at a sufficient depth will keep them from freezing during the entire winter. Wearing thick gloves will protect the fingers from possible frostbite. Also, a molten metal or plastic in a container will solidify on the inner surface if the container is not properly insulated.
- **Reducing Noise and Vibration** An added benefit of thermal insulation is its ability to dampen noise and vibrations (Fig. 7–34). The insulation materials differ in their ability to reduce noise and vibration, and the proper kind can be selected if noise reduction is an important consideration.

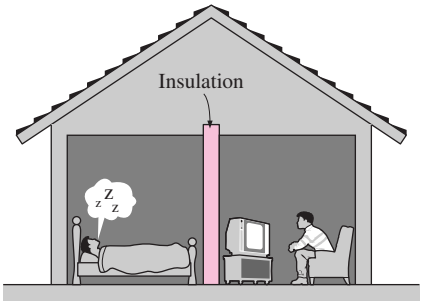


FIGURE 7–34
Insulation materials absorb vibration and sound waves, and are used to minimize sound transmission.

There are a wide variety of insulation materials available in the market, but most are primarily made of fiberglass, mineral wool, polyethylene, foam, or calcium silicate. They come in various trade names such as Ethafoam Polyethylene Foam Sheeting, Solimide Polimide Foam Sheets, FPC Fiberglass Reinforced Silicone Foam Sheeting, Silicone Sponge Rubber Sheets, fiberglass/mineral wool insulation blankets, wire-reinforced

mineral wool insulation, Reflect-All Insulation, granulated bulk mineral wool insulation, cork insulation sheets, foil-faced fiberglass insulation, blended sponge rubber sheeting, and numerous others.

Today various forms of *fiberglass insulation* are widely used in process industries and heating and air-conditioning applications because of their low cost, light weight, resiliency, and versatility. But they are not suitable for some applications because of their low resistance to moisture and fire and their limited maximum service temperature. Fiberglass insulations come in various forms such as unfaced fiberglass insulation, vinyl-faced fiberglass insulation, foil-faced fiberglass insulation, and fiberglass insulation sheets. The reflective foil-faced fiberglass insulation resists vapor penetration and retards radiation because of the aluminum foil on it and is suitable for use on pipes, ducts, and other surfaces.

Mineral wool is resilient, lightweight, fibrous, wool-like, thermally efficient, fire resistant up to 1100°C (2000°F), and forms a sound barrier. Mineral wool insulation comes in the form of blankets, rolls, or blocks. *Calcium silicate* is a solid material that is suitable for use at high temperatures, but it is more expensive. Also, it needs to be cut with a saw during installation, and thus it takes longer to install and there is more waste.

Superinsulators

You may be tempted to think that the most effective way to reduce heat transfer is to use insulating materials that are known to have very low thermal conductivities such as urethane or rigid foam ($k = 0.026$ W/m · °C) or fiberglass ($k = 0.035$ W/m · °C). After all, they are widely available, inexpensive, and easy to install. Looking at the thermal conductivities of materials, you may also notice that the thermal conductivity of air at room temperature is 0.026 W/m · °C, which is lower than the conductivities of practically all of the ordinary insulating materials. Thus you may think that a layer of enclosed air space is as effective as any of the common insulating materials of the same thickness. Of course, heat transfer through the air will probably be higher than what a pure conduction analysis alone would indicate because of the natural convection currents that are likely to occur in the air layer. Besides, air is transparent to radiation, and thus heat will also be transferred by radiation. The thermal conductivity of air is practically independent of pressure unless the pressure is extremely high or extremely low. Therefore, we can reduce the thermal conductivity of air and thus the conduction heat transfer through the air by evacuating the air space. In the limiting case of absolute vacuum, the thermal conductivity will be zero since there will be no particles in this case to "conduct" heat from one surface to the other, and thus the conduction heat transfer will be zero. Noting that the thermal conductivity cannot be negative, an absolute vacuum must be the ultimate insulator, right? Well, not quite.

The purpose of insulation is to reduce "total" heat transfer from a surface, not just conduction. A vacuum totally eliminates conduction but offers zero resistance to radiation, whose magnitude can be comparable to conduction or natural convection in gases (Fig. 7–35). Thus, a vacuum is

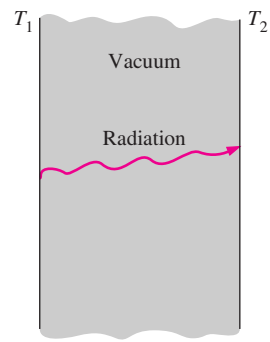


FIGURE 7–35
Evacuating the space between two surfaces completely eliminates heat transfer by conduction or convection but leaves the door wide open for radiation.

no more effective in reducing heat transfer than sealing off one of the lanes of a two-lane road is in reducing the flow of traffic on a one-way road.

Insulation against radiation heat transfer between two surfaces is achieved by placing "barriers" between the two surfaces, which are highly reflective thin metal sheets. Radiation heat transfer between two surfaces is inversely proportional to the number of such sheets placed between the surfaces. Very effective insulations are obtained by using closely packed layers of highly reflective thin metal sheets such as aluminum foil (usually 25 sheets per cm) separated by fibers made of insulating material such as glass fiber (Fig. 7–36). Further, the space between the layers is evacuated to form a vacuum under 0.000001 atm pressure to minimize conduction or convection heat transfer through the air space between the layers. The result is an insulating material whose apparent thermal conductivity is below 2×10^{-5} W/m · °C, which is one thousand times less than the conductivity of air or any common insulating material. These specially built insulators are called **superinsulators,** and they are commonly used in space applications and cryogenics, which is the branch of heat transfer dealing with temperatures below 100 K (−173°C) such as those encountered in the liquefaction, storage, and transportation of gases, with helium, hydrogen, nitrogen, and oxygen being the most common ones.

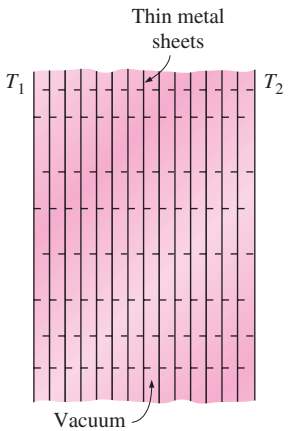


FIGURE 7–36
Superinsulators are built by closely packing layers of highly reflective thin metal sheets and evacuating the space between them.

The R-value of Insulation

The effectiveness of insulation materials is given by some manufacturers in terms of their ***R*-value,** which is the *thermal resistance* of the material *per unit surface area.* For *flat insulation* the *R*-value is obtained by simply dividing the thickness of the insulation by its thermal conductivity. That is,

$$R\text{-value} = \frac{L}{k} \qquad \text{(flat insulation)} \tag{7-50}$$

where L is the thickness and k is the thermal conductivity of the material. Note that doubling the thickness L doubles the R-value of flat insulation. For *pipe insulation,* the R-value is determined using the thermal resistance relation from

$$R\text{-value} = \frac{r_2}{k} \ln \frac{r_2}{r_1} \qquad \text{(pipe insulation)} \tag{7-51}$$

where r_1 is the inside radius of insulation and r_2 is the outside radius of insulation. Once the R-value is available, the rate of heat transfer through the insulation can be determined from

$$\dot{Q} = \frac{\Delta T}{R\text{-value}} \times \text{Area} \tag{7-52}$$

where ΔT is the temperature difference across the insulation and Area is the outer surface area for a cylinder.

In the United States, the R-values of insulation are expressed without any units, such as R-19 and R-30. These R-values are obtained by dividing the thickness of the material in *feet* by its thermal conductivity in the unit Btu/h · ft · °F so that the R-values actually have the unit h · ft^2 · °F/Btu. For example, the R-value of 6-in.-thick glass fiber insulation whose thermal conductivity is 0.025 Btu/h · ft · °F is (Fig. 7–37)

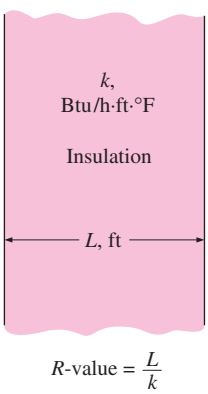


FIGURE 7–37
The R-value of an insulating material is simply the ratio of the thickness of the material to its thermal conductivity in proper units.

$$R\text{-value} = \frac{L}{k} = \frac{0.5\ \text{ft}}{0.025\ \text{Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}} = 20\ \text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F/Btu}$$

Thus, this 6-in.-thick glass fiber insulation would be referred to as *R*-20 insulation by the builders. The unit of *R*-value is $\text{m}^2 \cdot {}^\circ\text{C/W}$ in SI units, with the conversion relation $1\ \text{m}^2 \cdot {}^\circ\text{C/W} = 5.678\ \text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F/Btu}$. Therefore, a small *R*-value in SI corresponds to a large *R*-value in English units.

Optimum Thickness of Insulation

It should be realized that insulation does not eliminate heat transfer; it merely reduces it. The thicker the insulation, the lower the rate of heat transfer but also the higher the cost of insulation. Therefore, there should be an *optimum* thickness of insulation that corresponds to a minimum combined cost of insulation and heat lost. The determination of the optimum thickness of insulation is illustrated in Figure 7–38. Notice that the cost of insulation increases roughly linearly with thickness while the cost of heat loss decreases exponentially. The total cost, which is the sum of the insulation cost and the lost heat cost, decreases first, reaches a minimum, and then increases. The thickness corresponding to the minimum total cost is the optimum thickness of insulation, and this is the recommended thickness of insulation to be installed.

If you are mathematically inclined, you can determine the *optimum thickness* by obtaining an expression for the *total cost,* which is the sum of the expressions for the lost heat cost and insulation cost as a function of thickness; *differentiating* the total cost expression with respect to the thickness; and *setting* it equal to zero. The thickness value satisfying the resulting equation is the optimum thickness. The cost values can be determined from an annualized lifetime analysis or simply from the requirement that the insulation pay for itself within two or three years. Note that the optimum thickness of insulation depends on the fuel cost, and the higher the fuel cost, the larger the optimum thickness of insulation. Considering that insulation will be in service for many years and the fuel prices are likely to escalate, a reasonable increase in fuel prices must be assumed in calculations. Otherwise, what is optimum insulation today will be inadequate insulation in the years to come, and we may have to face the possibility of costly retrofitting projects. This is what happened in the 1970s and 1980s to insulations installed in the 1960s.

The discussion above on optimum thickness is valid when the type and manufacturer of insulation are already selected, and the only thing to be determined is the most economical thickness. But often there are several suitable insulations for a job, and the selection process can be rather confusing since each insulation can have a different thermal conductivity, different installation cost, and different service life. In such cases, a selection can be made by preparing an annualized cost versus thickness chart like Figure 7–39 for each insulation, and determining the one having the *lowest* minimum cost. The insulation with the lowest annual cost is obviously the most economical insulation, and the insulation thickness corresponding to the *minimum total cost* is the *optimum thickness.* When the optimum thickness falls between two commercially available thicknesses, it is a good practice to be conservative and choose the thicker insulation. The

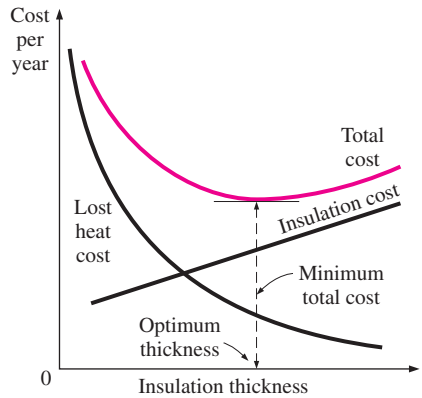


FIGURE 7–38
Determination of the optimum thickness of insulation on the basis of minimum total cost.

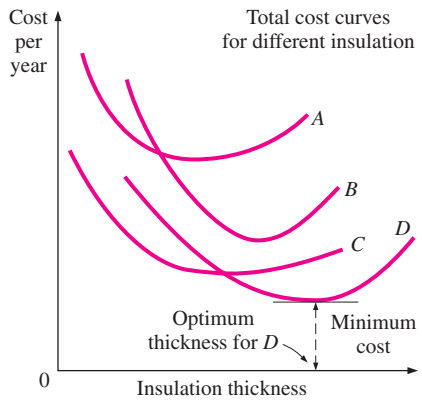


FIGURE 7–39
Determination of the most economical type of insulation and its optimum thickness.

extra thickness will provide a little safety cushion for any possible decline in performance over time and will help the environment by reducing the production of greenhouse gases such as CO_2.

The determination of the optimum thickness of insulation requires a heat transfer and economic analysis, which can be tedious and time-consuming. But a selection can be made in a few minutes using the tables and charts prepared by TIMA (Thermal Insulation Manufacturers Association) and member companies. The primary inputs required for using these tables or charts are the operating and ambient temperatures, pipe diameter (in the case of pipe insulation), and the unit fuel cost. Recommended insulation thicknesses for hot surfaces at specified temperatures are given in Table 7–4. Recommended thicknesses of *pipe insulations* as a function of service temperatures are 0.5 to 1 in. for 150°F, 1 to 2 in. for 250°F, 1.5 to 3 in. for 350°F, 2 to 4.5 in. for 450°F, 2.5 to 5.5 in. for 550°F, and 3 to 6 in. for 650°F for nominal pipe diameters of 0.5 to 36 in. The lower recommended insulation thicknesses are for pipes with small diameters, and the larger ones are for pipes with large diameters.

TABLE 7–4

Recommended insulation thicknesses for flat hot surfaces as a function of surface temperature (from TIMA *Energy Savings Guide*)

Surface temperature	Insulation thickness
150°F (66°C)	2″ (5.1 cm)
250°F (121°C)	3″ (7.6 cm)
350°F (177°C)	4″ (10.2 cm)
550°F (288°C)	6″ (15.2 cm)
750°F (400°C)	9″ (22.9 cm)
950°F (510°C)	10″ (25.44 cm)

EXAMPLE 7–8 Effect of Insulation on Surface Temperature

Hot water at $T_i = 120°C$ flows in a stainless steel pipe ($k = 15$ W/m · °C) whose inner diameter is 1.6 cm and thickness is 0.2 cm. The pipe is to be covered with adequate insulation so that the temperature of the outer surface of the insulation does not exceed 40°C when the ambient temperature is $T_o = 25°C$. Taking the heat transfer coefficients inside and outside the pipe to be $h_i = 70$ W/m^2 · °C and $h_o = 20$ W/m^2 · °C, respectively, determine the thickness of fiberglass insulation ($k = 0.038$ W/m · °C) that needs to be installed on the pipe.

SOLUTION A steam pipe is to be covered with enough insulation to reduce the exposed surface temperature. The thickness of insulation that needs to be installed is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 15$ W/m · °C for the steel pipe and $k = 0.038$ W/m · °C for fiberglass insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Figure 7–40. The inner radius of the pipe is $r_1 = 0.8$ cm and the outer radius of the pipe and thus the inner radius of the insulation is $r_2 = 1.0$ cm. Letting r_3 represent the outer radius of the insulation, the areas of the surfaces exposed to convection for an $L = 1$-m-long section of the pipe become

$$A_1 = 2\pi r_1 L = 2\pi(0.008\text{ m})(1\text{ m}) = 0.0503\text{ m}^2$$

$$A_3 = 2\pi r_3 L = 2\pi r_3\,(1\text{ m}) = 6.28 r_3\text{ m}^2$$

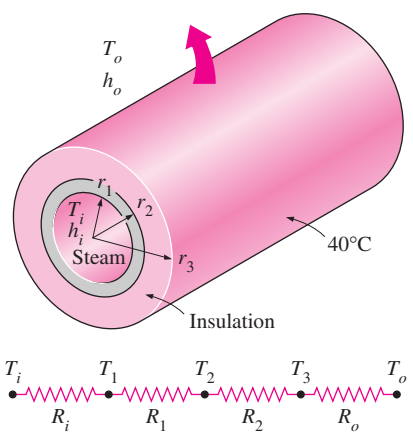


FIGURE 7–40
Schematic for Example 7–8.

Then the individual thermal resistances are determined to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_i A_1} = \frac{1}{(70\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.0503\ \text{m}^2)} = 0.284^\circ\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(0.01/0.008)}{2\pi(15\ \text{W/m} \cdot {}^\circ\text{C})(1\ \text{m})} = 0.0024^\circ\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(r_3/0.01)}{2\pi(0.038\ \text{W/m} \cdot {}^\circ\text{C})(1\ \text{m})}$$

$$= 4.188 \ln(r_3/0.01)^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_o A_3} = \frac{1}{(20\ \text{W/m}^2 \cdot {}^\circ\text{C})(6.28 r_3\ \text{m}^2)} = \frac{1}{125.6 r_3}\ {}^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_0$$
$$= [0.284 + 0.0024 + 4.188 \ln(r_3/0.01) + 1/125.6 r_3]^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_i - T_o}{R_{\text{total}}} = \frac{(120 - 125)^\circ\text{C}}{[0.284 + 0.0024 + 4.188 \ln(r_3/0.01) + 1/125.6 r_3]^\circ\text{C/W}}$$

Noting that the outer surface temperature of insulation is specified to be 40°C, the rate of heat loss can also be expressed as

$$\dot{Q} = \frac{T_3 - T_o}{R_o} = \frac{(40 - 25)^\circ\text{C}}{(1/125.6 r_3)^\circ\text{C/W}} = 1884 r_3$$

Setting the two relations above equal to each other and solving for r_3 gives $r_3 = 0.0170$ m. Then the minimum thickness of fiberglass insulation required is

$$t = r_3 - r_2 = 0.0170 - 0.0100 = 0.0070\ \text{m} = \mathbf{0.70\ cm}$$

Discussion Insulating the pipe with at least 0.70-cm-thick fiberglass insulation will ensure that the outer surface temperature of the pipe will be at 40°C or below.

EXAMPLE 7–9 Optimum Thickness of Insulation

During a plant visit, you notice that the outer surface of a cylindrical curing oven is very hot, and your measurements indicate that the average temperature of the exposed surface of the oven is 180°F when the surrounding air temperature is 75°F. You suggest to the plant manager that the oven should be insulated, but the manager does not think it is worth the expense. Then you propose to the manager to pay for the insulation yourself if he lets you keep the savings from the fuel bill for one year. That is, if the fuel bill is $5000/yr before insulation and drops to $2000/yr after insulation, you will get paid $3000. The manager agrees since he has nothing to lose, and a lot to gain. Is this a smart bet on your part?

The oven is 12 ft long and 8 ft in diameter, as shown in Figure 7–41. The plant operates 16 h a day 365 days a year, and thus 5840 h/yr. The insulation

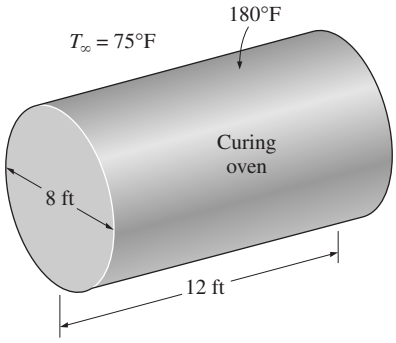


FIGURE 7–41
Schematic for Example 7–9.

to be used is fiberglass (k_{ins} = 0.024 Btu/h · ft · °F), whose cost is \$0.70/ft^2 per inch of thickness for materials, plus \$2.00/ft^2 for labor regardless of thickness. The combined heat transfer coefficient on the outer surface is estimated to be h_o = 3.5 Btu/h · ft^2 · °F. The oven uses natural gas, whose unit cost is \$0.75/therm input (1 therm = 100,000 Btu), and the efficiency of the oven is 80 percent. Disregarding any inflation or interest, determine how much money you will make out of this venture, if any, and the thickness of insulation (in whole inches) that will maximize your earnings.

SOLUTION A cylindrical oven is to be insulated to reduce heat losses. The optimum thickness of insulation and the potential earnings are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulation is one-dimensional. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** The surfaces of the cylindrical oven can be treated as plain surfaces since its diameter is greater than 3 ft.

Properties The thermal conductivity of insulation is given to be k = 0.024 Btu/h · ft · °F.

Analysis The exposed surface area of the oven is

$$A_s = 2A_{\text{base}} + A_{\text{side}} = 2\pi r^2 + 2\pi rL = 2\pi(4\text{ ft})^2 + 2\pi(4\text{ ft})(12\text{ ft}) = 402\text{ ft}^2$$

The rate of heat loss from the oven before the insulation is installed is determined from

$$\dot{Q} = h_o A_s(T_s - T_\infty) = (3.5\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(402\text{ ft}^2)(180 - 75)^\circ\text{F} = 147{,}700\text{ Btu/h}$$

Noting that the plant operates 5840 h/yr, the total amount of heat loss from the oven per year is

$$Q = \dot{Q}\Delta t = (147{,}700\text{ Btu/h})(5840\text{ h/yr}) = 0.863 \times 10^9\text{ Btu/yr}$$

The efficiency of the oven is given to be 80 percent. Therefore, to generate this much heat, the oven must consume energy (in the form of natural gas) at a rate of

$$\begin{aligned} Q_{\text{in}} &= Q/\eta_{\text{oven}} = (0.863 \times 10^9\text{ Btu/yr})/0.80 = 1.079 \times 10^9\text{ Btu/yr} \\ &= 10{,}790\text{ therms} \end{aligned}$$

since 1 therm = 100,000 Btu. Then the annual fuel cost of this oven before insulation becomes

$$\begin{aligned} \text{Annual cost} &= Q_{\text{in}} \times \text{Unit cost} \\ &= (10{,}790\text{ therm/yr})(\$0.75/\text{therm}) = \$8093/\text{yr} \end{aligned}$$

That is, the heat losses from the exposed surfaces of the oven are currently costing the plant over \$8000/yr.

When insulation is installed, the rate of heat transfer from the oven can be determined from

$$\dot{Q}_{\text{ins}} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = A_s\frac{T_s - T_\infty}{\dfrac{t_{\text{ins}}}{k_{\text{ins}}} + \dfrac{1}{h_o}}$$

We expect the surface temperature of the oven to increase and the heat transfer coefficient to decrease somewhat when insulation is installed. We assume

these two effects to counteract each other. Then the relation above for 1-in.-thick insulation gives the rate of heat loss to be

$$\dot{Q}_{\text{ins}} = \frac{A_s(T_s - T_\infty)}{\dfrac{t_{\text{ins}}}{k_{\text{ins}}} + \dfrac{1}{h_o}} = \frac{(402\ \text{ft}^2)(180 - 75)°\text{F}}{\dfrac{1/12\ \text{ft}}{0.024\ \text{Btu/h} \cdot \text{ft} \cdot °\text{F}} + \dfrac{1}{3.5\ \text{Btu/h} \cdot \text{ft}^2 \cdot °\text{F}}}$$
$$= 11{,}230\ \text{Btu/h}$$

Also, the total amount of heat loss from the oven per year and the amount and cost of energy consumption of the oven become

$$Q_{\text{ins}} = \dot{Q}_{\text{ins}}\,\Delta t = (11{,}230\ \text{Btu/h})(5840\ \text{h/yr}) = 0.6558 \times 10^8\ \text{Btu/yr}$$

$$Q_{\text{in, ins}} = Q_{\text{ins}}/\eta_{\text{oven}} = (0.6558 \times 10^8\ \text{Btu/yr})/0.80 = 0.820 \times 10^8\ \text{Btu/yr}$$
$$= 820\ \text{therms}$$

$$\text{Annual cost} = Q_{\text{in, ins}} \times \text{Unit cost}$$
$$= (820\ \text{therm/yr})(\$0.75/\text{therm}) = \$615/\text{yr}$$

Therefore, insulating the oven by 1-in.-thick fiberglass insulation will reduce the fuel bill by \$8093 − \$615 = \$7362 per year. The unit cost of insulation is given to be \$2.70/ft^2. Then the installation cost of insulation becomes

$$\text{Insulation cost} = (\text{Unit cost})(\text{Surface area}) = (\$2.70/\text{ft}^2)(402\ \text{ft}^2) = \$1085$$

The sum of the insulation and heat loss costs is

$$\text{Total cost} = \text{Insulation cost} + \text{Heat loss cost} = \$1085 + \$615 = \$1700$$

Then the net earnings will be

$$\text{Earnings} = \text{Income} - \text{Expenses} = \$8093 - \$1700 = \$6393$$

To determine the thickness of insulation that maximizes your earnings, we repeat the calculations above for 2-, 3-, 4-, and 5-in.-thick insulations, and list the results in Table 7–5. Note that the total cost of insulation decreases first with increasing insulation thickness, reaches a minimum, and then starts to increase.

TABLE 7–5

The variation of total insulation cost with insulation thickness

Insulation thickness	Heat loss, Btu/h	Lost fuel, therms/yr	Lost fuel cost, \$/yr	Insulation cost, \$	Total cost, \$
1 in.	11,230	820	615	1085	1700
2 in.	5838	426	320	1367	1687
3 in.	3944	288	216	1648	1864
4 in.	2978	217	163	1930	2093
5 in.	2392	175	131	2211	2342

We observe that the total insulation cost is a minimum at \$1687 for the case of **2-in.-thick** insulation. The earnings in this case are

$$\text{Maximum earnings} = \text{Income} - \text{Minimum expenses}$$
$$= \$8093 - \$1687 = \mathbf{\$6406}$$

which is not bad for a day's worth of work. The plant manager is also a big winner in this venture since the heat losses will cost him only $320/yr during the second and consequent years instead of $8093/yr. A thicker insulation could probably be justified in this case if the cost of insulation is annualized over the lifetime of insulation, say 20 years. Several energy conservation measures are being marketed as explained above by several power companies and private firms.

SUMMARY

The force a flowing fluid exerts on a body in the flow direction is called *drag*. The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* since it is caused by frictional effects, and the part that is due directly to pressure is called the *pressure drag* or *form drag* because of its strong dependence on the form or shape of the body.

The *drag coefficient* C_D is a dimensionless number that represents the drag characteristics of a body, and is defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho \mathcal{V}^2 A}$$

where A is the *frontal area* for blunt bodies, and surface area for parallel flow over flat plates or thin airfoils. For flow over a flat plate, the Reynolds number is

$$\mathrm{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\nu}$$

Transition from laminar to turbulent occurs at the *critical Reynolds* number of

$$\mathrm{Re}_{x,\,\mathrm{cr}} = \frac{\rho \mathcal{V} x_{\mathrm{cr}}}{\mu} = 5 \times 10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $$C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}} \qquad \mathrm{Re}_x < 5 \times 10^5$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6$$

Turbulent: $$C_{f,x} = \frac{0.0592}{\mathrm{Re}_x^{1/5}}, \qquad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $$C_f = \frac{1.328}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L < 5 \times 10^5$$

Turbulent: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

Combined: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

Rough surface, turbulent: $$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5}$$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Re}_L < 5 \times 10^5$$

Turbulent:

$$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

Combined:

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3}, \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1 - (\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1 - (\xi/x)^{3/4}]^{1/3}}$$

Turbulent: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1 - (\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1 - (\xi/x)^{9/10}]^{1/9}}$$

Laminar: $$h = \frac{2[1 - (\xi/x)^{3/4}]}{1 - \xi/L}\,h_{x=L}$$

Turbulent: $$h = \frac{5[1 - (\xi/x)^{9/10}}{(1 - \xi/L)}\,h_{x=L}$$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux*, the local Nusselt number is given by

Laminar: $\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$

Turbulent: $\mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for Re Pr > 0.2, and

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.

In tube banks, the Reynolds number is based on the maximum velocity $\mathcal{V}_{\max}$ that is related to the approach velocity $\mathcal{V}$ as

In-line and *Staggered* with $S_D < (S_T + D)/2$:

$$\mathcal{V}_{\max} = \frac{S_T}{S_T - D}\mathcal{V}$$

Staggered with $S_D < (S_T + D)/2$:

$$\mathcal{V}_{\max} = \frac{S_T}{2(S_D - D)}\mathcal{V}$$

where S_T the transverse pitch and S_D is the diagonal pitch. The average Nusselt number for cross flow over tube banks is expressed as

$$\mathrm{Nu}_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\mathrm{Pr}^n(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$$

where the values of the constants C, m, and n depend on value Reynolds number. Such correlations are given in Table 7–2. All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid defined as $T_m = (T_i + T_e)/2$.

The average Nusselt number for tube banks with less than 16 rows is expressed as

$$\mathrm{Nu}_{D,N_L} = F\mathrm{Nu}_D$$

where F is the *correction factor* whose values are given in Table 7-3. The heat transfer rate to or from a tube bank is determined from

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}C_p(T_e - T_i)$$

where $\Delta T_{\ln}$ is the logarithmic mean temperature difference defined as

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

and the exit temperature of the fluid T_e is

$$T_e = T_s - (T_s - T_i)\exp\left(\frac{A_s h}{\dot{m}C_p}\right)$$

where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid. The pressure drop ΔP for a tube bank is expressed as

$$\Delta P = N_L f\chi\frac{\rho\mathcal{V}_{\max}^2}{2}$$

where f is the friction factor and χ is the correction factor, both given in Figs. 7–27.

REFERENCES AND SUGGESTED READING

1. R. D. Blevin. *Applied Fluid Dynamics Handbook*. New York: Van Nostrand Reinhold, 1984.
2. S. W. Churchill and M. Bernstein. "A Correlating Equation for Forced Convection from Gases and Liquids to a Circular Cylinder in Cross Flow." *Journal of Heat Transfer* 99 (1977), pp. 300–306.
3. S. W. Churchill and H. Ozoe. "Correlations for Laminar Forced Convection in Flow over an Isothermal Flat Plate and in Developing and Fully Developed Flow in an Isothermal Tube." *Journal of Heat Transfer* 95 (Feb. 1973), pp. 78–84.
4. W. M. Edmunds. "Residential Insulation." *ASTM Standardization News* (Jan. 1989), pp. 36–39.
5. W. H. Giedt. "Investigation of Variation of Point Unit-Heat Transfer Coefficient around a Cylinder Normal to an Air Stream." *Transactions of the ASME* 71 (1949), pp. 375–381.
6. M. Jakob. *Heat Transfer.* Vol. l. New York: John Wiley & Sons, 1949.
7. W. M. Kays and M. E. Crawford. *Convective Heat and Mass Transfer.* 3rd ed. New York: McGraw-Hill, 1993.
8. F. Kreith and M. S. Bohn. *Principles of Heat Transfer,* 6th ed. Pacific Grove, CA: Brooks/Cole, 2001.
9. H. Schlichting. *Boundary Layer Theory,* 7th ed. New York, McGraw-Hill, 1979.

10. N. V. Suryanarayana. *Engineering Heat Transfer.* St. Paul, MN: West, 1995.

11. W. C. Thomas. "Note on the Heat Transfer Equation for Forced Convection Flow over a Flat Plate with and Unheated Starting Length." *Mechanical Engineering News,* 9, no.1 (1977), p. 361.

12. R. D. Willis. "Photographic Study of Fluid Flow Between Banks of Tubes." *Engineering* (1934), pp. 423–425.

13. A. Zukauskas, "Convection Heat Transfer in Cross Flow." In *Advances in Heat Transfer,* J. P. Hartnett and T. F. Irvine, Jr., Eds. New York: Academic Press, 1972, Vol. 8, pp. 93–106.

14. A. Zukauskas. "Heat Transfer from Tubes in Cross Flow." In *Advances in Heat Transfer,* ed. J. P. Hartnett and T. F. Irvine, Jr. Vol. 8. New York: Academic Press, 1972.

15. A. Zukauskas. "Heat Transfer from Tubes in Cross Flow." In *Handbook of Single Phase Convective Heat Transfer,* Eds. S. Kakac, R. K. Shah, and Win Aung. New York: Wiley Interscience, 1987.

16. A. Zukauskas and R. Ulinskas, "Efficiency Parameters for Heat Transfer in Tube Banks." *Heat Transfer Engineering* no. 2 (1985), pp. 19–25.

PROBLEMS*

Drag Force and Heat Transfer in External Flow

7–1C What is the difference between the upstream velocity and the free-stream velocity? For what types of flow are these two velocities equal to each other?

7–2C What is the difference between streamlined and blunt bodies? Is a tennis ball a streamlined or blunt body?

7–3C What is drag? What causes it? Why do we usually try to minimize it?

7–4C What is lift? What causes it? Does wall shear contribute to the lift?

7–5C During flow over a given body, the drag force, the upstream velocity, and the fluid density are measured. Explain how you would detennine the drag coefficient. What area would you use in calculations?

7–6C Define frontal area of a body subjected to external flow. When is it appropriate to use the frontal area in drag and lift calculations?

7–7C What is the difference between skin friction drag and pressure drag? Which is usually more significant for slender bodies such as airfoils?

7–8C What is the effect of surface roughness on the friction drag coefficient in laminar and turbulent flows?

7–9C What is the effect of streamlining on (*a*) friction drag and (*b*) pressure drag? Does the total drag acting on a body necessarily decrease as a result of streamlining? Explain.

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

7–10C What is flow separation? What causes it? What is the effect of flow separation on the drag coefficient?

Flow Over Flat Plates

7–11C What does the friction coefficient represent in flow over a flat plate? How is it related to the drag force acting on the plate?

7–12C Consider laminar flow over a flat plate. Will the friction coefficient change with distance from the leading edge? How about the heat transfer coefficient?

7–13C How are the average friction and heat transfer coefficients determined in flow over a flat plate?

7–14 Engine oil at 80°C flows over a 6-m-long flat plate whose temperature is 30°C with a velocity of 3 m/s. Determine the total drag force and the rate of heat transfer over the entire plate per unit width.

7–15 The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and at 30°C flows with a velocity of 6 m/s over a 2.5-m × 8-m flat plate whose temperature is 120°C. Determine the rate of heat transfer from the plate if the air flows parallel to the (*a*) 8-m-long side and (*b*) the 2.5-m side.

7–16 During a cold winter day, wind at 55 km/h is blowing parallel to a 4-m-high and 10-m-long wall of a house. If the air outside is at 5°C and the surface temperature of the wall is

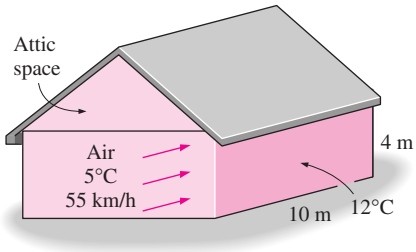


FIGURE P7–16

12°C, determine the rate of heat loss from that wall by convection. What would your answer be if the wind velocity was doubled? *Answers:* 9081 W, 16,200 W

7–17 Reconsider Problem 7–16. Using EES (or other) software, investigate the effects of wind velocity and outside air temperature on the rate of heat loss from the wall by convection. Let the wind velocity vary from 10 km/h to 80 km/h and the outside air temperature from 0ºC to 10ºC. Plot the rate of heat loss as a function of the wind velocity and of the outside temperature, and discuss the results.

7–18E Air at 60°F flows over a 10-ft-long flat plate at 7 ft/s. Determine the local friction and heat transfer coefficients at intervals of 1 ft, and plot the results against the distance from the leading edge.

7–19E Reconsider Problem 7–18. Using EES (or other) software, evaluate the local friction and heat transfer coefficients along the plate at intervals of 0.1 ft, and plot them against the distance from the leading edge.

7–20 Consider a hot automotive engine, which can be approximated as a 0.5-m-high, 0.40-m-wide, and 0.8-m-long rectangular block. The bottom surface of the block is at a temperature of 80°C and has an emissivity of 0.95. The ambient air is at 20°C, and the road surface is at 25°C. Determine the rate of heat transfer from the bottom surface of the engine block by convection and radiation as the car travels at a velocity of 80 km/h. Assume the flow to be turbulent over the entire surface because of the constant agitation of the engine block.

7–21 The forming section of a plastics plant puts out a continuous sheet of plastic that is 1.2 m wide and 2 mm thick at a rate of 15 m/min. The temperature of the plastic sheet is 90°C when it is exposed to the surrounding air, and the sheet is subjected to air flow at 30°C at a velocity of 3 m/s on both sides along its surfaces normal to the direction of motion of the sheet. The width of the air cooling section is such that a fixed point on the plastic sheet passes through that section in 2 s. Determine the rate of heat transfer from the plastic sheet to the air.

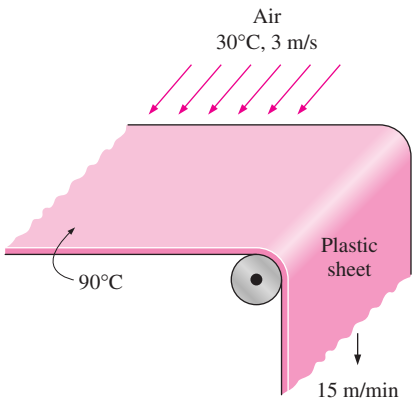


FIGURE P7–21

7–22 The top surface of the passenger car of a train moving at a velocity of 70 km/h is 2.8 m wide and 8 m long. The top surface is absorbing solar radiation at a rate of 200 W/m^2, and the temperature of the ambient air is 30°C. Assuming the roof of the car to be perfectly insulated and the radiation heat exchange with the surroundings to be small relative to convection, determine the equilibrium temperature of the top surface of the car. *Answer:* 35.1°C

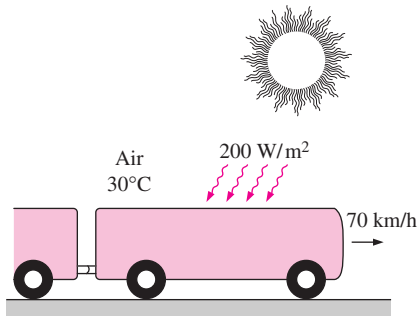


FIGURE P7–22

7–23 Reconsider Problem 7–22. Using EES (or other) software, investigate the effects of the train velocity and the rate of absorption of solar radiation on the equilibrium temperature of the top surface of the car. Let the train velocity vary from 10 km/h to 120 km/h and the rate of solar absorption from 100 W/m^2 to 500 W/m^2. Plot the equilibrium temperature as functions of train velocity and solar radiation absorption rate, and discuss the results.

7–24 A 15-cm × 15-cm circuit board dissipating 15 W of power uniformly is cooled by air, which approaches the circuit board at 20°C with a velocity of 5 m/s. Disregarding any heat transfer from the back surface of the board, determine the surface temperature of the electronic components (*a*) at the leading edge and (*b*) at the end of the board. Assume the flow to be turbulent since the electronic components are expected to act as turbulators.

7–25 Consider laminar flow of a fluid over a flat plate maintained at a constant temperature. Now the free-stream velocity of the fluid is doubled. Determine the change in the drag force on the plate and rate of heat transfer between the fluid and the plate. Assume the flow to remain laminar.

7–26E Consider a refrigeration truck traveling at 55 mph at a location where the air temperature is 80°F. The refrigerated compartment of the truck can be considered to be a 9-ft-wide, 8-ft-high, and 20-ft-long rectangular box. The refrigeration system of the truck can provide 3 tons of refrigeration (i.e., it can remove heat at a rate of 600 Btu/min). The outer surface of the truck is coated with a low-emissivity material, and thus radiation heat transfer is very small. Determine the average temperature of the outer surface of the refrigeration compartment of the truck if the refrigeration system is observed to be

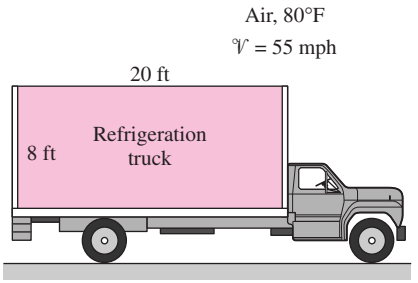


FIGURE P7–26E

operating at half the capacity. Assume the air flow over the entire outer surface to be turbulent and the heat transfer coefficient at the front and rear surfaces to be equal to that on side surfaces.

7–27 Solar radiation is incident on the glass cover of a solar collector at a rate of 700 W/m^2. The glass transmits 88 percent of the incident radiation and has an emissivity of 0.90. The entire hot water needs of a family in summer can be met by two collectors 1.2 m high and 1 m wide. The two collectors are attached to each other on one side so that they appear like a single collector 1.2 m × 2 m in size. The temperature of the glass cover is measured to be 35°C on a day when the surrounding air temperature is 25°C and the wind is blowing at 30 km/h. The effective sky temperature for radiation exchange between the glass cover and the open sky is −40°C. Water enters the tubes attached to the absorber plate at a rate of 1 kg/min. Assuming the back surface of the absorber plate to be heavily insulated and the only heat loss to occur through the glass cover, determine (*a*) the total rate of heat loss from the collector, (*b*) the collector efficiency, which is the ratio of the amount of heat transferred to the water to the solar energy incident on the collector, and (*c*) the temperature rise of water as it flows through the collector.

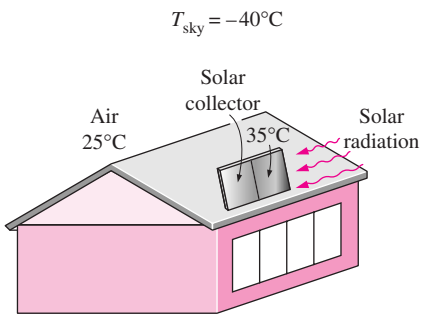


FIGURE P7–27

7–28 A transformer that is 10 cm long, 6.2 cm wide, and 5 cm high is to be cooled by attaching a 10 cm × 6.2 cm wide polished aluminum heat sink (emissivity = 0.03) to its top surface. The heat sink has seven fins, which are 5 mm high, 2 mm thick, and 10 cm long. A fan blows air at 25°C parallel to the passages between the fins. The heat sink is to dissipate 20 W of heat and the base temperature of the heat sink is not to exceed 60°C. Assuming the fins and the base plate to be nearly isothermal and the radiation heat transfer to be negligible, determine the minimum free-stream velocity the fan needs to supply to avoid overheating.

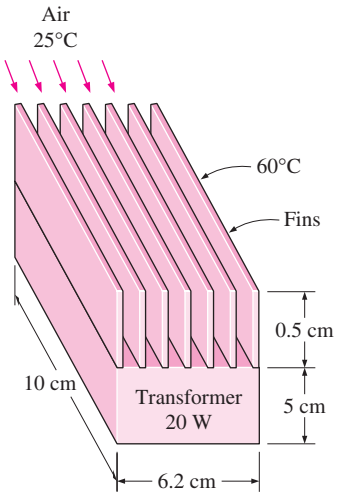


FIGURE P7–28

7–29 Repeat Problem 7–28 assuming the heat sink to be black-anodized and thus to have an effective emissivity of 0.90. Note that in radiation calculations the base area (10 cm × 6.2 cm) is to be used, not the total surface area.

7–30 An array of power transistors, dissipating 6 W of power each, are to be cooled by mounting them on a 25-cm × 25-cm square aluminum plate and blowing air at 35°C over the plate with a fan at a velocity of 4 m/s. The average temperature of the plate is not to exceed 65°C. Assuming the heat transfer from the back side of the plate to be negligible and disregarding radiation, determine the number of transistors that can be placed on this plate.

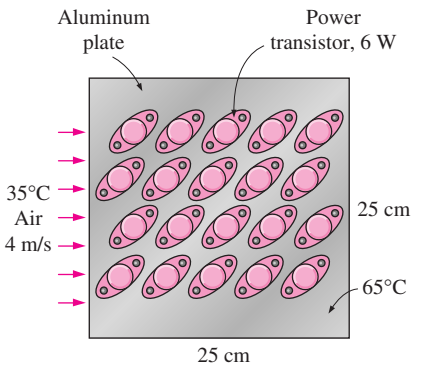


FIGURE P7–30

7–31 Repeat Problem 7–30 for a location at an elevation of 1610 m where the atmospheric pressure is 83.4 kPa.
Answer: 4

7–32 Air at 25°C and 1 atm is flowing over a long flat plate with a velocity of 8 m/s. Determine the distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location.

7–33 Repeat Problem 7–32 for water.

7–34 The weight of a thin flat plate 50 cm × 50 cm in size is balanced by a counterweight that has a mass of 2 kg, as shown in the figure. Now a fan is turned on, and air at 1 atm and 25°C flows downward over both surfaces of the plate with a free-stream velocity of 10 m/s. Determine the mass of the counterweight that needs to be added in order to balance the plate in this case.

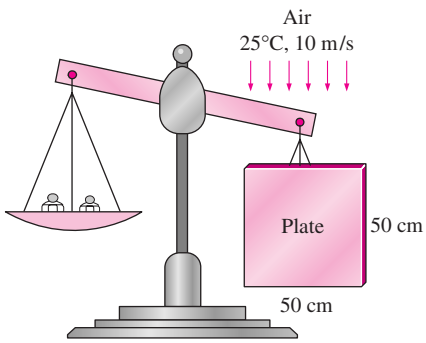


FIGURE P7–34

Flow across Cylinders and Spheres

7–35C Consider laminar flow of air across a hot circular cylinder. At what point on the cylinder will the heat transfer be highest? What would your answer be if the flow were turbulent?

7–36C In flow over cylinders, why does the drag coefficient suddenly drop when the flow becomes turbulent? Isn't turbulence supposed to increase the drag coefficient instead of decreasing it?

7–37C In flow over blunt bodies such as a cylinder, how does the pressure drag differ from the friction drag?

7–38C Why is flow separation in flow over cylinders delayed in turbulent flow?

7–39 A long 8-cm-diameter steam pipe whose external surface temperature is 90°C passes through some open area that is not protected against the winds. Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 7°C and the wind is blowing across the pipe at a velocity of 50 km/h.

7–40 A stainless steel ball (ρ = 8055 kg/m^3, C_p = 480 J/kg · °C) of diameter D = 15 cm is removed from the oven at a uniform temperature of 350°C. The ball is then subjected to the flow of air at 1 atm pressure and 30°C with a velocity of 6 m/s. The surface temperature of the ball eventually drops to 250°C. Determine the average convection heat transfer coefficient during this cooling process and estimate how long this process has taken.

7–41 Reconsider Problem 7–40. Using EES (or other) software, investigate the effect of air velocity on the average convection heat transfer coefficient and the cooling time. Let the air velocity vary from 1 m/s to 10 m/s. Plot the heat transfer coefficient and the cooling time as a function of air velocity, and discuss the results.

7–42E A person extends his uncovered arms into the windy air outside at 54°F and 20 mph in order to feel nature closely. Initially, the skin temperature of the arm is 86°F. Treating the arm as a 2-ft-long and 3-in.-diameter cylinder, determine the rate of heat loss from the arm.

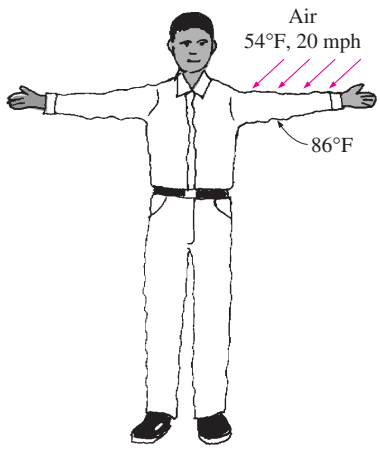


FIGURE P7–42E

7–43E Reconsider Problem 7–42E. Using EES (or other) software, investigate the effects of air temperature and wind velocity on the rate of heat loss from the arm. Let the air temperature vary from 20°F to 80°F and the wind velocity from 10 mph to 40 mph. Plot the rate of heat loss as a function of air temperature and of wind velocity, and discuss the results.

7–44 An average person generates heat at a rate of 84 W while resting. Assuming one-quarter of this heat is lost from the head and disregarding radiation, determine the average surface temperature of the head when it is not covered and is subjected to winds at 10°C and 35 km/h. The head can be approximated as a 30-cm-diameter sphere. *Answer:* 12.7°C

7–45 Consider the flow of a fluid across a cylinder maintained at a constant temperature. Now the free-stream velocity of the fluid is doubled. Determine the change in the drag force on the cylinder and the rate of heat transfer between the fluid and the cylinder.

7–46 A 6-mm-diameter electrical transmission line carries an electric current of 50 A and has a resistance of 0.002 ohm per meter length. Determine the surface temperature of the wire during a windy day when the air temperature is 10°C and the wind is blowing across the transmission line at 40 km/h.

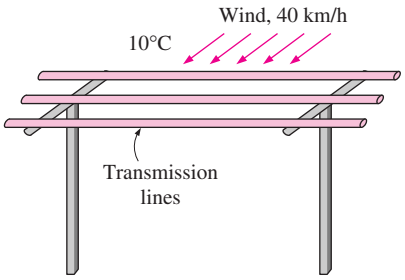


FIGURE P7–46

7–47 Reconsider Problem 7–46. Using EES (or other) software, investigate the effect of the wind velocity on the surface temperature of the wire. Let the wind velocity vary from 10 km/h to 80 km/h. Plot the surface temperature as a function of wind velocity, and discuss the results.

7–48 A heating system is to be designed to keep the wings of an aircraft cruising at a velocity of 900 km/h above freezing temperatures during flight at 12,200-m altitude where the standard atmospheric conditions are −55.4°C and 18.8 kPa. Approximating the wing as a cylinder of elliptical cross section whose minor axis is 30 cm and disregarding radiation, determine the average convection heat transfer coefficient on the wing surface and the average rate of heat transfer per unit surface area.

7–49 A long aluminum wire of diameter 3 mm is extruded at a temperature of 370°C. The wire is subjected to cross air flow at 30°C at a velocity of 6 m/s. Determine the rate of heat transfer from the wire to the air per meter length when it is first exposed to the air.

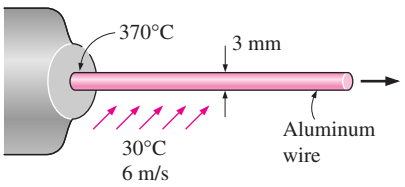


FIGURE P7–49

7–50E Consider a person who is trying to keep cool on a hot summer day by turning a fan on and exposing his entire body to air flow. The air temperature is 85°F and the fan is blowing air at a velocity of 6 ft/s. If the person is doing light work and generating sensible heat at a rate of 300 Btu/h, determine the average temperature of the outer surface (skin or clothing) of the person. The average human body can be treated as a 1-ft-diameter cylinder with an exposed surface area of 18 ft^2. Disregard any heat transfer by radiation. What would your answer be if the air velocity were doubled? *Answers:* 95.1°F, 91.6°F

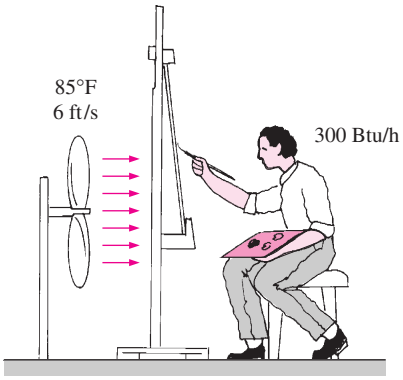


FIGURE P7–50E

7–51 An incandescent lightbulb is an inexpensive but highly inefficient device that converts electrical energy into light. It converts about 10 percent of the electrical energy it consumes into light while converting the remaining 90 percent into heat. (A fluorescent lightbulb will give the same amount of light while consuming only one-fourth of the electrical energy, and it will last 10 times longer than an incandescent lightbulb.) The glass bulb of the lamp heats up very quickly as a result of absorbing all that heat and dissipating it to the surroundings by convection and radiation.

Consider a 10-cm-diameter 100-W lightbulb cooled by a fan that blows air at 25°C to the bulb at a velocity of 2 m/s. The surrounding surfaces are also at 25°C, and the emissivity of the glass is 0.9. Assuming 10 percent of the energy passes through the glass bulb as light with negligible absorption and the rest of the energy is absorbed and dissipated by the bulb itself, determine the equilibrium temperature of the glass bulb.

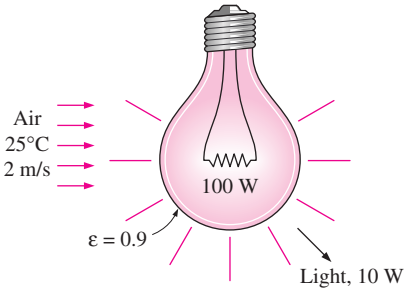


FIGURE P7–51

7–52 During a plant visit, it was noticed that a 12-m-long section of a 10-cm-diameter steam pipe is completely exposed to the ambient air. The temperature measurements indicate that

the average temperature of the outer surface of the steam pipe is 75°C when the ambient temperature is 5°C. There are also light winds in the area at 10 km/h. The emissivity of the outer surface of the pipe is 0.8, and the average temperature of the surfaces surrounding the pipe, including the sky, is estimated to be 0°C. Determine the amount of heat lost from the steam during a 10-h-long work day.

Steam is supplied by a gas-fired steam generator that has an efficiency of 80 percent, and the plant pays $0.54/therm of natural gas (1 therm = 105,500 kJ). If the pipe is insulated and 90 percent of the heat loss is saved, determine the amount of money this facility will save a year as a result of insulating the steam pipes. Assume the plant operates every day of the year for 10 h. State your assumptions.

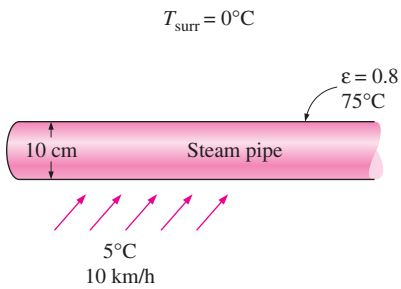


FIGURE P7–52

7–53 Reconsider Problem 7–52. There seems to be some uncertainty about the average temperature of the surfaces surrounding the pipe used in radiation calculations, and you are asked to determine if it makes any significant difference in overall heat transfer. Repeat the calculations for average surrounding and surface temperatures of −20°C and 25°C, respectively, and determine the change in the values obtained.

7–54E A 12-ft-long, 1.5-kW electrical resistance wire is made of 0.1-in.-diameter stainless steel (k = 8.7 Btu/h · ft · °F). The resistance wire operates in an environment at 85°F. Determine the surface temperature of the wire if it is cooled by a fan blowing air at a velocity of 20 ft/s.

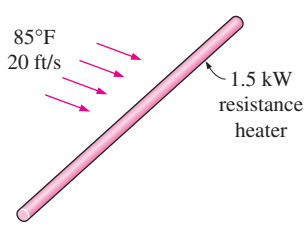


FIGURE P7–54E

7–55 The components of an electronic system are located in a 1.5-m-long horizontal duct whose cross section is 20 cm × 20 cm. The components in the duct are not allowed to come into direct contact with cooling air, and thus are cooled by air at 30°C flowing over the duct with a velocity of 200 m/min. If the surface temperature of the duct is not to exceed 65°C, determine the total power rating of the electronic devices that can be mounted into the duct. *Answer:* 640 W

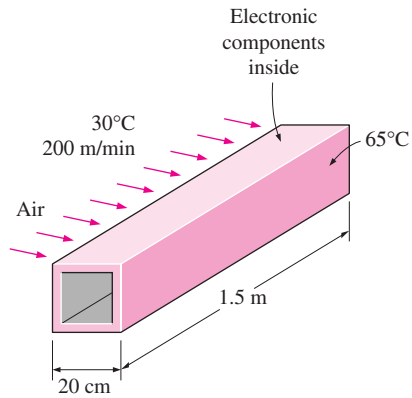


FIGURE P7–55

7–56 Repeat Problem 7–55 for a location at 4000-m altitude where the atmospheric pressure is 61.66 kPa.

7–57 A 0.4-W cylindrical electronic component with diameter 0.3 cm and length 1.8 cm and mounted on a circuit board is cooled by air flowing across it at a velocity of 150 m/min. If the air temperature is 40°C, determine the surface temperature of the component.

7–58 Consider a 50-cm-diameter and 95-cm-long hot water tank. The tank is placed on the roof of a house. The water inside the tank is heated to 80ºC by a flat-plate solar collector during the day. The tank is then exposed to windy air at 18ºC with an average velocity of 40 km/h during the night. Estimate the temperature of the tank after a 45-mm period. Assume the tank surface to be at the same temperature as the water inside, and the heat transfer coefficient on the top and bottom surfaces to be the same as that on the side surface.

7–59 Reconsider Problem 7–58. Using EES (or other) software, plot the temperature of the tank as a function of the cooling time as the time varies from 30 mm to 5 h, and discuss the results.

7–60 A 1.8-m-diameter spherical tank of negligible thickness contains iced water at 0ºC. Air at 25ºC flows over the tank with a velocity of 7 m/s. Determine the rate of heat transfer to the tank and the rate at which ice melts. The heat of fusion of water at 0ºC is 333.7 kJ/kg.

7-61 A 10-cm-diameter, 30-cm-high cylindrical bottle contains cold water at 3ºC. The bottle is placed in windy air at 27ºC. The water temperature is measured to be 11ºC after 45 minutes of cooling. Disregarding radiation effects and heat transfer from the top and bottom surfaces, estimate the average wind velocity.

Flow across Tube Banks

7–62C In flow across tube banks, why is the Reynolds number based on the maximum velocity instead of the uniform approach velocity?

7–63C In flow across tube banks, how does the heat transfer coefficient vary with the row number in the flow direction? How does it vary with in the transverse direction for a given row number?

7–64 Combustion air in a manufacturing facility is to be preheated before entering a furnace by hot water at 90ºC flowing through the tubes of a tube bank located in a duct. Air enters the duct at 15ºC and 1 atm with a mean velocity of 3.8 m/s, and flows over the tubes in normal direction. The outer diameter of the tubes is 2.1 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of $S_L = S_T = 5$ cm. There are eight rows in the flow direction with eight tubes in each row. Determine the rate of heat transfer per unit length of the tubes, and the pressure drop across the tube bank.

7–65 Repeat Problem 7–64 for staggered arrangement with $S_L = S_T = 5$ cm.

7–66 Air is to be heated by passing it over a bank of 3-m-long tubes inside which steam is condensing at 100ºC. Air approaches the tube bank in the normal direction at 20ºC and 1 atm with a mean velocity of 5.2 m/s. The outer diameter of the tubes is 1.6 cm, and the tubes are arranged staggered with longitudinal and transverse pitches of $S_L = S_T = 4$ cm. There are 20 rows in the flow direction with 10 tubes in each row. Determine (*a*) the rate of heat transfer, (*b*) and pressure drop across the tube bank, and (*c*) the rate of condensation of steam inside the tubes.

7–67 Repeat Problem 7–66 for in-line arrangement with $S_L = S_T = 5$ cm.

7–68 Exhaust gases at 1 atm and 300ºC are used to preheat water in an industrial facility by passing them over a bank of tubes through which water is flowing at a rate of 6 kg/s. The mean tube wall temperature is 80ºC. Exhaust gases approach the tube bank in normal direction at 4.5 m/s. The outer diameter of the tubes is 2.1 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of $S_L = S_T = 8$ cm. There are 16 rows in the flow direction with eight tubes in each row. Using the properties of air for exhaust gases, determine (*a*) the rate of heat transfer per unit length of tubes, (*b*) and pressure drop across the tube bank, and (*c*) the temperature rise of water flowing through the tubes per unit length of tubes.

7–69 Water at 15ºC is to be heated to 65ºC by passing it over a bundle of 4-m-long 1-cm-diameter resistance heater rods maintained at 90ºC. Water approaches the heater rod bundle in normal direction at a mean velocity of 0.8 m/s. The rods arc arranged in-line with longitudinal and transverse pitches of $S_L = 4$ cm and $S_T = 3$ cm. Determine the number of tube rows N_L in the flow direction needed to achieve the indicated temperature rise.

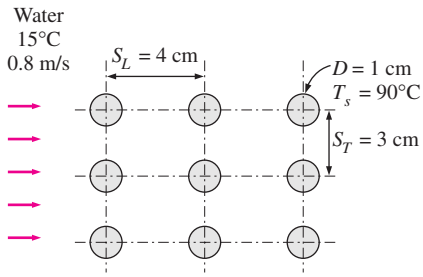


FIGURE P7–69

7–70 Air is to be cooled in the evaporator section of a refrigerator by passing it over a bank of 0.8-cm-outer-diameter and 0.4-m-long tubes inside which the refrigerant is evaporating at −20ºC. Air approaches the tube bank in the normal direction at 0ºC and 1 atm with a mean velocity of 4 m/s. The tubes are arranged in-line with longitudinal and transverse pitches of $S_L = S_T = 1.5$ cm. There are 30 rows in the flow direction with 15 tubes in each row. Determine (*a*) the refrigeration capacity of this system and (*b*) and pressure drop across the tube bank.

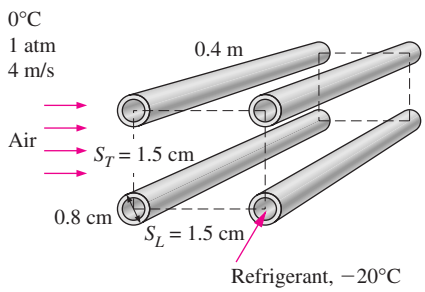


FIGURE P7–70

7–71 Repeat Problem 7–70 by solving it for staggered arrangement with $S_L = S_T = 1.5$ cm, and compare the performance of the evaporator for the in-line and staggered arrangements.

7–72 A tube bank consists of 300 tubes at a distance of 6 cm between the centerlines of any two adjacent tubes. Air approaches the tube bank in the normal direction at 40ºC and 1 atm with a mean velocity of 7 m/s. There are 20 rows in the flow direction with 15 tubes in each row with an average surface temperature of 140ºC. For an outer tube diameter of 2 cm, determine the average heat transfer coefficient.

Special Topic: Thermal Insulation

7–73C What is thermal insulation? How does a thermal insulator differ in purpose from an electrical insulator and from a sound insulator?

7–74C Does insulating cold surfaces save energy? Explain.

7–75C What is the R-value of insulation? How is it determined? Will doubling the thickness of flat insulation double its R-value?

7–76C How does the R-value of an insulation differ from its thermal resistance?

7–77C Why is the thermal conductivity of superinsulation orders of magnitude lower than the thermal conductivities of ordinary insulations?

7–78C Someone suggests that one function of hair is to insulate the head. Do you agree with this suggestion?

7–79C Name five different reasons for using insulation in industrial facilities.

7–80C What is optimum thickness of insulation? How is it determined?

7–81 What is the thickness of flat R-8 (in SI units) insulation whose thermal conductivity is 0.04 W/m · °C?

7–82E What is the thickness of flat R-20 (in English units) insulation whose thermal conductivity is 0.02 Btu/h · ft · °F?

7–83 Hot water at 110°C flows in a cast iron pipe (k = 52 W/m · °C) whose inner radius is 2.0 cm and thickness is 0.3 cm. The pipe is to be covered with adequate insulation so that the temperature of the outer surface of the insulation does not exceed 30°C when the ambient temperature is 22°C. Taking the heat transfer coefficients inside and outside the pipe to be h_i = 80 W/m² · °C and h_o = 22 W/m² · °C, respectively, determine the thickness of fiber glass insulation (k = 0.038 W/m · °C) that needs to be installed on the pipe.

Answer: 1.32 cm

7–84 Reconsider Problem 7–83. Using EES (or other) software, plot the thickness of the insulation as a function of the maximum temperature of the outer surface of insulation in the range of 24ºC to 48ºC. Discuss the results.

7–85 Consider a furnace whose average outer surface temperature is measured to be 90°C when the average surrounding air temperature is 27°C. The furnace is 6 m long and 3 m in diameter. The plant operates 80 h per week for 52 weeks per year. You are to insulate the furnace using fiberglass insulation (k_{ins} = 0.038 W/m · °C) whose cost is \$10/m² per cm of thickness for materials, plus \$30/m² for labor regardless of thickness. The combined heat transfer coefficient on the outer surface is estimated to be h_o = 30 W/m² · °C. The furnace uses natural gas whose unit cost is \$0.50/therm input (1 therm = 105,500 kJ), and the efficiency of the furnace is 78 percent. The management is willing to authorize the installation of the thickest insulation (in whole cm) that will pay for itself (materials and labor) in one year. That is, the total cost of insulation should be roughly equal to the drop in the fuel cost of the furnace for one year. Determine the thickness of insulation to be used and the money saved per year. Assume the surface temperature of the furnace and the heat transfer coefficient are to remain constant.

Answer: 14 cm

7–85 Repeat Problem 7–85 for an outer surface temperature of 75°C for the furnace.

7–87E Steam at 400°F is flowing through a steel pipe (k = 8.7 Btu/h · ft · °F) whose inner and outer diameters are 3.5 in. and 4.0 in., respectively, in an environment at 60°F. The pipe is insulated with 1-in.-thick fiberglass insulation (k = 0.020 Btu/h · ft · °F), and the heat transfer coefficients on the inside and the outside of the pipe are 30 Btu/h · ft² · °F and 5 Btu/h · ft² · °F, respectively. It is proposed to add another 1-in.-thick layer of fiberglass insulation on top of the existing one to reduce the heat losses further and to save energy and money. The total cost of new insulation is \$7 per ft length of the pipe, and the net fuel cost of energy in the steam is \$0.01 per 1000 Btu (therefore, each 1000 Btu reduction in the heat loss will save the plant \$0.01). The policy of the plant is to implement energy conservation measures that pay for themselves within two years. Assuming continuous operation (8760 h/year), determine if the proposed additional insulation is justified.

7–88 The plumbing system of a plant involves a section of a plastic pipe (k = 0.16 W/m · °C) of inner diameter 6 cm and outer diameter 6.6 cm exposed to the ambient air. You are to insulate the pipe with adequate weather-jacketed fiberglass insulation (k = 0.035 W/m · °C) to prevent freezing of water in the pipe. The plant is closed for the weekends for a period of 60 h, and the water in the pipe remains still during that period. The ambient temperature in the area gets as low as −10°C in winter, and the high winds can cause heat transfer coefficients as high as 30 W/m² · °C. Also, the water temperature in the pipe can be as cold as 15°C, and water starts freezing when its temperature drops to 0°C. Disregarding the convection resistance inside the pipe, determine the thickness of insulation that will protect the water from freezing under worst conditions.

7–89 Repeat Problem 7–88 assuming 20 percent of the water in the pipe is allowed to freeze without jeopardizing safety.

Answer: 27.9 cm

Review Problems

7–90 Consider a house that is maintained at 22°C at all times. The walls of the house have R-3.38 insulation in SI units (i.e., an L/k value or a thermal resistance of 3.38 m² · °C/W). During a cold winter night, the outside air temperature is 4°C and wind at 50 km/h is blowing parallel to a 3-m-high and 8-m-long wall of the house. If the heat transfer coefficient on the interior surface of the wall is 8 W/m² · °C, determine the rate of heat loss from that wall of the house. Draw the thermal resistance network and disregard radiation heat transfer.

Answer: 122 W

7–91 An automotive engine can be approximated as a 0.4-m-high, 0.60-m-wide, and 0.7-m-long rectangular block. The bottom surface of the block is at a temperature of 75°C and has an emissivity of 0.92. The ambient air is at 5°C, and the road surface is at 10°C. Determine the rate of heat transfer from the bottom surface of the engine block by convection and radiation

as the car travels at a velocity of 60 km/h. Assume the flow to be turbulent over the entire surface because of the constant agitation of the engine block. How will the heat transfer be affected when a 2-mm-thick gunk (k = 3 W/m · °C) has formed at the bottom surface as a result of the dirt and oil collected at that surface over time? Assume the metal temperature under the gunk still to be 75°C.

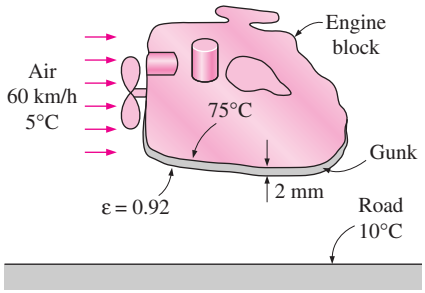


FIGURE P7–91

7–92E The passenger compartment of a minivan traveling at 60 mph can be modeled as a 3.2-ft-high, 6-ft-wide, and 11-ft-long rectangular box whose walls have an insulating value of R-3 (i.e., a wall thickness–to–thermal conductivity ratio of 3 h · ft² · °F/Btu). The interior of a minivan is maintained at an average temperature of 70°F during a trip at night while the outside air temperature is 90°F. The average heat transfer coefficient on the interior surfaces of the van is 1.2 Btu/h · ft² · °F. The air flow over the exterior surfaces can be assumed to be turbulent because of the intense vibrations involved, and the heat transfer coefficient on the front and back surfaces can be taken to be equal to that on the top surface. Disregarding any heat gain or loss by radiation, determine the rate of heat transfer from the ambient air to the van.

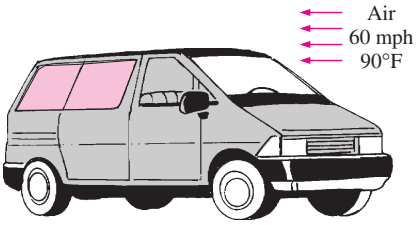


FIGURE P7–92E

7–93 Consider a house that is maintained at a constant temperature of 22°C. One of the walls of the house has three single-pane glass windows that are 1.5 m high and 1.2 m long. The glass (k = 0.78 W/m · °C) is 0.5 cm thick, and the heat transfer coefficient on the inner surface of the glass is 8 W/m² · C. Now winds at 60 km/h start to blow parallel to the surface of this wall. If the air temperature outside is −2°C, determine the rate of heat loss through the windows of this wall. Assume radiation heat transfer to be negligible.

7–94 Consider a person who is trying to keep cool on a hot summer day by turning a fan on and exposing his body to air flow. The air temperature is 32°C, and the fan is blowing air at a velocity of 5 m/s. The surrounding surfaces are at 40°C, and the emissivity of the person can be taken to be 0.9. If the person is doing light work and generating sensible heat at a rate of 90 W, determine the average temperature of the outer surface (skin or clothing) of the person. The average human body can be treated as a 30-cm-diameter cylinder with an exposed surface area of 1.7 m². *Answer:* 36.2°C

7–95 Four power transistors, each dissipating 12 W, are mounted on a thin vertical aluminum plate (k = 237 W/m · °C) 22 cm × 22 cm in size. The heat generated by the transistors is to be dissipated by both surfaces of the plate to the surrounding air at 20°C, which is blown over the plate by a fan at a velocity of 250 m/min. The entire plate can be assumed to be nearly isothermal, and the exposed surface area of the transistor can be taken to be equal to its base area. Determine the temperature of the aluminum plate.

7–96 A 3-m-internal-diameter spherical tank made of 1-cm-thick stainless steel (k = 15 W/m · °C) is used to store iced water at 0°C. The tank is located outdoors at 30°C and is subjected to winds at 25 km/h. Assuming the entire steel tank to be at 0°C and thus its thermal resistance to be negligible, determine (a) the rate of heat transfer to the iced water in the tank and (b) the amount of ice at 0°C that melts during a 24-h period. The heat of fusion of water at atmospheric pressure is h_{if} = 333.7 kJ/kg. Disregard any heat transfer by radiation.

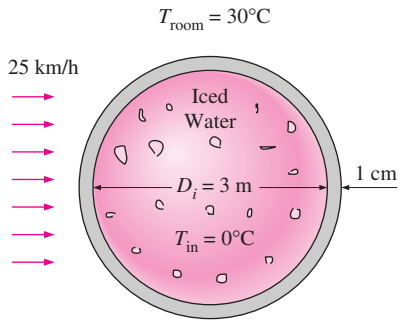


FIGURE P7–96

7–97 Repeat Problem 7–96, assuming the inner surface of the tank to be at 0°C but by taking the thermal resistance of the tank and heat transfer by radiation into consideration. Assume the average surrounding surface temperature for radiation exchange to be 15°C and the outer surface of the tank to have an emissivity of 0.9. *Answers:* (a) 9630 W, (b) 2493 kg

7–98E A transistor with a height of 0.25 in. and a diameter of 0.22 in. is mounted on a circuit board. The transistor is cooled by air flowing over it at a velocity of 500 ft/min. If the air temperature is 120°F and the transistor case temperature is not to exceed 180°F, determine the amount of power this transistor can dissipate safely.

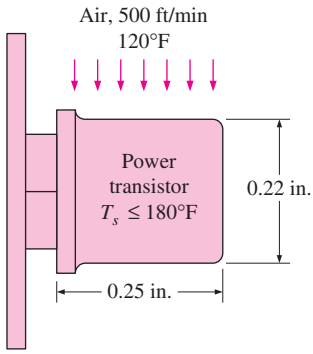


FIGURE P7–98E

7–99 The roof of a house consists of a 15-cm-thick concrete slab (k = 2 W/m · °C) 15 m wide and 20 m long. The convection heat transfer coefficient on the inner surface of the roof is 5 W/m^2 · °C. On a clear winter night, the ambient air is reported to be at 10°C, while the night sky temperature is 100 K. The house and the interior surfaces of the wall are maintained at a constant temperature of 20°C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfer, determine the rate of heat transfer through the roof when wind at 60 km/h is blowing over the roof.

If the house is heated by a furnace burning natural gas with an efficiency of 85 percent, and the price of natural gas is $0.60/therm (1 therm = 105,500 kJ of energy content), determine the money lost through the roof that night during a 14-h period. *Answers:* 28 kW, $9.44

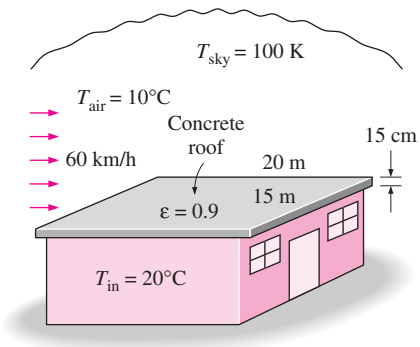


FIGURE P7–99

7–100 Steam at 250°C flows in a stainless steel pipe (k = 15 W/m · °C) whose inner and outer diameters are 4 cm and 4.6 cm, respectively. The pipe is covered with 3.5-cm-thick glass wool insulation (k = 0.038 W/m · °C) whose outer surface has an emissivity of 0.3. Heat is lost to the surrounding air and surfaces at 3°C by convection and radiation. Taking the heat transfer coefficient inside the pipe to be 80 W/m^2 · °C, determine the rate of heat loss from the steam per unit length of the pipe when air is flowing across the pipe at 4 m/s.

7–101 The boiling temperature of nitrogen at atmospheric pressure at sea level (1 atm pressure) is −196°C. Therefore, nitrogen is commonly used in low-temperature scientific studies, since the temperature of liquid nitrogen in a tank open to the atmosphere will remain constant at −196°C until it is depleted. Any heat transfer to the tank will result in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg and a density of 810 kg/m^3 at 1 atm.

Consider a 4-m-diameter spherical tank that is initially filled with liquid nitrogen at 1 atm and −196°C. The tank is exposed to 20°C ambient air and 40 km/h winds. The temperature of the thin-shelled spherical tank is observed to be almost the same as the temperature of the nitrogen inside. Disregarding any radiation heat exchange, determine the rate of evaporation of the liquid nitrogen in the tank as a result of heat transfer from the ambient air if the tank is (*a*) not insulated, (*b*) insulated with 5-cm-thick fiberglass insulation (k = 0.035 W/m · °C), and (*c*) insulated with 2-cm-thick superinsulation that has an effective thermal conductivity of 0.00005 W/m · °C.

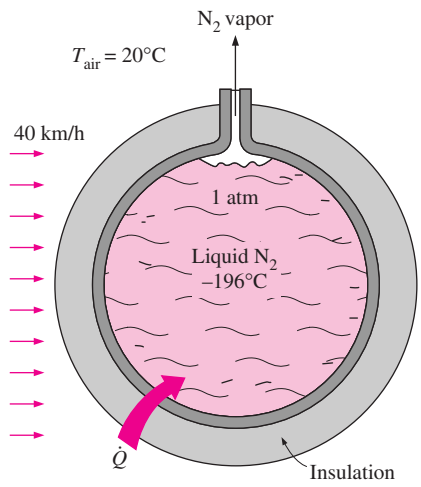


FIGURE P7–101

7–102 Repeat Problem 7–101 for liquid oxygen, which has a boiling temperature of −183°C, a heat of vaporization of 213 kJ/kg, and a density of 1140 kg/m^3 at 1 atm pressure.

7–103 A 0.3-cm-thick, 12-cm-high, and 18-cm-long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.06 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 16 W/m · °C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to the ambient air at 30°C, which is forced to flow over the surface by a fan at a free-stream velocity of 400 m/min. Determine the temperatures on the two sides of the circuit board.

7–104E It is well known that cold air feels much colder in windy weather than what the thermometer reading indicates because of the "chilling effect" of the wind. This effect is due to the increase in the convection heat transfer coefficient with increasing air velocities. The *equivalent windchill temperature* in °F is given by (1993 *ASHRAE Handbook of Fundamentals,* Atlanta, GA, p. 8.15)

$$T_{\text{equiv}} = 91.4 - (91.4 - T_{\text{ambient}})(0.475 - 0.0203\mathcal{V} + 0.304\sqrt{\mathcal{V}})$$

where $\mathcal{V}$ is the wind velocity in mph and T_{ambient} is the ambient air temperature in °F in calm air, which is taken to be air with light winds at speeds up to 4 mph. The constant 91.4°F in the above equation is the mean skin temperature of a resting person in a comfortable environment. Windy air at a temperature T_{ambient} and velocity $\mathcal{V}$ will feel as cold as calm air at a temperature T_{equiv}. The equation above is valid for winds up to 43 mph. Winds at higher velocities produce little additional chilling effect. Determine the equivalent wind chill temperature of an environment at 10°F at wind speeds of 10, 20, 30, and 40 mph. Exposed flesh can freeze within one minute at a temperature below −25°F in calm weather. Does a person need to be concerned about this possibility in any of the cases above?

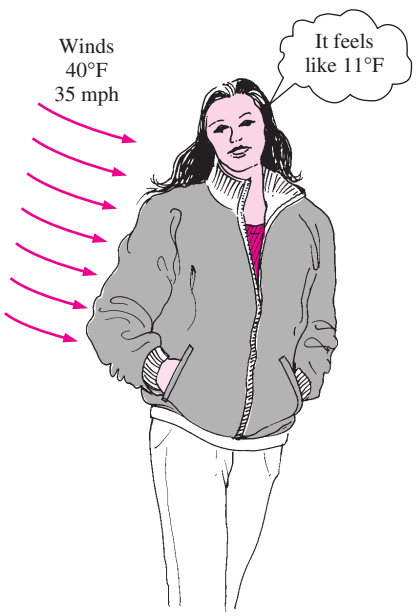


FIGURE P7–104E

7–105E Reconsider Problem 7–104E. Using EES (or other) software, plot the equivalent wind chill temperatures in ºF as a function of wind velocity in the range of 4 mph to 100 mph for ambient temperatures of 20ºF, 40ºF and 60ºF. Discuss the results.

Design and Essay Problems

7–106 On average, superinsulated homes use just 15 percent of the fuel required to heat the same size conventional home built before the energy crisis in the 1970s. Write an essay on superinsulated homes, and identify the features that make them so energy efficient as well as the problems associated with them. Do you think superinsulated homes will be economically attractive in your area?

7–107 Conduct this experiment to determine the heat loss coefficient of your house or apartment in W/ºC or But/h · ºF. First make sure that the conditions in the house are steady and the house is at the set temperature of the thermostat. Use an outdoor thermometer to monitor outdoor temperature. One evening, using a watch or timer, determine how long the heater was on during a 3-h period and the average outdoor temperature during that period. Then using the heat output rating of your heater, determine the amount of heat supplied. Also, estimate the amount of heat generation in the house during that period by noting the number of people, the total wattage of lights that were on, and the heat generated by the appliances and equipment. Using that information, calculate the average rate of heat loss from the house and the heat loss coefficient.

7–108 The decision of whether to invest in an energy-saving measure is made on the basis of the length of time for it to pay for itself in projected energy (and thus cost) savings. The easiest way to reach a decision is to calculate the simple payback period by simply dividing the installed cost of the measure by the annual cost savings and comparing it to the lifetime of the installation. This approach is adequate for short payback periods (less than 5 years) in stable economies with low interest rates (under 10 percent) since the error involved is no larger than the uncertainties. However, if the payback period is long, it may be necessary to consider the *interest rate* if the money is to be borrowed, or the *rate of return* if the money is invested elsewhere instead of the energy conservation measure. For example, a simple payback period of five years corresponds to 5.0, 6.12, 6.64, 7.27, 8.09, 9.919, 10.84, and 13.91 for an interest rate (or return on investment) of 0, 6, 8, 10, 12, 14, 16, and 18 percent, respectively. Finding out the proper relations from engineering economics books, determine the payback periods for the interest rates given above corresponding to simple payback periods of 1 through 10 years.

7–109 Obtain information on frostbite and the conditions under which it occurs. Using the relation in Problem 7–104E, prepare a table that shows how long people can stay in cold and windy weather for specified temperatures and wind speeds before the exposed flesh is in danger of experiencing frostbite.

7–110 Write an article on forced convection cooling with air, helium, water, and a dielectric liquid. Discuss the advantages and disadvantages of each fluid in heat transfer. Explain the circumstances under which a certain fluid will be most suitable for the cooling job.

CHAPTER 8

INTERNAL FORCED CONVECTION

Liquid or gas flow through pipes or ducts is commonly used in heating and cooling applications. The fluid in such applications is forced to flow by a fan or pump through a tube that is sufficiently long to accomplish the desired heat transfer. In this chapter we will pay particular attention to the determination of the *friction factor* and *convection coefficient* since they are directly related to the *pressure drop* and *heat transfer rate,* respectively. These quantities are then used to determine the pumping power requirement and the required tube length.

There is a fundamental difference between external and internal flows. In *external flow,* considered in Chapter 7, the fluid has a free surface, and thus the boundary layer over the surface is free to grow indefinitely. In *internal flow,* however, the fluid is completely confined by the inner surfaces of the tube, and thus there is a limit on how much the boundary layer can grow.

We start this chapter with a general physical description of internal flow, and the *mean velocity* and *mean temperature.* We continue with the discussion of the *hydrodynamic* and *thermal entry lengths, developing flow,* and *fully developed flow.* We then obtain the velocity and temperature profiles for fully developed laminar flow, and develop relations for the friction factor and Nusselt number. Finally we present empirical relations for developing and fully developed flows, and demonstrate their use.

CONTENTS

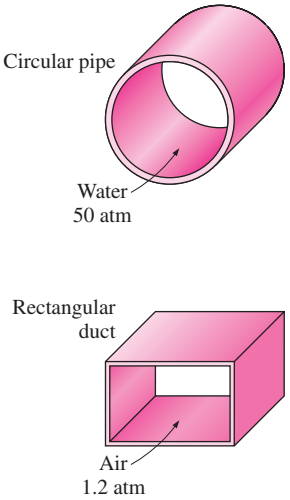


FIGURE 8–1
Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but the noncircular pipes cannot.

8–1 ▪ INTRODUCTION

You have probably noticed that most fluids, especially liquids, are transported in circular pipes. This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing any distortion. Noncircular pipes are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small and the manufacturing and installation costs are lower (Fig. 8–1). For a fixed surface area, the circular tube gives the most heat transfer for the least pressure drop, which explains the overwhelming popularity of circular tubes in heat transfer equipment.

The terms *pipe, duct, tube,* and *conduit* are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as *pipes* (especially when the fluid is a liquid), and the flow sections of noncircular cross section as *ducts* (especially when the fluid is a gas). Small diameter pipes are usually referred to as *tubes.* Given this uncertainty, we will use more descriptive phrases (such as *a circular pipe* or *a rectangular duct*) whenever necessary to avoid any misunderstandings.

Although the theory of fluid flow is reasonably well understood, theoretical solutions are obtained only for a few simple cases such as the fully developed laminar flow in a circular pipe. Therefore, we must rely on the experimental results and the empirical relations obtained for most fluid flow problems rather than closed form analytical solutions. Noting that the experimental results are obtained under carefully controlled laboratory conditions, and that no two systems are exactly alike, we must not be so naive as to view the results obtained as "exact." An error of 10 percent (or more) in friction or convection coefficient calculated using the relations in this chapter is the "norm" rather than the "exception."

Perhaps we should mention that the friction between the fluid layers in a tube may cause a slight rise in fluid temperature as a result of mechanical energy being converted to thermal energy. But this *frictional heating* is too small to warrant any consideration in calculations, and thus is disregarded. For example, in the absence of any heat transfer, no noticeable difference will be detected between the inlet and exit temperatures of a fluid flowing in a tube. The primary consequence of friction in fluid flow is pressure drop. Thus, it is reasonable to assume that any temperature change in the fluid is due to heat transfer. But frictional heating must be considered for flows that involve highly viscous fluids with large velocity gradients.

In most practical applications, the flow of a fluid through a pipe or duct can be approximated to be *one-dimensional,* and thus the properties can be assumed to vary in *one* direction only (the direction of flow). As a result, all properties are *uniform* at any cross section normal to the flow direction, and the properties are assumed to have *bulk average values* over the cross section. But the values of the properties at a cross section *may* change with time unless the flow is steady.

8–2 ▪ MEAN VELOCITY AND MEAN TEMPERATURE

In external flow, the free-stream velocity served as a convenient reference velocity for use in the evaluation of the Reynolds number and the friction

coefficient. In internal flow, there is no free stream and thus we need an alternative. The fluid velocity in a tube changes from zero at the surface because of the no-slip condition, to a maximum at the tube center. Therefore, it is convenient to work with an **average** or **mean velocity** $\mathcal{V}_m$, which remains constant for incompressible flow when the cross sectional area of the tube is constant.

The mean velocity in actual heating and cooling applications may change somewhat because of the changes in density with temperature. But, in practice, we evaluate the fluid properties at some average temperature and treat them as constants. The convenience in working with constant properties usually more than justifies the slight loss in accuracy.

The value of the mean velocity $\mathcal{V}_m$ in a tube is determined from the requirement that the *conservation of mass principle* be satisfied (Fig. 8–2). That is,

$$\dot{m} = \rho \mathcal{V}_m A_c = \int_{A_c} \rho \mathcal{V}(r, x) dA_c \tag{8-1}$$

where $\dot{m}$ is the mass flow rate, ρ is the density, A_c is the cross sectional area, and $\mathcal{V}(r, x)$ is the velocity profile. Then the mean velocity for incompressible flow in a circular tube of radius R can be expressed as

$$\mathcal{V}_m = \frac{\int_{A_c} \rho \mathcal{V}(r, x) dA_c}{\rho A_c} = \frac{\int_0^R \rho \mathcal{V}(r, x) 2\pi r dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R \mathcal{V}(r, x) r dr \tag{8-2}$$

Therefore, when we know the mass flow rate or the velocity profile, the mean velocity can be determined easily.

When a fluid is heated or cooled as it flows through a tube, the temperature of the fluid at any cross section changes from T_s at the surface of the wall to some maximum (or minimum in the case of heating) at the tube center. In fluid flow it is convenient to work with an **average** or **mean temperature** T_m that remains uniform at a cross section. Unlike the mean velocity, the mean temperature T_m *will change* in the flow direction whenever the fluid is heated or cooled.

The value of the mean temperature T_m is determined from the requirement that the *conservation of energy* principle be satisfied. That is, the energy transported by the fluid through a cross section in actual flow must be equal to the energy that would be transported through the same cross section if the fluid were at a constant temperature T_m. This can be expressed mathematically as (Fig. 8–3)

$$\dot{E}_{\text{fluid}} = \dot{m} C_p T_m = \int_{\dot{m}} C_p T \delta \dot{m} = \int_{A_c} \rho C_p T \mathcal{V} dA_c \tag{8-3}$$

where C_p is the specific heat of the fluid. Note that the product $\dot{m} C_p T_m$ at any cross section along the tube represents the *energy flow* with the fluid at that cross section. Then the mean temperature of a fluid with constant density and specific heat flowing in a circular pipe of radius R can be expressed as

$$T_m = \frac{\int_{\dot{m}} C_p T \delta \dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T (\rho \mathcal{V} 2\pi r dr)}{\rho \mathcal{V}_m (\pi R^2) C_p} = \frac{2}{\mathcal{V}_m R^2} \int_0^R T(r, x)\, \mathcal{V}(r, x)\, r dr \tag{8-4}$$

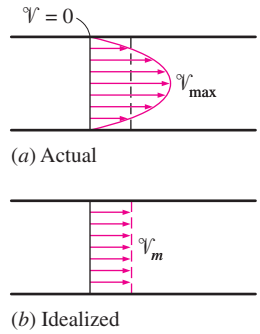


FIGURE 8–2
Actual and idealized velocity profiles for flow in a tube (the mass flow rate of the fluid is the same for both cases).

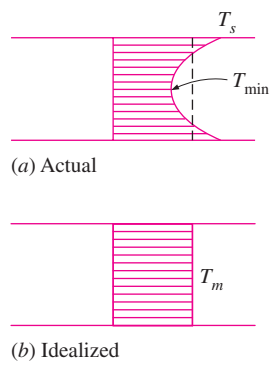


FIGURE 8–3
Actual and idealized temperature profiles for flow in a tube (the rate at which energy is transported with the fluid is the same for both cases).

Note that the mean temperature T_m of a fluid changes during heating or cooling. Also, the fluid properties in internal flow are usually evaluated at the *bulk mean fluid temperature,* which is the arithmetic average of the mean temperatures at the inlet and the exit. That is, $T_b = (T_{m,\,i} + T_{m,\,e})/2$.

Laminar and Turbulent Flow In Tubes

Flow in a tube can be laminar or turbulent, depending on the flow conditions. Fluid flow is streamlined and thus laminar at low velocities, but turns turbulent as the velocity is increased beyond a critical value. Transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some range of velocity where the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most pipe flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small diameter tubes or narrow passages.

For flow in a circular tube, the Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu} \tag{8-5}$$

where $\mathcal{V}_m$ is the mean fluid velocity, D is the diameter of the tube, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid.

For flow through noncircular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the **hydraulic diameter** D_h defined as (Fig. 8–4)

$$D_h = \frac{4A_c}{p} \tag{8-6}$$

where A_c is the cross sectional area of the tube and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular tubes since

Circular tubes: $$D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D$$

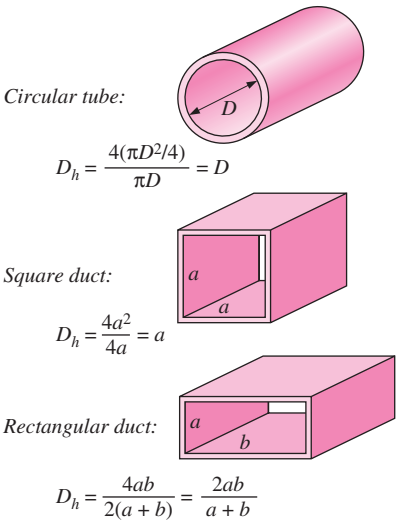


FIGURE 8–4
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

It certainly is desirable to have precise values of Reynolds numbers for laminar, transitional, and turbulent flows, but this is not the case in practice. This is because the transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and the *fluctuations in the flow.* Under most practical conditions, the flow in a tube is laminar for Re < 2300, turbulent for Re > 10,000, and transitional in between. That is,

$\mathrm{Re} < 2300$	laminar flow
$2300 \le \mathrm{Re} \le 10{,}000$	transitional flow
$\mathrm{Re} > 10{,}000$	turbulent flow

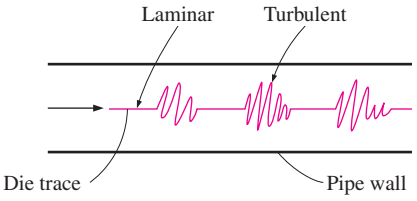


FIGURE 8–5
In the transitional flow region of $2300 \le \mathrm{Re} \le 4000$, the flow switches between laminar and turbulent randomly.

In transitional flow, the flow switches between laminar and turbulent randomly (Fig. 8–5). It should be kept in mind that laminar flow can be maintained at much higher Reynolds numbers in very smooth pipes by avoiding flow disturbances and tube vibrations. In such carefully controlled

experiments, laminar flow has been maintained at Reynolds numbers of up to 100,000.

8–3 ▪ THE ENTRANCE REGION

Consider a fluid entering a circular tube at a uniform velocity. As in external flow, the fluid particles in the layer in contact with the surface of the tube will come to a complete stop. This layer will also cause the fluid particles in the adjacent layers to slow down gradually as a result of friction. To make up for this velocity reduction, the velocity of the fluid at the midsection of the tube will have to increase to keep the mass flow rate through the tube constant. As a result, a *velocity boundary layer* develops along the tube. The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube, as shown in Figure 8–6.

The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the **hydrodynamic entrance region,** and the length of this region is called the **hydrodynamic entry length** L_h. Flow in the entrance region is called *hydrodynamically developing flow* since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the **hydrodynamically fully developed region.** The velocity profile in the fully developed region is *parabolic* in laminar flow and somewhat *flatter* in turbulent flow due to eddy motion in radial direction.

Now consider a fluid at a uniform temperature entering a circular tube whose surface is maintained at a different temperature. This time, the fluid particles in the layer in contact with the surface of the tube will assume the surface temperature. This will initiate convection heat transfer in the tube and the development of a *thermal boundary layer* along the tube. The thickness of this boundary layer also increases in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube, as shown in Figure 8–7.

The region of flow over which the thermal boundary layer develops and reaches the tube center is called the **thermal entrance region,** and the length of this region is called the **thermal entry length** L_t. Flow in the thermal entrance region is called *thermally developing flow* since this is the region where the temperature profile develops. The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_s - T)/(T_s - T_m)$ remains unchanged is called the **thermally fully developed region.** The region in which the flow is both hydrodynamically and thermally developed and thus both the velocity and dimensionless temperature profiles remain unchanged is called *fully developed flow.* That is,

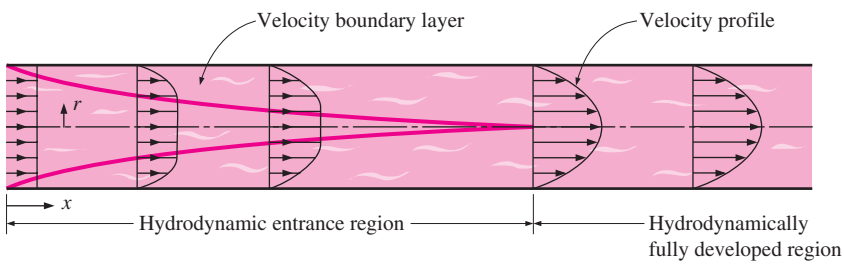


FIGURE 8–6
The development of the velocity boundary layer in a tube. (The developed mean velocity profile will be parabolic in laminar flow, as shown, but somewhat blunt in turbulent flow.)

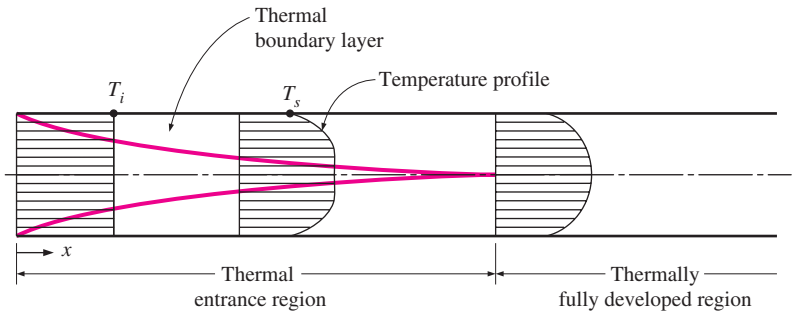


FIGURE 8–7
The development of the thermal boundary layer in a tube. (The fluid in the tube is being cooled.)

Hydrodynamically fully developed:
$$\frac{\partial \mathcal{V}(r, x)}{\partial x} = 0 \longrightarrow \mathcal{V} = \mathcal{V}(r) \tag{8-7}$$

Thermally fully developed:
$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right] = 0 \tag{8-8}$$

The friction factor is related to the shear stress at the surface, which is related to the slope of the velocity profile at the surface. Noting that the velocity profile remains unchanged in the hydrodynamically fully developed region, the friction factor also remains constant in that region. A similar argument can be given for the heat transfer coefficient in the thermally fully developed region.

In a thermally fully developed region, the derivative of $(T_s - T)/(T_s - T_m)$ with respect to x is zero by definition, and thus $(T_s - T)/(T_s - T_m)$ is independent of x. Then the derivative of $(T_s - T)/(T_s - T_m)$ with respect r must also be independent of x. That is,

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=R} = \frac{-(\partial T/\partial r)|_{r=R}}{T_s - T_m} \neq f(x) \tag{8-9}$$

Surface heat flux can be expressed as

$$\dot{q}_s = h_x(T_s - T_m) = k\frac{\partial T}{\partial r}\bigg|_{r=R} \longrightarrow h_x = \frac{k(\partial T/\partial r)|_{r=R}}{T_s - T_m} \tag{8-10}$$

which, from Eq. 8–9, is independent of x. Thus we conclude that *in the thermally fully developed region of a tube, the local convection coefficient is constant* (does not vary with x). Therefore, both *the friction and convection coefficients remain constant in the fully developed region of a tube.*

Note that the *temperature profile* in the thermally fully developed region may vary with x in the flow direction. That is, unlike the velocity profile, the temperature profile can be different at different cross sections of the tube in the developed region, and it usually is. However, the dimensionless temperature profile defined above remains unchanged in the thermally developed region when the temperature or heat flux at the tube surface remains constant.

During laminar flow in a tube, the magnitude of the dimensionless Prandtl number Pr is a measure of the relative growth of the velocity and thermal boundary layers. For fluids with $\text{Pr} \approx 1$, such as gases, the two boundary layers essentially coincide with each other. For fluids with $\text{Pr} \gg 1$, such as oils, the velocity boundary layer outgrows the thermal boundary layer. As a result,

the hydrodynamic entry length is smaller than the thermal entry length. The opposite is true for fluids with Pr ≪ 1 such as liquid metals.

Consider a fluid that is being heated (or cooled) in a tube as it flows through it. The friction factor and the heat transfer coefficient are *highest* at the tube inlet where the thickness of the boundary layers is zero, and decrease gradually to the fully developed values, as shown in Figure 8–8. Therefore, the pressure drop and heat flux are *higher* in the entrance regions of a tube, and the effect of the entrance region is always to *enhance* the average friction and heat transfer coefficients for the entire tube. This enhancement can be significant for short tubes but negligible for long ones.

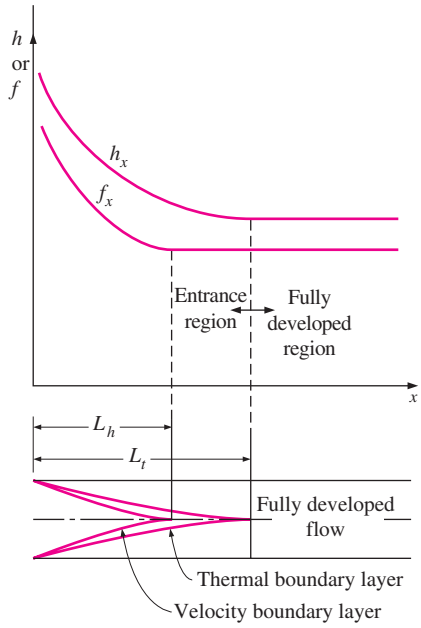


FIGURE 8–8
Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube (Pr > 1).

Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the tube entrance where the friction coefficient reaches within about 2 percent of the fully developed value. In *laminar flow,* the hydrodynamic and thermal entry lengths are given approximately as [see Kays and Crawford (1993), Ref. 13, and Shah and Bhatti (1987), Ref. 25]

$$L_{h,\ \text{laminar}} \approx 0.05\ \text{Re}\ D \tag{8-11}$$

$$L_{t,\ \text{laminar}} \approx 0.05\ \text{Re Pr}\ D = \text{Pr}\ L_{h,\ \text{laminar}} \tag{8-12}$$

For Re = 20, the hydrodynamic entry length is about the size of the diameter, but increases linearly with the velocity. In the limiting case of Re = 2300, the hydrodynamic entry length is 115*D*.

In *turbulent flow,* the intense mixing during random fluctuations usually overshadows the effects of momentum and heat diffusion, and therefore the hydrodynamic and thermal entry lengths are of about the same size and independent of the Prandtl number. Also, *the friction factor and the heat transfer coefficient remain constant in fully developed laminar or turbulent flow* since the velocity and normalized temperature profiles do not vary in the flow direction. The hydrodynamic entry length for turbulent flow can be determined from [see Bhatti and Shah (1987), Ref. 1, and Zhi-qing (1982), Ref. 31]

$$L_{h,\ \text{turbulent}} = 1.359\ \text{Re}^{1/4} \tag{8-13}$$

The hydrodynamic entry length is much shorter in turbulent flow, as expected, and its dependence on the Reynolds number is weaker. It is 11*D* at Re = 10,000, and increases to 43*D* at Re = 10^5. In practice, it is generally agreed that the entrance effects are confined within a tube length of 10 diameters, and the hydrodynamic and thermal entry lengths are approximately taken to be

$$L_{h,\ \text{turbulent}} \approx L_{t,\ \text{turbulent}} \approx 10D \tag{8-14}$$

The variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux is given in Figure 8–9 for the range of Reynolds numbers encountered in heat transfer equipment. We make these important observations from this figure:

- The Nusselt numbers and thus the convection heat transfer coefficients are much higher in the entrance region.

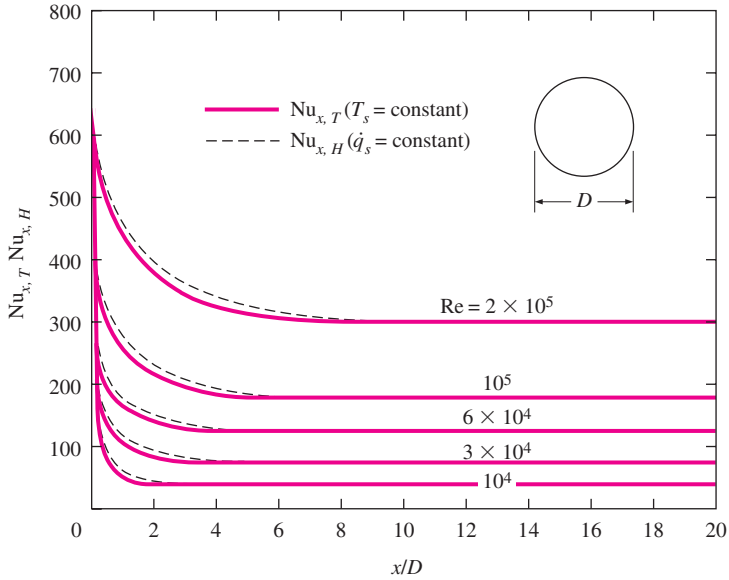


FIGURE 8–9
Variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux [Deissler (1953), Ref. 4].

- The Nusselt number reaches a constant value at a distance of less than 10 diameters, and thus the flow can be assumed to be fully developed for $x > 10D$.
- The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions. Therefore, Nusselt number is insensitive to the type of thermal boundary condition, and the turbulent flow correlations can be used for either type of boundary condition.

Precise correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature. However, the tubes used in practice in forced convection are usually several times the length of either entrance region, and thus the flow through the tubes is often assumed to be fully developed for the entire length of the tube. This simplistic approach gives *reasonable* results for long tubes and *conservative* results for short ones.

8–4 ▪ GENERAL THERMAL ANALYSIS

You will recall that in the absence of any work interactions (such as electric resistance heating), the conservation of energy equation for the steady flow of a fluid in a tube can be expressed as (Fig. 8–10)

$$\dot{Q} = \dot{m}C_p(T_e - T_i) \qquad \text{(W)} \tag{8-15}$$

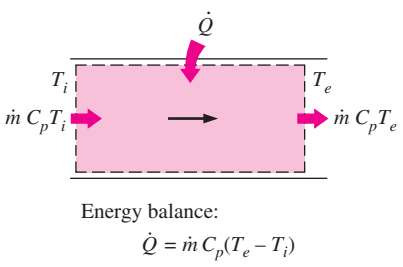


FIGURE 8–10
The heat transfer to a fluid flowing in a tube is equal to the increase in the energy of the fluid.

where T_i and T_e are the mean fluid temperatures at the inlet and exit of the tube, respectively, and $\dot{Q}$ is the rate of heat transfer to or from the fluid. Note that the temperature of a fluid flowing in a tube remains constant in the absence of any energy interactions through the wall of the tube.

The thermal conditions at the surface can usually be approximated with reasonable accuracy to be *constant surface temperature* (T_s = constant) or *constant surface heat flux* ($\dot{q}_s$ = constant). For example, the constant surface

temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube. The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

Surface heat flux is expressed as

$$\dot{q}_s = h_x(T_s - T_m) \qquad (\text{W/m}^2) \tag{8-16}$$

where h_x is the *local* heat transfer coefficient and T_s and T_m are the surface and the mean fluid temperatures at that location. Note that the mean fluid temperature T_m of a fluid flowing in a tube must change during heating or cooling. Therefore, when $h_x = h =$ constant, the surface temperature T_s must change when $\dot{q}_s =$ constant, and the surface heat flux $\dot{q}_s$ must change when $T_s =$ constant. Thus we may have either $T_s =$ constant or $\dot{q}_s =$ constant at the surface of a tube, but not both. Next we consider convection heat transfer for these two common cases.

Constant Surface Heat Flux ($\dot{q}_s$ = constant)

In the case of $\dot{q}_s =$ constant, the rate of heat transfer can also be expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i) \qquad (\text{W}) \tag{8-17}$$

Then the mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p} \tag{8-18}$$

Note that the mean fluid temperature increases *linearly* in the flow direction in the case of constant surface heat flux, since the surface area increases linearly in the flow direction (A_s is equal to the perimeter, which is constant, times the tube length).

The surface temperature in the case of constant surface heat flux $\dot{q}_s$ can be determined from

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h} \tag{8-19}$$

In the fully developed region, the surface temperature T_s will also increase linearly in the flow direction since h is constant and thus $T_s - T_m =$ constant (Fig. 8–11). Of course this is true when the fluid properties remain constant during flow.

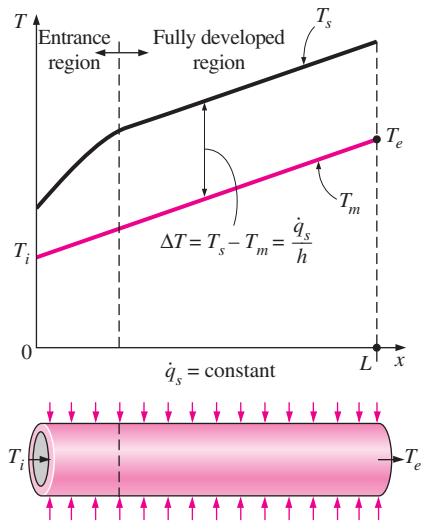


FIGURE 8–11
Variation of the *tube surface* and the *mean fluid* temperatures along the tube for the case of constant surface heat flux.

The slope of the mean fluid temperature T_m on a T-x diagram can be determined by applying the steady-flow energy balance to a tube slice of thickness dx shown in Figure 8–12. It gives

$$\dot{m} C_p \, dT_m = \dot{q}_s (p\,dx) \longrightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant} \tag{8-20}$$

where p is the perimeter of the tube.

Noting that both $\dot{q}_s$ and h are constants, the differentiation of Eq. 8–19 with respect to x gives

$$\frac{dT_m}{dx} = \frac{dT_s}{dx} \tag{8-21}$$

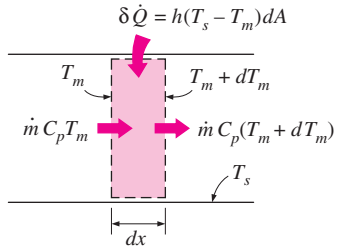


FIGURE 8–12
Energy interactions for a differential control volume in a tube.

Also, the requirement that the dimensionless temperature profile remains unchanged in the fully developed region gives

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \longrightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0 \longrightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx} \tag{8-22}$$

since $T_s - T_m$ = constant. Combining Eqs. 8–20, 8–21, and 8–22 gives

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant} \tag{8-23}$$

Then we conclude that *in fully developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x and thus the shape of the temperature profile does not change along the tube* (Fig. 8–13).

For a circular tube, $p = 2\pi R$ and $\dot{m} = \rho \mathcal{V}_m A_c = \rho \mathcal{V}_m (\pi R^2)$, and Eq. 8–23 becomes

Circular tube:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho \mathcal{V}_m C_p R} = \text{constant} \tag{8-24}$$

where $\mathcal{V}_m$ is the mean velocity of the fluid.

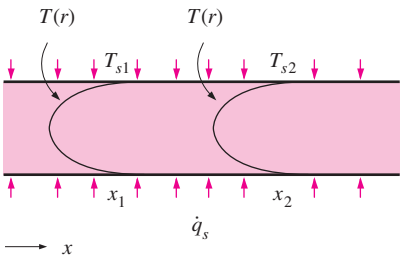


FIGURE 8–13
The shape of the temperature profile remains unchanged in the fully developed region of a tube subjected to constant surface heat flux.

Constant Surface Temperature (T_s = constant)

From Newton's law of cooling, the rate of heat transfer to or from a fluid flowing in a tube can be expressed as

$$\dot{Q} = hA_s \Delta T_{\text{ave}} = hA_s (T_s - T_m)_{\text{ave}} \qquad \text{(W)} \tag{8-25}$$

where h is the average convection heat transfer coefficient, A_s is the heat transfer surface area (it is equal to πDL for a circular pipe of length L), and ΔT_{ave} is some appropriate *average* temperature difference between the fluid and the surface. Below we discuss two suitable ways of expressing ΔT_{ave}.

In the constant surface temperature (T_s = constant) case, ΔT_{ave} can be expressed *approximately* by the **arithmetic mean temperature difference** ΔT_{am} as

$$\Delta T_{\text{ave}} \approx \Delta T_{\text{am}} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2}$$
$$= T_s - T_b \tag{8-26}$$

where $T_b = (T_i + T_e)/2$ is the *bulk mean fluid temperature,* which is the *arithmetic average* of the mean fluid temperatures at the inlet and the exit of the tube.

Note that the *arithmetic mean temperature difference* ΔT_{am} is simply the *average* of the *temperature differences* between the surface and the fluid at the inlet and the exit of the tube. Inherent in this definition is the assumption that the mean fluid temperature varies linearly along the tube, which is hardly ever the case when T_s = constant. This simple approximation often gives acceptable results, but not always. Therefore, we need a better way to evaluate ΔT_{ave}.

Consider the heating of a fluid in a tube of constant cross section whose inner surface is maintained at a constant temperature of T_s. We know that the

mean temperature of the fluid T_m will increase in the flow direction as a result of heat transfer. The energy balance on a differential control volume shown in Figure 8–12 gives

$$\dot{m}C_p\,dT_m = h(T_s - T_m)dA_s \tag{8-27}$$

That is, the increase in the energy of the fluid (represented by an increase in its mean temperature by dT_m) is equal to the heat transferred to the fluid from the tube surface by convection. Noting that the differential surface area is $dA_s = p\,dx$, where p is the perimeter of the tube, and that $dT_m = -d(T_s - T_m)$, since T_s is constant, the relation above can be rearranged as

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m}C_p}\,dx \tag{8-28}$$

Integrating from $x = 0$ (tube inlet where $T_m = T_i$) to $x = L$ (tube exit where $T_m = T_e$) gives

$$\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}C_p} \tag{8-29}$$

where $A_s = pL$ is the surface area of the tube and h is the constant *average* convection heat transfer coefficient. Taking the exponential of both sides and solving for T_e gives the following relation which is very useful for the determination of the *mean fluid temperature at the tube exit:*

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p) \tag{8-30}$$

This relation can also be used to determine the mean fluid temperature $T_m(x)$ at any x by replacing $A_s = pL$ by px.

Note that the temperature difference between the fluid and the surface *decays exponentially* in the flow direction, and the rate of decay depends on the magnitude of the exponent $hA_x/\dot{m}C_p$, as shown in Figure 8–14. This dimensionless parameter is called the *number of transfer units,* denoted by NTU, and is a measure of the effectiveness of the heat transfer systems. For NUT > 5, the exit temperature of the fluid becomes almost equal to the surface temperature, $T_e \approx T_s$ (Fig. 8–15). Noting that the fluid temperature can approach the surface temperature but cannot cross it, an NTU of about 5 indicates that the limit is reached for heat transfer, and the heat transfer will not increase no matter how much we extend the length of the tube. A small value of NTU, on the other hand, indicates more opportunities for heat transfer, and the heat transfer will continue increasing as the tube length is increased. A large NTU and thus a large heat transfer surface area (which means a large tube) may be desirable from a heat transfer point of view, but it may be unacceptable from an economic point of view. The selection of heat transfer equipment usually reflects a compromise between heat transfer performance and cost.

Solving Eq. 8–29 for $\dot{m}C_p$ gives

$$\dot{m}C_p = -\frac{hA_s}{\ln[(T_s - T_e)/(T_s - T_i)]} \tag{8-31}$$

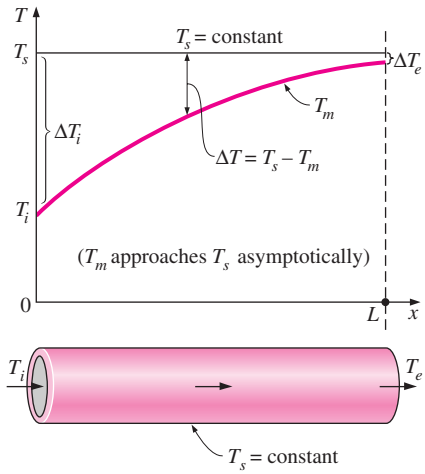
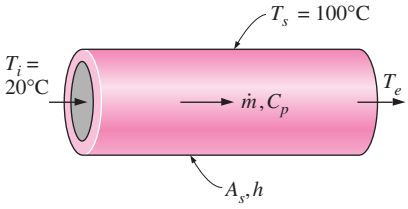


FIGURE 8–14
The variation of the *mean fluid* temperature along the tube for the case of constant temperature.



NTU = $hA_s/\dot{m}C_p$	T_e, °C
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

FIGURE 8–15
An NTU greater than 5 indicates that the fluid flowing in a tube will reach the surface temperature at the exit regardless of the inlet temperature.

Substituting this into Eq. 8–17, we obtain

$$\dot{Q} = hA_s\Delta T_{\ln} \tag{8-32}$$

where

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \tag{8-33}$$

is the **logarithmic mean temperature difference.** Note that $\Delta T_i = T_s - T_i$ and $\Delta T_e = T_s - T_e$ are the temperature differences between the surface and the fluid at the inlet and the exit of the tube, respectively. This $\Delta T_{\ln}$ relation appears to be prone to misuse, but it is practically fail-safe, since using T_i in place of T_e and vice versa in the numerator and/or the denominator will, at most, affect the sign, not the magnitude. Also, it can be used for both heating ($T_s > T_i$ and T_e) and cooling ($T_s < T_i$ and T_e) of a fluid in a tube.

The logarithmic mean temperature difference $\Delta T_{\ln}$ is obtained by tracing the actual temperature profile of the fluid along the tube, and is an *exact* representation of the *average temperature difference* between the fluid and the surface. It truly reflects the exponential decay of the local temperature difference. When ΔT_e differs from ΔT_i by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_e differs from ΔT_i by greater amounts. Therefore, we should always use the logarithmic mean temperature difference when determining the convection heat transfer in a tube whose surface is maintained at a constant temperature T_s.

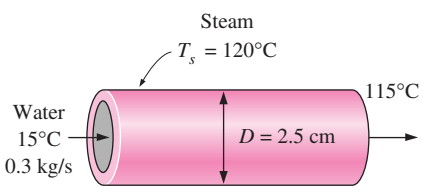


FIGURE 8–16
Schematic for Example 8–1.

EXAMPLE 8–1 **Heating of Water in a Tube by Steam**

Water enters a 2.5-cm-internal-diameter thin copper tube of a heat exchanger at 15°C at a rate of 0.3 kg/s, and is heated by steam condensing outside at 120°C. If the average heat transfer coefficient is 800 W/m^2 · C, determine the length of the tube required in order to heat the water to 115°C (Fig. 8–16).

SOLUTION Water is heated by steam in a circular tube. The tube length required to heat the water to a specified temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Fluid properties are constant. **3** The convection heat transfer coefficient is constant. **4** The conduction resistance of copper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam.

Properties The specific heat of water at the bulk mean temperature of (15 + 115)/2 = 65°C is 4187 J/kg · °C. The heat of condensation of steam at 120°C is 2203 kJ/kg (Table A-9).

Analysis Knowing the inlet and exit temperatures of water, the rate of heat transfer is determined to be

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (0.3\text{ kg/s})(4.187\text{ kJ/kg}\cdot\text{°C})(115\text{°C} - 15\text{°C}) = 125.6\text{ kW}$$

The logarithmic mean temperature difference is

$$\Delta T_e = T_s - T_e = 120°\text{C} - 115°\text{C} = 5°\text{C}$$
$$\Delta T_i = T_s - T_i = 120°\text{C} - 15°\text{C} = 105°\text{C}$$
$$\Delta T_{\ln} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} = \frac{5 - 105}{\ln(5/105)} = 32.85°\text{C}$$

The heat transfer surface area is

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\ln}} = \frac{125.6\text{ kW}}{(0.8\text{ kW/m}^2\cdot°\text{C})(32.85°\text{C})} = 4.78\text{ m}^2$$

Then the required length of tube becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{4.78\text{ m}^2}{\pi(0.025\text{ m})} = \mathbf{61\text{ m}}$$

Discussion The bulk mean temperature of water during this heating process is 65°C, and thus the *arithmetic* mean temperature difference is $\Delta T_{am} = 120 - 65 = 55°\text{C}$. Using ΔT_{am} instead of $\Delta T_{\ln}$ would give $L = 36$ m, which is grossly in error. This shows the importance of using the logarithmic mean temperature in calculations.

8–5 ▪ LAMINAR FLOW IN TUBES

We mentioned earlier that flow in tubes is laminar for Re < 2300, and that the flow is fully developed if the tube is sufficiently long (relative to the entry length) so that the entrance effects are negligible. In this section we consider the *steady laminar flow* of an *incompressible fluid* with *constant properties* in the *fully developed region* of a *straight circular tube*. We obtain the momentum and energy equations by applying momentum and energy balances to a differential volume element, and obtain the velocity and temperature profiles by solving them. Then we will use them to obtain relations for the friction factor and the Nusselt number. An important aspect of the analysis below is that it is one of the few available for viscous flow and forced convection.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $\mathcal{V}(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component v in the direction normal to flow is everywhere zero. There is no acceleration since the flow is steady.

Now consider a ring-shaped differential volume element of radius r, thickness dr, and length dx oriented coaxially with the tube, as shown in Figure 8–17. The pressure force acting on a submerged plane surface is the product of the pressure at the centroid of the surface and the surface area.

The volume element involves only pressure and viscous effects, and thus the pressure and shear forces must balance each other. A force balance on the volume element in the flow direction gives

$$(2\pi rdrP)_x - (2\pi rdrP)_{x+dx} + (2\pi rdx\tau)_r - (2\pi rdx\tau)_{r+dr} = 0 \qquad \textbf{(8-34)}$$

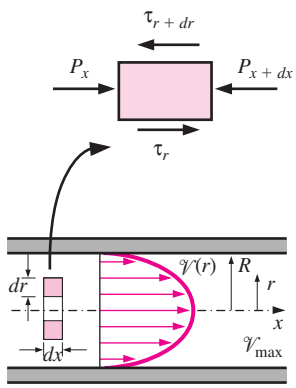


FIGURE 8–17
Free body diagram of a cylindrical fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal tube in fully developed steady flow.

which indicates that in fully developed flow in a tube, the viscous and pressure forces balance each other. Dividing by $2\pi drdx$ and rearranging,

$$r\frac{P_{x+dx}-P_x}{dx}+\frac{(r\tau)_{x+dr}-(r\tau)_r}{dr}=0 \tag{8-35}$$

Taking the limit as $dr, dx \to 0$ gives

$$r\frac{dP}{dx}+\frac{d(r\tau)}{dr}=0 \tag{8-36}$$

Substituting $\tau = -\mu(d\mathcal{V}/dr)$ and rearranging gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{d\mathcal{V}}{dr}\right)=\frac{dP}{dx} \tag{8-37}$$

The quantity $d\mathcal{V}/dr$ is negative in tube flow, and the negative sign is included to obtain positive values for τ. (Or, $d\mathcal{V}/dr = -d\mathcal{V}/dy$ since $y = R - r$.) The left side of this equation is a function of r and the right side is a function of x. The equality must hold for any value of r and x, and an equality of the form $f(r) = g(x)$ can happen only if both $f(r)$ and $g(x)$ are equal to constants. Thus we conclude that $dP/dx =$ constant. This can be verified by writing a force balance on a volume element of radius R and thickness dx (a slice of the tube), which gives $dP/dx = -2\tau_s/R$. Here τ_s is constant since the viscosity and the velocity profile are constants in the fully developed region. Therefore, $dP/dx =$ constant.

Equation 8–37 can be solved by rearranging and integrating it twice to give

$$\mathcal{V}(r)=\frac{1}{4\mu}\left(\frac{dP}{dx}\right)+C_1\ln r+C_2 \tag{8-38}$$

The velocity profile $\mathcal{V}(r)$ is obtained by applying the boundary conditions $\partial\mathcal{V}/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $\mathcal{V} = 0$ at $r = R$ (the no-slip condition at the tube surface). We get

$$\mathcal{V}(r)=-\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right) \tag{8-39}$$

Therefore, the velocity profile in fully developed laminar flow in a tube is *parabolic* with a maximum at the centerline and minimum at the tube surface. Also, the axial velocity $\mathcal{V}$ is positive for any r, and thus the axial pressure gradient dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).

The mean velocity is determined from its definition by substituting Eq. 8–39 into Eq. 8–2, and performing the integration. It gives

$$\mathcal{V}_m=\frac{2}{R^2}\int_0^R \mathcal{V}rdr=\frac{-2}{R^2}\int_0^R\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right)rdr=-\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right) \tag{8-40}$$

Combining the last two equations, the velocity profile is obtained to be

$$\mathcal{V}(r)=2\mathcal{V}_m\left(1-\frac{r^2}{R^2}\right) \tag{8-41}$$

This is a convenient form for the velocity profile since $\mathcal{V}_m$ can be determined easily from the flow rate information.

The maximum velocity occurs at the centerline, and is determined from Eq. 8–39 by substituting $r = 0$,

$$\mathcal{V}_{\max} = 2\mathcal{V}_m \tag{8-42}$$

Therefore, *the mean velocity is one-half of the maximum velocity.*

Pressure Drop

A quantity of interest in the analysis of tube flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx = constant, and integrate it from $x = 0$ where the pressure is P_1 to $x = L$ where the pressure is P_2. We get

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} = -\frac{\Delta P}{L} \tag{8-43}$$

Note that in fluid mechanics, the pressure drop ΔP is a positive quantity, and is defined as $\Delta P = P_1 - P_2$. Substituting Eq. 8–43 into the $\mathcal{V}_m$ expression in Eq. 8–40, the *pressure drop* can be expressed as

Laminar flow:

$$\Delta P = \frac{8\mu L\,\mathcal{V}_m}{R^2} = \frac{32\mu L\,\mathcal{V}_m}{D^2} \tag{8-44}$$

In practice, it is found convenient to express the pressure drop for all types of internal flows (laminar or turbulent flows, circular or noncircular tubes, smooth or rough surfaces) as (Fig. 8–18)

$$\Delta P = f\frac{L}{D}\frac{\mu\mathcal{V}_m^2}{2} \tag{8-45}$$

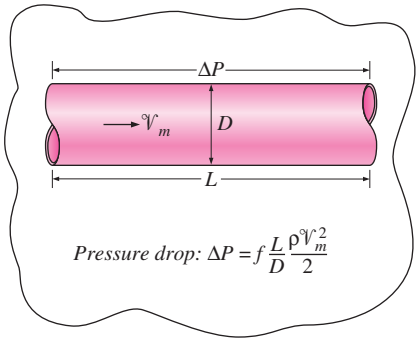


FIGURE 8–18
The relation for pressure drop is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and smooth or rough surfaces.

where the dimensionless quantity f is the **friction factor** (also called the *Darcy friction factor* after French engineer Henry Darcy, 1803–1858, who first studied experimentally the effects of roughness on tube resistance). It should not be confused with the *friction coefficient* C_f (also called the *Fanning friction factor*), which is defined as $C_f = \tau_s(\rho\mathcal{V}_m^2/2) = f/4$.

Equation 8–45 gives the pressure drop for a flow section of length L provided that (1) the flow section is horizontal so that there are no hydrostatic or gravity effects, (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure, and (3) the cross sectional area of the flow section is constant and thus the mean flow velocity is constant.

Setting Eqs. 8–44 and 8–45 equal to each other and solving for f gives the friction factor for the *fully developed laminar flow in a circular tube* to be

Circular tube, laminar:

$$f = \frac{64\mu}{\rho D\mathcal{V}_m} = \frac{64}{\mathrm{Re}} \tag{8-46}$$

This equation shows that *in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the tube*

surface. Once the pressure drop is available, the required pumping power is determined from

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P \tag{8-47}$$

where $\dot{V}$ is the volume flow rate of flow, which is expressed as

$$\dot{V} = \mathcal{V}_{\text{ave}} A_c = \frac{\Delta P R^2}{8\mu L}\pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi D^4 \Delta P}{128\mu L} \tag{8-48}$$

This equation is known as the **Poiseuille's Law,** and this flow is called the *Hagen–Poiseuille flow* in honor of the works of G. Hagen (1797–1839) and J. Poiseuille (1799–1869) on the subject. Note from Eq. 8–48 that *for a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the tube and the viscosity of the fluid, but it is inversely proportional to the fourth power of the radius (or diameter) of the tube.* Therefore, the pumping power requirement for a piping system can be reduced by a factor of 16 by doubling the tube diameter (Fig. 8–19). Of course the benefits of the reduction in the energy costs must be weighed against the increased cost of construction due to using a larger diameter tube.

The pressure drop is caused by viscosity, and it is directly related to the wall shear stress. For the ideal inviscid flow, the pressure drop is zero since there are no viscous effects. Again, Eq. 8–47 is valid for both laminar and turbulent flows in circular and noncircular tubes.

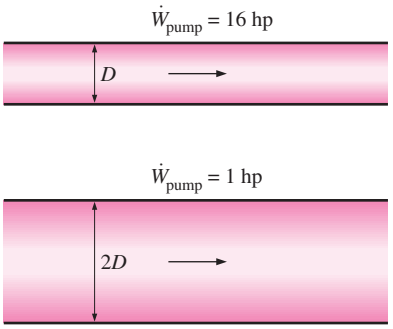


FIGURE 8–19
The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

Temperature Profile and the Nusselt Number

In the analysis above, we have obtained the velocity profile for fully developed flow in a circular tube from a momentum balance applied on a volume element, determined the friction factor and the pressure drop. Below we obtain the energy equation by applying the energy balance to a differential volume element, and solve it to obtain the temperature profile for the constant surface temperature and the constant surface heat flux cases.

Reconsider steady laminar flow of a fluid in a circular tube of radius R. The fluid properties ρ, k, and C_p are constant, and the work done by viscous stresses is negligible. The fluid flows along the x-axis with velocity $\mathcal{V}$. The flow is fully developed so that $\mathcal{V}$ is independent of x and thus $\mathcal{V} = \mathcal{V}(r)$. Noting that energy is transferred by mass in the x-direction, and by conduction in the r-direction (heat conduction in the x-direction is assumed to be negligible), the steady-flow energy balance for a cylindrical shell element of thickness dr and length dx can be expressed as (Fig. 8–20)

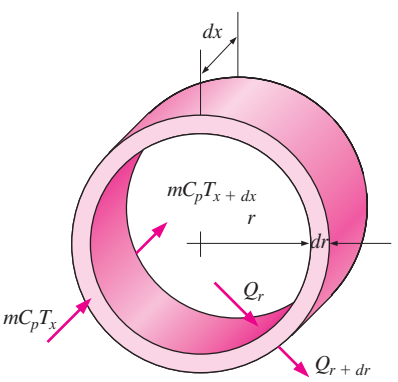


FIGURE 8–20
The differential volume element used in the derivation of energy balance relation.

$$\dot{m}C_pT_x - \dot{m}C_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0 \tag{8-49}$$

where $\dot{m} = \rho\mathcal{V}A_c = \rho\mathcal{V}(2\pi r dr)$. Substituting and dividing by $2\pi r dr dx$ gives, after rearranging,

$$\rho C_p \mathcal{V}\frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx}\frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr} \tag{8-50}$$

or

$$\mathcal{V}\frac{\partial T}{\partial x} = -\frac{1}{2\rho C_p \pi r dx}\frac{\partial \dot{Q}}{\partial r} \tag{8-51}$$

where we used the definition of derivative. But

$$\frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-k2\pi r dx\frac{\partial T}{\partial r}\right) = -2\pi k dx\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{8-52}$$

Substituting and using $\alpha = k/\rho C_p$ gives

$$\mathcal{V}\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{dr}\left(r\frac{\partial T}{\partial r}\right) \tag{8-53}$$

which states that *the rate of net energy transfer to the control volume by mass flow is equal to the net rate of heat conduction in the radial direction.*

Constant Surface Heat Flux

For fully developed flow in a circular pipe subjected to constant surface heat flux, we have, from Eq. 8–24,

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho \mathcal{V}_m C_p R} = \text{constant} \tag{8-54}$$

If heat conduction in the x-direction were considered in the derivation of Eq. 8–53, it would give an additional term $\alpha\partial^2 T/\partial x^2$, which would be equal to zero since $\partial T/\partial x =$ constant and thus $T = T(r)$. Therefore, the assumption that there is no axial heat conduction is satisfied exactly in this case.

Substituting Eq. 8–54 and the relation for velocity profile (Eq. 8–41) into Eq. 8–53 gives

$$\frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \tag{8-55}$$

which is a second-order ordinary differential equation. Its general solution is obtained by separating the variables and integrating twice to be

$$T = \frac{\dot{q}_s}{kR}\left(r^2 - \frac{r^2}{4R^2}\right) + C_1 r + C_2 \tag{8-56}$$

The desired solution to the problem is obtained by applying the boundary conditions $\partial T/\partial x = 0$ at $r = 0$ (because of symmetry) and $T = T_s$ at $r = R$. We get

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right) \tag{8-57}$$

The bulk mean temperature T_m is determined by substituting the velocity and temperature profile relations (Eqs. 8–41 and 8–57) into Eq. 8–4 and performing the integration. It gives

$$T_m = T_s - \frac{11}{24}\frac{\dot{q}_s R}{k} \tag{8-58}$$

Combining this relation with $\dot{q}_s = h(T_s - T_m)$ gives

$$h = \frac{24}{11}\frac{k}{R} = \frac{48}{11}\frac{k}{D} = 4.36\,\frac{k}{D} \tag{8-59}$$

or

Circular tube, laminar ($\dot{q}_x$ = constant): $$\mathrm{Nu} = \frac{hD}{k} = 4.36 \tag{8-60}$$

Therefore, for fully developed laminar flow in a circular tube subjected to constant surface heat flux, the Nusselt number is a constant. There is no dependence on the Reynolds or the Prandtl numbers.

Constant Surface Temperature

A similar analysis can be performed for fully developed laminar flow in a circular tube for the case of constant surface temperature T_s. The solution procedure in this case is more complex as it requires iterations, but the Nusselt number relation obtained is equally simple (Fig. 8–21):

Circular tube, laminar (T_s = constant): $$\mathrm{Nu} = \frac{hD}{k} = 3.66 \tag{8-61}$$

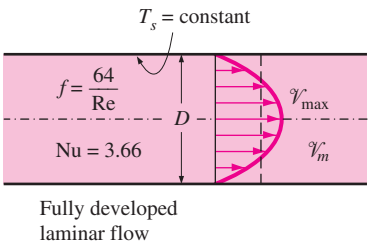


FIGURE 8–21
In laminar flow in a tube with constant surface temperature, both the *friction factor* and the *heat transfer coefficient* remain constant in the fully developed region.

The thermal conductivity k for use in the Nu relations above should be evaluated at the bulk mean fluid temperature, which is the arithmetic average of the mean fluid temperatures at the inlet and the exit of the tube. For laminar flow, the effect of *surface roughness* on the friction factor and the heat transfer coefficient is negligible.

Laminar Flow in Noncircular Tubes

The friction factor f and the Nusselt number relations are given in Table 8–1 for *fully developed laminar flow* in tubes of various cross sections. The Reynolds and Nusselt numbers for flow in these tubes are based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross sectional area of the tube and p is its perimeter. Once the Nusselt number is available, the convection heat transfer coefficient is determined from $h = k\mathrm{Nu}/D_h$.

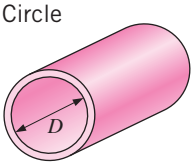
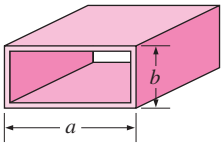
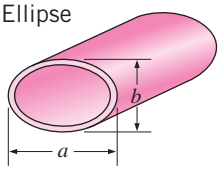
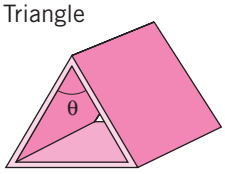
Developing Laminar Flow in the Entrance Region

For a circular tube of length L subjected to constant surface temperature, the average Nusselt number for the *thermal entrance region* can be determined from (Edwards et al., 1979)

Entry region, laminar: $$\mathrm{Nu} = 3.66 + \frac{0.065\,(D/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.04[(D/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}} \tag{8-62}$$

TABLE 8–1

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\text{Re} = \mathcal{V}_m D_h/\nu$, and $\text{Nu} = hD_h/k$)

Tube Geometry	a/b or $\theta°$	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Note that the average Nusselt number is larger at the entrance region, as expected, and it approaches asymptotically to the fully developed value of 3.66 as $L \to \infty$. This relation assumes that the flow is hydrodynamically developed when the fluid enters the heating section, but it can also be used approximately for flow developing hydrodynamically.

When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature. The average Nusselt number for developing laminar flow in a circular tube in that case can be determined from [Sieder and Tate (1936), Ref. 26]

$$\text{Nu} = 1.86\left(\frac{\text{Re Pr } D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \tag{8-63}$$

All properties are evaluated at the bulk mean fluid temperature, except for μ_s, which is evaluated at the surface temperature.

The average Nusselt number for the thermal entrance region of flow between *isothermal parallel plates* of length L is expressed as (Edwards et al., 1979)

Entry region, laminar:
$$\mathrm{Nu} = 7.54 + \frac{0.03\,(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.016[(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}} \qquad \textbf{(8-64)}$$

where D_h is the hydraulic diameter, which is twice the spacing of the plates. This relation can be used for $\mathrm{Re} \le 2800$.

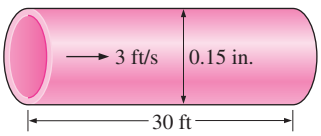


FIGURE 8–22
Schematic for Example 8–2.

EXAMPLE 8–2 **Pressure Drop in a Pipe**

Water at 40°F (ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft · h) is flowing in a 0.15-in.-diameter 30-ft-long pipe steadily at an average velocity of 3 ft/s (Fig. 8–22). Determine the pressure drop and the pumping power requirement to overcome this pressure drop.

SOLUTION The average flow velocity in a pipe is given. The pressure drop and the required pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors.

Properties The density and dynamic viscosity of water are given to be ρ = 62.42 lbm/ft^3 and μ = 3.74 lbm/ft · h = 0.00104 lbm/ft · s.

Analysis First we need to determine the flow regime. The Reynolds number is

$$\mathrm{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{(62.42\ \mathrm{lbm/ft^3})(3\ \mathrm{ft/s})(0.12/12\ \mathrm{ft})}{3.74\ \mathrm{lbm/ft \cdot h}}\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{1803} = 0.0355$$

$$\Delta P = f\frac{L}{D}\frac{\rho \mathcal{V}_m^2}{2} = 0.0355\,\frac{30\ \mathrm{ft}}{0.12/12\ \mathrm{ft}}\,\frac{(62.42\ \mathrm{lbm/ft^3})(3\ \mathrm{ft/s})^2}{2}\left(\frac{1\ \mathrm{lbf}}{32.174\ \mathrm{lbm \cdot ft/s^2}}\right)$$
$$= \mathbf{930\ lbf/ft^2 = 6.46\ psi}$$

The volume flow rate and the pumping power requirements are

$$\dot{V} = \mathcal{V}_m A_c = \mathcal{V}_m(\pi D^2/4) = (3\ \mathrm{ft/s})[\pi(0.12/12\ \mathrm{ft})^2/4] = 0.000236\ \mathrm{ft^3/s}$$
$$\dot{W}_{\mathrm{pump}} = \dot{V}\Delta P = (0.000236\ \mathrm{ft^3/s})(930\ \mathrm{lbf/ft^2})\left(\frac{1\ \mathrm{W}}{0.737\ \mathrm{lbf \cdot ft/s}}\right) = \mathbf{0.30\ W}$$

Therefore, mechanical power input in the amount of 0.30 W is needed to overcome the frictional losses in the flow due to viscosity.

EXAMPLE 8–3 **Flow of Oil in a Pipeline through a Lake**

Consider the flow of oil at 20°C in a 30-cm-diameter pipeline at an average velocity of 2 m/s (Fig. 8–23). A 200-m-long section of the pipeline passes through icy waters of a lake at 0°C. Measurements indicate that the surface temperature of the pipe is very nearly 0°C. Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe.

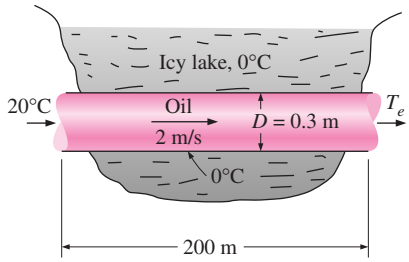


FIGURE 8–23
Schematic for Example 8–3.

SOLUTION Oil flows in a pipeline that passes through icy waters of a lake at 0°C. The exit temperature of the oil, the rate of heat loss, and the pumping power needed to overcome pressure losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is very nearly 0°C. **3** The thermal resistance of the pipe is negligible. **4** The inner surfaces of the pipeline are smooth. **5** The flow is hydrodynamically developed when the pipeline reaches the lake.

Properties We do not know the exit temperature of the oil, and thus we cannot determine the bulk mean temperature, which is the temperature at which the properties of oil are to be evaluated. The mean temperature of the oil at the inlet is 20°C, and we expect this temperature to drop somewhat as a result of heat loss to the icy waters of the lake. We evaluate the properties of the oil at the inlet temperature, but we will repeat the calculations, if necessary, using properties at the evaluated bulk mean temperature. At 20°C we read (Table A-14)

$$\rho = 888 \text{ kg/m}^3 \qquad \nu = 901 \times 10^{-6} \text{ m}^2\text{/s}$$
$$k = 0.145 \text{ W/m} \cdot {}^\circ\text{C} \qquad C_p = 1880 \text{ J/kg} \cdot {}^\circ\text{C}$$
$$\text{Pr} = 10{,}400$$

Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(2 \text{ m/s})(0.3 \text{ m})}{901 \times 10^{-6} \text{ m}^2\text{/s}} = 666$$

which is less than the critical Reynolds number of 2300. Therefore, the flow is laminar, and the thermal entry length in this case is roughly

$$L_t \approx 0.05 \text{ Re Pr } D = 0.05 \times 666 \times 10{,}400 \times (0.3 \text{ m}) \approx 104{,}000 \text{ m}$$

which is much greater than the total length of the pipe. This is typical of fluids with high Prandtl numbers. Therefore, we assume thermally developing flow and determine the Nusselt number from

$$\text{Nu} = \frac{hD}{k} = 3.66 + \frac{0.065\,(D/L)\,\text{Re Pr}}{1 + 0.04\,[(D/L)\,\text{Re Pr}]^{2/3}}$$
$$= 3.66 + \frac{0.065(0.3/200) \times 666 \times 10{,}400}{1 + 0.04[(0.3/200) \times 666 \times 10{,}400]^{2/3}}$$
$$= 37.3$$

Note that this Nusselt number is considerably higher than the fully developed value of 3.66. Then,

$$h = \frac{k}{D}\,\text{Nu} = \frac{0.145\ \text{W/m}}{0.3\ \text{m}}(37.3) = 18.0\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Also,

$$A_s = pL = \pi DL = \pi(0.3\ \text{m})(200\ \text{m}) = 188.5\ \text{m}^2$$
$$\dot{m} = \rho A_c \mathcal{V}_m = (888\ \text{kg/m}^3)[\tfrac{1}{4}\pi(0.3\ \text{m})^2](2\ \text{m/s}) = 125.5\ \text{kg/s}$$

Next we determine the exit temperature of oil from

$$\begin{aligned} T_e &= T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p) \\ &= 0^\circ\text{C} - [(0-20)^\circ\text{C}]\exp\left[-\frac{(18.0\ \text{W/m}^2\cdot{}^\circ\text{C})(188.5\ \text{m}^2)}{(125.5\ \text{kg/s})(1880\ \text{J/kg}\cdot{}^\circ\text{C})}\right] \\ &= \mathbf{19.71^\circ C} \end{aligned}$$

Thus, the mean temperature of oil drops by a mere 0.29°C as it crosses the lake. This makes the bulk mean oil temperature 19.86°C, which is practically identical to the inlet temperature of 20°C. Therefore, we do not need to re-evaluate the properties.

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln\dfrac{T_s - T_e}{T_s - T_i}} = \frac{20 - 19.71}{\ln\dfrac{0 - 19.71}{0 - 20}} = -19.85^\circ\text{C}$$
$$\dot{Q} = hA_s\,\Delta T_{\ln} = (18.0\ \text{W/m}^2\cdot{}^\circ\text{C})(188.5\ \text{m}^2)(-19.85^\circ\text{C}) = \mathbf{-6.74\times10^4}$$

Therefore, the oil will lose heat at a rate of 67.4 kW as it flows through the pipe in the icy waters of the lake. Note that $\Delta T_{\ln}$ is identical to the arithmetic mean temperature in this case, since $\Delta T_i \approx \Delta T_e$.

(*c*) The laminar flow of oil is hydrodynamically developed. Therefore, the friction factor can be determined from

$$f = \frac{64}{\text{Re}} = \frac{64}{666} = 0.0961$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho\mathcal{V}_m^2}{2} = 0.0961\,\frac{200\ \text{m}}{0.3\ \text{m}}\,\frac{(888\ \text{kg/m}^3)(2\ \text{m/s})^2}{2} = 1.14\times10^5\ \text{N/m}^2$$
$$\dot{W}_{\text{pump}} = \frac{\dot{m}\,\Delta P}{\rho} = \frac{(125.5\ \text{kg/s})(1.14\times10^5\ \text{N/m}^2)}{888\ \text{kg/m}^3} = \mathbf{16.1\ kW}$$

Discussion We will need a 16.1-kW pump just to overcome the friction in the pipe as the oil flows in the 200-m-long pipe through the lake.

8–6 ▪ TURBULENT FLOW IN TUBES

We mentioned earlier that flow in smooth tubes is fully turbulent for Re > 10,000. Turbulent flow is commonly utilized in practice because of the higher heat transfer coefficients associated with it. Most correlations for the friction and heat transfer coefficients in turbulent flow are based on experimental studies because of the difficulty in dealing with turbulent flow theoretically.

For *smooth* tubes, the friction factor in turbulent flow can be determined from the explicit *first Petukhov equation* [Petukhov (1970), Ref. 21] given as

Smooth tubes: $$f = (0.790 \ln \mathrm{Re} - 1.64)^{-2} \qquad 10^4 < \mathrm{Re} < 10^6 \tag{8-65}$$

The Nusselt number in turbulent flow is related to the friction factor through the *Chilton–Colburn analogy* expressed as

$$\mathrm{Nu} = 0.125\, f\, \mathrm{Re}\mathrm{Pr}^{1/3} \tag{8-66}$$

Once the friction factor is available, this equation can be used conveniently to evaluate the Nusselt number for both smooth and rough tubes.

For fully developed turbulent flow in *smooth tubes,* a simple relation for the Nusselt number can be obtained by substituting the simple power law relation $f = 0.184\ \mathrm{Re}^{-0.2}$ for the friction factor into Eq. 8–66. It gives

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{pmatrix} \tag{8-67}$$

which is known as the *Colburn equation.* The accuracy of this equation can be improved by modifying it as

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{n} \tag{8-68}$$

where $n = 0.4$ for *heating* and 0.3 for *cooling* of the fluid flowing through the tube. This equation is known as the *Dittus–Boelter equation* [Dittus and Boelter (1930), Ref. 6] and it is preferred to the Colburn equation.

The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. When the temperature difference between the fluid and the wall is very large, it may be necessary to use a correction factor to account for the different viscosities near the wall and at the tube center.

The Nusselt number relations above are fairly simple, but they may give errors as large as 25 percent. This error can be reduced considerably to less than 10 percent by using more complex but accurate relations such as the *second Petukhov equation* expressed as

$$\mathrm{Nu} = \frac{(f/8)\ \mathrm{Re}\ \mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 10^4 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix} \tag{8-69}$$

The accuracy of this relation at lower Reynolds numbers is improved by modifying it as [Gnielinski (1976), Ref. 8]

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\ \mathrm{Pr}}{1 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix} \tag{8-70}$$

where the friction factor f can be determined from an appropriate relation such as the first Petukhov equation. Gnielinski's equation should be preferred in calculations. Again properties should be evaluated at the bulk mean fluid temperature.

The relations above are not very sensitive to the *thermal conditions* at the tube surfaces and can be used for both T_s = constant and $\dot{q}_s$ = constant cases. Despite their simplicity, the correlations already presented give sufficiently accurate results for most engineering purposes. They can also be used to obtain rough estimates of the friction factor and the heat transfer coefficients in the transition region $2300 \leq \text{Re} \leq 10{,}000$, especially when the Reynolds number is closer to 10,000 than it is to 2300.

The relations given so far do not apply to liquid metals because of their very low Prandtl numbers. For liquid metals ($0.004 < \text{Pr} < 0.01$), the following relations are recommended by Sleicher and Rouse (1975, Ref. 27) for $10^4 < \text{Re} < 10^6$:

Liquid metals, T_s = constant: $$\text{Nu} = 4.8 + 0.0156\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93} \quad \textbf{(8-71)}$$

Liquid metals, $\dot{q}_s$ = constant: $$\text{Nu} = 6.3 + 0.0167\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93} \quad \textbf{(8-72)}$$

where the subscript s indicates that the Prandtl number is to be evaluated at the surface temperature.

Rough Surfaces

Any irregularity or roughness on the surface disturbs the laminar sublayer, and affects the flow. Therefore, unlike laminar flow, the friction factor and the convection coefficient in turbulent flow are strong functions of surface roughness.

The friction factor in fully developed turbulent flow depends on the Reynolds number and the *relative roughness* ε/D. In 1939, C. F. Colebrook (Ref. 3) combined all the friction factor data for transition and turbulent flow in smooth as well as rough pipes into the following implicit relation known as the **Colebrook equation.**

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)} \quad \textbf{(8-73)}$$

In 1944, L. F. Moody (Ref. 17) plotted this formula into the famous **Moody chart** given in the Appendix. It presents the friction factors for pipe flow as a function of the Reynolds number and ε/D over a wide range. For smooth tubes, the agreement between the Petukhov and Colebrook equations is very good. The friction factor is minimum for a smooth pipe (but still not zero because of the no-slip condition), and increases with roughness (Fig. 8–24).

Although the Moody chart is developed for circular pipes, it can also be used for noncircular pipes by replacing the diameter by the hydraulic diameter. At very large Reynolds numbers (to the right of the dashed line on the chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number. In calculations, we should make sure that we use the internal diameter of the pipe, which may be different than the nominal diameter. For example, the internal diameter of a steel pipe whose nominal diameter is 1 in. is 1.049 in. (Table 8–2).

Relative Roughness, ε/L	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6, and are calculated from Eq. 8–73.

FIGURE 8–24
The friction factor is minimum for a smooth pipe and increases with roughness.

TABLE 8–2
Standard sizes for Schedule 40 steel pipes

Nominal Size, in.	Actual Inside Diameter, in.
⅛	0.269
¼	0.364
⅜	0.493
½	0.622
¾	0.824
1	1.049
1½	1.610
2	2.067
2½	2.469
3	3.068
5	5.047
10	10.02

Commercially available pipes differ from those used in the experiments in that the roughness of pipes in the market is not uniform, and it is difficult to give a precise description of it. Equivalent roughness values for some commercial pipes are given in Table 8–3, as well as on the Moody chart. But it should be kept in mind that these values are for new pipes, and the relative roughness of pipes may increase with use as a result of corrosion, scale buildup, and precipitation. As a result, the friction factor may increase by a factor of 5 to 10. Actual operating conditions must be considered in the design of piping systems. Also, the Moody chart and its equivalent Colebrook equation involve several uncertainties (the roughness size, experimental error, curve fitting of data, etc.), and thus the results obtained should not be treated as "exact." It is usually considered to be accurate to ±15 percent over the entire range in the figure.

The Colebrook equation is implicit in f, and thus the determination of the friction factor requires tedious iteration unless an equation solver is used. An approximate explicit relation for f is given by S. E. Haaland in 1983 (Ref. 9) as

$$\frac{1}{\sqrt{f}} \approx -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \tag{8-74}$$

The results obtained from this relation are within 2 percent of those obtained from Colebrook equation, and we recommend using this relation rather than the Moody chart to avoid reading errors.

In turbulent flow, wall roughness increases the heat transfer coefficient h by a factor of 2 or more [Dipprey and Sabersky (1963), Ref. 5]. The convection heat transfer coefficient for rough tubes can be calculated approximately from the Nusselt number relations such as Eq. 8–70 by using the friction factor determined from the Moody chart or the Colebrook equation. However, this approach is not very accurate since there is no further increase in h with f for $f > 4f_{\text{smooth}}$ [Norris (1970), Ref. 20] and correlations developed specifically for rough tubes should be used when more accuracy is desired.

TABLE 8–3

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

*The uncertainty in these values can be as much as ±60 percent.

Developing Turbulent Flow in the Entrance Region

The entry lengths for turbulent flow are typically short, often just 10 tube diameters long, and thus the Nusselt number determined for fully developed turbulent flow can be used approximately for the entire tube. This simple approach gives reasonable results for pressure drop and heat transfer for long tubes and conservative results for short ones. Correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature for better accuracy.

Turbulent Flow in Noncircular Tubes

The velocity and temperature profiles in turbulent flow are nearly straight lines in the core region, and any significant velocity and temperature gradients occur in the viscous sublayer (Fig. 8–25). Despite the small thickness of laminar sublayer (usually much less than 1 percent of the pipe diameter), the characteristics of the flow in this layer are very important since they set the stage for flow in the rest of the pipe. Therefore, pressure drop and heat transfer characteristics of turbulent flow in tubes are dominated by the very thin

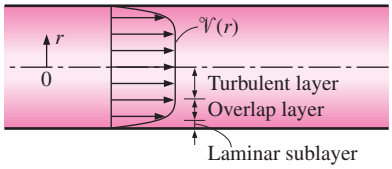


FIGURE 8–25
In turbulent flow, the velocity profile is nearly a straight line in the core region, and any significant velocity gradients occur in the viscous sublayer.

viscous sublayer next to the wall surface, and the shape of the core region is not of much significance. Consequently, the turbulent flow relations given above for circular tubes can also be used for noncircular tubes with reasonable accuracy by replacing the diameter D in the evaluation of the Reynolds number by the hydraulic diameter $D_h = 4A_c/p$.

Flow through Tube Annulus

Some simple heat transfer equipments consist of two concentric tubes, and are properly called *double-tube heat exchangers* (Fig. 8–26). In such devices, one fluid flows through the tube while the other flows through the annular space. The governing differential equations for both flows are identical. Therefore, steady laminar flow through an annulus can be studied analytically by using suitable boundary conditions.

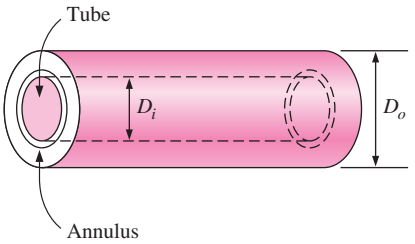


FIGURE 8–26
A double-tube heat exchanger that consists of two concentric tubes.

Consider a concentric annulus of inner diameter D_i and outer diameter D_o. The hydraulic diameter of annulus is

$$D_h = \frac{4A_c}{p} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)} = D_o - D_i \tag{8-75}$$

Annular flow is associated with two Nusselt numbers—Nu_i on the inner tube surface and Nu_o on the outer tube surface—since it may involve heat transfer on both surfaces. The Nusselt numbers for fully developed laminar flow with one surface isothermal and the other adiabatic are given in Table 8–4. When Nusselt numbers are known, the convection coefficients for the inner and the outer surfaces are determined from

$$\text{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \text{Nu}_o = \frac{h_o D_h}{k} \tag{8-76}$$

TABLE 8–4

Nusselt number for fully developed laminar flow in an annulus with one surface isothermal and the other adiabatic (Kays and Perkins, Ref. 14)

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

For fully developed turbulent flow, the inner and outer convection coefficients are approximately equal to each other, and the tube annulus can be treated as a noncircular duct with a hydraulic diameter of $D_h = D_o - D_i$. The Nusselt number in this case can be determined from a suitable turbulent flow relation such as the Gnielinski equation. To improve the accuracy of Nusselt numbers obtained from these relations for annular flow, Petukhov and Roizen (1964, Ref. 22) recommend multiplying them by the following correction factors when one of the tube walls is adiabatic and heat transfer is through the other wall:

$$F_i = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16} \quad \text{(outer wall adiabatic)} \tag{8-77}$$

$$F_o = 0.86\left(\frac{D_i}{D_o}\right)^{-0.16} \quad \text{(inner wall adiabatic)} \tag{8-78}$$

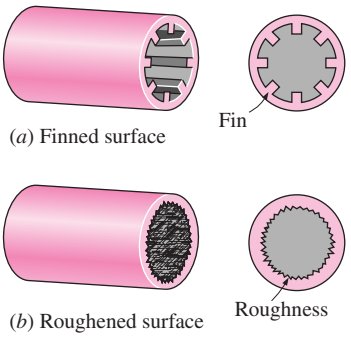


FIGURE 8–27
Tube surfaces are often *roughened, corrugated,* or *finned* in order to *enhance* convection heat transfer.

Heat Transfer Enhancement

Tubes with rough surfaces have much higher heat transfer coefficients than tubes with smooth surfaces. Therefore, tube surfaces are often intentionally *roughened, corrugated,* or *finned* in order to *enhance* the convection heat transfer coefficient and thus the convection heat transfer rate (Fig. 8–27). Heat transfer in turbulent flow in a tube has been increased by as much as

400 percent by roughening the surface. Roughening the surface, of course, also increases the friction factor and thus the power requirement for the pump or the fan.

The convection heat transfer coefficient can also be increased by inducing pulsating flow by pulse generators, by inducing swirl by inserting a twisted tape into the tube, or by inducing secondary flows by coiling the tube.

EXAMPLE 8–4 Pressure Drop in a Water Pipe

Water at 60°F (ρ = 62.36 lbm/ft^3 and μ = 2.713 lbm/ft · h) is flowing steadily in a 2-in.-diameter horizontal pipe made of stainless steel at a rate of 0.2 ft^3/s (Fig. 8–28). Determine the pressure drop and the required pumping power input for flow through a 200-ft-long section of the pipe.

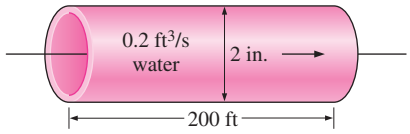


FIGURE 8–28
Schematic for Example 8–4.

SOLUTION The flow rate through a specified water pipe is given. The pressure drop and the pumping power requirements are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as a pump or a turbine.

Properties The density and dynamic viscosity of water are given by ρ = 62.36 lbm/ft^3 and μ = 2.713 lbm/ft · h = 0.0007536 lbm/ft · s, respectively.

Analysis First we calculate the mean velocity and the Reynolds number to determine the flow regime:

$$\mathcal{V} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.2\ \text{ft}^3/\text{s}}{\pi(2/12\ \text{ft})^2/4} = 9.17\ \text{ft/s}$$

$$\text{Re} = \frac{\rho \mathcal{V} D}{\mu} = \frac{(62.36\ \text{lbm/ft}^3)(9.17\ \text{ft/s})(2/12\ \text{ft})}{2.713\ \text{lbm/ft} \cdot \text{h}}\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 126{,}400$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.000007\ \text{ft}}{2/12\ \text{ft}} = 0.000042$$

The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.000042}{3.7} + \frac{2.51}{126{,}400\sqrt{f}}\right)$$

Using an equation solver or an iterative scheme, the friction factor is determined to be f = 0.0174. Then the pressure drop and the required power input become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathcal{V}^2}{2} = 0.0174\,\frac{200\ \text{ft}}{2/12\ \text{ft}}\,\frac{(62.36\ \text{lbm/ft}^3)(9.17\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.2\ \text{lbm} \cdot \text{ft/s}^2}\right)$$

$$= \mathbf{1700\ lbf/ft^2 = 11.8\ psi}$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.2\ \text{ft}^3/\text{s})(1700\ \text{lbf/ft}^2)\left(\frac{1\ \text{W}}{0.737\ \text{lbf} \cdot \text{ft/s}}\right) = \mathbf{461\ W}$$

Therefore, power input in the amount of 461 W is needed to overcome the frictional losses in the pipe.

Discussion The friction factor also could be determined easily from the explicit Haaland relation. It would give $f = 0.0172$, which is sufficiently close to 0.0174. Also, the friction factor corresponding to $\varepsilon = 0$ in this case is 0.0171, which indicates that stainless steel pipes can be assumed to be smooth with negligible error.

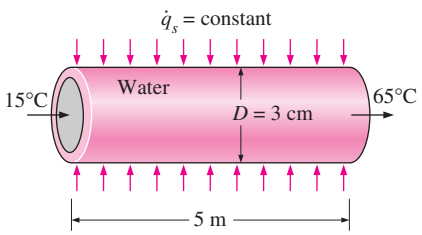


FIGURE 8–29
Schematic for Example 8–5.

EXAMPLE 8–5 **Heating of Water by Resistance Heaters in a Tube**

Water is to be heated from 15°C to 65°C as it flows through a 3-cm-internal-diameter 5-m-long tube (Fig. 8–29). The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.

SOLUTION Water is to be heated in a tube equipped with an electric resistance heater on its surface. The power rating of the heater and the inner surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the bulk mean temperature of $T_b = (T_i + T_e)/2 = (15 + 65)/2 = 40°C$ are (Table A-9).

$$\rho = 992.1 \text{ kg/m}^3 \qquad C_p = 4179 \text{ J/kg} \cdot °\text{C}$$
$$k = 0.631 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 4.32$$
$$\nu = \mu/\rho = 0.658 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis The cross sectional and heat transfer surface areas are

$$A_c = \tfrac{1}{4}\pi D^2 = \tfrac{1}{4}\pi(0.03 \text{ m})^2 = 7.069 \times 10^{-4} \text{ m}^2$$
$$A_s = pL = \pi DL = \pi(0.03 \text{ m})(5 \text{ m}) = 0.471 \text{ m}^2$$

The volume flow rate of water is given as $\dot{V} = 10$ L/min $= 0.01$ m³/min. Then the mass flow rate becomes

$$\dot{m} = \rho\dot{V} = (992.1 \text{ kg/m}^3)(0.01 \text{ m}^3/\text{min}) = 9.921 \text{ kg/min} = 0.1654 \text{ kg/s}$$

To heat the water at this mass flow rate from 15°C to 65°C, heat must be supplied to the water at a rate of

$$\begin{aligned}\dot{Q} &= \dot{m}C_p(T_e - T_i)\\ &= (0.1654 \text{ kg/s})(4.179 \text{ kJ/kg} \cdot °\text{C})(65 - 15)°\text{C}\\ &= 34.6 \text{ kJ/s} = 34.6 \text{ kW}\end{aligned}$$

All of this energy must come from the resistance heater. Therefore, the power rating of the heater must be **34.6 kW.**

The surface temperature T_s of the tube at any location can be determined from

$$\dot{q}_s = h(T_s - T_m) \quad \rightarrow \quad T_s = T_m + \frac{\dot{q}_s}{h}$$

where h is the heat transfer coefficient and T_m is the mean temperature of the fluid at that location. The surface heat flux is constant in this case, and its value can be determined from

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{34.6 \text{ kW}}{0.471 \text{ m}^2} = 73.46 \text{ kW/m}^2$$

To determine the heat transfer coefficient, we first need to find the mean velocity of water and the Reynolds number:

$$\mathcal{V}_m = \frac{\dot{V}}{A_c} = \frac{0.010 \text{ m}^3/\text{min}}{7.069 \times 10^{-4} \text{ m}^2} = 14.15 \text{ m/min} = 0.236 \text{ m/s}$$

$$\text{Re} = \frac{\mathcal{V}_m D}{\nu} = \frac{(0.236 \text{ m/s})(0.03 \text{ m})}{0.658 \times 10^{-6} \text{ m}^2/\text{s}} = 10{,}760$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry length is roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.03 = 0.3 \text{ m}$$

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe and determine the Nusselt number from

$$\text{Nu} = \frac{hD}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4} = 0.023(10{,}760)^{0.8} (4.34)^{0.4} = 69.5$$

Then,

$$h = \frac{k}{D} \text{Nu} = \frac{0.631 \text{ W/m} \cdot {}^\circ\text{C}}{0.03 \text{ m}} (69.5) = 1462 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

and the surface temperature of the pipe at the exit becomes

$$T_s = T_m + \frac{\dot{q}_s}{h} = 65^\circ\text{C} + \frac{73{,}460 \text{ W/m}^2}{1462 \text{ W/m}^2 \cdot {}^\circ\text{C}} = \mathbf{115^\circ C}$$

Discussion Note that the inner surface temperature of the pipe will be 50°C higher than the mean water temperature at the pipe exit. This temperature difference of 50°C between the water and the surface will remain constant throughout the fully developed flow region.

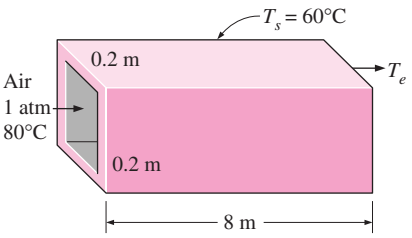


FIGURE 8–30
Schematic for Example 8–6.

EXAMPLE 8–6 Heat Loss from the Ducts of a Heating System

Hot air at atmospheric pressure and 80°C enters an 8–m-long uninsulated square duct of cross section 0.2 m × 0.2 m that passes through the attic of a house at a rate of 0.15 m³/s (Fig. 8–30). The duct is observed to be nearly isothermal at 60°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the attic space.

SOLUTION Heat loss from uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas.

Properties We do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk mean temperature of air, which is the temperature at which the properties are to be determined. The temperature of air at the inlet is 80°C and we expect this temperature to drop somewhat as a result of heat loss through the duct whose surface is at 60°C. At 80°C and 1 atm we read (Table A-15)

$$\rho = 0.9994 \text{ kg/m}^3 \qquad C_p = 1008 \text{ J/kg} \cdot °\text{C}$$
$$k = 0.02953 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7154$$
$$\nu = 2.097 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The characteristic length (which is the hydraulic diameter), the mean velocity, and the Reynolds number in this case are

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2 \text{ m}$$
$$\mathcal{V}_m = \frac{\dot{V}}{A_c} = \frac{0.15 \text{ m}^3/\text{s}}{(0.2 \text{ m})^2} = 3.75 \text{ m/s}$$
$$\text{Re} = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(3.75 \text{ m/s})(0.2 \text{ m})}{2.097 \times 10^{-5} \text{ m}^2/\text{s}} = 35{,}765$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10 \times 0.2 \text{ m} = 2 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct and determine the Nusselt number from

$$\text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.3} = 0.023(35{,}765)^{0.8} (0.7154)^{0.3} = 91.4$$

Then,

$$h = \frac{k}{D_h}\text{Nu} = \frac{0.02953\ \text{W/m}\cdot{}^\circ\text{C}}{0.2\ \text{m}}(91.4) = 13.5\ \text{W/m}^2\cdot{}^\circ\text{C}$$
$$A_s = pL = 4aL = 4\times(0.2\ \text{m})(8\ \text{m}) = 6.4\ \text{m}^2$$
$$\dot{m} = \rho\dot{V} = (1.009\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 0.151\ \text{kg/s}$$

Next, we determine the exit temperature of air from

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$
$$= 60^\circ\text{C} - [(60-80)^\circ\text{C}]\exp\left[-\frac{(13.5\ \text{W/m}^2\cdot{}^\circ\text{C})(6.4\ \text{m}^2)}{(0.151\ \text{kg/s})(1008\ \text{J/kg}\cdot{}^\circ\text{C})}\right]$$
$$= \mathbf{71.3^\circ C}$$

Then the logarithmic mean temperature difference and the rate of heat loss from the air become

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln\dfrac{T_s - T_e}{T_s - T_i}} = \frac{80 - 71.3}{\ln\dfrac{60-71.3}{60-80}} = -15.2^\circ\text{C}$$
$$\dot{Q} = hA_s\,\Delta T_{\ln} = (13.5\ \text{W/m}^2\cdot{}^\circ\text{C})(6.4\ \text{m}^2)(-15.2^\circ\text{C}) = \mathbf{-1313\ W}$$

Therefore, air will lose heat at a rate of 1313 W as it flows through the duct in the attic.

Discussion The average fluid temperature is (80 + 71.3)/2 = 75.7°C, which is sufficiently close to 80°C at which we evaluated the properties of air. Therefore, it is not necessary to re-evaluate the properties at this temperature and to repeat the calculations.

SUMMARY

Internal flow is characterized by the fluid being completely confined by the inner surfaces of the tube. The mean velocity and mean temperature for a circular tube of radius R are expressed as

$$\mathcal{V}_m = \frac{2}{R^2}\int_0^R \mathcal{V}(r,x)r\,dr \qquad \text{and} \qquad T_m = \frac{2}{\mathcal{V}_m R^2}\int_0^R \mathcal{V}Tr\,dr$$

The Reynolds number for internal flow and the hydraulic diameter are defined as

$$\text{Re} = \frac{\rho\mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu} \qquad \text{and} \qquad D_h = \frac{4A_c}{p}$$

The flow in a tube is laminar for Re < 2300, turbulent for Re > 10,000, and transitional in between.

The length of the region from the tube inlet to the point at which the boundary layer merges at the centerline is the *hydrodynamic entry length* L_h. The region beyond the entrance region in which the velocity profile is fully developed is the *hydrodynamically fully developed region.* The length of the region of flow over which the thermal boundary layer develops and reaches the tube center is the *thermal entry length* L_t. The region in which the flow is both hydrodynamically and thermally developed is the *fully developed flow region.* The entry lengths are given by

$$L_{h,\ \text{laminar}} \approx 0.05\ \text{Re}\ D$$
$$L_{t,\ \text{laminar}} \approx 0.05\ \text{Re Pr}\ D = \text{Pr}\ L_{h,\ \text{laminar}}$$
$$L_{h,\ \text{turbulent}} \approx L_{t,\ \text{turbulent}} \approx 10D$$

For $\dot{q}_s$ = constant, the rate of heat transfer is expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m}C_p(T_e - T_i)$$

For T_s = constant, we have

$$\dot{Q} = hA_s \Delta T_{\ln} = \dot{m} C_p (T_e - T_i)$$
$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m} C_p)$$
$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

The pressure drop and required pumping power for a volume flow rate of $\dot{V}$ are

$$\Delta P = \frac{L}{D}\frac{\rho \mathcal{V}_m^2}{2} \qquad \text{and} \qquad \dot{W}_{\text{pump}} = \dot{V}\Delta P$$

For *fully developed laminar flow* in a circular pipe, we have:

$$\mathcal{V}(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right) = \mathcal{V}_{\max}\left(1 - \frac{r^2}{R^2}\right)$$
$$f = \frac{64\mu}{\rho D \mathcal{V}_m} = \frac{64}{\text{Re}}$$
$$\dot{V} = \mathcal{V}_{\text{ave}} A_c = \frac{\Delta P R^2}{8\mu L}\pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi R^4 \Delta P}{128\mu L}$$

Circular tube, laminar ($\dot{q}_s$ = constant): $\text{Nu} = \dfrac{hD}{k} = 4.36$

Circular tube, laminar (T_s = constant): $\text{Nu} = \dfrac{hD}{k} = 3.66$

For *developing laminar flow* in the entrance region with constant surface temperature, we have

Circular tube: $\text{Nu} = 3.66 + \dfrac{0.065(D/L)\,\text{Re Pr}}{1 + 0.04[(D/L)\,\text{Re Pr}]^{2/3}}$

Circular tube: $\text{Nu} = 1.86\left(\dfrac{\text{Re Pr}\,D}{L}\right)^{1/3}\left(\dfrac{\mu_b}{\mu_s}\right)^{0.14}$

Parallel plates: $\text{Nu} = 7.54 + \dfrac{0.03(D_h/L)\,\text{Re Pr}}{1 + 0.016[(D_h/L)\,\text{Re Pr}]^{2/3}}$

For *fully developed turbulent flow with smooth surfaces,* we have

$$f = (0.790 \ln \text{Re} - 1.64)^{-2} \qquad 10^4 < \text{Re} < 10^6$$
$$\text{Nu} = 0.125 f\,\text{Re Pr}^{1/3}$$
$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le \text{Pr} \le 160 \\ \text{Re} > 10{,}000 \end{pmatrix}$$

$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^n$ with $n = 0.4$ for *heating* and 0.3 for *cooling* of fluid

$$\text{Nu} = \frac{(f/8)(\text{Re} - 1000)\,\text{Pr}}{1 + 12.7(f/8)^{0.5}(\text{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \text{Pr} \le 2000 \\ 3 \times 10^3 < \text{Re} < 5 \times 10^6 \end{pmatrix}$$

The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. For liquid metal flow in the range of $10^4 < \text{Re} < 10^6$ we have:

T_s = constant: $\text{Nu} = 4.8 + 0.0156\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$

$\dot{q}_s$ = constant: $\text{Nu} = 6.3 + 0.0167\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$

For *fully developed turbulent flow with rough surfaces,* the friction factor f is determined from the Moody chart or

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \approx -1.8 \log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

For a *concentric annulus,* the hydraulic diameter is $D_h = D_o - D_i$, and the Nusselt numbers are expressed as

$$\text{Nu}_i = \frac{h_i D_h}{k} \qquad \text{and} \qquad \text{Nu}_o = \frac{h_o D_h}{k}$$

where the values for the Nusselt numbers are given in Table 8–4.

REFERENCES AND SUGGESTED READING

1. M. S. Bhatti and R. K. Shah. "Turbulent and Transition Flow Convective Heat Transfer in Ducts." In *Handbook of Single-Phase Convective Heat Transfer,* ed. S. Kakaç, R. K. Shah, and W. Aung. New York: Wiley Interscience, 1987.
2. A. P. Colburn. *Transactions of the AIChE* 26 (1933), p. 174.
3. C. F. Colebrook. "Turbulent flow in Pipes, with Particular Reference to the Transition between the Smooth and

Rough Pipe Laws." *Journal of the Institute of Civil Engineers London.* 11 (1939), pp. 133–156.

4. R. G. Deissler. "Analysis of Turbulent Heat Transfer and Flow in the Entrance Regions of Smooth Passages." 1953. Referred to in *Handbook of Single-Phase Convective Heat Transfer,* ed. S. Kakaç, R. K. Shah, and W. Aung. New York: Wiley Interscience, 1987.
5. D. F. Dipprey and D. H. Sabersky. "Heat and Momentum Transfer in Smooth and Rough Tubes at Various Prandtl Numbers." *International Journal of Heat Mass Transfer* 6 (1963), pp. 329–353.
6. F. W. Dittus and L. M. K. Boelter. *University of California Publications on Engineering* 2 (1930), p. 433.
7. D. K. Edwards, V. E. Denny, and A. F. Mills. *Transfer Processes.* 2nd ed. Washington, DC: Hemisphere, 1979.
8. V. Gnielinski. "New Equations for Heat and Mass Transfer in Turbulent Pipe and Channel Flow." *International Chemical Engineering* 16 (1976), pp. 359–368.
9. S. E. Haaland. "Simple and Explicit Formulas for the Friction Factor in Turbulent Pipe Flow." *Journal of Fluids Engineering* (March 1983), pp. 89–90.
10. J. P. Holman. *Heat Transfer.* 8th ed. New York: McGraw-Hill, 1997.
11. F. P. Incropera and D. P. DeWitt. *Introduction to Heat Transfer.* 3rd ed. New York: John Wiley & Sons, 1996.
12. S. Kakaç, R. K. Shah, and W. Aung, eds. *Handbook of Single-Phase Convective Heat Transfer.* New York: Wiley Interscience, 1987.
13. W. M. Kays and M. E. Crawford. *Convective Heat and Mass Transfer.* 3rd ed. New York: McGraw-Hill, 1993.
14. W. M. Kays and H. C. Perkins. Chapter 7. In *Handbook of Heat Transfer,* ed. W. M. Rohsenow and J. P. Hartnett. New York: McGraw-Hill, 1972.
15. F. Kreith and M. S. Bohn. *Principles of Heat Transfer.* 6th ed. Pacific Grove, CA: Brooks/Cole, 2001.
16. A. F. Mills. *Basic Heat and Mass Transfer.* 2nd ed. Upper Saddle River, NJ: Prentice Hall, 1999.
17. L. F. Moody. "Friction Factors for Pipe Flows." *Transactions of the ASME* 66 (1944), pp. 671–684.
18. M. Molki and E. M. Sparrow. "An Empirical Correlation for the Average Heat Transfer Coefficient in Circular Tubes." *Journal of Heat Transfer* 108 (1986), pp. 482–484.
19. B. R. Munson, D. F. Young, and T. Okiishi. *Fundamentals of Fluid Mechanics.* 4th ed. New York: Wiley, 2002.
20. R. H. Norris. "Some Simple Approximate Heat Transfer Correlations for Turbulent Flow in Ducts with Rough Surfaces." In *Augmentation of Convective Heat Transfer,* ed. A. E. Bergles and R. L. Webb. New York: ASME, 1970.
21. B. S. Petukhov. "Heat Transfer and Friction in Turbulent Pipe Flow with Variable Physical Properties." In *Advances in Heat Transfer,* ed. T. F. Irvine and J. P. Hartnett, Vol. 6. New York: Academic Press, 1970.
22. B. S. Petukhov and L. I. Roizen. "Generalized Relationships for Heat Transfer in a Turbulent Flow of a Gas in Tubes of Annular Section." *High Temperature* (USSR) 2 (1964), pp. 65–68.
23. O. Reynolds. "On the Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and the Law of Resistance in Parallel Channels." *Philosophical Transactions of the Royal Society of London* 174 (1883), pp. 935–982.
24. H. Schlichting. *Boundary Layer Theory.* 7th ed. New York: McGraw-Hill, 1979.
25. R. K. Shah and M. S. Bhatti. "Laminar Convective Heat Transfer in Ducts." In *Handbook of Single-Phase Convective Heat Transfer,* ed. S. Kakaç, R. K. Shah, and W. Aung. New York: Wiley Interscience, 1987.
26. E. N. Sieder and G. E. Tate. "Heat Transfer and Pressure Drop of Liquids in Tubes." *Industrial Engineering Chemistry* 28 (1936), pp. 1429–1435.
27. C. A. Sleicher and M. W. Rouse. "A Convenient Correlation for Heat Transfer to Constant and Variable Property Fluids in Turbulent Pipe Flow." *International Journal of Heat Mass Transfer* 18 (1975), pp. 1429–1435.
28. N. V. Suryanarayana. *Engineering Heat Transfer.* St. Paul, MN: West, 1995.
29. F. M. White. *Heat and Mass Transfer.* Reading, MA: Addison-Wesley, 1988.
30. S. Whitaker. "Forced Convection Heat Transfer Correlations for Flow in Pipes, Past Flat Plates, Single Cylinders, and for Flow in Packed Beds and Tube Bundles." *AIChE Journal* 18 (1972), pp. 361–371.
31. W. Zhi-qing. "Study on Correction Coefficients of Laminar and Turbulent Entrance Region Effects in Round Pipes." *Applied Mathematical Mechanics* 3 (1982), p. 433.

PROBLEMS*

General Flow Analysis

8–1C Why are liquids usually transported in circular pipes?

8–2C Show that the Reynolds number for flow in a circular tube of diameter D can be expressed as $\text{Re} = 4\dot{m}/(\pi D\mu)$.

8–3C Which fluid at room temperature requires a larger pump to move at a specified velocity in a given tube: water or engine oil? Why?

8–4C What is the generally accepted value of the Reynolds number above which the flow in smooth pipes is turbulent?

8–5C What is hydraulic diameter? How is it defined? What is it equal to for a circular tube of diameter?

8–6C How is the hydrodynamic entry length defined for flow in a tube? Is the entry length longer in laminar or turbulent flow?

8–7C Consider laminar flow in a circular tube. Will the friction factor be higher near the inlet of the tube or near the exit? Why? What would your response be if the flow were turbulent?

8–8C How does surface roughness affect the pressure drop in a tube if the flow is turbulent? What would your response be if the flow were laminar?

8–9C How does the friction factor f vary along the flow direction in the fully developed region in (*a*) laminar flow and (*b*) turbulent flow?

8–10C What fluid property is responsible for the development of the velocity boundary layer? For what kinds of fluids will there be no velocity boundary layer in a pipe?

8–11C What is the physical significance of the number of transfer units $\text{NTU} = hA/\dot{m}C_p$? What do small and large NTU values tell about a heat transfer system?

8–12C What does the logarithmic mean temperature difference represent for flow in a tube whose surface temperature is constant? Why do we use the logarithmic mean temperature instead of the arithmetic mean temperature?

8–13C How is the thermal entry length defined for flow in a tube? In what region is the flow in a tube fully developed?

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

8–14C Consider laminar forced convection in a circular tube. Will the heat flux be higher near the inlet of the tube or near the exit? Why?

8–15C Consider turbulent forced convection in a circular tube. Will the heat flux be higher near the inlet of the tube or near the exit? Why?

8–16C In the fully developed region of flow in a circular tube, will the velocity profile change in the flow direction? How about the temperature profile?

8–17C Consider the flow of oil in a tube. How will the hydrodynamic and thermal entry lengths compare if the flow is laminar? How would they compare if the flow were turbulent?

8–18C Consider the flow of mercury (a liquid metal) in a tube. How will the hydrodynamic and thermal entry lengths compare if the flow is laminar? How would they compare if the flow were turbulent?

8–19C What do the mean velocity $\mathcal{V}_m$ and the mean temperature T_m represent in flow through circular tubes of constant diameter?

8–20C Consider fluid flow in a tube whose surface temperature remains constant. What is the appropriate temperature difference for use in Newton's law of cooling with an average heat transfer coefficient?

8–21 Air enters a 20-cm-diameter 12-m-long underwater duct at 50°C and 1 atm at a mean velocity of 7 m/s, and is cooled by the water outside. If the average heat transfer coefficient is 85 W/m² · °C and the tube temperature is nearly equal to the water temperature of 5°C, determine the exit temperature of air and the rate of heat transfer.

8–22 Cooling water available at 10°C is used to condense steam at 30°C in the condenser of a power plant at a rate of 0.15 kg/s by circulating the cooling water through a bank of 5-m-long 1.2-cm-internal-diameter thin copper tubes. Water enters the tubes at a mean velocity of 4 m/s, and leaves at a temperature of 24°C. The tubes are nearly isothermal at 30°C. Determine the average heat transfer coefficient between the water and the tubes, and the number of tubes needed to achieve the indicated heat transfer rate in the condenser.

8–23 Repeat Problem 8–22 for steam condensing at a rate of 0.60 kg/s.

8–24 Combustion gases passing through a 3-cm-internal-diameter circular tube are used to vaporize waste water at atmospheric pressure. Hot gases enter the tube at 115 kPa and 250°C at a mean velocity of 5 m/s, and leave at 150°C. If the average heat transfer coefficient is 120 W/m² · °C and the inner surface temperature of the tube is 110°C, determine (*a*) the tube length and (*b*) the rate of evaporation of water.

8–25 Repeat Problem 8–24 for a heat transfer coefficient of 60 W/m² · °C.

Laminar and Turbulent Flow in Tubes

8–26C How is the friction factor for flow in a tube related to the pressure drop? How is the pressure drop related to the pumping power requirement for a given mass flow rate?

8–27C Someone claims that the shear stress at the center of a circular pipe during fully developed laminar flow is zero. Do you agree with this claim? Explain.

8–28C Someone claims that in fully developed turbulent flow in a tube, the shear stress is a maximum at the tube surface. Do you agree with this claim? Explain.

8–29C Consider fully developed flow in a circular pipe with negligible entrance effects. If the length of the pipe is doubled, the pressure drop will (*a*) double, (*b*) more than double, (*c*) less than double, (*d*) reduce by half, or (*e*) remain constant.

8–30C Someone claims that the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross sectional area, and dividing the result by 2. Do you agree? Explain.

8–31C Someone claims that the average velocity in a circular pipe in fully developed laminar flow can be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). Do you agree? Explain.

8–32C Consider fully developed laminar flow in a circular pipe. If the diameter of the pipe is reduced by half while the flow rate and the pipe length are held constant, the pressure drop will (*a*) double, (*b*) triple, (*c*) quadruple, (*d*) increase by a factor of 8, or (*e*) increase by a factor of 16.

8–33C Consider fully developed laminar flow in a circular pipe. If the viscosity of the fluid is reduced by half by heating while the flow rate is held constant, how will the pressure drop change?

8–34C How does surface roughness affect the heat transfer in a tube if the fluid flow is turbulent? What would your response be if the flow in the tube were laminar?

8–35 Water at 15°C ($\rho = 999.1$ kg/m^3 and $\mu = 1.138 \times 10^{-3}$ kg/m · s) is flowing in a 4-cm-diameter and 30-m long horizontal pipe made of stainless steel steadily at a rate of 5 L/s. Determine (*a*) the pressure drop and (*b*) the pumping power requirement to overcome this pressure drop.

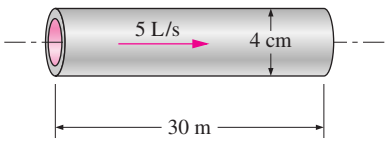


FIGURE P8–35

8–36 In fully developed laminar flow in a circular pipe, the velocity at $R/2$ (midway between the wall surface and the centerline) is measured to be 6 m/s. Determine the velocity at the center of the pipe. *Answer:* 8 m/s

8–37 The velocity profile in fully developed laminar flow in a circular pipe of inner radius $R = 2$ cm, in m/s, is given by $\mathcal{V}(r) = 4(1 - r^2/R^2)$. Determine the mean and maximum velocities in the pipe, and the volume flow rate.

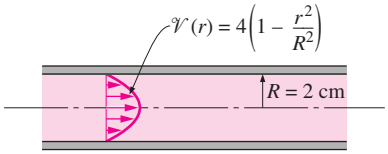


FIGURE P8–37

8–38 Repeat Problem 8–37 for a pipe of inner radius 5 cm.

8–39 Water at 10°C ($\rho = 999.7$ kg/m^3 and $\mu = 1.307 \times 10^{-3}$ kg/m · s) is flowing in a 0.20-cm-diameter 15-m-long pipe steadily at an average velocity of 1.2 m/s. Determine (*a*) the pressure drop and (*b*) the pumping power requirement to overcome this pressure drop.

Answers: (*a*) 188 kPa, (*b*) 0.71 W

8–40 Water is to be heated from 10°C to 80°C as it flows through a 2-cm-internal-diameter, 7-m-long tube. The tube is equipped with an electric resistance heater, which provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 8 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.

8–41 Hot air at atmospheric pressure and 85°C enters a 10-m-long uninsulated square duct of cross section 0.15 m × 0.15 m that passes through the attic of a house at a rate of 0.10 m^3/s. The duct is observed to be nearly isothermal at 70°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the air space in the attic.

Answers: 75.7°C, 941 W

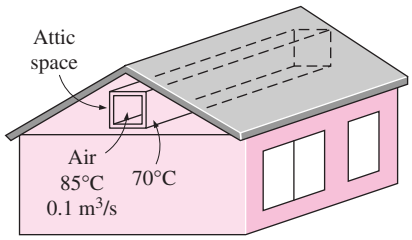


FIGURE P8–41

8–42 Reconsider Problem 8–41. Using EES (or other) software, investigate the effect of the volume flow rate of air on the exit temperature of air and the rate of heat loss. Let the flow rate vary from 0.05 m^3/s to 0.15 m^3/s.

Plot the exit temperature and the rate of heat loss as a function of flow rate, and discuss the results.

8–43 Consider an air solar collector that is 1 m wide and 5 m long and has a constant spacing of 3 cm between the glass cover and the collector plate. Air enters the collector at 30°C at a rate of 0.15 m^3/s through the 1-m-wide edge and flows along the 5-m-long passage way. If the average temperatures of the glass cover and the collector plate are 20°C and 60°C, respectively, determine (*a*) the net rate of heat transfer to the air in the collector and (*b*) the temperature rise of air as it flows through the collector.

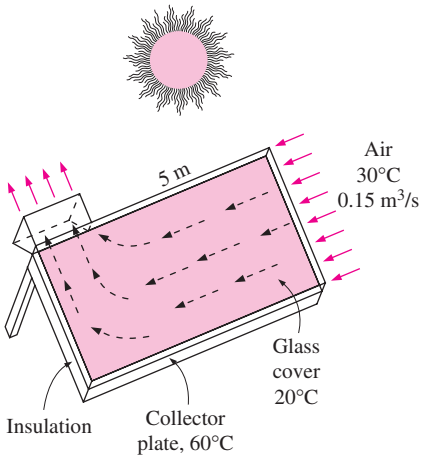


FIGURE P8–43

8–44 Consider the flow of oil at 10°C in a 40-cm-diameter pipeline at an average velocity of 0.5 m/s. A 300-m-long section of the pipeline passes through icy waters of a lake at 0°C. Measurements indicate that the surface temperature of the pipe is very nearly 0°C. Disregarding the thermal resistance of the pipe material, determine (*a*) the temperature of the oil when the pipe leaves the lake, (*b*) the rate of heat transfer from the oil, and (*c*) the pumping power required to overcome the pressure losses and to maintain the flow oil in the pipe.

8–45 Consider laminar flow of a fluid through a square channel maintained at a constant temperature. Now the mean velocity of the fluid is doubled. Determine the change in the pressure drop and the change in the rate of heat transfer between the fluid and the walls of the channel. Assume the flow regime remains unchanged.

8–46 Repeat Problem 8–45 for turbulent flow.

8–47E The hot water needs of a household are to be met by heating water at 55°F to 200°F by a parabolic solar collector at a rate of 4 lbm/s. Water flows through a 1.25-in.-diameter thin aluminum tube whose outer surface is blackanodized in order to maximize its solar absorption ability. The centerline of the tube coincides with the focal line of the collector, and a glass

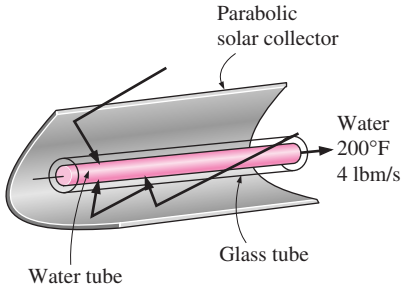


FIGURE P8–47E

sleeve is placed outside the tube to minimize the heat losses. If solar energy is transferred to water at a net rate of 350 Btu/h per ft length of the tube, determine the required length of the parabolic collector to meet the hot water requirements of this house. Also, determine the surface temperature of the tube at the exit.

8–48 A 15-cm × 20-cm printed circuit board whose components are not allowed to come into direct contact with air for reliability reasons is to be cooled by passing cool air through a 20-cm-long channel of rectangular cross section 0.2 cm × 14 cm drilled into the board. The heat generated by the electronic components is conducted across the thin layer of the board to the channel, where it is removed by air that enters the channel at 15°C. The heat flux at the top surface of the channel can be considered to be uniform, and heat transfer through other surfaces is negligible. If the velocity of the air at the inlet of the channel is not to exceed 4 m/s and the surface temperature of the channel is to remain under 50°C, determine the maximum total power of the electronic components that can safely be mounted on this circuit board.

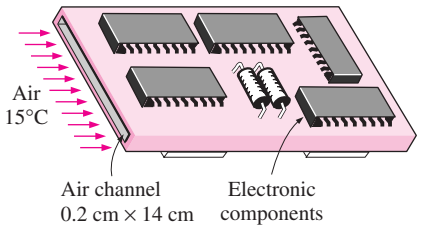


FIGURE P8–48

8–49 Repeat Problem 8–48 by replacing air with helium, which has six times the thermal conductivity of air.

8–50 Reconsider Problem 8–48. Using EES (or other) software, investigate the effects of air velocity at the inlet of the channel and the maximum surface temperature on the maximum total power dissipation of electronic components. Let the air velocity vary from 1 m/s to 10 m/s and the surface temperature from 30°C to 90°C. Plot the power dissipation as functions of air velocity and surface temperature, and discuss the results.

8–51 Air enters a 7-m-long section of a rectangular duct of cross section 15 cm × 20 cm at 50°C at an average velocity of 7 m/s. If the walls of the duct are maintained at 10°C, determine (*a*) the outlet temperature of the air, (*b*) the rate of heat transfer from the air, and (*c*) the fan power needed to overcome the pressure losses in this section of the duct.

Answers: (*a*) 32.8°C, (*b*) 3674 W, (*c*) 4.2 W

8–52 Reconsider Problem 8–51. Using EES (or other) software, investigate the effect of air velocity on the exit temperature of air, the rate of heat transfer, and the fan power. Let the air velocity vary from 1 m/s to 10 m/s. Plot the exit temperature, the rate of heat transfer, and the fan power as a function of the air velocity, and discuss the results.

8–53 Hot air at 60°C leaving the furnace of a house enters a 12-m-long section of a sheet metal duct of rectangular cross section 20 cm × 20 cm at an average velocity of 4 m/s. The thermal resistance of the duct is negligible, and the outer surface of the duct, whose emissivity is 0.3, is exposed to the cold air at 10°C in the basement, with a convection heat transfer coefficient of 10 W/m^2 · °C. Taking the walls of the basement to be at 10°C also, determine (*a*) the temperature at which the hot air will leave the basement and (*b*) the rate of heat loss from the hot air in the duct to the basement.

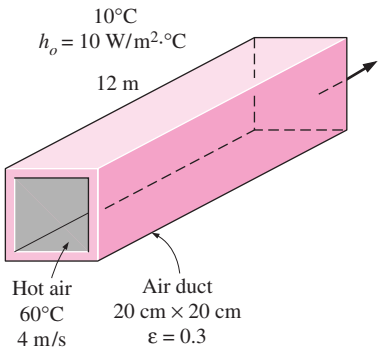


FIGURE P8–53

8–54 Reconsider Problem 8–53. Using EES (or other) software, investigate the effects of air velocity and the surface emissivity on the exit temperature of air and the rate of heat loss. Let the air velocity vary from 1 m/s to 10 m/s and the emissivity from 0.1 to 1.0. Plot the exit temperature and the rate of heat loss as functions of air velocity and emissivity, and discuss the results.

8–55 The components of an electronic system dissipating 90 W are located in a 1-m-long horizontal duct whose cross section is 16 cm × 16 cm. The components in the duct are cooled by forced air, which enters at 32°C at a rate of 0.65 m^3/min. Assuming 85 percent of the heat generated inside is transferred to air flowing through the duct and the remaining 15 percent is lost through the outer surfaces of the duct, determine (*a*) the exit temperature of air and (*b*) the highest component surface temperature in the duct.

8–56 Repeat Problem 8–55 for a circular horizontal duct of 15-cm diameter.

8–57 Consider a hollow-core printed circuit board 12 cm high and 18 cm long, dissipating a total of 20 W. The width of the air gap in the middle of the PCB is 0.25 cm. The cooling air enters the 12-cm-wide core at 32°C at a rate of 0.8 L/s. Assuming the heat generated to be uniformly distributed over the two side surfaces of the PCB, determine (*a*) the temperature at which the air leaves the hollow core and (*b*) the highest temperature on the inner surface of the core.

Answers: (*a*) 54.0°C, (*b*) 72.8°C

8–58 Repeat Problem 8–57 for a hollow-core PCB dissipating 35 W.

8–59E Water at 54°F is heated by passing it through 0.75-in.-internal-diameter thin-walled copper tubes. Heat is supplied to the water by steam that condenses outside the copper tubes at 250°F. If water is to be heated to 140°F at a rate of 0.7 lbm/s, determine (*a*) the length of the copper tube that needs to be used and (*b*) the pumping power required to overcome pressure losses. Assume the entire copper tube to be at the steam temperature of 250°F.

8–60 A computer cooled by a fan contains eight PCBs, each dissipating 10 W of power. The height of the PCBs is 12 cm and the length is 18 cm. The clearance between the tips of the components on the PCB and the back surface of the adjacent PCB is 0.3 cm. The cooling air is supplied by a 10-W fan mounted at the inlet. If the temperature rise of air as it flows through the case of the computer is not to exceed 10°C, determine (*a*) the flow rate of the air that the fan needs to deliver, (*b*) the fraction of the temperature rise of air that is due to the heat generated by the fan and its motor, and (*c*) the highest allowable inlet air temperature if the surface temperature of the

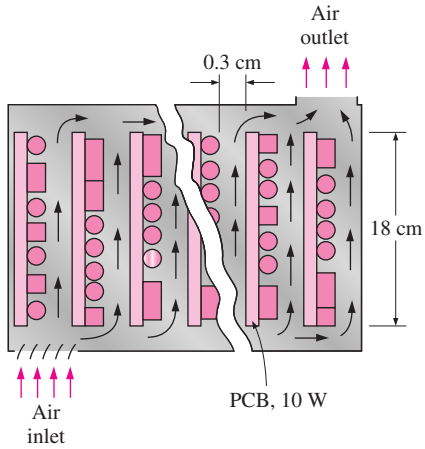


FIGURE P8–60

components is not to exceed 70°C anywhere in the system. Use air properties at 25°C.

Review Problems

8–61 A geothermal district heating system involves the transport of geothermal water at 110°C from a geothermal well to a city at about the same elevation for a distance of 12 km at a rate of 1.5 m^3/s in 60-cm-diameter stainless steel pipes. The fluid pressures at the wellhead and the arrival point in the city are to be the same. The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. (*a*) Assuming the pump-motor efficiency to be 65 percent, determine the electric power consumption of the system for pumping. (*b*) Determine the daily cost of power consumption of the system if the unit cost of electricity is \$0.06/kWh. (*c*) The temperature of geothermal water is estimated to drop 0.5°C during this long flow. Determine if the frictional heating during flow can make up for this drop in temperature.

8–62 Repeat Problem 8–61 for cast iron pipes of the same diameter.

8–63 The velocity profile in fully developed laminar flow in a circular pipe, in m/s, is given by $\mathcal{V}(r) = 6(1 - 100r^2)$ where r is the radial distance from the centerline of the pipe in m. Determine (*a*) the radius of the pipe, (*b*) the mean velocity through the pipe, and (*c*) the maximum velocity in the pipe.

8–64E The velocity profile in fully developed laminar flow of water at 40°F in a 80-ft-long horizontal circular pipe, in ft/s, is given by $\mathcal{V}(r) = 0.8(1 - 625r^2)$ where r is the radial distance from the centerline of the pipe in ft. Determine (*a*) the volume flow rate of water through the pipe, (*b*) the pressure drop across the pipe, and (*c*) the useful pumping power required to overcome this pressure drop.

8–65 The compressed air requirements of a manufacturing facility are met by a 150-hp compressor located in a room that is maintained at 20°C. In order to minimize the compressor work, the intake port of the compressor is connected to the outside through an 11-m-long, 20-cm-diameter duct made of thin aluminum sheet. The compressor takes in air at a rate of 0.27 m^3/s at the outdoor conditions of 10°C and 95 kPa. Disregarding the thermal resistance of the duct and taking the heat transfer coefficient on the outer surface of the duct to be 10 $W/m^2 \cdot °C$, determine (*a*) the power used by the compressor to overcome the pressure drop in this duct, (*b*) the rate of heat transfer to the incoming cooler air, and (*c*) the temperature rise of air as it flows through the duct.

8–66 A house built on a riverside is to be cooled in summer by utilizing the cool water of the river, which flows at an average temperature of 15°C. A 15-m-long section of a circular duct of 20-cm diameter passes through the water. Air enters the underwater section of the duct at 25°C at a velocity of 3 m/s. Assuming the surface of the duct to be at the temperature of the water, determine the outlet temperature of air as it leaves the underwater portion of the duct. Also, for an overall fan efficiency of 55 percent, determine the fan power input needed to overcome the flow resistance in this section of the duct.

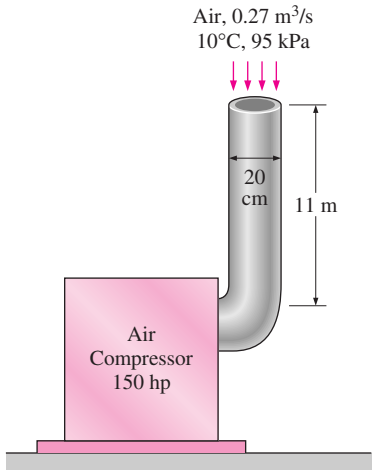


FIGURE P8–65

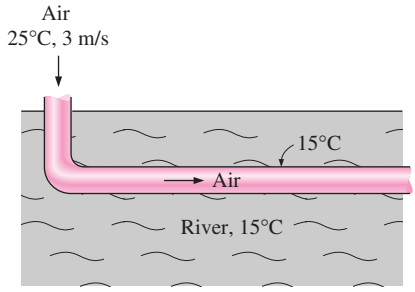


FIGURE P8–66

8–67 Repeat Problem 8–66 assuming that a 0.15-mm-thick layer of mineral deposit (k = 3 W/m · °C) formed on the inner surface of the pipe.

8–68E The exhaust gases of an automotive engine leave the combustion chamber and enter a 8-ft-long and 3.5-in.-diameter thin-walled steel exhaust pipe at 800°F and 15.5 psia at a rate of 0.2 lbm/s. The surrounding ambient air is at a temperature of 80°F, and the heat transfer coefficient on the outer surface of the exhaust pipe is 3 $Btu/h \cdot ft^2 \cdot °F$. Assuming the exhaust gases to have the properties of air, determine (*a*) the velocity of the exhaust gases at the inlet of the exhaust pipe and (*b*) the temperature at which the exhaust gases will leave the pipe and enter the air.

8–69 Hot water at 90°C enters a 15-m section of a cast iron pipe (k = 52 W/m · °C) whose inner and outer diameters are 4 and 4.6 cm, respectively, at an average velocity of 0.8 m/s. The outer surface of the pipe, whose emissivity is 0.7, is exposed to the cold air at 10°C in a basement, with a convection heat

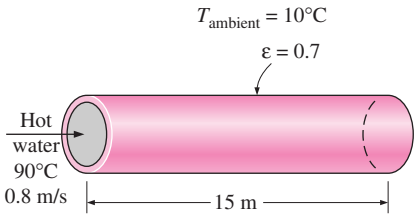


FIGURE P8–69

transfer coefficient of 15 W/m^2 · °C. Taking the walls of the basement to be at 10°C also, determine (*a*) the rate of heat loss from the water and (*b*) the temperature at which the water leaves the basement.

8–70 Repeat Problem 8–69 for a pipe made of copper (k = 386 W/m · °C) instead of cast iron.

8–71 D. B. Tuckerman and R. F. Pease of Stanford University demonstrated in the early 1980s that integrated circuits can be cooled very effectively by fabricating a series of microscopic channels 0.3 mm high and 0.05 mm wide in the back of the substrate and covering them with a plate to confine the fluid flow within the channels. They were able to dissipate 790 W of power generated in a 1-cm^2 silicon chip at a junction-to-ambient temperature difference of 71°C using water as the coolant flowing at a rate of 0.01 L/s through 100 such channels under a 1-cm × 1-cm silicon chip. Heat is transferred primarily through the base area of the channel, and it was found that the increased surface area and thus the fin effect are of lesser importance. Disregarding the entrance effects and ignoring any heat transfer from the side and cover surfaces, determine (*a*) the temperature rise of water as it flows through the microchannels and (*b*) the average surface temperature of the base of the microchannels for a power dissipation of 50 W. Assume the water enters the channels at 20°C.

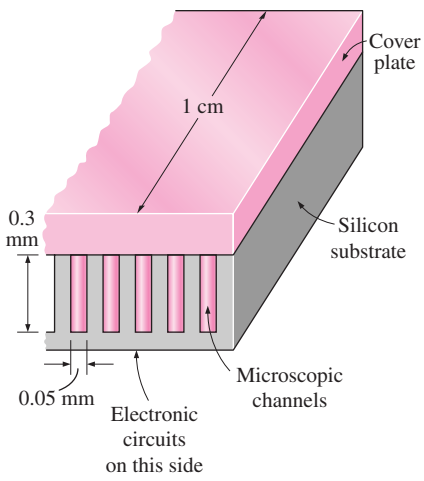


FIGURE P8–71

8–72 Liquid-cooled systems have high heat transfer coefficients associated with them, but they have the inherent disadvantage that they present potential leakage problems. Therefore, air is proposed to be used as the microchannel coolant. Repeat Problem 8–71 using air as the cooling fluid instead of water, entering at a rate of 0.5 L/s.

8–73 Hot exhaust gases leaving a stationary diesel engine at 450°C enter a 15-cm-diameter pipe at an average velocity of 3.6 m/s. The surface temperature of the pipe is 180°C. Determine the pipe length if the exhaust gases are to leave the pipe at 250°C after transferring heat to water in a heat recovery unit. Use properties of air for exhaust gases.

8–74 Geothermal steam at 165°C condenses in the shell side of a heat exchanger over the tubes through which water flows. Water enters the 4-cm-diameter, 14-m-long tubes at 20°C at a rate of 0.8 kg/s. Determine the exit temperature of water and the rate of condensation of geothermal steam.

8–75 Cold air at 5°C enters a 12-cm-diameter 20-m-long isothermal pipe at a velocity of 2.5 m/s and leaves at 19°C. Estimate the surface temperature of the pipe.

8–76 Oil at 10°C is to be heated by saturated steam at 1 atm in a double-pipe heat exchanger to a temperature of 30°C. The inner and outer diameters of the annular space are 3 cm and 5 cm, respectively, and oil enters at with a mean velocity of 0.8 m/s. The inner tube may be assumed to be isothermal at 100°C, and the outer tube is well insulated. Assuming fully developed flow for oil, determine the tube length required to heat the oil to the indicated temperature. In reality, will you need a shorter or longer tube? Explain.

Design and Essay Problems

8–77 Electronic boxes such as computers are commonly cooled by a fan. Write an essay on forced air cooling of electronic boxes and on the selection of the fan for electronic devices.

8–78 Design a heat exchanger to pasteurize milk by steam in a dairy plant. Milk is to flow through a bank of 1.2-cm internal diameter tubes while steam condenses outside the tubes at 1 atm. Milk is to enter the tubes at 4°C, and it is to be heated to 72°C at a rate of 15 L/s. Making reasonable assumptions, you are to specify the tube length and the number of tubes, and the pump for the heat exchanger.

8–79 A desktop computer is to be cooled by a fan. The electronic components of the computer consume 80 W of power under full-load conditions. The computer is to operate in environments at temperatures up to 50°C and at elevations up to 3000 m where the atmospheric pressure is 70.12 kPa. The exit temperature of air is not to exceed 60°C to meet the reliability requirements. Also, the average velocity of air is not to exceed 120 m/min at the exit of the computer case, where the fan is installed to keep the noise level down. Specify the flow rate of the fan that needs to be installed and the diameter of the casing of the fan.

CHAPTER 9

NATURAL CONVECTION

In Chapters 7 and 8, we considered heat transfer by *forced convection,* where a fluid was *forced* to move over a surface or in a tube by external means such as a pump or a fan. In this chapter, we consider *natural convection,* where any fluid motion occurs by natural means such as buoyancy. The fluid motion in forced convection is quite *noticeable,* since a fan or a pump can transfer enough momentum to the fluid to move it in a certain direction. The fluid motion in natural convection, however, is often not noticeable because of the low velocities involved.

The convection heat transfer coefficient is a strong function of *velocity:* the higher the velocity, the higher the convection heat transfer coefficient. The fluid velocities associated with natural convection are low, typically less than 1 m/s. Therefore, the heat transfer coefficients encountered in natural convection are usually much lower than those encountered in forced convection. Yet several types of heat transfer equipment are designed to operate under natural convection conditions instead of forced convection, because natural convection does not require the use of a fluid mover.

We start this chapter with a discussion of the physical mechanism of *natural convection* and the *Grashof number.* We then present the correlations to evaluate heat transfer by natural convection for various geometries, including finned surfaces and enclosures. Finally, we discuss simultaneous forced and natural convection.

CONTENTS

9–1 ▪ PHYSICAL MECHANISM OF NATURAL CONVECTION

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipment such as power transistors, TVs, and VCRs; heat transfer from electric baseboard heaters or steam radiators; heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings. Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

We know that a hot boiled egg (or a hot baked potato) on a plate eventually cools to the surrounding air temperature (Fig. 9–1). The egg is cooled by transferring heat by convection to the air and by radiation to the surrounding surfaces. Disregarding heat transfer by radiation, the physical mechanism of cooling a hot egg (or any hot object) in a cooler environment can be explained as follows:

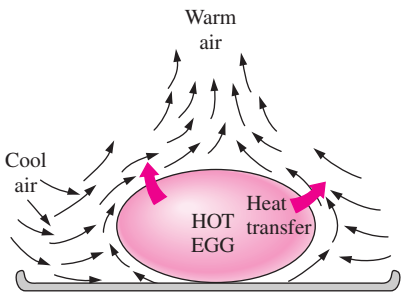


FIGURE 9–1
The cooling of a boiled egg in a cooler environment by natural convection.

As soon as the hot egg is exposed to cooler air, the temperature of the outer surface of the egg shell will drop somewhat, and the temperature of the air adjacent to the shell will rise as a result of heat conduction from the shell to the air. Consequently, the egg will soon be surrounded by a thin layer of warmer air, and heat will then be transferred from this warmer layer to the outer layers of air. The cooling process in this case would be rather slow since the egg would always be blanketed by warm air, and it would have no direct contact with the cooler air farther away. We may not notice any air motion in the vicinity of the egg, but careful measurements indicate otherwise.

The temperature of the air adjacent to the egg is higher, and thus its density is lower, since at constant pressure the density of a gas is inversely proportional to its temperature. Thus, we have a situation in which some low-density or "light" gas is surrounded by a high-density or "heavy" gas, and the natural laws dictate that *the light gas rise.* This is no different than the oil in a vinegar-and-oil salad dressing rising to the top (since $\rho_{oil} < \rho_{vinegar}$). This phenomenon is characterized incorrectly by the phrase "heat rises," which is understood to mean *heated air rises.* The space vacated by the warmer air in the vicinity of the egg is replaced by the cooler air nearby, and the presence of cooler air in the vicinity of the egg speeds up the cooling process. The rise of warmer air and the flow of cooler air into its place continues until the egg is cooled to the temperature of the surrounding air. The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a **natural convection current,** and the heat transfer that is enhanced as a result of this natural convection current is called **natural convection heat transfer.** Note that in the absence of natural convection currents, heat transfer from the egg to the air surrounding it would be by conduction only, and the rate of heat transfer from the egg would be much lower.

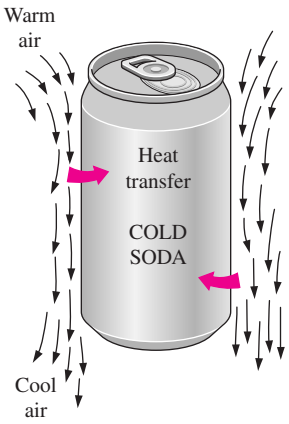


FIGURE 9–2
The warming up of a cold drink in a warmer environment by natural convection.

Natural convection is just as effective in the heating of cold surfaces in a warmer environment as it is in the cooling of hot surfaces in a cooler environment, as shown in Figure 9–2. Note that the direction of fluid motion is reversed in this case.

In a gravitational field, there is a net force that pushes upward a light fluid placed in a heavier fluid. The upward force exerted by a fluid on a body

completely or partially immersed in it is called the **buoyancy force.** The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body. That is,

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g V_{\text{body}} \tag{9-1}$$

where ρ_{fluid} is the average density of the *fluid* (not the body), g is the gravitational acceleration, and V_{body} is the volume of the portion of the body immersed in the fluid (for bodies completely immersed in the fluid, it is the total volume of the body). In the absence of other forces, the net vertical force acting on a body is the difference between the weight of the body and the buoyancy force. That is,

$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}}\, g V_{\text{body}} - \rho_{\text{fluid}}\, g V_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}})\, g V_{\text{body}} \end{aligned} \tag{9-2}$$

Note that this force is proportional to the difference in the densities of the fluid and the body immersed in it. Thus, a body immersed in a fluid will experience a "weight loss" in an amount equal to the weight of the fluid it displaces. This is known as *Archimedes' principle.*

To have a better understanding of the buoyancy effect, consider an egg dropped into water. If the average density of the egg is greater than the density of water (a sign of freshness), the egg will settle at the bottom of the container. Otherwise, it will rise to the top. When the density of the egg equals the density of water, the egg will settle somewhere in the water while remaining completely immersed, acting like a "weightless object" in space. This occurs when the upward buoyancy force acting on the egg equals the weight of the egg, which acts downward.

The *buoyancy effect* has far-reaching implications in life. For one thing, without buoyancy, heat transfer between a hot (or cold) surface and the fluid surrounding it would be by *conduction* instead of by *natural convection.* The natural convection currents encountered in the oceans, lakes, and the atmosphere owe their existence to buoyancy. Also, light boats as well as heavy warships made of steel float on water because of buoyancy (Fig. 9–3). Ships are designed on the basis of the principle that the entire weight of a ship and its contents is equal to the weight of the water that the submerged volume of the ship can contain. The "chimney effect" that induces the upward flow of hot combustion gases through a chimney is also due to the buoyancy effect, and the upward force acting on the gases in the chimney is proportional to the difference between the densities of the hot gases in the chimney and the cooler air outside. Note that there is *no gravity* in space, and thus there can be no natural convection heat transfer in a spacecraft, even if the spacecraft is filled with atmospheric air.

In heat transfer studies, the primary variable is *temperature,* and it is desirable to express the net buoyancy force (Eq. 9-2) in terms of temperature differences. But this requires expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the *variation of the density of a fluid with temperature at constant pressure. The property that provides that information is the* **volume expansion coefficient** β, defined as (Fig. 9–4)

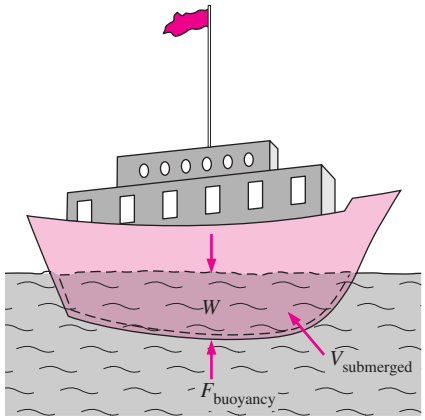


FIGURE 9–3
It is the buoyancy force that keeps the ships afloat in water ($W = F_{\text{buoyancy}}$ for floating objects).

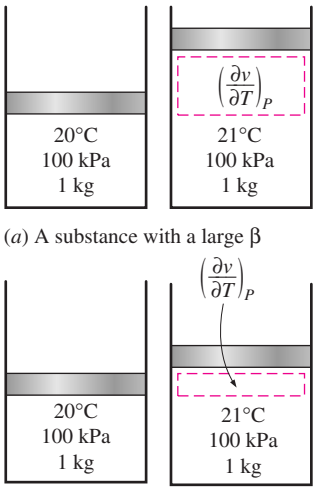


FIGURE 9–4
The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\mathrm{K}) \tag{9-3}$$

In natural convection studies, the condition of the fluid sufficiently far from the hot or cold surface is indicated by the subscript "infinity" to serve as a reminder that this is the value at a distance where the presence of the surface is not felt. In such cases, the volume expansion coefficient can be expressed approximately by replacing differential quantities by differences as

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad \text{(at constant } P\text{)} \tag{9-4}$$

or

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad \text{(at constant } P\text{)} \tag{9-5}$$

where ρ_∞ is the density and T_∞ is the temperature of the quiescent fluid away from the surface.

We can show easily that the volume expansion coefficient β of an *ideal gas* ($P = \rho RT$) at a temperature T is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\mathrm{K}) \tag{9-6}$$

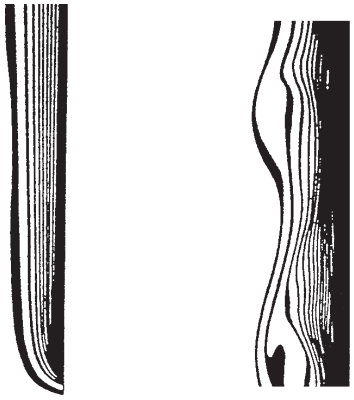
where T is the *absolute* temperature. Note that a large value of β for a fluid means a large change in density with temperature, and that the product $\beta\,\Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT at constant pressure. Also note that the buoyancy force is proportional to the *density difference,* which is proportional to the *temperature difference* at constant pressure. Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

The magnitude of the natural convection heat transfer between a surface and a fluid is directly related to the *flow rate* of the fluid. The higher the flow rate, the higher the heat transfer rate. In fact, it is the very high flow rates that increase the heat transfer coefficient by orders of magnitude when forced convection is used. In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally. The flow rate in this case is established by the dynamic balance of *buoyancy* and *friction.*

As we have discussed earlier, the buoyancy force is caused by the density difference between the heated (or cooled) fluid adjacent to the surface and the fluid surrounding it, and is proportional to this density difference and the volume occupied by the warmer fluid. It is also well known that whenever two bodies in contact (solid–solid, solid–fluid, or fluid–fluid) move relative to each other, a *friction force* develops at the contact surface in the direction opposite to that of the motion. This opposing force slows down the fluid and thus reduces the flow rate of the fluid. Under steady conditions, the air flow rate driven by buoyancy is established at the point where these two effects *balance* each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting the fluid flow and heat transfer. For that reason, heat sinks with closely spaced fins are not suitable for natural convection cooling.

Most heat transfer correlations in natural convection are based on experimental measurements. The instrument often used in natural convection

experiments is the *Mach–Zehnder interferometer,* which gives a plot of isotherms in the fluid in the vicinity of a surface. The operation principle of interferometers is based on the fact that at low pressure, the lines of constant temperature for a gas correspond to the lines of constant density, and that the index of refraction of a gas is a function of its density. Therefore, the degree of refraction of light at some point in a gas is a measure of the temperature gradient at that point. An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature* as shown in Figure 9–5. The smooth and parallel lines in (*a*) indicate that the flow is *laminar,* whereas the eddies and irregularities in (*b*) indicate that the flow is *turbulent.* Note that the lines are closest near the surface, indicating a *higher temperature gradient.*



(*a*) Laminar flow (*b*) Turbulent flow

FIGURE 9–5
Isotherms in natural convection over a hot plate in air.

9–2 ▪ EQUATION OF MOTION AND THE GRASHOF NUMBER

In this section we derive the equation of motion that governs the natural convection flow in laminar boundary layer. The conservation of mass and energy equations derived in Chapter 6 for forced convection are also applicable for natural convection, but the momentum equation needs to be modified to incorporate buoyancy.

Consider a vertical hot flat plate immersed in a quiescent fluid body. We assume the natural convection flow to be steady, laminar, and two-dimensional, and the fluid to be Newtonian with constant properties, including density, with one exception: the density difference $\rho - \rho_\infty$ is to be considered since it is this density difference between the inside and the outside of the boundary layer that gives rise to buoyancy force and sustains flow. (This is known as the *Boussinesq approximation.*) We take the upward direction along the plate to be x, and the direction normal to surface to be y, as shown in Figure 9–6. Therefore, gravity acts in the $-x$-direction. Noting that the flow is steady and two-dimensional, the x- and y-components of velocity within boundary layer are $u = u(x, y)$ and $v = v(x, y)$, respectively.

The velocity and temperature profiles for natural convection over a vertical hot plate are also shown in Figure 9–6. Note that as in forced convection, the thickness of the boundary layer increases in the flow direction. Unlike forced convection, however, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate. This is expected since the fluid beyond the boundary layer is motionless. Thus, the fluid velocity increases with distance from the surface, reaches a maximum, and gradually decreases to zero at a distance sufficiently far from the surface. At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface, as shown in the figure. In the case of *cold surfaces,* the shape of the velocity and temperature profiles remains the same but their direction is reversed.

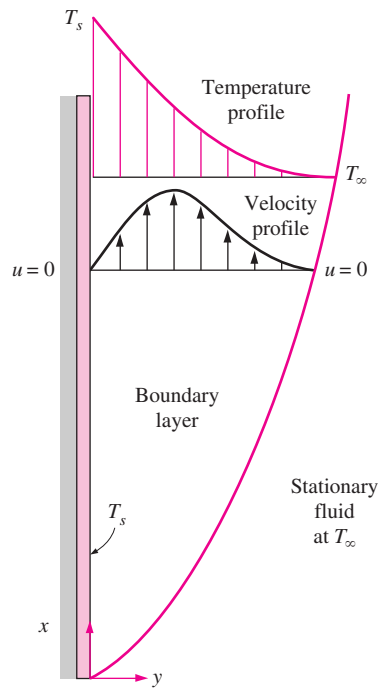


FIGURE 9–6
Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

Consider a differential volume element of height dx, length dy, and unit depth in the z-direction (normal to the paper) for analysis. The forces acting on this volume element are shown in Figure 9–7. Newton's second law of motion for this control volume can be expressed as

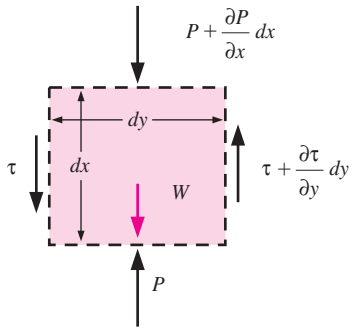


FIGURE 9–7
Forces acting on a differential control volume in the natural convection boundary layer over a vertical flat plate.

$$\delta m \cdot a_x = F_x \tag{9-7}$$

where $\delta m = \rho(dx \cdot dy \cdot 1)$ is the mass of the fluid element within the control volume. The acceleration in the x-direction is obtained by taking the total differential of $u(x, y)$, which is $du = (\partial u/\partial x)dx + (\partial u/\partial y)dy$, and dividing it by dt. We get

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \tag{9-8}$$

The forces acting on the differential volume element in the vertical direction are the pressure forces acting on the top and bottom surfaces, the shear stresses acting on the side surfaces (the normal stresses acting on the top and bottom surfaces are small and are disregarded), and the force of gravity acting on the entire volume element. Then the net surface force acting in the x-direction becomes

$$\begin{aligned} F_x &= \left(\frac{\partial \tau}{\partial y} dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x} dx\right)(dy \cdot 1) - \rho g(dx \cdot dy \cdot 1) \\ &= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1) \end{aligned} \tag{9-9}$$

since $\tau = \mu(\partial u/\partial y)$. Substituting Eqs. 9-8 and 9-9 into Eq. 9-7 and dividing by $\rho \cdot dx \cdot dy \cdot 1$ gives the *conservation of momentum* in the x-direction as

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g \tag{9-10}$$

The x-momentum equation in the quiescent fluid outside the boundary layer can be obtained from the relation above as a special case by setting $u = 0$. It gives

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g \tag{9-11}$$

which is simply the relation for the variation of hydrostatic pressure in a quiescent fluid with height, as expected. Also, noting that $v \ll u$ in the boundary layer and thus $\partial v/\partial x \approx \partial v/\partial y \approx 0$, and that there are no body forces (including gravity) in the y-direction, the force balance in that direction gives $\partial P/\partial y = 0$. That is, the variation of pressure in the direction normal to the surface is negligible, and for a given x the pressure in the boundary layer is equal to the pressure in the quiescent fluid. Therefore, $P = P(x) = P_\infty(x)$ and $\partial P/\partial x = \partial P_\infty/\partial x = -\rho_\infty g$. Substituting into Eq. 9-10,

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g \tag{9-12}$$

The last term represents the net upward force per unit volume of the fluid (the difference between the buoyant force and the fluid weight). This is the force that initiates and sustains convection currents.

From Eq. 9-5, we have $\rho_\infty - \rho = \rho\beta(T - T_\infty)$. Substituting it into the last equation and dividing both sides by ρ gives the desired form of the x-momentum equation,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{9-13}$$

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy. Note that the momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

The set of three partial differential equations (the continuity, momentum, and the energy equations) that govern natural convection flow over vertical isothermal plates can be reduced to a set of two ordinary nonlinear differential equations by the introduction of a similarity variable. But the resulting equations must still be solved numerically [Ostrach (1953), Ref. 27]. Interested reader is referred to advanced books on the topic for detailed discussions [e.g., Kays and Crawford (1993), Ref. 23].

The Grashof Number

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities: all lengths by a characteristic length L_c, all velocities by an arbitrary reference velocity $\mathcal{V}$ (which, from the definition of Reynolds number, is taken to be $\mathcal{V} = \mathrm{Re}_L\, \nu/L_c$), and temperature by a suitable temperature difference (which is taken to be $T_s - T_\infty$) as

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{\mathcal{V}} \qquad v^* = \frac{v}{\mathcal{V}} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

where asterisks are used to denote nondimensional variables. Substituting them into the momentum equation and simplifying give

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right]\frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{9-14}$$

The dimensionless parameter in the brackets represents the natural convection effects, and is called the **Grashof number** Gr_L,

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \tag{9-15}$$

where

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s

We mentioned in the preceding chapters that the flow regime in forced convection is governed by the dimensionless *Reynolds number,* which represents the ratio of inertial forces to viscous forces acting on the fluid. The flow regime in natural convection is governed by the dimensionless *Grashof number,* which represents the ratio of the *buoyancy force* to the *viscous force* acting on the fluid (Fig. 9–8).

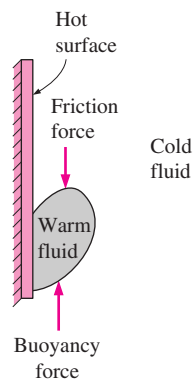


FIGURE 9–8
The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about 10^9. Therefore, the flow regime on a vertical plate becomes *turbulent* at Grashof numbers greater than 10^9.

When a surface is subjected to external flow, the problem involves both natural and forced convection. The relative importance of each mode of heat transfer is determined by the value of the coefficient $\mathrm{Gr}_L/\mathrm{Re}_L^2$: Natural convection effects are negligible if $\mathrm{Gr}_L/\mathrm{Re}_L^2 \ll 1$, free convection dominates and the forced convection effects are negligible if $\mathrm{Gr}_L/\mathrm{Re}_L^2 \gg 1$, and both effects are significant and must be considered if $\mathrm{Gr}_L/\mathrm{Re}_L^2 \approx 1$.

9–3 ▪ NATURAL CONVECTION OVER SURFACES

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid involved.

Although we understand the mechanism of natural convection well, the complexities of fluid motion make it very difficult to obtain simple analytical relations for heat transfer by solving the governing equations of motion and energy. Some analytical solutions exist for natural convection, but such solutions lack generality since they are obtained for simple geometries under some simplifying assumptions. Therefore, with the exception of some simple cases, heat transfer relations in natural convection are based on experimental studies. Of the numerous such correlations of varying complexity and claimed accuracy available in the literature for any given geometry, we present here the ones that are best known and widely used.

The simple empirical correlations for the average *Nusselt number* Nu in natural convection are of the form (Fig. 9–9)

$$\mathrm{Nu} = \frac{hL_c}{k} = C(\mathrm{Gr}_L\,\mathrm{Pr})^n = C\,\mathrm{Ra}_L^n \tag{9-16}$$

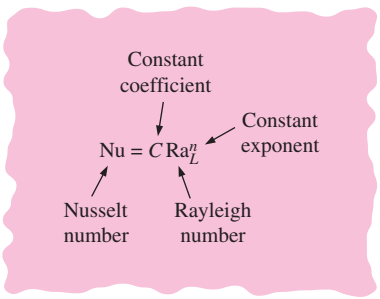


FIGURE 9–9
Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant n multiplied by another constant C, both of which are determined experimentally.

where Ra_L is the **Rayleigh number,** which is the product of the Grashof and Prandtl numbers:

$$\mathrm{Ra}_L = \mathrm{Gr}_L\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\mathrm{Pr} \tag{9-17}$$

The values of the constants C and n depend on the *geometry* of the surface and the *flow regime,* which is characterized by the range of the Rayleigh number. The value of n is usually $\frac{1}{4}$ for laminar flow and $\frac{1}{3}$ for turbulent flow. The value of the constant C is normally less than 1.

Simple relations for the average Nusselt number for various geometries are given in Table 9–1, together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature $T_f = \frac{1}{2}(T_s + T_\infty)$.

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid

surface at a uniform temperature T_s to the surrounding fluid is expressed by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad (\text{W}) \tag{9-18}$$

where A_s is the heat transfer surface area and h is the average heat transfer coefficient on the surface.

Vertical Plates (T_s = constant)

For a vertical flat plate, the characteristic length is the plate height L. In Table 9–1 we give three relations for the average Nusselt number for an isothermal vertical plate. The first two relations are very simple. Despite its complexity, we suggest using the third one (Eq. 9-21) recommended by Churchill and Chu (1975, Ref. 13) since it is applicable over the entire range of Rayleigh number. This relation is most accurate in the range of $10^{-1} < \text{Ra}_L < 10^9$.

Vertical Plates ($\dot{q}_s$ = constant)

In the case of constant surface heat flux, the rate of heat transfer is known (it is simply $\dot{Q} = \dot{q}_s A_s$), but the surface temperature T_s is not. In fact, T_s increases with height along the plate. It turns out that the Nusselt number relations for the constant surface temperature and constant surface heat flux cases are nearly identical [Churchill and Chu (1975), Ref. 13]. Therefore, the relations for isothermal plates can also be used for plates subjected to uniform heat flux, provided that the plate midpoint temperature $T_{L/2}$ is used for T_s in the evaluation of the film temperature, Rayleigh number, and the Nusselt number. Noting that $h = \dot{q}_s/(T_{L/2} - T_\infty)$, the average Nusselt number in this case can be expressed as

$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \tag{9-27}$$

The midpoint temperature $T_{L/2}$ is determined by iteration so that the Nusselt numbers determined from Eqs. 9-21 and 9-27 match.

Vertical Cylinders

An outer surface of a vertical cylinder can be treated as a vertical plate when the diameter of the cylinder is sufficiently large so that the curvature effects are negligible. This condition is satisfied if

$$D \geq \frac{35L}{\text{Gr}_L^{1/4}} \tag{9-28}$$

When this criteria is met, the relations for vertical plates can also be used for vertical cylinders. Nusselt number relations for slender cylinders that do not meet this criteria are available in the literature [e.g., Cebeci (1974), Ref. 8].

Inclined Plates

Consider an inclined hot plate that makes an angle θ from the vertical, as shown in Figure 9–10, in a cooler environment. The net force $F = g(\rho_\infty - \rho)$ (the difference between the buoyancy and gravity) acting on a unit volume of the fluid in the boundary layer is always in the vertical direction. In the case

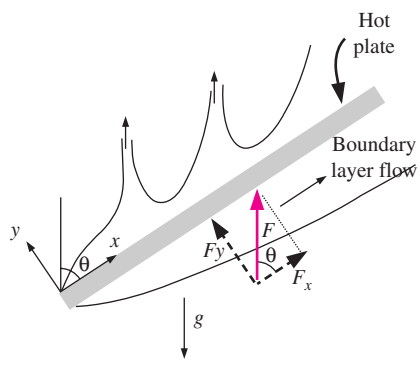
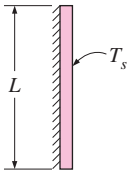
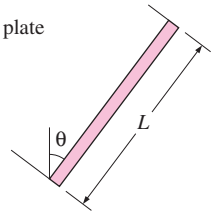
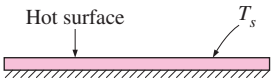
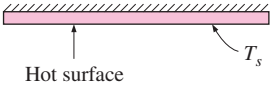
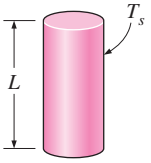
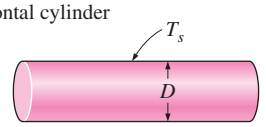
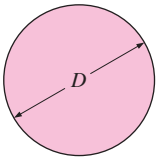


FIGURE 9–10
Natural convection flows on the upper and lower surfaces of an inclined hot plate.

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20) $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\text{Gr}_L^{1/4}}$
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \dfrac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ ($\text{Pr} \geq 0.7$)	$\text{Nu} = 2 + \dfrac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26)

of inclined plate, this force can be resolved into two components: $F_y = F \cos \theta$ parallel to the plate that drives the flow along the plate, and $F_y = F \sin \theta$ normal to the plate. Noting that the force that drives the motion is reduced, we expect the convection currents to be weaker, and the rate of heat transfer to be lower relative to the vertical plate case.

The experiments confirm what we suspect for the lower surface of a hot plate, but the opposite is observed on the upper surface. The reason for this curious behavior for the upper surface is that the force component F_y initiates upward motion in addition to the parallel motion along the plate, and thus the boundary layer breaks up and forms plumes, as shown in the figure. As a result, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation.

In the case of a cold plate in a warmer environment, the opposite occurs as expected: The boundary layer on the upper surface remains intact with weaker boundary layer flow and thus lower rate of heat transfer, and the boundary layer on the lower surface breaks apart (the colder fluid falls down) and thus enhances heat transfer.

When the boundary layer remains intact (the lower surface of a hot plate or the upper surface of a cold plate), the Nusselt number can be determined from the vertical plate relations provided that g in the Rayleigh number relation is replaced by $g \cos \theta$ for $\theta < 60^\circ$. Nusselt number relations for the other two surfaces (the upper surface of a hot plate or the lower surface of a cold plate) are available in the literature [e.g., Fujiii and Imura (1972), Ref. 18].

Horizontal Plates

The rate of heat transfer to or from a horizontal surface depends on whether the surface is facing upward or downward. For a hot surface in a cooler environment, the net force acts upward, forcing the heated fluid to rise. If the hot surface is facing upward, the heated fluid rises freely, inducing strong natural convection currents and thus effective heat transfer, as shown in Figure 9–11. But if the hot surface is facing downward, the plate will block the heated fluid that tends to rise (except near the edges), impeding heat transfer. The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in this case acts downward, and the cooled fluid near the plate tends to descend.

The average Nusselt number for horizontal surfaces can be determined from the simple power-law relations given in Table 9–1. The characteristic length for horizontal surfaces is calculated from

$$L_c = \frac{A_s}{p} \tag{9-29}$$

where A_s is the surface area and p is the perimeter. Note that $L_c = a/4$ for a horizontal square surface of length a, and $D/4$ for a horizontal circular surface of diameter D.

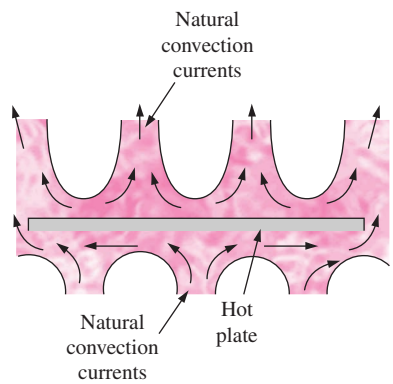


FIGURE 9–11
Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

Horizontal Cylinders and Spheres

The boundary layer over a hot horizontal cylinder start to develop at the bottom, increasing in thickness along the circumference, and forming a rising

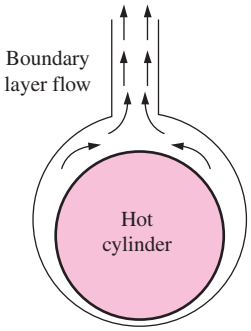


FIGURE 9–12
Natural convection flow over a horizontal hot cylinder.

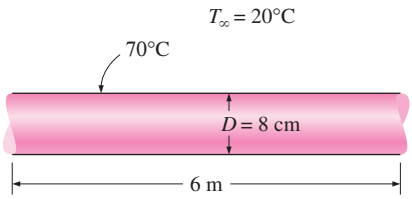


FIGURE 9–13
Schematic for Example 9–1.

plume at the top, as shown in Figure 9–12. Therefore, the local Nusselt number is highest at the bottom, and lowest at the top of the cylinder when the boundary layer flow remains laminar. The opposite is true in the case of a cold horizontal cylinder in a warmer medium, and the boundary layer in this case starts to develop at the top of the cylinder and ending with a descending plume at the bottom.

The average Nusselt number over the entire surface can be determined from Eq. 9-26 [Churchill and Chu (1975), Ref. 13] for an isothermal horizontal cylinder, and from Eq. 9-27 for an isothermal sphere [Churchill (1983}, Ref. 11] both given in Table 9–1.

EXAMPLE 9–1 **Heat Loss from Hot Water Pipes**

A 6-m-long section of an 8-cm-diameter horizontal hot water pipe shown in Figure 9–13 passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

SOLUTION A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45°C$ and 1 atm are (Table A–15)

$$k = 0.02699 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7241$$

$$\nu = 1.749 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = \frac{1}{318 \text{ K}}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.08$ m. Then the Rayleigh number becomes

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(318 \text{ K})](70 - 20 \text{ K})(0.08 \text{ m})^3}{(1.749 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.7241) = 1.869 \times 10^6$$

The natural convection Nusselt number in this case can be determined from Eq. 9-25 to be

$$\text{Nu} = \left\{0.6 + \frac{0.387 \text{ Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1869 \times 10^6)^{1/6}}{[1 + (0.559/0.7241)^{9/16}]^{8/27}}\right\}^2$$

$$= 17.40$$

Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.02699 \text{ W/m} \cdot °\text{C}}{0.08 \text{ m}}(17.40) = 5.869 \text{ W/m} \cdot °\text{C}$$

$$A_s = \pi DL = \pi(0.08 \text{ m})(6 \text{ m}) = 1.508 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.869 \text{ W/m}^2 \cdot °\text{C})(1.508 \text{ m}^2)(70 - 20)°\text{C} = \mathbf{443 \text{ W}}$$

Therefore, the pipe will lose heat to the air in the room at a rate of 443 W by natural convection.

Discussion The pipe will lose heat to the surroundings by radiation as well as by natural convection. Assuming the outer surface of the pipe to be black (emissivity $\varepsilon = 1$) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer is determined to be (Fig. 9–14)

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4)\\ &= (1)(1.508\ \text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(70 + 273\ \text{K})^4 - (20 + 273\ \text{K})^4]\\ &= 553\ \text{W}\end{aligned}$$

which is larger than natural convection. The emissivity of a real surface is less than 1, and thus the radiation heat transfer for a real surface will be less. But radiation will still be significant for most systems cooled by natural convection. Therefore, a radiation analysis should normally accompany a natural convection analysis unless the emissivity of the surface is low.

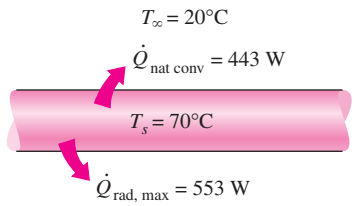


FIGURE 9–14
Radiation heat transfer is usually comparable to natural convection in magnitude and should be considered in heat transfer analysis.

EXAMPLE 9–2 Cooling of a Plate in Different Orientations

Consider a 0.6-m × 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Figure 9–15. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down.

SOLUTION A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60°C$ and 1 atm are (Table A-15)

$$k = 0.02808\ \text{W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7202$$

$$\nu = 1.896 \times 10^{-5}\ \text{m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = \frac{1}{333\ \text{K}}$$

Analysis (*a*) *Vertical.* The characteristic length in this case is the height of the plate, which is $L = 0.6$ m. The Rayleigh number is

$$\begin{aligned}\text{Ra}_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr}\\ &= \frac{(9.81\ \text{m/s}^2)[1/(333\ \text{K})](90 - 30\ \text{K})(0.6\ \text{m})^3}{(1.896 \times 10^{-5}\ \text{m}^2/\text{s})^2}(0.722) = 7.656 \times 10^8\end{aligned}$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

$$\begin{aligned}\text{Nu} &= \left\{0.825 + \frac{0.387\ \text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2\\ &= \left\{0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.4\end{aligned}$$

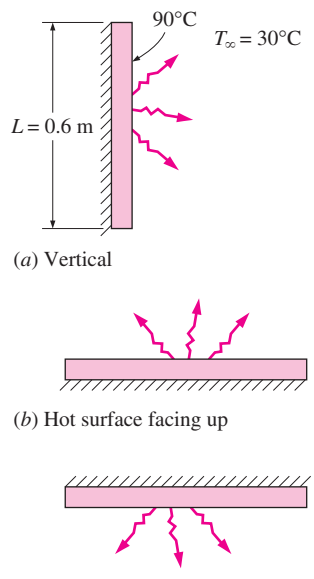


FIGURE 9–15
Schematic for Example 9–2.

Note that the simpler relation Eq. 9-19 would give $\text{Nu} = 0.59\ \text{Ra}_L^{1/4} = 98.14$, which is 13 percent lower. Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.02808\ \text{W/m}\cdot{}^\circ\text{C}}{0.6\ \text{m}}(113.4) = 5.306\ \text{W/m}^2\cdot{}^\circ\text{C}$$
$$A_s = L^2 = (0.6\ \text{m})^2 = 0.36\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306\ \text{W/m}^2\cdot{}^\circ\text{C})(0.36\ \text{m}^2)(90 - 30)^\circ\text{C} = \mathbf{115\ W}$$

(*b*) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6\ \text{m}}{4} = 0.15\ \text{m}$$
$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81\ \text{m/s}^2)[1/(333\ \text{K})](90 - 30\ \text{K})(0.15\ \text{m})^3}{(1.896\times 10^{-5}\ \text{m}^2/\text{s})^2}(0.7202) = 1.196\times 10^7$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$\text{Nu} = 0.54\ \text{Ra}_L^{1/4} = 0.54(1.196\times 10^7)^{1/4} = 31.76$$

Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.0280\ \text{W/m}\cdot{}^\circ\text{C}}{0.15\ \text{m}}(31.76) = 5.946\ \text{W/m}^2\cdot{}^\circ\text{C}$$
$$A_s = L^2 = (0.6\ \text{m})^2 = 0.36\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946\ \text{W/m}^2\cdot{}^\circ\text{C})(0.36\ \text{m}^2)(90 - 30)^\circ\text{C} = \mathbf{128\ W}$$

(*c*) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (*b*). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$\text{Nu} = 0.27\ \text{Ra}_L^{1/4} = 0.27(1.196\times 10^7)^{1/4} = 15.86$$

Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.02808\ \text{W/m}\cdot{}^\circ\text{C}}{0.15\ \text{m}}(15.86) = 2.973\ \text{W/m}^2\cdot{}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973\ \text{W/m}^2\cdot{}^\circ\text{C})(0.36\ \text{m}^2)(90 - 30)^\circ\text{C} = \mathbf{64.2\ W}$$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is "trapped" under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer.

Discussion The plate will lose heat to the surroundings by radiation as well as by natural convection. Assuming the surface of the plate to be black (emissivity

$\varepsilon = 1$) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer in this case is determined to be

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4)\\ &= (1)(0.36\ \text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(90 + 273\ \text{K})^4 - (30 + 273\ \text{K})^4]\\ &= 182\ \text{W}\end{aligned}$$

which is larger than that for natural convection heat transfer for each case. Therefore, radiation can be significant and needs to be considered in surfaces cooled by natural convection.

9–4 ▪ NATURAL CONVECTION FROM FINNED SURFACES AND PCBs

Natural convection flow through a channel formed by two parallel plates as shown in Figure 9–16 is commonly encountered in practice. When the plates are hot ($T_s > T_\infty$), the ambient fluid at T_∞ enters the channel from the lower end, rises as it is heated under the effect of buoyancy, and the heated fluid leaves the channel from the upper end. The plates could be the fins of a finned heat sink, or the PCBs (printed circuit boards) of an electronic device. The plates can be approximated as being isothermal (T_s = constant) in the first case, and isoflux ($\dot{q}_s$ = constant) in the second case.

Boundary layers start to develop at the lower ends of opposing surfaces, and eventually merge at the midplane if the plates are vertical and sufficiently long. In this case, we will have fully developed channel flow after the merger of the boundary layers, and the natural convection flow is analyzed as channel flow. But when the plates are short or the spacing is large, the boundary layers of opposing surfaces never reach each other, and the natural convection flow on a surface is not affected by the presence of the opposing surface. In that case, the problem should be analyzed as natural convection from two independent plates in a quiescent medium, using the relations given for surfaces, rather than natural convection flow through a channel.

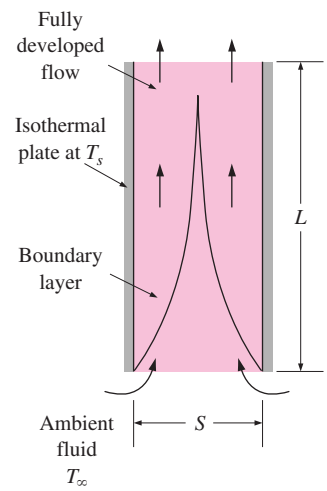


FIGURE 9–16
Natural convection flow through a channel between two isothermal vertical plates.

Natural Convection Cooling of Finned Surfaces (T_s = constant)

Finned surfaces of various shapes, called *heat sinks,* are frequently used in the cooling of electronic devices. Energy dissipated by these devices is transferred to the heat sinks by conduction and from the heat sinks to the ambient air by natural or forced convection, depending on the power dissipation requirements. Natural convection is the preferred mode of heat transfer since it involves no moving parts, like the electronic components themselves. However, in the natural convection mode, the components are more likely to run at a higher temperature and thus undermine reliability. A properly selected heat sink may considerably lower the operation temperature of the components and thus reduce the risk of failure.

Natural convection from vertical finned surfaces of rectangular shape has been the subject of numerous studies, mostly experimental. Bar-Cohen and

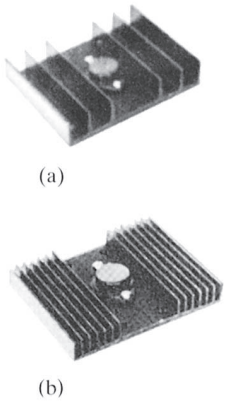


FIGURE 9–17
Heat sinks with (*a*) widely spaced and (*b*) closely packed fins (courtesy of Vemaline Products).

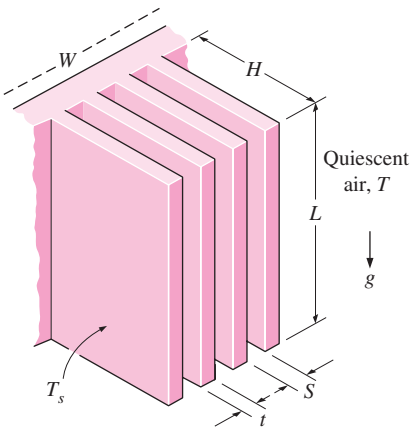


FIGURE 9–18
Various dimensions of a finned surface oriented vertically.

Rohsenow (1984, Ref. 5) have compiled the available data under various boundary conditions, and developed correlations for the Nusselt number and optimum spacing. The characteristic length for vertical parallel plates used as fins is usually taken to be the spacing between adjacent fins *S,* although the fin height *L* could also be used. The Rayleigh number is expressed as

$$\mathrm{Ra}_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2}\mathrm{Pr} \quad \text{and} \quad \mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \mathrm{Ra}_S\frac{L^3}{S^3} \tag{9-30}$$

The recommended relation for the average Nusselt number for vertical isothermal parallel plates is

$$T_s = \text{constant:} \qquad \mathrm{Nu} = \frac{hS}{k} = \left[\frac{576}{(\mathrm{Ra}_S S/L)^2} + \frac{2.873}{(\mathrm{Ra}_S S/L)^{0.5}}\right]^{-0.5} \tag{9-31}$$

A question that often arises in the selection of a heat sink is whether to select one with *closely packed* fins or *widely spaced* fins for a given base area (Fig. 9–17). A heat sink with closely packed fins will have greater surface area for heat transfer but a smaller heat transfer coefficient because of the extra resistance the additional fins introduce to fluid flow through the interfin passages. A heat sink with widely spaced fins, on the other hand, will have a higher heat transfer coefficient but a smaller surface area. Therefore, there must be an *optimum spacing* that maximizes the natural convection heat transfer from the heat sink for a given base area *WL,* where *W* and *L* are the width and height of the base of the heat sink, respectively, as shown in Figure 9–18. When the fins are essentially isothermal and the fin thickness *t* is small relative to the fin spacing *S,* the optimum fin spacing for a vertical heat sink is determined by Bar-Cohen and Rohsenow to be

$$T_s = \text{constant:} \qquad S_{\text{opt}} = 2.714\left(\frac{S^3L}{\mathrm{Ra}_S}\right)^{0.25} = 2.714\,\frac{L}{\mathrm{Ra}_L^{0.25}} \tag{9-32}$$

It can be shown by combining the three equations above that when $S = S_{\text{opt}}$, the Nusselt number is a constant and its value is 1.307,

$$S = S_{\text{opt}}: \qquad \mathrm{Nu} = \frac{hS_{\text{opt}}}{k} = 1.307 \tag{9-33}$$

The rate of heat transfer by natural convection from the fins can be determined from

$$\dot{Q} = h(2nLH)(T_s - T_\infty) \tag{9-34}$$

where $n = W/(S + t) \approx W/S$ is the number of fins on the heat sink and T_s is the surface temperature of the fins. All fluid properties are to be evaluated at the average temperature $T_{\text{ave}} = (T_s + T_\infty)/2$.

Natural Convection Cooling of Vertical PCBs ($\dot{q}_s$ = constant)

Arrays of printed circuit boards used in electronic systems can often be modeled as parallel plates subjected to uniform heat flux $\dot{q}_s$ (Fig. 9–19). The plate temperature in this case increases with height, reaching a maximum at the

upper edge of the board. The modified Rayleigh number for uniform heat flux on both plates is expressed as

$$\mathrm{Ra}_S^* = \frac{g\beta \dot{q}_s S^4}{k\nu^2}\,\mathrm{Pr} \tag{9-35}$$

The Nusselt number at the upper edge of the plate where maximum temperature occurs is determined from [Bar-Cohen and Rohsenow (1984), Ref. 5]

$$\mathrm{Nu}_L = \frac{h_L S}{k} = \left[\frac{48}{\mathrm{Ra}_S^* S/L} + \frac{2.51}{(\mathrm{Ra}_L^* S/L)^{0.4}}\right]^{-0.5} \tag{9-36}$$

The optimum fin spacing for the case of uniform heat flux on both plates is given as

$$\dot{q}_s = \text{constant:} \qquad S_{\text{opt}} = 2.12\left(\frac{S^4 L}{\mathrm{Ra}_S^*}\right)^{0.2} \tag{9-37}$$

The total rate of heat transfer from the plates is

$$\dot{Q} = \dot{q}_s A_s = \dot{q}_s(2nLH) \tag{9-38}$$

where $n = W/(S + t) \approx W/S$ is the number of plates. The critical surface temperature T_L occurs at the upper edge of the plates, and it can be determined from

$$\dot{q}_s = h_L(T_L - T_\infty) \tag{9-39}$$

All fluid properties are to be evaluated at the average temperature $T_{\text{ave}} = (T_L + T_\infty)/2$.

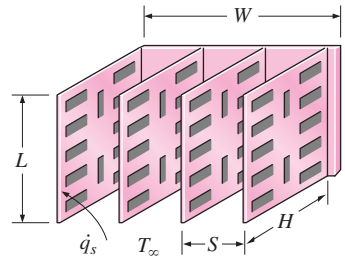


FIGURE 9–19
Arrays of vertical printed circuit boards (PCBs) cooled by natural convection.

Mass Flow Rate through the Space between Plates

As we mentioned earlier, the magnitude of the natural convection heat transfer is directly related to the mass flow rate of the fluid, which is established by the dynamic balance of two opposing effects: *buoyancy* and *friction.*

The fins of a heat sink introduce both effects: *inducing extra buoyancy* as a result of the elevated temperature of the fin surfaces and *slowing down the fluid* by acting as an added obstacle on the flow path. As a result, increasing the number of fins on a heat sink can either enhance or reduce natural convection, depending on which effect is dominant. The buoyancy-driven fluid flow rate is established at the point where these two effects balance each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting fluid flow and heat transfer. Under some conditions, the increase in friction may more than offset the increase in buoyancy. This in turn will tend to reduce the flow rate and thus the heat transfer. For that reason, heat sinks with closely spaced fills are not suitable for natural convection cooling.

When the heat sink involves closely spaced fins, the narrow channels formed tend to block or "suffocate" the fluid, especially when the heat sink is long. As a result, the blocking action produced overwhelms the extra buoyancy and downgrades the heat transfer characteristics of the heat sink. Then, at a fixed power setting, the heat sink runs at a higher temperature relative to the no-shroud case. When the heat sink involves widely spaced fins, the

shroud does not introduce a significant increase in resistance to flow, and the buoyancy effects dominate. As a result, heat transfer by natural convection may improve, and at a fixed power level the heat sink may run at a lower temperature.

When extended surfaces such as fins are used to enhance natural convection heat transfer between a solid and a fluid, the flow rate of the fluid in the vicinity of the solid adjusts itself to incorporate the changes in buoyancy and friction. It is obvious that this enhancement technique will work to advantage only when the increase in buoyancy is greater than the additional friction introduced. One does not need to be concerned with pressure drop or pumping power when studying natural convection since no pumps or blowers are used in this case. Therefore, an enhancement technique in natural convection is evaluated on heat transfer performance alone.

The failure rate of an electronic component increases almost exponentially with operating temperature. The cooler the electronic device operates, the more reliable it is. A rule of thumb is that the semiconductor failure rate is halved for each 10°C reduction in junction operating temperature. The desire to lower the operating temperature without having to resort to forced convection has motivated researchers to investigate enhancement techniques for natural convection. Sparrow and Prakash (Ref. 31) have demonstrated that, under certain conditions, the use of discrete plates in lieu of continuous plates of the same surface area increases heat transfer considerably. In other experimental work, using transistors as the heat source, Çengel and Zing (Ref. 9) have demonstrated that temperature recorded on the transistor case dropped by as much as 30°C when a shroud was used, as opposed to the corresponding no-shroud case.

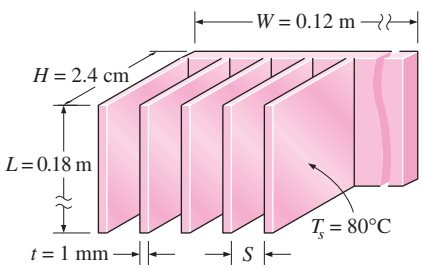


FIGURE 9–20
Schematic for Example 9–3.

EXAMPLE 9–3 Optimum Fin Spacing of a Heat Sink

A 12-cm-wide and 18-cm-high vertical hot surface in 30°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile (Fig. 9–20). The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.

SOLUTION A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The atmospheric pressure at that location is 1 atm. **4** The thickness t of the fins is very small relative to the fin spacing S so that Eq. 9-32 for optimum fin spacing is applicable. **5** All fin surfaces are isothermal at base temperature.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (80 + 30)/2 = 55°C$ and 1 atm pressure are (Table A-15)

$$k = 0.02772 \text{ W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 0.7215$$
$$\nu = 1.846 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = 1/T_f = 1/328 \text{ K}$$

Analysis We take the characteristic length to be the length of the fins in the vertical direction (since we do not know the fin spacing). Then the Rayleigh number becomes

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr}$$
$$= \frac{(981\ \mathrm{m/s^2})[1/(328\ \mathrm{K})](80 - 30\ \mathrm{K})(0.18\ \mathrm{m})^3}{(1.846 \times 10^{-5}\ \mathrm{m^2/s})^2}(0.7215) = 1.846 \times 10^7$$

The optimum fin spacing is determined from Eq. 7-32 to be

$$S_{\mathrm{opt}} = 2.714\frac{L}{\mathrm{Ra}_L^{0.25}} = 2.714\frac{0.8\ \mathrm{m}}{(1.846 \times 10^7)^{0.25}} = 7.45 \times 10^{-3}\ \mathrm{m} = \mathbf{7.45\ mm}$$

which is about seven times the thickness of the fins. Therefore, the assumption of negligible fin thickness in this case is acceptable. The number of fins and the heat transfer coefficient for this optimum fin spacing case are

$$n = \frac{W}{S + t} = \frac{0.12\ \mathrm{m}}{(0.00745 + 0.0001)\ \mathrm{m}} \approx 15\ \mathrm{fins}$$

The convection coefficient for this optimum in spacing case is, from Eq. 9-33,

$$h = \mathrm{Nu}_{\mathrm{opt}}\frac{k}{S_{\mathrm{opt}}} = 1.307\frac{0.02772\ \mathrm{W/m \cdot ^\circ C}}{0.00745\ \mathrm{m}} = 0.2012\ \mathrm{W/m^2 \cdot ^\circ C}$$

Then the rate of natural convection heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = h(2nLH)(T_s - T_\infty)$$
$$= (0.2012\ \mathrm{W/m^2 \cdot ^\circ C})[2 \times 15(0.18\ \mathrm{m})(0.024\ \mathrm{m})](80 - 30)^\circ\mathrm{C} = \mathbf{1.30\ W}$$

Therefore, this heat sink can dissipate heat by natural convection at a rate of 1.30 W.

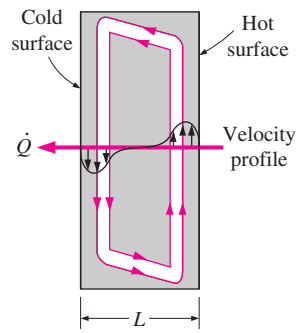


FIGURE 9–21
Convective currents in a vertical rectangular enclosure.

9–5 ▪ NATURAL CONVECTION INSIDE ENCLOSURES

A considerable portion of heat loss from a typical residence occurs through the windows. We certainly would insulate the windows, if we could, in order to conserve energy. The problem is finding an insulating material that is transparent. An examination of the thermal conductivities of the insulting materials reveals that *air* is a *better insulator* than most common insulating materials. Besides, it is transparent. Therefore, it makes sense to insulate the windows with a layer of air. Of course, we need to use another sheet of glass to trap the air. The result is an *enclosure,* which is known as a *double-pane window* in this case. Other examples of enclosures include wall cavities, solar collectors, and cryogenic chambers involving concentric cylinders or spheres.

Enclosures are frequently encountered in practice, and heat transfer through them is of practical interest. Heat transfer in enclosed spaces is complicated by the fact that the fluid in the enclosure, in general, does not remain stationary. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotationary motion within the enclosure that enhances heat transfer through the enclosure. Typical flow patterns in vertical and horizontal rectangular enclosures are shown in Figures 9–21 and 9–22.

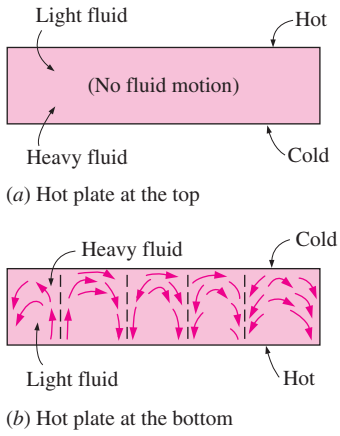


FIGURE 9–22
Convective currents in a horizontal enclosure with (*a*) hot plate at the top and (*b*) hot plate at the bottom.

The characteristics of heat transfer through a horizontal enclosure depend on whether the hotter plate is at the top or at the bottom, as shown in Figure 9–22. When the hotter plate is at the *top,* no convection currents will develop in the enclosure, since the lighter fluid will always be on top of the heavier fluid. Heat transfer in this case will be by *pure conduction,* and we will have Nu = 1. When the hotter plate is at the *bottom,* the heavier fluid will be on top of the lighter fluid, and there will be a tendency for the lighter fluid to topple the heavier fluid and rise to the top, where it will come in contact with the cooler plate and cool down. Until that happens, however, the heat transfer is still by *pure conduction* and Nu = 1. When Ra > 1708, the buoyant force overcomes the fluid resistance and initiates natural convection currents, which are observed to be in the form of hexagonal cells called *Bénard cells.* For Ra > 3×10^5, the cells break down and the fluid motion becomes turbulent.

The Rayleigh number for an enclosure is determined from

$$\mathrm{Ra}_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\,\mathrm{Pr} \tag{9-40}$$

where the characteristic length L_c is the distance between the hot and cold surfaces, and T_1 and T_2 are the temperatures of the hot and cold surfaces, respectively. All fluid properties are to be evaluated at the average fluid temperature $T_{\text{ave}} = (T_1 + T_2)/2$.

Effective Thermal Conductivity

When the Nusselt number is known, the rate of heat transfer through the enclosure can be determined from

$$\dot{Q} = hA_s(T_1 - T_2) = k\mathrm{Nu}A_s\frac{T_1 - T_2}{L_c} \tag{9-41}$$

since $h = k\mathrm{Nu}/L$. The rate of steady heat conduction across a layer of thickness L_c, area A_s, and thermal conductivity k is expressed as

$$\dot{Q}_{\text{cond}} = kA_s\frac{T_1 - T_2}{L_c} \tag{9-42}$$

where T_1 and T_2 are the temperatures on the two sides of the layer. A comparison of this relation with Eq. 9-41 reveals that the convection heat transfer in an enclosure is analogous to heat conduction across the fluid layer in the enclosure provided that the thermal conductivity k is replaced by kNu. That is, *the fluid in an enclosure behaves like a fluid whose thermal conductivity is kNu as a result of convection currents.* Therefore, the quantity kNu is called the **effective thermal conductivity** of the enclosure. That is,

$$k_{\text{eff}} = k\mathrm{Nu} \tag{9-43}$$

Note that for the special case of Nu = 1, the effective thermal conductivity of the enclosure becomes equal to the conductivity of the fluid. This is expected since this case corresponds to pure conduction (Fig. 9–23).

Natural convection heat transfer in enclosed spaces has been the subject of many experimental and numerical studies, and numerous correlations for the Nusselt number exist. Simple power-law type relations in the form of

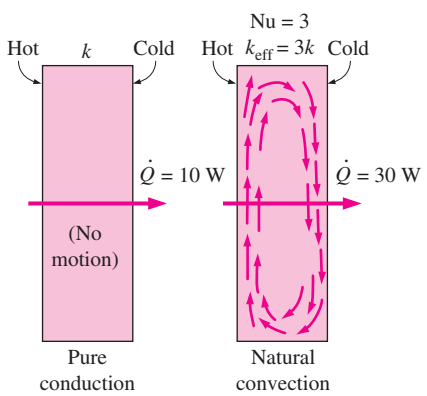


FIGURE 9–23
A Nusselt number of 3 for an enclosure indicates that heat transfer through the enclosure by *natural convection* is three times that by *pure conduction.*

$\text{Nu} = C\text{Ra}_L^n$, where C and n are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios. The relations that are more comprehensive are naturally more complex. Next we present some widely used relations for various types of enclosures.

Horizontal Rectangular Enclosures

We need no Nusselt number relations for the case of the hotter plate being at the top, since there will be no convection currents in this case and heat transfer will be downward by conduction (Nu = 1). When the hotter plate is at the bottom, however, significant convection currents set in for $\text{Ra}_L > 1708$, and the rate of heat transfer increases (Fig. 9–24).

For horizontal enclosures that contain air, Jakob (1949, Ref. 22) recommends the following simple correlations

$$\text{Nu} = 0.195\text{Ra}_L^{1/4} \qquad 10^4 < \text{Ra}_L < 4 \times 10^5 \tag{9-44}$$

$$\text{Nu} = 0.068\text{Ra}_L^{1/3} \qquad 4 \times 10^5 < \text{Ra}_L < 10^7 \tag{9-45}$$

These relations can also be used for other gases with $0.5 < \text{Pr} < 2$. Using water, silicone oil, and mercury in their experiments, Globe and Dropkin (1959) obtained this correlation for horizontal enclosures heated from below,

$$\text{Nu} = 0.069\text{Ra}_L^{1/3}\,\text{Pr}^{0.074} \qquad 3 \times 10^5 < \text{Ra}_L < 7 \times 10^9 \tag{9-46}$$

Based on experiments with air, Hollands et al (1976, Ref. 19) recommend this correlation for horizontal enclosures,

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}_L}\right]^+ + \left[\frac{\text{Ra}_L^{1/3}}{18} - 1\right]^+ \qquad \text{Ra}_L < 10^8 \tag{9-47}$$

The notation $[\ ]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. This relation also correlates data well for liquids with moderate Prandtl numbers for $\text{Ra}_L < 10^5$, and thus it can also be used for water.

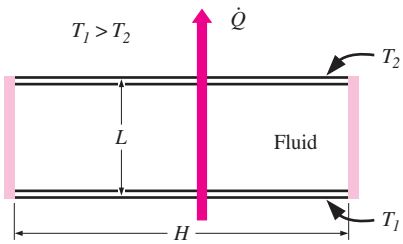


FIGURE 9–24
A horizontal rectangular enclosure with isothermal surfaces.

Inclined Rectangular Enclosures

Air spaces between two inclined parallel plates are commonly encountered in flat-plate solar collectors (between the glass cover and the absorber plate) and the double-pane skylights on inclined roofs. Heat transfer through an inclined enclosure depends on the **aspect ratio** H/L as well as the tilt angle θ from the horizontal (Fig. 9–25).

For large aspect ratios ($H/L \geq 12$), this equation [Hollands et al., 1976, Ref. 19] correlates experimental data extremely well for tilt angles up to 70°,

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}_L \cos\theta}\right]^+\left(1 - \frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}_L \cos\theta}\right) + \left[\frac{(\text{Ra}_L \cos\theta)^{1/3}}{18} - 1\right]^+ \tag{9-48}$$

for $\text{Ra}_L < 10^5$, $0 < \theta < 70°$, and $H/L \geq 12$. Again any quantity in $[\ ]^+$ should be set equal to zero if it is negative. This is to ensure that Nu = 1 for $\text{Ra}_L \cos\theta < 1708$. Note that this relation reduces to Eq. 9-47 for horizontal enclosures for $\theta = 0°$, as expected.

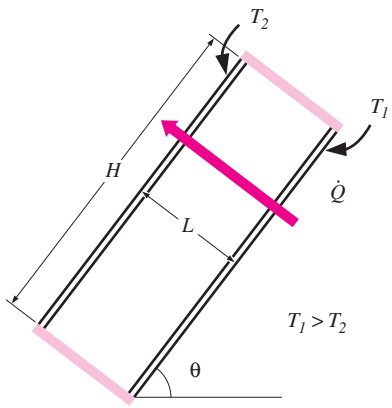


FIGURE 9–25
An inclined rectangular enclosure with isothermal surfaces.

For enclosures with smaller aspect ratios ($H/L < 12$), the next correlation can be used provided that the tilt angle is less than the critical value θ_{cr} listed in Table 9–2 [Catton (1978), Ref. 7]

$$\mathrm{Nu} = \mathrm{Nu}_{\theta=0^\circ}\left(\frac{\mathrm{Nu}_{\theta=90^\circ}}{\mathrm{Nu}_{\theta=0^\circ}}\right)^{\theta/\theta_{cr}}(\sin\theta_{cr})^{\theta/(4\theta_{cr})} \qquad 0^\circ < \theta < \theta_{cr} \tag{9-49}$$

For tilt angles greater than the critical value ($\theta_{cr} < \theta < 90^\circ$), the Nusselt number can be obtained by multiplying the Nusselt number for a vertical enclosure by $(\sin\theta)^{1/4}$ [Ayyaswamy and Catton (1973), Ref. 3],

$$\mathrm{Nu} = \mathrm{Nu}_{\theta=90^\circ}(\sin\theta)^{1/4} \qquad \theta_{cr} < \theta < 90^\circ, \text{ any } H/L \tag{9-50}$$

For enclosures tilted more than 90°, the recommended relation is [Arnold et al., (1974), Ref. 2]

$$\mathrm{Nu} = 1 + (\mathrm{Nu}_{\theta=90^\circ} - 1)\sin\theta \qquad 90^\circ < \theta < 180^\circ, \text{ any } H/L \tag{9-51}$$

More recent but more complex correlations are also available in the literature [e.g., and ElSherbiny et al. (1982), Ref. 17].

TABLE 9–2

Critical angles for inclined rectangular enclosures

Aspect ratio, H/L	Critical angle, θ_{cr}
1	25°
3	53°
6	60°
12	67°
> 12	70°

Vertical Rectangular Enclosures

For vertical enclosures (Fig. 9–26), Catton (1978, Ref. 7) recommends these two correlations due to Berkovsky and Polevikov (1977, Ref. 6),

$$\mathrm{Nu} = 0.18\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29} \qquad \begin{array}{c} 1 < H/L < 2 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L\,\mathrm{Pr}/(0.2+\mathrm{Pr}) > 10^3 \end{array} \tag{9-52}$$

$$\mathrm{Nu} = 0.22\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \qquad \begin{array}{c} 2 < H/L < 10 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L < 10^{10} \end{array} \tag{9-53}$$

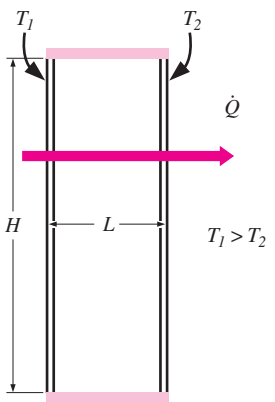


FIGURE 9–26
A vertical rectangular enclosure with isothermal surfaces.

For vertical enclosures with larger aspect ratios, the following correlations can be used [MacGregor and Emery (1969), Ref. 26]

$$\mathrm{Nu} = 0.42\,\mathrm{Ra}_L^{1/4}\,\mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} \qquad \begin{array}{c} 10 < H/L < 40 \\ 1 < \mathrm{Pr} < 2\times10^4 \\ 10^4 < \mathrm{Ra}_L < 10^7 \end{array} \tag{9-54}$$

$$\mathrm{Nu} = 0.46\mathrm{Ra}_L^{1/3} \qquad \begin{array}{c} 1 < H/L < 40 \\ 1 < \mathrm{Pr} < 20 \\ 10^6 < \mathrm{Ra}_L < 10^9 \end{array} \tag{9-55}$$

Again all fluid properties are to be evaluated at the average temperature $(T_1 + T_2)/2$.

Concentric Cylinders

Consider two long concentric horizontal cylinders maintained at uniform but different temperatures of T_i and T_o, as shown in Figure 9–27. The diameters of the inner and outer cylinders are D_i and D_o, respectively, and the characteristic length is the spacing between the cylinders, $L_c = (D_o - D_i)/2$. The rate of heat transfer through the annular space between the natural convection unit is expressed as

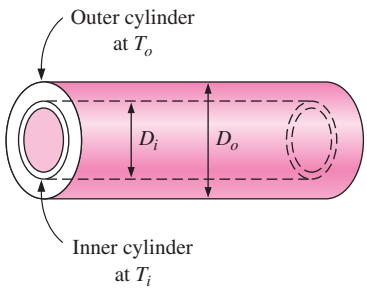


FIGURE 9–27
Two concentric horizontal isothermal cylinders.

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) \qquad \text{(W/m)} \tag{9-56}$$

The recommended relation for effective thermal conductivity is [Raithby and Hollands (1975), Ref. 28]

$$\frac{k_{\text{eff}}}{\text{k}} = 0.386\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4} \tag{9-57}$$

where the geometric factor for concentric cylinders F_{cyl} is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} \tag{9-58}$$

The k_{eff} relation in Eq. 9-57 is applicable for $0.70 \le \text{Pr} \le 6000$ and $10^2 \le F_{\text{cyl}}\text{Ra}_L \le 10^7$. For $F_{\text{cyl}}\text{Ra}_L < 100$, natural convection currents are negligible and thus $k_{\text{eff}} = k$. Note that k_{eff} cannot be less than k, and thus we should set $k_{\text{eff}} = k$ if $k_{\text{eff}}/k < 1$. The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

Concentric Spheres

For concentric isothermal spheres, the rate of heat transfer through the gap between the spheres by natural convection is expressed as (Fig. 9–28)

$$\dot{Q} = k_{\text{eff}}\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o) \qquad \text{(W)} \tag{9-59}$$

where $L_c = (D_o - D_i)/2$ is the characteristic length. The recommended relation for effective thermal conductivity is [Raithby and Hollands (1975), Ref. 28]

$$\frac{k_{\text{eff}}}{\text{k}} = 0.74\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4} \tag{9-60}$$

where the geometric factor for concentric spheres F_{sph} is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5} \tag{9-61}$$

The k_{eff} relation in Eq. 9-60 is applicable for $0.70 \le \text{Pr} \le 4200$ and $10^2 \le F_{\text{sph}}\text{Ra}_L \le 10^4$. If $k_{\text{eff}}/k < 1$, we should set $k_{\text{eff}} = k$.

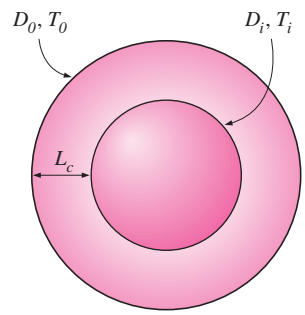


FIGURE 9–28
Two concentric isothermal spheres.

Combined Natural Convection and Radiation

Gases are nearly transparent to radiation, and thus heat transfer through a gas layer is by simultaneous convection (or conduction, if the gas is quiescent) and radiation. Natural convection heat transfer coefficients are typically very low compared to those for forced convection. Therefore, radiation is usually disregarded in forced convection problems, but it must be considered in natural convection problems that involve a gas. This is especially the case for surfaces with high emissivities. For example, about half of the heat transfer through the air space of a double pane window is by radiation. The total rate of heat transfer is determined by adding the convection and radiation components,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \tag{9-62}$$

Radiation heat transfer from a surface at temperature T_s surrounded by surfaces at a temperature T_{surr} (both in absolute temperature unit K) is determined from

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) \qquad \text{(W)} \tag{9-63}$$

where ε is the emissivity of the surface, A_s is the surface area, and $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$ K^4 is the Stefan–Boltzmann constant.

When the end effects are negligible, radiation heat transfer between two large parallel plates at absolute temperatures T_1 and T_2 is expressed as (see Chapter 12 for details)

$$\dot{Q}_{rad} = \frac{\sigma A_s(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \varepsilon_{effective}\,\sigma A_s(T_1^4 - T_2^4) \qquad \text{(W)} \tag{9-64}$$

where ε_1 and ε_2 are the emissivities of the plates, and $\varepsilon_{effective}$ is the *effective emissivity* defined as

$$\varepsilon_{effective} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \tag{9-65}$$

The emissivity of an ordinary glass surface, for example, is 0.84. Therefore, the effective emissivity of two parallel glass surfaces facing each other is 0.72. Radiation heat transfer between concentric cylinders and spheres is discussed in Chapter 12.

Note that in some cases the temperature of the surrounding medium may be below the surface temperature ($T_\infty < T_s$), while the temperature of the surrounding surfaces is above the surface temperature ($T_{surr} > T_s$). In such cases, convection and radiation heat transfers are subtracted from each other instead of being added since they are in opposite directions. Also, for a metal surface, the radiation effect can be reduced to negligible levels by polishing the surface and thus lowering the surface emissivity to a value near zero.

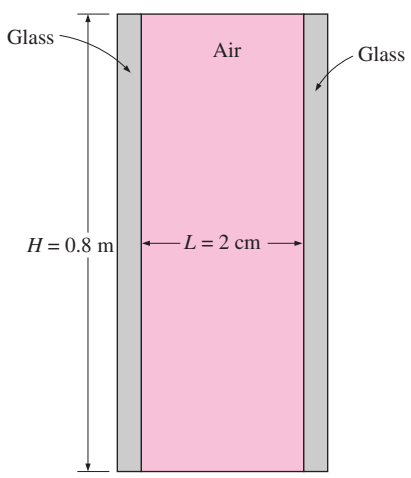


FIGURE 9–29
Schematic for Example 9–4.

EXAMPLE 9–4 Heat Loss through a Double-Pane Window

The vertical 0.8-m-high, 2-m-wide double-pane window shown in Fig. 9–29 consists of two sheets of glass separated by a 2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 12°C and 2°C, determine the rate of heat transfer through the window.

SOLUTION Two glasses of a double-pane window are maintained at specified temperatures. The rate of heat transfer through the window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{ave} = (T_1 + T_2)/2 = (12 + 2)/2 = 7$°C and 1 atm pressure are (Table A-15)

$$k = 0.02416 \text{ W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 0.7344$$

$$\nu = 1.399 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_{ave}} = \frac{1}{280 \text{ K}}$$

Analysis We have a rectangular enclosure filled with air. The characteristic length in this case is the distance between the two glasses, $L = 0.02$ m. Then the Rayleigh number becomes

$$\mathrm{Ra}_L = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}$$
$$= \frac{(9.81\ \mathrm{m/s^2})[1/(280\ \mathrm{K})](12 - 2\ \mathrm{K})(0.02\ \mathrm{m})^3}{(1.399 \times 10^{-5}\mathrm{m^2/s})^2}(0.7344) = 1.051 \times 10^4$$

The aspect ratio of the geometry is $H/L = 0.8/0.02 = 40$. Then the Nusselt number in this case can be determined from Eq. 9-54 to be

$$\mathrm{Nu} = 0.42\mathrm{Ra}_L^{1/4}\,\mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$
$$= 0.42(1.051 \times 10^4)^{1/4}(0.7344)^{0.012}\left(\frac{0.8}{0.02}\right)^{-0.3} = 1.401$$

Then,

$$A_s = H \times W = (0.8\ \mathrm{m})(2\ \mathrm{m}) = 1.6\ \mathrm{m}^2$$

and

$$\dot{Q} = hA_s(T_1 - T_2) = k\mathrm{Nu}A_s\frac{T_1 - T_2}{L}$$
$$= (0.02416\ \mathrm{W/m \cdot {}^\circ C})(1.401)(1.6\ \mathrm{m}^2)\frac{(12 - 2)^\circ\mathrm{C}}{0.02\ \mathrm{m}} = \mathbf{27.1\ W}$$

Therefore, heat will be lost through the window at a rate of 27.1 W.

Discussion Recall that a Nusselt number of Nu = 1 for an enclosure corresponds to pure conduction heat transfer through the enclosure. The air in the enclosure in this case remains still, and no natural convection currents occur in the enclosure. The Nusselt number in our case is 1.32, which indicates that heat transfer through the enclosure is 1.32 times that by pure conduction. The increase in heat transfer is due to the natural convection currents that develop in the enclosure.

EXAMPLE 9–5 Heat Transfer through a Spherical Enclosure

The two concentric spheres of diameters $D_i = 20$ cm and $D_o = 30$ cm shown in Fig. 9–30 are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_i = 320$ K and $T_o = 280$ K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

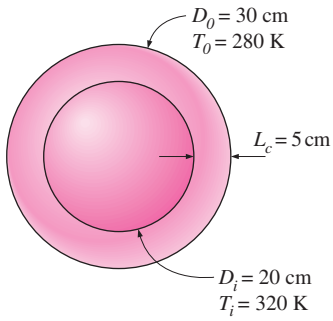


FIGURE 9–30
Schematic for Example 9–5.

SOLUTION Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{\mathrm{ave}} = (T_i + T_o)/2 = (320 + 280)/2 = 300$ K = 27°C and 1 atm pressure are (Table A-15)

$$k = 0.02566\ \mathrm{W/m \cdot {}^\circ C} \qquad \mathrm{Pr} = 0.7290$$
$$\nu = 1.580 \times 10^{-5}\ \mathrm{m^2/s} \qquad \beta = \frac{1}{T_{\mathrm{ave}}} = \frac{1}{300\ \mathrm{K}}$$

Analysis We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05 \text{ m}$$

The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81 \text{ m/s}^2)[1/(300 \text{ K})](320 - 280 \text{ K})(0.05 \text{ m})^3}{(1.58 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.729) = 4.776 \times 10^5$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5}$$
$$= \frac{0.05 \text{ m}}{[(0.2 \text{ m})(0.3 \text{ m})]^4[(0.2 \text{ m}^{-7/5} + (0.3 \text{ m})^{-7/5}]^5} = 0.005229$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}$$
$$= 0.74(0.02566 \text{ W/m} \cdot {}^\circ\text{C})\left(\frac{0.729}{0.861 + 0.729}\right)(0.005229 \times 4.776 \times 10^5)^{1/4}$$
$$= 0.1104 \text{ W/m} \cdot {}^\circ\text{C}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{\text{eff}}\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o)$$
$$= (0.1104 \text{ W/m} \cdot {}^\circ\text{C})\pi\left(\frac{(0.2 \text{ m})(0.3 \text{ m})}{0.05 \text{ m}}\right)(320 - 280)\text{K} = \mathbf{16.7\ W}$$

Therefore, heat will be lost from the inner sphere to the outer one at a rate of 16.7 W.

Discussion Note that the air in the spherical enclosure will act like a stationary fluid whose thermal conductivity is $k_{\text{eff}}/k = 0.1104/0.02566 = 4.3$ times that of air as a result of natural convection currents. Also, radiation heat transfer between spheres is usually very significant, and should be considered in a complete analysis.

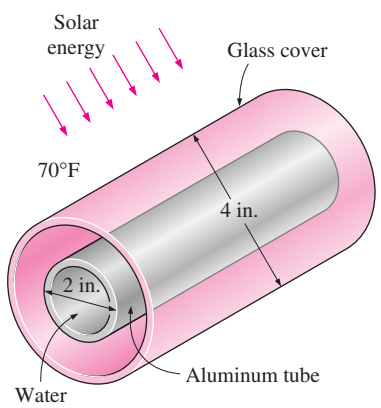


FIGURE 9–31
Schematic for Example 9–6.

EXAMPLE 9–6 Heating Water in a Tube by Solar Energy

A solar collector consists of a horizontal aluminum tube having an outer diameter of 2 in. enclosed in a concentric thin glass tube of 4-in.-diameter (Fig. 9–31). Water is heated as it flows through the tube, and the annular space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 30 Btu/h per foot length, and the temperature of the ambient air outside is 70°F. Disregarding any heat loss by radiation, determine the temperature of the aluminum tube when steady operation is established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).

SOLUTION The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** Heat loss by radiation is negligible.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 110°F, and use properties at an anticipated average temperature of (70 + 110)/2 = 90°F (Table A-15E),

$$k = 0.01505 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F} \qquad \text{Pr} = 0.7275$$

$$\nu = 0.6310 \text{ ft}^2/\text{h} = 1.753 \times 10^{-4} \text{ ft}^2/\text{s} \qquad \beta = \frac{1}{T_{\text{ave}}} = \frac{1}{550 \text{ K}}$$

Analysis We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \qquad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{\text{glass}} = (\pi D_o L) = \pi(4/12 \text{ ft})(1 \text{ ft}) = 1.047 \text{ ft}^2 \qquad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial-and-error approach. Assuming the glass cover temperature to be 100°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\text{Ra}_{D_o} = \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2}\text{Pr}$$
$$= \frac{(32.2 \text{ ft/s}^2)[1/(550 \text{ R})](110 - 70 \text{ R})(4/12 \text{ ft})^3}{(1.753 \times 10^{-4} \text{ ft}^2/\text{s})^2}(0.7275) = 2.054 \times 10^6$$

$$\text{Nu} = \left\{0.6 + \frac{0.387 \text{ Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.054 \times 10^6)^{1/6}}{[1 + (0.559/0.7275)^{9/16}]^{8/27}}\right\}^2$$
$$= 17.89$$

$$h_o = \frac{k}{D_0}\text{Nu} = \frac{0.0150 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}}{4/12 \text{ ft}}(17.89) = 0.8075 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

$$\dot{Q}_o = h_o A_o(T_o - T_\infty) = (0.8075 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(1.047 \text{ ft}^2)(110 - 70)^\circ\text{F}$$
$$= 33.8 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 110°F for the glass cover is high. Repeating the calculations with lower temperatures, the glass cover temperature corresponding to 30 Btu/h is determined to be 106°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (4 - 2)/2 = 1 \text{ in.} = 1/12 \text{ ft}$$

We start the calculations by assuming the tube temperature to be 200°F, and thus an average temperature of (106 + 200)/2 = 154°F = 614 R. This gives

$$\begin{aligned}\text{Ra}_L &= \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr}\\ &= \frac{(32.2 \text{ ft/s}^2)[1/614 \text{ R}](200 - 106 \text{ R})(1/12 \text{ ft})^3}{(2.117 \times 10^{-4} \text{ ft}^2/\text{s})^2}(0.7184) = 4.579 \times 10^4\end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned}F_{\text{cyl}} &= \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}\\ &= \frac{[\ln(4/2)]^4}{(1/12 \text{ ft})^3[(2/12 \text{ ft})^{-3/5} + (4/12 \text{ ft})^{-3/5}]^5} = 0.1466\end{aligned}$$

$$\begin{aligned}k_{\text{eff}} &= 0.386k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4}\\ &= 0.386(0.01653 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F})\left(\frac{0.7184}{0.861 + 0.7184}\right)(0.1466 \times 4.579 \times 10^4)^{1/4}\\ &= 0.04743 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}\end{aligned}$$

Then the rate of heat transfer between the cylinders becomes

$$\begin{aligned}\dot{Q} &= \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o)\\ &= \frac{2\pi(0.04743 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F})}{\ln(4/2)}(200 - 106)^\circ\text{F} = 40.4 \text{ Btu/h}\end{aligned}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 200°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **180°F.** Therefore, the tube will reach an equilibrium temperature of 180°F when the pump fails.

Discussion Note that we have not considered heat loss by radiation in the calculations, and thus the tube temperature determined above is probably too high. This problem is considered again in Chapter 12 by accounting for the effect of radiation heat transfer.

9–6 ▪ COMBINED NATURAL AND FORCED CONVECTION

The presence of a temperature gradient in a fluid in a gravity field always gives rise to natural convection currents, and thus heat transfer by natural convection. Therefore, forced convection is always accompanied by natural convection.

We mentioned earlier that the convection heat transfer coefficient, natural or forced, is a strong function of the fluid velocity. Heat transfer coefficients encountered in forced convection are typically much higher than those encountered in natural convection because of the higher fluid velocities associated with forced convection. As a result, we tend to ignore natural convection in heat transfer analyses that involve forced convection, although we recognize that natural convection always accompanies forced convection. The error involved in ignoring natural convection is negligible at high velocities but may be considerable at low velocities associated with forced convection. Therefore, it is desirable to have a criterion to assess the relative magnitude of natural convection in the presence of forced convection.

For a given fluid, it is observed that the parameter Gr/Re^2 represents the importance of natural convection relative to forced convection. This is not surprising since the convection heat transfer coefficient is a strong function of the Reynolds number Re in forced convection and the Grashof number Gr in natural convection.

A plot of the nondimensionalized heat transfer coefficient for combined natural and forced convection on a vertical plate is given in Fig. 9–32 for different fluids. We note from this figure that natural convection is negligible when $Gr/Re^2 < 0.1$, forced convection is negligible when $Gr/Re^2 > 10$, and neither is negligible when $0.1 < Gr/Re^2 < 10$. Therefore, both natural and forced convection must be considered in heat transfer calculations when the Gr and Re^2 are of the same order of magnitude (one is within a factor of 10 times the other). Note that forced convection is small relative to natural convection only in the rare case of extremely low forced flow velocities.

Natural convection may *help* or *hurt* forced convection heat transfer, depending on the relative directions of *buoyancy-induced* and the *forced convection* motions (Fig. 9–33):

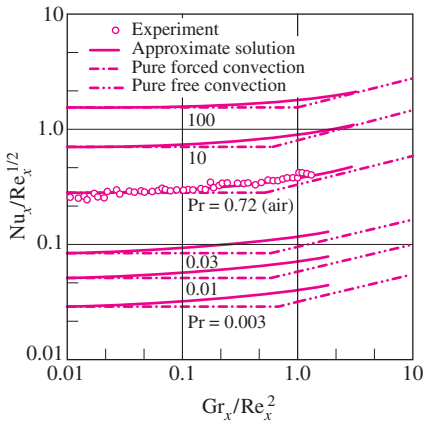


FIGURE 9–32

Variation of the local Nusselt number NU_x for combined natural and forced convection from a hot isothermal vertical plate (from Lloyd and Sparrow, Ref. 25).

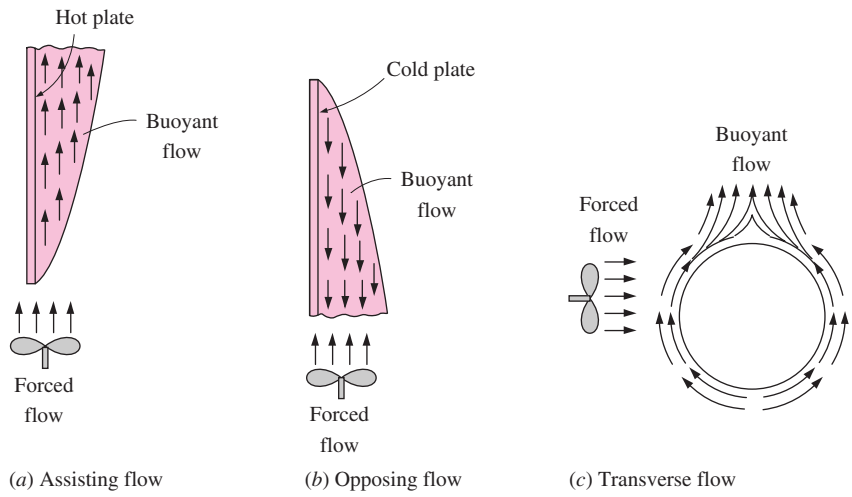


FIGURE 9–33

Natural convection can *enhance* or *inhibit* heat transfer, depending on the relative directions of *buoyancy-induced motion* and the *forced convection motion.*

1. In *assisting flow,* the buoyant motion is in the *same* direction as the forced motion. Therefore, natural convection assists forced convection and *enhances* heat transfer. An example is upward forced flow over a hot surface.
2. In *opposing flow,* the buoyant motion is in the *opposite* direction to the forced motion. Therefore, natural convection resists forced convection and *decreases* heat transfer. An example is upward forced flow over a cold surface.
3. In *transverse flow,* the buoyant motion is *perpendicular* to the forced motion. Transverse flow enhances fluid mixing and thus *enhances* heat transfer. An example is horizontal forced flow over a hot or cold cylinder or sphere.

When determining heat transfer under combined natural and forced convection conditions, it is tempting to add the contributions of natural and forced convection in assisting flows and to subtract them in opposing flows. However, the evidence indicates differently. A review of experimental data suggests a correlation of the form

$$\mathrm{Nu}_{\text{combined}} = (\mathrm{Nu}^n_{\text{forced}} \pm \mathrm{Nu}^n_{\text{natural}})^{1/n} \tag{9–41}$$

where $\mathrm{Nu}_{\text{forced}}$ and $\mathrm{Nu}_{\text{natural}}$ are determined from the correlations for *pure forced* and *pure natural convection,* respectively. The plus sign is for *assisting* and *transverse* flows and the minus sign is for *opposing* flows. The value of the exponent n varies between 3 and 4, depending on the geometry involved. It is observed that $n = 3$ correlates experimental data for vertical surfaces well. Larger values of n are better suited for horizontal surfaces.

A question that frequently arises in the cooling of heat-generating equipment such as electronic components is whether to use a fan (or a pump if the cooling medium is a liquid)—that is, whether to utilize *natural* or *forced* convection in the cooling of the equipment. The answer depends on the maximum allowable operating temperature. Recall that the convection heat transfer rate from a surface at temperature T_s in a medium at T_∞ is given by

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where h is the convection heat transfer coefficient and A_s is the surface area. Note that for a fixed value of power dissipation and surface area, h and T_s are *inversely proportional.* Therefore, the device will operate at a *higher* temperature when h is low (typical of natural convection) and at a *lower* temperature when h is high (typical of forced convection).

Natural convection is the preferred mode of heat transfer since no blowers or pumps are needed and thus all the problems associated with these, such as noise, vibration, power consumption, and malfunctioning, are avoided. Natural convection is adequate for cooling *low-power-output* devices, especially when they are attached to extended surfaces such as heat sinks. For *high-power-output* devices, however, we have no choice but to use a blower or a pump to keep the operating temperature below the maximum allowable level. For *very-high-power-output* devices, even forced convection may not be sufficient to keep the surface temperature at the desirable levels. In such cases, we may have to use *boiling* and *condensation* to take advantage of the very high heat transfer coefficients associated with phase change processes.

TOPIC OF SCPECIAL INTEREST*

Heat Transfer Through Windows

Windows are *glazed apertures* in the building envelope that typically consist of single or multiple *glazing* (glass or plastic), *framing,* and *shading*. In a building envelope, windows offer the *least resistance* to heat flow. In a typical house, about *one-third* of the total heat loss in winter occurs through the windows. Also, most air infiltration occurs at the edges of the windows. The solar heat gain through the windows is responsible for much of the cooling load in summer. The net effect of a window on the heat balance of a building depends on the characteristics and orientation of the window as well as the solar and weather data. Workmanship is very important in the construction and installation of windows to provide effective sealing around the edges while allowing them to be opened and closed easily.

Despite being so undesirable from an energy conservation point of view, windows are an essential part of any building envelope since they enhance the appearance of the building, allow *daylight* and *solar heat* to come in, and allow people to view and observe outside without leaving their home. For low-rise buildings, windows also provide easy exit areas during emergencies such as fire. Important considerations in the selection of windows are *thermal comfort* and *energy conservation.* A window should have a good light transmittance while providing effective resistance to heat flow. The lighting requirements of a building can be minimized by maximizing the use of natural daylight. Heat loss in winter through the windows can be minimized by using airtight double- or triple-pane windows with spectrally selective films or coatings, and letting in as much solar radiation as possible. Heat gain and thus cooling load in summer can be minimized by using effective internal or external shading on the windows.

Even in the absence of solar radiation and air infiltration, heat transfer through the windows is more complicated than it appears to be. This is because the structure and properties of the frame are quite different than the glazing. As a result, heat transfer through the frame and the edge section of the glazing adjacent to the frame is two-dimensional. Therefore, it is customary to consider the window in three regions when analyzing heat transfer through it: (1) the *center-of-glass,* (2) the *edge-of-glass,* and (3) the *frame* regions, as shown in Figure 9–34. Then the total rate of heat transfer through the window is determined by adding the heat transfer through each region as

$$\begin{aligned}\dot{Q}_{\text{window}} &= \dot{Q}_{\text{center}} + \dot{Q}_{\text{edge}} + \dot{Q}_{\text{frame}} \\ &= U_{\text{window}} A_{\text{window}} (T_{\text{indoors}} - T_{\text{outdoors}})\end{aligned} \tag{9-67}$$

where

$$U_{\text{window}} = (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}})/A_{\text{window}} \tag{9-68}$$

is the ***U*-factor** or the **overall heat transfer coefficient** of the window; A_{window} is the window area; A_{center}, A_{edge}, and A_{frame} are the areas of the

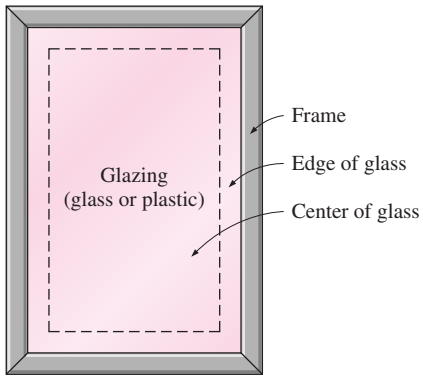


FIGURE 9–34
The three regions of a window considered in heat transfer analysis.

*This section can be skipped without a loss of continuity.

center, edge, and frame sections of the window, respectively; and U_{center}, U_{edge}, and U_{frame} are the heat transfer coefficients for the center, edge, and frame sections of the window. Note that $A_{window} = A_{center} + A_{edge} + A_{frame}$, and the overall U-factor of the window is determined from the area-weighed U-factors of each region of the window. Also, the inverse of the U-factor is the R-value, which is the unit thermal resistance of the window (thermal resistance for a unit area).

Consider steady one-dimensional heat transfer through a single-pane glass of thickness L and thermal conductivity k. The thermal resistance network of this problem consists of surface resistances on the inner and outer surfaces and the conduction resistance of the glass in series, as shown in Figure 9–35, and the total resistance on a unit area basis can be expressed as

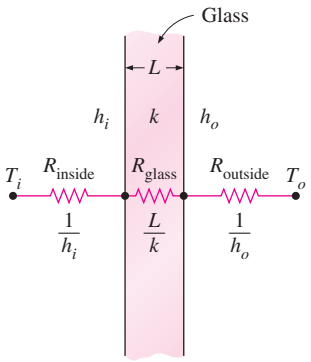


FIGURE 9–35
The thermal resistance network for heat transfer through a single glass.

$$R_{total} = R_{inside} + R_{glass} + R_{outside} = \frac{1}{h_i} + \frac{L_{glass}}{k_{glass}} + \frac{1}{h_o} \tag{9-69}$$

Using common values of 3 mm for the thickness and 0.92 W/m · °C for the thermal conductivity of the glass and the winter design values of 8.29 and 34.0 W/m² · °C for the inner and outer surface heat transfer coefficients, the thermal resistance of the glass is determined to be

$$R_{total} = \frac{1}{8.29\ \text{W/m}^2 \cdot {}^\circ\text{C}} + \frac{0.003\ \text{m}}{0.92\ \text{W/m} \cdot {}^\circ\text{C}} + \frac{1}{34.0\ \text{W/m}^2 \cdot {}^\circ\text{C}}$$
$$= 0.121 + 0.003 + 0.029 = 0.153\ \text{m}^2 \cdot {}^\circ\text{C/W}$$

Note that the ratio of the glass resistance to the total resistance is

$$\frac{R_{glass}}{R_{total}} = \frac{0.003\ \text{m}^2 \cdot {}^\circ\text{C/W}}{0.153\ \text{m}^2 \cdot {}^\circ\text{C/W}} = 2.0\%$$

That is, the glass layer itself contributes about 2 percent of the total thermal resistance of the window, which is negligible. The situation would not be much different if we used acrylic, whose thermal conductivity is 0.19 W/m · °C, instead of glass. Therefore, we cannot reduce the heat transfer through the window effectively by simply increasing the thickness of the glass. But we can reduce it by trapping still air between two layers of glass. The result is a **double-pane window,** which has become the norm in window construction.

The thermal conductivity of air at room temperature is $k_{air} = 0.025$ W/m · °C, which is one-thirtieth that of glass. Therefore, the thermal resistance of 1-cm-thick still air is equivalent to the thermal resistance of a 30-cm-thick glass layer. Disregarding the thermal resistances of glass layers, the thermal resistance and U-factor of a double-pane window can be expressed as (Fig. 9–36)

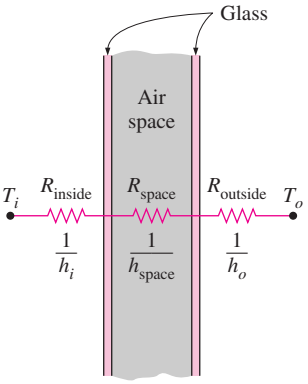


FIGURE 9–36
The thermal resistance network for heat transfer through the center section of a double-pane window (the resistances of the glasses are neglected).

$$\frac{1}{U_{\text{double-pane (center region)}}} \cong \frac{1}{h_i} + \frac{1}{h_{space}} + \frac{1}{h_o} \tag{9-70}$$

where $h_{space} = h_{rad,\ space} + h_{conv,\ space}$ is the combined radiation and convection heat transfer coefficient of the space trapped between the two glass layers.

Roughly half of the heat transfer through the air space of a double-pane window is by radiation and the other half is by conduction (or convection,

if there is any air motion). Therefore, there are two ways to minimize h_{space} and thus the rate of heat transfer through a double-pane window:

1. *Minimize radiation heat transfer through the air space.* This can be done by reducing the emissivity of glass surfaces by coating them with low-emissivity (or "low-e" for short) material. Recall that the *effective emissivity* of two parallel plates of emissivities ε_1 and ε_2 is given by

$$\varepsilon_{effective} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \tag{9-71}$$

The emissivity of an ordinary glass surface is 0.84. Therefore, the effective emissivity of two parallel glass surfaces facing each other is 0.72. But when the glass surfaces are coated with a film that has an emissivity of 0.1, the effective emissivity reduces to 0.05, which is one-fourteenth of 0.72. Then for the same surface temperatures, radiation heat transfer will also go down by a factor of 14. Even if only one of the surfaces is coated, the overall emissivity reduces to 0.1, which is the emissivity of the coating. Thus it is no surprise that about one-fourth of all windows sold for residences have a low-e coating. The heat transfer coefficient h_{space} for the air space trapped between the two vertical parallel glass layers is given in Table 9–3 for 13-mm- ($\frac{1}{2}$-in.) and 6-mm- ($\frac{1}{4}$-in.) thick air spaces for various effective emissivities and temperature differences.

It can be shown that coating just one of the two parallel surfaces facing each other by a material of emissivity ε reduces the effective emissivity nearly to ε. Therefore, it is usually more economical to coat only one of the facing surfaces. Note from Figure 9–37 that coating one of the interior surfaces of a double-pane window with a material having an emissivity of 0.1

TABLE 9–3

The heat transfer coefficient h_{space} for the air space trapped between the two vertical parallel glass layers for 13-mm- and 6-mm-thick air spaces (from Building Materials and Structures, Report 151, U.S. Dept. of Commerce).

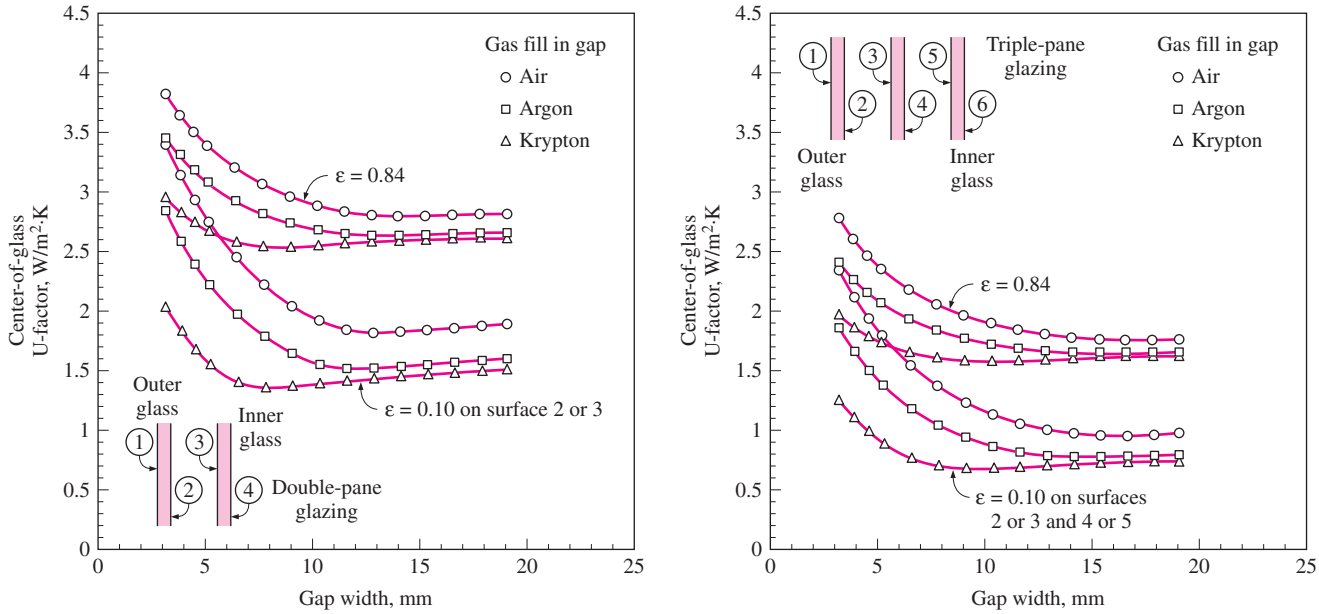
(*a*) Air space thickness = 13 mm

T_{ave}, °C	ΔT, °C	h_{space}, W/m² · °C* $\varepsilon_{effective}$ 0.72	0.4	0.2	0.1
0	5	5.3	3.8	2.9	2.4
0	15	5.3	3.8	2.9	2.4
0	30	5.5	4.0	3.1	2.6
10	5	5.7	4.1	3.0	2.5
10	15	5.7	4.1	3.1	2.5
10	30	6.0	4.3	3.3	2.7
30	5	5.7	4.6	3.4	2.7
30	15	5.7	4.7	3.4	2.8
30	30	6.0	4.9	3.6	3.0

(*b*) Air space thickness = 6 mm

T_{ave}, °C	ΔT, °C	h_{space}, W/m² · °C* $\varepsilon_{effective}$ 0.72	0.4	0.2	0.1
0	5	7.2	5.7	4.8	4.3
0	50	7.2	5.7	4.8	4.3
10	5	7.7	6.0	5.0	4.5
10	50	7.7	6.1	5.0	4.5
30	5	8.8	6.8	5.5	4.9
30	50	8.8	6.8	5.5	4.9
50	5	10.0	7.5	6.0	5.2
50	50	10.0	7.5	6.0	5.2

*Multiply by 0.176 to convert to Btu/h · ft² · °F.



(*a*) Double-pane window (*b*) Triple-pane window

FIGURE 9–37

The variation of the *U*-factor for the center section of double- and triple-pane windows with uniform spacing between the panes (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 27, Fig. 1).

reduces the rate of heat transfer through the center section of the window by half.

2. *Minimize conduction heat transfer through air space.* This can be done by *increasing* the distance *d* between the two glasses. However, this cannot be done indefinitely since increasing the spacing beyond a critical value initiates convection currents in the enclosed air space, which increases the heat transfer coefficient and thus defeats the purpose. Besides, increasing the spacing also increases the thickness of the necessary framing and the cost of the window. Experimental studies have shown that when the spacing *d* is less than about 13 mm, there is no convection, and heat transfer through the air is by conduction. But as the spacing is increased further, convection currents appear in the air space, and the increase in heat transfer coefficient offsets any benefit obtained by the thicker air layer. As a result, the heat transfer coefficient remains nearly constant, as shown in Figure 9–37. Therefore, it makes no sense to use an air space thicker than 13 mm in a double-pane window unless a thin polyester film is used to divide the air space into two halves to suppress convection currents. The film provides added insulation without adding much to the weight or cost of the double-pane window. The thermal resistance of the window can be increased further by using triple- or quadruple-pane windows whenever it is economical to do so. Note that using a triple-pane window instead of a double-pane reduces the rate of heat transfer through the center section of the window by about one-third.

Another way of reducing conduction heat transfer through a double-pane window is to use a *less-conducting fluid* such as argon or krypton to fill the gap between the glasses instead of air. The gap in this case needs to be well sealed to prevent the gas from leaking outside. Of course, another alternative is to evacuate the gap between the glasses completely, but it is not practical to do so.

Edge-of-Glass *U*-Factor of a Window

The glasses in double- and triple-pane windows are kept apart from each other at a uniform distance by **spacers** made of metals or insulators like aluminum, fiberglass, wood, and butyl. Continuous spacer strips are placed around the glass perimeter to provide an edge seal as well as uniform spacing. However, the spacers also serve as undesirable "thermal bridges" between the glasses, which are at different temperatures, and this short-circuiting may increase heat transfer through the window considerably. Heat transfer in the edge region of a window is two-dimensional, and lab measurements indicate that the edge effects are limited to a 6.5-cm-wide band around the perimeter of the glass.

The *U*-factor for the edge region of a window is given in Figure 9–38 relative to the *U*-factor for the center region of the window. The curve would be a straight diagonal line if the two *U*-values were equal to each other. Note that this is almost the case for insulating spacers such as wood and fiberglass. But the *U*-factor for the edge region can be twice that of the center region for conducting spacers such as those made of aluminum. Values for steel spacers fall between the two curves for metallic and insulating spacers. The edge effect is not applicable to single-pane windows.

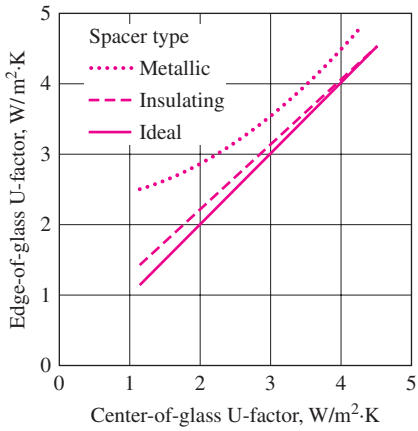


FIGURE 9–38
The edge-of-glass *U*-factor relative to the center-of-glass *U*-factor for windows with various spacers (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 27, Fig. 2).

Frame *U*-Factor

The framing of a window consists of the entire window except the glazing. Heat transfer through the framing is difficult to determine because of the different window configurations, different sizes, different constructions, and different combination of materials used in the frame construction. The type of glazing such as single pane, double pane, and triple pane affects the thickness of the framing and thus heat transfer through the frame. Most frames are made of *wood, aluminum, vinyl,* or *fiberglass.* However, using a combination of these materials (such as aluminum-clad wood and vinyl-clad aluminum) is also common to improve appearance and durability.

Aluminum is a popular framing material because it is inexpensive, durable, and easy to manufacture, and does not rot or absorb water like wood. However, from a heat transfer point of view, it is the least desirable framing material because of its high thermal conductivity. It will come as no surprise that the *U*-factor of solid aluminum frames is the highest, and thus a window with aluminum framing will lose much more heat than a comparable window with wood or vinyl framing. Heat transfer through the aluminum framing members can be reduced by using plastic inserts between components to serve as thermal barriers. The thickness of these inserts greatly affects heat transfer through the frame. For aluminum frames without the plastic strips, the primary resistance to heat transfer is due to the interior surface heat transfer coefficient. The *U*-factors for various

TABLE 9–4

Representative frame *U*-factors for fixed vertical windows (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 27, Table 2)

Frame material	*U*-factor, W/m² · °C*
Aluminum:	
Single glazing (3 mm)	10.1
Double glazing (18 mm)	10.1
Triple glazing (33 mm)	10.1
Wood or vinyl:	
Single glazing (3 mm)	2.9
Double glazing (18 mm)	2.8
Triple glazing (33 mm)	2.7

*Multiply by 0.176 to convert to Btu/h · ft² · °F

TABLE 9–5

Combined convection and radiation heat transfer coefficient h_i at the inner surface of a vertical glass under still air conditions (in W/m² · °C)*

T_i, °C	T_g, °C	Glass emissivity, ε_g 0.05	0.20	0.84
20	17	2.6	3.5	7.1
20	15	2.9	3.8	7.3
20	10	3.4	4.2	7.7
20	5	3.7	4.5	7.9
20	0	4.0	4.8	8.1
20	−5	4.2	5.0	8.2
20	−10	4.4	5.1	8.3

*Multiply by 0.176 to convert to Btu/h · ft² · °F.

frames are listed in Table 9–4 as a function of spacer materials and the glazing unit thicknesses. Note that the *U*-factor of metal framing and thus the rate of heat transfer through a metal window frame is more than three times that of a wood or vinyl window frame.

Interior and Exterior Surface Heat Transfer Coefficients

Heat transfer through a window is also affected by the convection and radiation heat transfer coefficients between the glass surfaces and surroundings. The effects of convection and radiation on the inner and outer surfaces of glazings are usually combined into the combined convection and radiation heat transfer coefficients h_i and h_o, respectively. Under still air conditions, the combined heat transfer coefficient at the inner surface of a vertical window can be determined from

$$h_i = h_{\text{conv}} + h_{\text{rad}} = 1.77(T_g - T_i)^{0.25} + \frac{\varepsilon_g \sigma (T_g^4 - T_i^4)}{T_g - T_i} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C}) \tag{9-72}$$

where T_g = glass temperature in K, T_i = indoor air temperature in K, ε_g = emissivity of the inner surface of the glass exposed to the room (taken to be 0.84 for uncoated glass), and $\sigma = 5.67 \times 10^{-8}$ W/m² · K⁴ is the Stefan–Boltzmann constant. Here the temperature of the interior surfaces facing the window is assumed to be equal to the indoor air temperature. This assumption is reasonable when the window faces mostly interior walls, but it becomes questionable when the window is exposed to heated or cooled surfaces or to other windows. The commonly used value of h_i for peak load calculation is

$$h_i = 8.29 \text{ W/m}^2 \cdot {}^\circ\text{C} = 1.46 \text{ Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} \qquad \text{(winter and summer)}$$

which corresponds to the winter design conditions of $T_i = 22°\text{C}$ and $T_g = -7°\text{C}$ for uncoated glass with $\varepsilon_g = 0.84$. But the same value of h_i can also be used for summer design conditions as it corresponds to summer conditions of $T_i = 24°\text{C}$ and $T_g = 32°\text{C}$. The values of h_i for various temperatures and glass emissivities are given in Table 9–5. The commonly used values of h_o for peak load calculations are the same as those used for outer wall surfaces (34.0 W/m² · °C for winter and 22.7 W/m² · °C for summer).

Overall *U*-Factor of Windows

The overall *U*-factors for various kinds of windows and skylights are evaluated using computer simulations and laboratory testing for winter design conditions; representative values are given in Table 9–6. Test data may provide more accurate information for specific products and should be preferred when available. However, the values listed in the table can be used to obtain satisfactory results under various conditions in the absence of product-specific data. The *U*-factor of a fenestration product that differs considerably from the ones in the table can be determined by (1) determining the fractions of the area that are frame, center-of-glass, and edge-of-glass (assuming a 65-mm-wide band around the perimeter of each glazing),

TABLE 9–6

Overall *U*-factors (heat transfer coefficients) for various windows and skylights in W/m^2 · °C (from ASHRAE *Handbook of Fundamentals,* Ref. 1, Chap. 27, Table 5)

	Glass section (glazing) only			*Aluminum frame (without thermal break)*			*Wood or vinyl frame*					
Type →	*Center-of-glass*	*Edge-of-glass*		*Fixed*	*Double door*	*Sloped skylight*	*Fixed*		*Double door*		*Sloped skylight*	
Frame width →	*(Not applicable)*			32 mm ($1\frac{1}{4}$ in.)	53 mm (2 in.)	19 mm ($\frac{3}{4}$ in.)	41 mm ($1\frac{5}{8}$ in.)		88 mm ($3\frac{7}{18}$ in.)		23 mm ($\frac{7}{8}$ in.)	
Spacer type →	—	Metal	Insul.	All	All	All	Metal	Insul.	Metal	Insul.	Metal	Insul.
Glazing Type												
Single Glazing												
3 mm ($\frac{1}{8}$ in.) glass	6.30	6.30	—	6.63	7.16	9.88	5.93	—	5.57	—	7.57	—
6.4 mm ($\frac{1}{4}$ in.) acrylic	5.28	5.28	—	5.69	6.27	8.86	5.02	—	4.77	—	6.57	—
3 mm ($\frac{1}{8}$ in.) acrylic	5.79	5.79	—	6.16	6.71	9.94	5.48	—	5.17	—	7.63	—
Double Glazing (no coating)												
6.4 mm air space	3.24	3.71	3.34	3.90	4.55	6.70	3.26	3.16	3.20	3.09	4.37	4.22
12.7 mm air space	2.78	3.40	2.91	3.51	4.18	6.65	2.88	2.76	2.86	2.74	4.32	4.17
6.4 mm argon space	2.95	3.52	3.07	3.66	4.32	6.47	3.03	2.91	2.98	2.87	4.14	3.97
12.7 mm argon space	2.61	3.28	2.76	3.36	4.04	6.47	2.74	2.61	2.73	2.60	4.14	3.97
Double Glazing [$\varepsilon = 0.1$, coating on one of the surfaces of air space (surface 2 or 3, counting from the outside toward inside)]												
6.4 mm air space	2.44	3.16	2.60	3.21	3.89	6.04	2.59	2.46	2.60	2.47	3.73	3.53
12.7 mm air space	1.82	2.71	2.06	2.67	3.37	6.04	2.06	1.92	2.13	1.99	3.73	3.53
6.4 mm argon space	1.99	2.83	2.21	2.82	3.52	5.62	2.21	2.07	2.26	2.12	3.32	3.09
12.7 mm argon space	1.53	2.49	1.83	2.42	3.14	5.71	1.82	1.67	1.91	1.78	3.41	3.19
Triple Glazing (no coating)												
6.4 mm air space	2.16	2.96	2.35	2.97	3.66	5.81	2.34	2.18	2.36	2.21	3.48	3.24
12.7 mm air space	1.76	2.67	2.02	2.62	3.33	5.67	2.01	1.84	2.07	1.91	3.34	3.09
6.4 mm argon space	1.93	2.79	2.16	2.77	3.47	5.57	2.15	1.99	2.19	2.04	3.25	3.00
12.7 mm argon space	1.65	2.58	1.92	2.52	3.23	5.53	1.91	1.74	1.98	1.82	3.20	2.95
Triple Glazing [$\varepsilon = 0.1$, coating on one of the surfaces of air spaces (surfaces 3 and 5, counting from the outside toward inside)]												
6.4 mm air space	1.53	2.49	1.83	2.42	3.14	5.24	1.81	1.64	1.89	1.73	2.92	2.66
12.7 mm air space	0.97	2.05	1.38	1.92	2.66	5.10	1.33	1.15	1.46	1.30	2.78	2.52
6.4 mm argon space	1.19	2.23	1.56	2.12	2.85	4.90	1.52	1.35	1.64	1.47	2.59	2.33
12.7 mm argon space	0.80	1.92	1.25	1.77	2.51	4.86	1.18	1.01	1.33	1.17	2.55	2.28

Notes:

(1) Multiply by 0.176 to obtain *U*-factors in Btu/h · ft^2 · °F.

(2) The *U*-factors in this table include the effects of surface heat transfer coefficients and are based on winter conditions of −18°C outdoor air and 21°C indoor air temperature, with 24 km/h (15 mph) winds outdoors and zero solar flux. Small changes in indoor and outdoor temperatures will not affect the overall *U*-factors much. Windows are assumed to be vertical, and the skylights are tilted 20° from the horizontal with upward heat flow. Insulation spacers are wood, fiberglass, or butyl. Edge-of-glass effects are assumed to extend the 65-mm band around perimeter of each glazing. The product sizes are 1.2 m × 1.8 m for fixed windows, 1.8 m × 2.0 m for double-door windows, and 1.2 m × 0.6 m for the skylights, but the values given can also be used for products of similar sizes. All data are based on 3-mm ($\frac{1}{8}$-in.) glass unless noted otherwise.

(2) determining the U-factors for each section (the center-of-glass and edge-of-glass U-factors can be taken from the first two columns of Table 9–6 and the frame U-factor can be taken from Table 9–5 or other sources), and (3) multiplying the area fractions and the U-factors for each section and adding them up (or from Eq. 9-68 for U_{window}).

Glazed wall systems can be treated as fixed windows. Also, the data for double-door windows can be used for single-glass doors. Several observations can be made from the data in the table:

1. Skylight U-factors are considerably greater than those of vertical windows. This is because the skylight area, including the curb, can be 13 to 240 percent greater than the rough opening area. The slope of the skylight also has some effect.
2. The U-factor of multiple-glazed units can be reduced considerably by filling cavities with argon gas instead of dry air. The performance of CO_2-filled units is similar to those filled with argon. The U-factor can be reduced even further by filling the glazing cavities with krypton gas.
3. Coating the glazing surfaces with low-e (low-emissivity) films reduces the U-factor significantly. For multiple-glazed units, it is adequate to coat one of the two surfaces facing each other.
4. The thicker the air space in multiple-glazed units, the lower the U-factor, for a thickness of up to 13 mm ($\frac{1}{2}$ in.) of air space. For a specified number of glazings, the window with thicker air layers will have a lower U-factor. For a specified overall thickness of glazing, the higher the number of glazings, the lower the U-factor. Therefore, a triple-pane window with air spaces of 6.4 mm (two such air spaces) will have a lower U-value than a double-pane window with an air space of 12.7 mm.
5. Wood or vinyl frame windows have a considerably lower U-value than comparable metal-frame windows. Therefore, wood or vinyl frame windows are called for in energy-efficient designs.

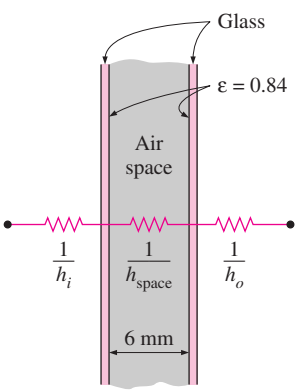


FIGURE 9–39
Schematic of Example 9–7.

EXAMPLE 9–7 *U*-Factor for Center-of-Glass Section of Windows

Determine the U-factor for the center-of-glass section of a double-pane window with a 6-mm air space for winter design conditions (Fig. 9–39). The glazings are made of clear glass that has an emissivity of 0.84. Take the average air space temperature at design conditions to be 0°C.

SOLUTION The U-factor for the center-of-glass section of a double-pane window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** The thermal resistance of glass sheets is negligible.

Properties The emissivity of clear glass is 0.84.

Analysis Disregarding the thermal resistance of glass sheets, which are small, the U-factor for the center region of a double-pane window is determined from

$$\frac{1}{U_{\text{center}}} \cong \frac{1}{h_i} + \frac{1}{h_{\text{space}}} + \frac{1}{h_o}$$

where h_i, h_{space}, and h_o are the heat transfer coefficients at the inner surface of the window, the air space between the glass layers, and the outer surface of the window, respectively. The values of h_i and h_o for winter design conditions were given earlier to be $h_i = 8.29$ W/m² · °C and $h_o = 34.0$ W/m² · °C. The effective emissivity of the air space of the double-pane window is

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.84 + 1/0.84 - 1} = 0.72$$

For this value of emissivity and an average air space temperature of 0°C, we read $h_{space} = 7.2$ W/m² · °C from Table 9–3 for 6-mm-thick air space. Therefore,

$$\frac{1}{U_{\text{center}}} = \frac{1}{8.29} + \frac{1}{7.2} + \frac{1}{34.0} \quad \rightarrow \quad U_{\text{center}} = \mathbf{3.46\ W/m^2 \cdot {}^\circ C}$$

Discussion The center-of-glass *U*-factor value of 3.24 W/m² · °C in Table 9–6 (fourth row and second column) is obtained by using a standard value of h_o = 29 W/m² · °C (instead of 34.0 W/m² · °C) and h_{space} = 6.5 W/m² · °C at an average air space temperature of −15°C.

EXAMPLE 9-8 Heat Loss through Aluminum Framed Windows

A fixed aluminum-framed window with glass glazing is being considered for an opening that is 4 ft high and 6 ft wide in the wall of a house that is maintained at 72°F (Fig. 9–40). Determine the rate of heat loss through the window and the inner surface temperature of the window glass facing the room when the outdoor air temperature is 15°F if the window is selected to be (*a*) $\frac{1}{8}$-in. single glazing, (*b*) double glazing with an air space of $\frac{1}{2}$ in., and (*c*) low-e-coated triple glazing with an air space of $\frac{1}{2}$ in.

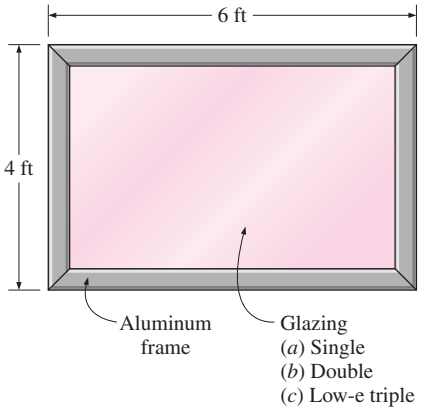


FIGURE 9–40
Schematic for Example 9–8.

SOLUTION The rate of heat loss through an aluminum framed window and the inner surface temperature are to be determined from the cases of single-pane, double-pane, and low-e triple-pane windows.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant.

Properties The U-factors of the windows are given in Table 9–6.

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}}(T_i - T_o)$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively; $U_{overall}$ is the *U*-factor (the overall heat transfer coefficient) of the window; and A_{window} is the window area, which is determined to be

$$A_{\text{window}} = \text{Height} \times \text{Width} = (4\ \text{ft})(6\ \text{ft}) = 24\ \text{ft}^2$$

The *U*-factors for the three cases can be determined directly from Table 9–6 to be 6.63, 3.51, and 1.92 W/m² · °C, respectively, to be multiplied by the factor 0.176 to convert them to Btu/h · ft² · °F. Also, the inner surface temperature of the window glass can be determined from Newton's law

$$\dot{Q}_{\text{window}} = h_i A_{\text{window}} (T_i - T_{\text{glass}}) \quad \rightarrow \quad T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}}$$

where h_i is the heat transfer coefficient on the inner surface of the window, which is determined from Table 9-5 to be $h_i = 8.3\ \text{W/m}^2 \cdot {}^\circ\text{C} = 1.46\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$. Then the rate of heat loss and the interior glass temperature for each case are determined as follows:

(*a*) Single glazing:

$$\dot{Q}_{\text{window}} = (6.63 \times 0.176\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(24\ \text{ft}^2)(72 - 15)^\circ\text{F} = \mathbf{1596\ Btu/h}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 72^\circ\text{F} - \frac{1596\ \text{Btu/h}}{(1.46\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(24\ \text{ft}^2)} = \mathbf{26.5^\circ F}$$

(*b*) Double glazing ($\frac{1}{2}$ in. air space):

$$\dot{Q}_{\text{window}} = (3.51 \times 0.176\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(24\ \text{ft}^2)(72 - 15)^\circ\text{F} = \mathbf{845\ Btu/h}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 72^\circ\text{F} - \frac{845\ \text{Btu/h}}{(1.46\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(24\ \text{ft}^2)} = \mathbf{47.9^\circ F}$$

(*c*) Triple glazing ($\frac{1}{2}$ in. air space, low-e coated):

$$\dot{Q}_{\text{window}} = (1.92 \times 0.176\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(24\ \text{ft}^2)(72 - 15)^\circ\text{F} = \mathbf{462\ Btu/h}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 72^\circ\text{F} - \frac{462\ \text{Btu/h}}{(1.46\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})(24\ \text{ft}^2)} = \mathbf{58.8^\circ F}$$

Therefore, heat loss through the window will be reduced by 47 percent in the case of double glazing and by 71 percent in the case of triple glazing relative to the single-glazing case. Also, in the case of single glazing, the low inner-glass surface temperature will cause considerable discomfort in the occupants because of the excessive heat loss from the body by radiation. It is raised from 26.5°F, which is below freezing, to 47.9°F in the case of double glazing and to 58.8°F in the case of triple glazing.

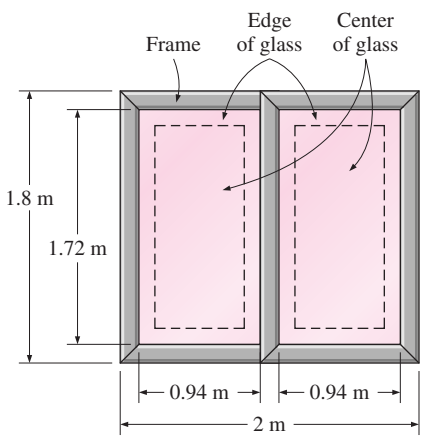


FIGURE 9–41
Schematic for Example 9–9.

EXAMPLE 9–9 *U*-Factor of a Double-Door Window

Determine the overall *U*-factor for a double-door-type, wood-framed double-pane window with metal spacers, and compare your result to the value listed in Table 9–6. The overall dimensions of the window are 1.80 m × 2.00 m, and the dimensions of each glazing are 1.72 m × 0.94 m (Fig. 9–41).

SOLUTION The overall U-factor for a double-door type window is to be determined, and the result is to be compared to the tabulated value.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional.

Properties The *U*-factors for the various sections of windows are given in Tables 9–4 and 9–6.

Analysis The areas of the window, the glazing, and the frame are

$$A_{\text{window}} = \text{Height} \times \text{Width} = (1.8\ \text{m})(2.0\ \text{m}) = 3.60\ \text{m}^2$$

$$A_{\text{glazing}} = 2 \times (\text{Height} \times \text{Width}) = 2(1.72\text{ m})(0.94\text{ m}) = 3.23\text{ m}^2$$
$$A_{\text{frame}} = A_{\text{window}} - A_{\text{glazing}} = 3.60 - 3.23 = 0.37\text{ m}^2$$

The edge-of-glass region consists of a 6.5-cm-wide band around the perimeter of the glazings, and the areas of the center and edge sections of the glazing are determined to be

$$A_{\text{center}} = 2 \times (\text{Height} \times \text{Width}) = 2(1.72 - 0.13\text{ m})(0.94 - 0.13\text{ m}) = 2.58\text{ m}^2$$
$$A_{\text{edge}} = A_{\text{glazing}} - A_{\text{center}} = 3.23 - 2.58 = 0.65\text{ m}^2$$

The U-factor for the frame section is determined from Table 9–4 to be $U_{\text{frame}} = 2.8\text{ W/m}^2 \cdot {}^\circ\text{C}$. The U-factors for the center and edge sections are determined from Table 9–6 (fifth row, second and third columns) to be $U_{\text{center}} = 3.24\text{ W/m}^2 \cdot {}^\circ\text{C}$ and $U_{\text{edge}} = 3.71\text{ W/m}^2 \cdot {}^\circ\text{C}$. Then the overall U-factor of the entire window becomes

$$\begin{aligned} U_{\text{window}} &= (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}})/A_{\text{window}} \\ &= (3.24 \times 2.58 + 3.71 \times 0.65 + 2.8 \times 0.37)/3.60 \\ &= \mathbf{3.28\ W/m^2 \cdot {}^\circ C} \end{aligned}$$

The overall U-factor listed in Table 9–6 for the specified type of window is $3.20\text{ W/m}^2 \cdot {}^\circ\text{C}$, which is sufficiently close to the value obtained above.

SUMMARY

In this chapter, we have considered *natural convection* heat transfer where any fluid motion occurs by natural means such as buoyancy. The *volume expansion coefficient* of a substance represents the variation of the density of that substance with temperature at constant pressure, and for an ideal gas, it is expressed as $\beta = 1/T$, where T is the absolute temperature in K or R.

The flow regime in natural convection is governed by a dimensionless number called the *Grashof number,* which represents the ratio of the buoyancy force to the viscous force acting on the fluid and is expressed as

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where L_c is the *characteristic length,* which is the height L for a vertical plate and the diameter D for a horizontal cylinder. The correlations for the Nusselt number $\text{Nu} = hL_c/k$ in natural convection are expressed in terms of the *Rayleigh number* defined as

$$\text{Ra}_L = \text{Gr}_L\,\text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\text{Pr}$$

Nusselt number relations for various surfaces are given in Table 9–1. All fluid properties are evaluated at the film temperature of $T_f = \frac{1}{2}(T_s + T_\infty)$. The outer surface of a vertical cylinder can be treated as a vertical plate when the curvature effects are negligible. The characteristic length for a horizontal surface is $L_c = A_s/p$, where A_s is the surface area and p is the perimeter.

The average Nusselt number for vertical isothermal *parallel plates* of spacing S and height L is given as

$$\text{Nu} = \frac{hS}{k} = \left[\frac{576}{(\text{Ra}_S S/L)^2} + \frac{2.873}{(\text{Ra}_S S/L)^{0.5}}\right]^{-0.5}$$

The optimum fin spacing for a vertical heat sink and the Nusselt number for optimally spaced fins is

$$S_{\text{opt}} = 2.714\left(\frac{S^3 L}{\text{Ra}_S}\right)^{0.25} = 2.714\,\frac{L}{\text{Ra}_L^{0.25}} \text{ and } \text{Nu} = \frac{hS_{\text{opt}}}{k} = 1.307$$

In a *horizontal rectangular enclosure* with the hotter plate at the top, heat transfer is by pure conduction and $\text{Nu} = 1$. When the hotter plate is at the bottom, the Nusselt is

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}_L}\right]^+ + \left[\frac{\text{Ra}_L^{1/3}}{18} - 1\right]^+ \qquad \text{Ra}_L < 10^8$$

The notation $[\ ]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. For *vertical horizontal enclosures,* the Nusselt number can be determined from

$$\mathrm{Nu} = 0.18\left(\frac{\mathrm{Pr}}{0.2 + \mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29} \qquad \begin{array}{c} 1 < H/L < 2 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L\,\mathrm{Pr}/(0.2 + \mathrm{Pr}) > 10^3 \end{array}$$

$$\mathrm{Nu} = 0.22\left(\frac{\mathrm{Pr}}{0.2 + \mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \qquad \begin{array}{c} 2 < H/L < 10 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L < 10^{10} \end{array}$$

For aspect ratios greater than 10, Eqs. 9-54 and 9-55 should be used. For inclined enclosures, Eqs. 9-48 through 9-51 should be used.

For *concentric horizontal cylinders,* the rate of heat transfer through the annular space between the cylinders by natural convection per unit length is

$$\dot{Q} = \frac{2\pi k_{\mathrm{eff}}}{\ln(D_o/D_i)}(T_i - T_o)$$

where

$$\frac{k_{\mathrm{eff}}}{k} = 0.386\left(\frac{\mathrm{Pr}}{0.861 + \mathrm{Pr}}\right)^{1/4}(F_{\mathrm{cyl}}\mathrm{Ra}_L)^{1/4}$$

and

$$F_{\mathrm{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

For a *spherical enclosure,* the rate of heat transfer through the space between the spheres by natural convection is expressed as

$$\dot{Q} = k_{\mathrm{eff}}\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o)$$

where

$$\frac{k_{\mathrm{eff}}}{k} = 0.74\left(\frac{\mathrm{Pr}}{0.861 + \mathrm{Pr}}\right)^{1/4}(F_{\mathrm{sph}}\mathrm{Ra}_L)^{1/4}$$

$$L_c = (D_o - D_i)/2$$

$$F_{\mathrm{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5}$$

The quantity $k\mathrm{Nu}$ is called the *effective thermal conductivity* of the enclosure, since a fluid in an enclosure behaves like a quiescent fluid whose thermal conductivity is $k\mathrm{Nu}$ as a result of convection currents. The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

For a given fluid, the parameter $\mathrm{Gr}/\mathrm{Re}^2$ represents the importance of natural convection relative to forced convection. Natural convection is negligible when $\mathrm{Gr}/\mathrm{Re}^2 < 0.1$, forced convection is negligible when $\mathrm{Gr}/\mathrm{Re}^2 > 10$, and neither is negligible when $0.1 < \mathrm{Gr}/\mathrm{Re}^2 < 10$.

REFERENCES AND SUGGESTED READING

1. American Society of Heating, Refrigeration, and Air Conditioning Engineers. *Handbook of Fundamentals.* Atlanta: ASHRAE, 1993.
2. J. N Arnold, I. Catton, and D. K. Edwards. "Experimental Investigation of Natural Convection in Inclined Rectangular Region of Differing Aspects Ratios." ASME Paper No. 75-HT-62, 1975.
3. P. S. Ayyaswamy and I. Catton. "The Boundary-Layer Regime for Natural Convection in a Differently Heated Tilted Rectangular Cavity." *Journal of Heat Transfer* 95 (1973), p. 543.
4. A. Bar-Cohen. "Fin Thickness for an Optimized Natural Convection Array of Rectangular Fins." *Journal of Heat Transfer* 101 (1979), pp. 564–566.
5. A. Bar-Cohen and W. M. Rohsenow. "Thermally Optimum Spacing of Vertical Natural Convection Cooled Parallel Plates." *Journal of Heat Transfer* 106 (1984), p. 116.
6. B. M. Berkovsky and V. K. Polevikov. "Numerical Study of Problems on High-Intensive Free Convection." In *Heat Transfer and Turbulent Buoyant Convection,* ed. D. B. Spalding and N. Afgan, pp. 443–445. Washington, DC: Hemisphere, 1977.
7. I. Catton. "Natural Convection in Enclosures." *Proceedings of Sixth International Heat Transfer Conference,* Toronto, Canada, 1978, Vol. 6, pp. 13–31.
8. T. Cebeci. "Laminar Free Convection Heat Transfer from the Outer Surface of a Vertical Slender Circular Cylinder." *Proceedings Fifth International Heat Transfer Conference* paper NCI.4, 1974 pp. 15–19.
9. Y. A. Çengel and P. T. L. Zing. "Enhancement of Natural Convection Heat Transfer from Heat Sinks by Shrouding." *Proceedings of ASME/JSME Thermal Engineering Conference,* Honolulu, HA, 1987, Vol. 3, pp. 451–475.
10. S. W. Churchill. "A Comprehensive Correlating Equation for Laminar Assisting Forced and Free Convection." *AIChE Journal* 23 (1977), pp. 10–16.
11. S. W. Churchill. "Free Convection Around Immersed Bodies." In *Heat Exchanger Design Handbook,* ed. E. U. Schlünder, Section 2.5.7. New York: Hemisphere, 1983.

12. S. W. Churchill. "Combined Free and Forced Convection around Immersed Bodies." In *Heat Exchanger Design Handbook,* Section 2.5.9. New York: Hemisphere Publishing, 1986.

13. S. W. Churchill and H. H. S. Chu. "Correlating Equations for Laminar and Turbulent Free Convection from a Horizontal Cylinder." *International Journal of Heat Mass Transfer* 18 (1975), p. 1049.

14. S. W. Churchill and H. H. S. Chu. "Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate." *International Journal of Heat Mass Transfer* 18 (1975), p. 1323.

15. E. R. G. Eckert and E. Soehngen. "Studies on Heat Transfer in Laminar Free Convection with Zehnder–Mach Interferometer." USAF Technical Report 5747, December 1948.

16. E. R. G. Eckert and E. Soehngen. "Interferometric Studies on the Stability and Transition to Turbulence of a Free Convection Boundary Layer." *Proceedings of General Discussion, Heat Transfer ASME-IME,* London, 1951.

17. S. M. ElSherbiny, G. D. Raithby, and K. G. T. Hollands. "Heat Transfer by Natural Convection Across Vertical and Inclined Air Layers. *Journal of Heat Transfer* 104 (1982), pp. 96–102.

18. T. Fujiii and H. Imura. "Natural Convection Heat Transfer from a Plate with Arbitrary Inclination." *International Journal of Heat Mass Transfer* 15 (1972), p. 755.

19. K. G. T. Hollands, T. E. Unny, G. D. Raithby, and L. Konicek. "Free Convective Heat Transfer Across Inclined Air Layers." *Journal of Heat Transfer* 98 (1976), pp. 189–193.

20. J. P. Holman. *Heat Transfer.* 7th ed. New York: McGraw-Hill, 1990.

21. F. P. Incropera and D. P. DeWitt. *Introduction to Heat Transfer.* 3rd ed. New York: John Wiley & Sons, 1996.

22. M. Jakob. *Heat Transfer.* New York: Wiley, 1949.

23. W. M. Kays and M. E. Crawford. *Convective Heat and Mass Transfer.* 3rd ed. New York: McGraw-Hill, 1993.

24. F. Kreith and M. S. Bohn. *Principles of Heat Transfer.* 6th ed. Pacific Grove, CA: Brooks/Cole, 2001.

25. J. R. Lloyd and E. M. Sparrow. "Combined Forced and Free Convection Flow on Vertical Surfaces." *International Journal of Heat Mass Transfer* 13 (1970), p. 434.

26. R. K. Macgregor and A. P. Emery. "Free Convection Through Vertical Plane Layers: Moderate and High Prandtl Number Fluids." *Journal of Heat Transfer* 91 (1969), p. 391.

27. S. Ostrach. "An Analysis of Laminar Free Convection Flow and Heat Transfer About a Flat Plate Parallel to the Direction of the Generating Body Force." National Advisory Committee for Aeronautics, Report 1111, 1953.

28. G. D. Raithby and K. G. T. Hollands. "A General Method of Obtaining Approximate Solutions to Laminar and Turbulent Free Convection Problems." In *Advances in Heat Transfer,* ed. F. Irvine and J. P. Hartnett, Vol. II, pp. 265–315. New York: Academic Press, 1975.

29. E. M. Sparrow and J. L. Gregg. "Laminar Free Convection from a Vertical Flat Plate." *Transactions of the ASME* 78 (1956), p. 438.

30. E. M. Sparrow and J. L. Gregg. "Laminar Free Convection Heat Transfer from the Outer Surface of a Vertical Circular Cylinder." *ASME* 78 (1956), p. 1823.

31. E. M. Sparrow and C. Prakash. "Enhancement of Natural Convection Heat Transfer by a Staggered Array of Vertical Plates." *Journal of Heat Transfer* 102 (1980), pp. 215–220.

32. E. M. Sparrow and S. B. Vemuri. "Natural Convection/Radiation Heat Transfer from Highly Populated Pin Fin Arrays." *Journal of Heat Transfer* 107 (1985), pp. 190–197.

PROBLEMS*

Physical Mechanism of Natural Convection

9–1C What is natural convection? How does it differ from forced convection? What force causes natural convection currents?

9–2C In which mode of heat transfer is the convection heat transfer coefficient usually higher, natural convection or forced convection? Why?

9–3C Consider a hot boiled egg in a spacecraft that is filled with air at atmospheric pressure and temperature at all times. Will the egg cool faster or slower when the spacecraft is in space instead of on the ground? Explain.

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with an EES-CD icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD. Problems with a computer-EES icon are comprehensive in nature, and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

9–4C What is buoyancy force? Compare the relative magnitudes of the buoyancy force acting on a body immersed in these mediums: (*a*) air, (*b*) water, (*c*) mercury, and (*d*) an evacuated chamber.

9–5C When will the hull of a ship sink in water deeper: when the ship is sailing in fresh water or in sea water? Why?

9–6C A person weighs himself on a waterproof spring scale placed at the bottom of a 1-m-deep swimming pool. Will the person weigh more or less in water? Why?

9–7C Consider two fluids, one with a large coefficient of volume expansion and the other with a small one. In what fluid will a hot surface initiate stronger natural convection currents? Why? Assume the viscosity of the fluids to be the same.

9–8C Consider a fluid whose volume does not change with temperature at constant pressure. What can you say about natural convection heat transfer in this medium?

9–9C What do the lines on an interferometer photograph represent? What do closely packed lines on the same photograph represent?

9–10C Physically, what does the Grashof number represent? How does the Grashof number differ from the Reynolds number?

9–11 Show that the volume expansion coefficient of an ideal gas is $\beta = 1/T$, where T is the absolute temperature.

Natural Convection over Surfaces

9–12C How does the Rayleigh number differ from the Grashof number?

9–13C Under what conditions can the outer surface of a vertical cylinder be treated as a vertical plate in natural convection calculations?

9–14C Will a hot horizontal plate whose back side is insulated cool faster or slower when its hot surface is facing down instead of up?

9–15C Consider laminar natural convection from a vertical hot plate. Will the heat flux be higher at the top or at the bottom of the plate? Why?

9–16 A 10-m-long section of a 6-cm-diameter horizontal hot water pipe passes through a large room whose temperature is 22°C. If the temperature and the emissivity of the outer surface of the pipe are 65°C and 0.8, respectively, determine the rate of heat loss from the pipe by (*a*) natural convection and (*b*) radiation.

9–17 Consider a wall-mounted power transistor that dissipates 0.18 W of power in an environment at 35°C. The transistor is 0.45 cm long and has a diameter of 0.4 cm. The emissivity of the outer surface of the transistor is 0.1, and the average temperature of the surrounding surfaces is 25°C. Disregarding any heat transfer from the base surface, determine the surface temperature of the transistor. Use air properties at 100°C. *Answer:* 183°C

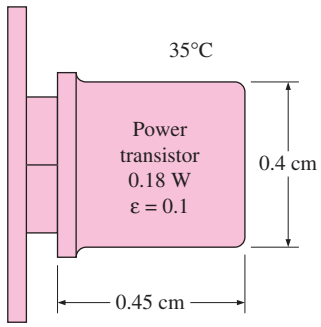


FIGURE P9–17

9–18 Reconsider Problem 9–17. Using EES (or other) software, investigate the effect of ambient temperature on the surface temperature of the transistor. Let the environment temperature vary from 10°C to 40°C and assume that the surrounding surfaces are 10°C colder than the environment temperature. Plot the surface temperature of the transistor versus the environment temperature, and discuss the results.

9–19E Consider a 2-ft × 2-ft thin square plate in a room at 75°F. One side of the plate is maintained at a temperature of 130°F, while the other side is insulated. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down.

9–20E Reconsider Problem 9–19E. Using EES (or other) software, plot the rate of natural convection heat transfer for different orientations of the plate as a function of the plate temperature as the temperature varies from 80°F to 180°F, and discuss the results.

9–21 A 400-W cylindrical resistance heater is 1 m long and 0.5 cm in diameter. The resistance wire is placed horizontally in a fluid at 20°C. Determine the outer surface temperature of the resistance wire in steady operation if the fluid is (*a*) air and (*b*) water. Ignore any heat transfer by radiation. Use properties at 500°C for air and 40°C for water.

9–22 Water is boiling in a 12-cm-deep pan with an outer diameter of 25 cm that is placed on top of a stove. The ambient air and the surrounding surfaces are at a temperature of 25°C, and the emissivity of the outer surface of the pan is 0.95. Assuming the entire pan to be at an average temperature of 98°C, determine the rate of heat loss from the cylindrical side surface of the pan to the surroundings by (*a*) natural convection and (*b*) radiation. (*c*) If water is boiling at a rate of 2 kg/h at 100°C,

determine the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water. The heat of vaporization of water at 100°C is 2257 kJ/kg.

Answers: 46.2 W, 56.1 W, 0.082

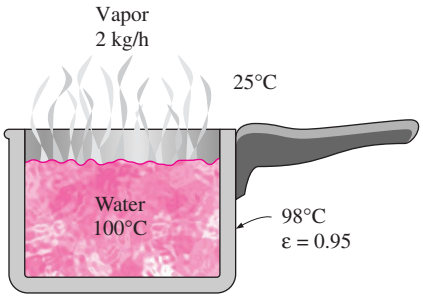


FIGURE P9–22

9–23 Repeat Problem 9–22 for a pan whose outer surface is polished and has an emissivity of 0.1.

9–24 In a plant that manufactures canned aerosol paints, the cans are temperature-tested in water baths at 55°C before they are shipped to ensure that they will withstand temperatures up to 55°C during transportation and shelving. The cans, moving on a conveyor, enter the open hot water bath, which is 0.5 m deep, 1 m wide, and 3.5 m long, and move slowly in the hot water toward the other end. Some of the cans fail the test and explode in the water bath. The water container is made of sheet metal, and the entire container is at about the same temperature as the hot water. The emissivity of the outer surface of the container is 0.7. If the temperature of the surrounding air and surfaces is 20°C, determine the rate of heat loss from the four side surfaces of the container (disregard the top surface, which is open).

The water is heated electrically by resistance heaters, and the cost of electricity is $0.085/kWh. If the plant operates 24 h a day 365 days a year and thus 8760 h a year, determine the annual cost of the heat losses from the container for this facility.

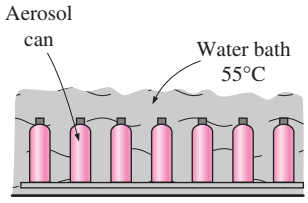


FIGURE P9–24

9–25 Reconsider Problem 9–24. In order to reduce the heating cost of the hot water, it is proposed to insulate the side and bottom surfaces of the container with 5-cm-thick fiberglass insulation ($k = 0.035$ W/m · °C) and to wrap the insulation with aluminum foil ($\varepsilon = 0.1$) in order to minimize the heat loss by radiation. An estimate is obtained from a local insulation contractor, who proposes to do the insulation job for $350, including materials and labor. Would you support this proposal? How long will it take for the insulation to pay for itself from the energy it saves?

9–26 Consider a 15-cm × 20-cm printed circuit board (PCB) that has electronic components on one side. The board is placed in a room at 20°C. The heat loss from the back surface of the board is negligible. If the circuit board is dissipating 8 W of power in steady operation, determine the average temperature of the hot surface of the board, assuming the board is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down. Take the emissivity of the surface of the board to be 0.8 and assume the surrounding surfaces to be at the same temperature as the air in the room. *Answers:* (*a*) 46.6°C, (*b*) 42.6°C, (*c*) 50.7°C

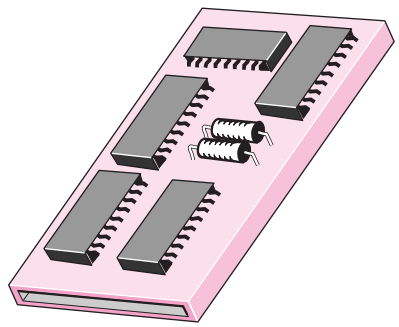


FIGURE P9–26

9–27 Reconsider Problem 9–26. Using EES (or other) software, investigate the effects of the room temperature and the emissivity of the board on the temperature of the hot surface of the board for different orientations of the board. Let the room temperature vary from 5°C to 35°C and the emissivity from 0.1 to 1.0. Plot the hot surface temperature for different orientations of the board as the functions of the room temperature and the emissivity, and discuss the results.

9–28 A manufacturer makes absorber plates that are 1.2 m × 0.8 m in size for use in solar collectors. The back side of the plate is heavily insulated, while its front surface is coated with black chrome, which has an absorptivity of 0.87 for solar radiation and an emissivity of 0.09. Consider such a plate placed horizontally outdoors in calm air at 25°C. Solar radiation is incident on the plate at a rate of 700 W/m^2. Taking the effective sky temperature to be 10°C, determine the equilibrium temperature of the absorber plate. What would your answer be if the absorber plate is made of ordinary aluminum plate that has a solar absorptivity of 0.28 and an emissivity of 0.07?

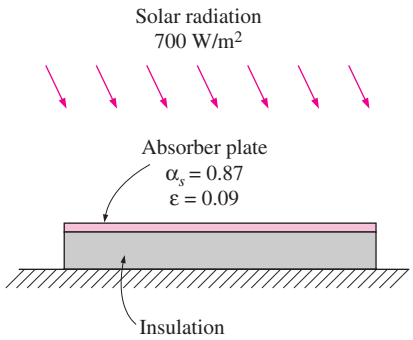


FIGURE P9–28

9–29 Repeat Problem 9–28 for an aluminum plate painted flat black (solar absorptivity 0.98 and emissivity 0.98) and also for a plate painted white (solar absorptivity 0.26 and emissivity 0.90).

9–30 The following experiment is conducted to determine the natural convection heat transfer coefficient for a horizontal cylinder that is 80 cm long and 2 cm in diameter. A 80-cm-long resistance heater is placed along the centerline of the cylinder, and the surfaces of the cylinder are polished to minimize the radiation effect. The two circular side surfaces of the cylinder are well insulated. The resistance heater is turned on, and the power dissipation is maintained constant at 40 W. If the average surface temperature of the cylinder is measured to be 120°C in the 20°C room air when steady operation is reached, determine the natural convection heat transfer coefficient. If the emissivity of the outer surface of the cylinder is 0.1 and a 5 percent error is acceptable, do you think we need to do any correction for the radiation effect? Assume the surrounding surfaces to be at 20°C also.

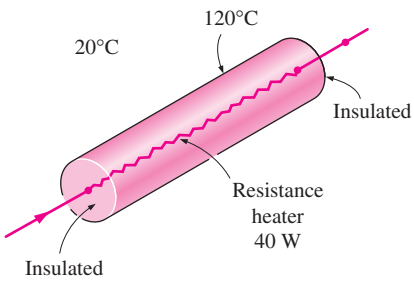


FIGURE P9–30

9–31 Thick fluids such as asphalt and waxes and the pipes in which they flow are often heated in order to reduce the viscosity of the fluids and thus to reduce the pumping costs. Consider the flow of such a fluid through a 100-m-long pipe of outer diameter 30 cm in calm ambient air at 0°C. The pipe is heated electrically, and a thermostat keeps the outer surface temperature of the pipe constant at 25°C. The emissivity of the outer surface of the pipe is 0.8, and the effective sky temperature is −30°C, Determine the power rating of the electric resistance heater, in kW, that needs to be used. Also, determine the cost of electricity associated with heating the pipe during a 10-h period under the above conditions if the price of electricity is $0.09/kWh. *Answers:* 29.1 kW, $26.2

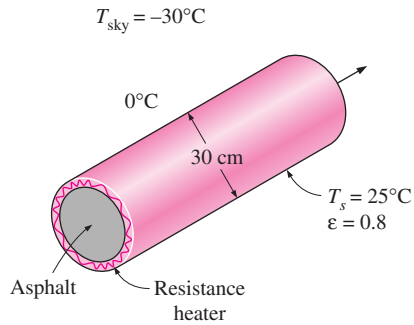


FIGURE P9–31

9–32 Reconsider Problem 9–31. To reduce the heating cost of the pipe, it is proposed to insulate it with sufficiently thick fiberglass insulation (k = 0.035 W/m · °C) wrapped with aluminum foil (ε = 0.1) to cut down the heat losses by 85 percent. Assuming the pipe temperature to remain constant at 25°C, determine the thickness of the insulation that needs to be used. How much money will the insulation save during this 10-h period? *Answers:* 1.3 cm, $22.3

9–33E Consider an industrial furnace that resembles a 13-ft-long horizontal cylindrical enclosure 8 ft in diameter whose end surfaces are well insulated. The furnace burns natural gas at a rate of 48 therms/h (1 therm = 100,000 Btu). The combustion efficiency of the furnace is 82 percent (i.e., 18 percent of the chemical energy of the fuel is lost through the flue gases as a result of incomplete combustion and the flue gases leaving the furnace at high temperature). If the heat loss from the outer surfaces of the furnace by natural convection and radiation is not to exceed 1 percent of the heat generated inside, determine the highest allowable surface temperature of the furnace. Assume the air and wall surface temperature of the room to be 75°F, and take the emissivity of the outer surface of the furnace to be 0.85. If the cost of natural gas is $0.65/therm and the furnace operates 2800 h per year, determine the annual cost of this heat loss to the plant.

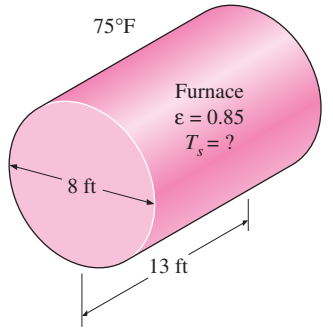


FIGURE P9–33

9–34 Consider a 1.2-m-high and 2-m-wide glass window with a thickness of 6 mm, thermal conductivity $k = 0.78$ W/m · °C, and emissivity $\varepsilon = 0.9$. The room and the walls that face the window are maintained at 25°C, and the average temperature of the inner surface of the window is measured to be 5°C. If the temperature of the outdoors is −5°C, determine (*a*) the convection heat transfer coefficient on the inner surface of the window, (*b*) the rate of total heat transfer through the window, and (*c*) the combined natural convection and radiation heat transfer coefficient on the outer surface of the window. Is it reasonable to neglect the thermal resistance of the glass in this case?

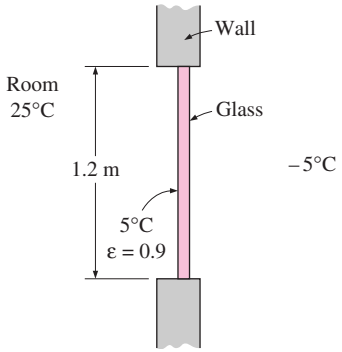


FIGURE P9–34

9–35 A 3-mm-diameter and 12-m-long electric wire is tightly wrapped with a 1.5-mm-thick plastic cover whose thermal conductivity and emissivity are $k = 0.15$ W/m · °C and $\varepsilon = 0.9$. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to calm atmospheric air at $T_\infty = 30$°C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Take the surrounding surfaces to be at about the same temperature as the air.

9–36 During a visit to a plastic sheeting plant, it was observed that a 60-m-long section of a 2-in. nominal (6.03-cm outer-diameter) steam pipe extended from one end of the plant to the other with no insulation on it. The temperature measurements at several locations revealed that the average temperature of the exposed surfaces of the steam pipe was 170°C, while the temperature of the surrounding air was 20°C. The outer surface of the pipe appeared to be oxidized, and its emissivity can be taken to be 0.7. Taking the temperature of the surrounding surfaces to be 20°C also, determine the rate of heat loss from the steam pipe.

Steam is generated in a gas furnace that has an efficiency of 78 percent, and the plant pays $0.538 per therm (1 therm = 105,500 kJ) of natural gas. The plant operates 24 h a day 365 days a year, and thus 8760 h a year. Determine the annual cost of the heat losses from the steam pipe for this facility.

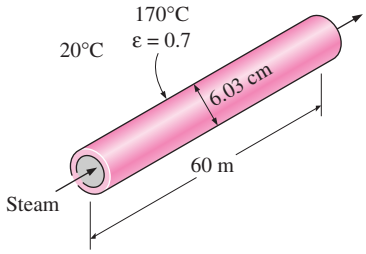


FIGURE P9–36

9–37 Reconsider Problem 9–36. Using EES (or other) software, investigate the effect of the surface temperature of the steam pipe on the rate of heat loss from the pipe and the annual cost of this heat loss. Let the surface temperature vary from 100°C to 200°C. Plot the rate of heat loss and the annual cost as a function of the surface temperature, and discuss the results.

9–38 Reconsider Problem 9–36. In order to reduce heat losses, it is proposed to insulate the steam pipe with 5-cm-thick fiberglass insulation ($k = 0.038$ W/m · °C) and to wrap it with aluminum foil ($\varepsilon = 0.1$) in order to minimize the radiation losses. Also, an estimate is obtained from a local insulation contractor, who proposed to do the insulation job for $750, including materials and labor. Would you support this proposal? How long will it take for the insulation to pay for itself from the energy it saves? Assume the temperature of the steam pipe to remain constant at 170°C.

9–39 A 30-cm × 30-cm circuit board that contains 121 square chips on one side is to be cooled by combined natural convection and radiation by mounting it on a vertical surface in a room at 25°C. Each chip dissipates 0.05 W of power, and the emissivity of the chip surfaces is 0.7. Assuming the heat transfer from the back side of the circuit board to be negligible, and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the surface temperature of the chips. *Answer:* 33.4°C

9–40 Repeat Prob. 9–35 assuming the circuit board to be positioned horizontally with (*a*) chips facing up and (*b*) chips facing down.

9–41 The side surfaces of a 2-m-high cubic industrial furnace burning natural gas are not insulated, and the temperature at the outer surface of this section is measured to be 110°C. The temperature of the furnace room, including its surfaces, is 30°C, and the emissivity of the outer surface of the furnace is 0.7. It is proposed that this section of the furnace wall be insulated with glass wool insulation ($k = 0.038$ W/m · °C) wrapped by a reflective sheet ($\varepsilon = 0.2$) in order to reduce the heat loss by 90 percent. Assuming the outer surface temperature of the metal section still remains at about 110°C, determine the thickness of the insulation that needs to be used.

The furnace operates continuously throughout the year and has an efficiency of 78 percent. The price of the natural gas is

$0.55/therm (1 therm = 105,500 kJ of energy content). If the installation of the insulation will cost $550 for materials and labor, determine how long it will take for the insulation to pay for itself from the energy it saves.

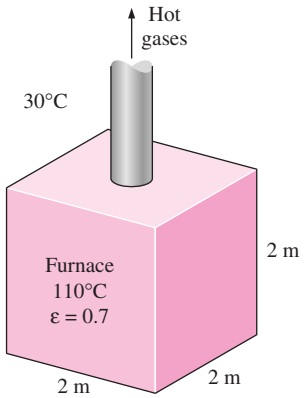


FIGURE P9–41

9–42 A 1.5-m-diameter, 5-m-long cylindrical propane tank is initially filled with liquid propane, whose density is 581 kg/m^3. The tank is exposed to the ambient air at 25°C in calm weather. The outer surface of the tank is polished so that the radiation heat transfer is negligible. Now a crack develops at the top of the tank, and the pressure inside drops to 1 atm while the temperature drops to −42°C, which is the boiling temperature of propane at 1 atm. The heat of vaporization of propane at 1 atm is 425 kJ/kg. The propane is slowly vaporized as a result of the heat transfer from the ambient air into the tank, and the propane vapor escapes the tank at −42°C through the crack. Assuming the propane tank to be at about the same temperature as the propane inside at all times, determine how long it will take for the tank to empty if it is not insulated.

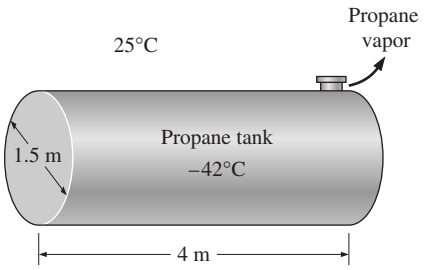


FIGURE P9–42

9–43E An average person generates heat at a rate of 287 Btu/h while resting in a room at 77°F. Assuming one-quarter of this heat is lost from the head and taking the emissivity of the skin to be 0.9, determine the average surface temperature of the head when it is not covered. The head can be approximated as a 12-in.-diameter sphere, and the interior surfaces of the room can be assumed to be at the room temperature.

9–44 An incandescent lightbulb is an inexpensive but highly inefficient device that converts electrical energy into light. It converts about 10 percent of the electrical energy it consumes into light while converting the remaining 90 percent into heat. The glass bulb of the lamp heats up very quickly as a result of absorbing all that heat and dissipating it to the surroundings by convection and radiation. Consider an 8-cm-diameter 60-W light bulb in a room at 25°C. The emissivity of the glass is 0.9. Assuming that 10 percent of the energy passes through the glass bulb as light with negligible absorption and the rest of the energy is absorbed and dissipated by the bulb itself by natural convection and radiation, determine the equilibrium temperature of the glass bulb. Assume the interior surfaces of the room to be at room temperature. *Answer:* 169°C

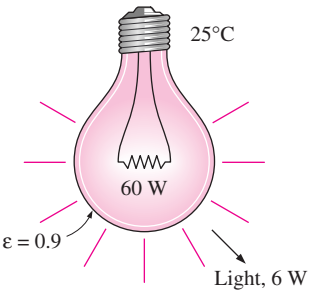


FIGURE P9–44

9–45 A 40-cm-diameter, 110-cm-high cylindrical hot water tank is located in the bathroom of a house maintained at 20°C. The surface temperature of the tank is measured to be 44°C and its emissivity is 0.4. Taking the surrounding surface temperature to be also 20°C, determine the rate of heat loss from all surfaces of the tank by natural convection and radiation.

9–46 A 28-cm-high, 18-cm-long, and 18-cm-wide rectangular container suspended in a room at 24°C is initially filled with cold water at 2°C. The surface temperature of the container is observed to be nearly the same as the water temperature inside. The emissivity of the container surface is 0.6, and the temperature of the surrounding surfaces is about the same as the air temperature. Determine the water temperature in the container after 3 h, and the average rate of heat transfer to the water. Assume the heat transfer coefficient on the top and bottom surfaces to be the same as that on the side surfaces.

9–47 Reconsider Problem 9–46. Using EES (or other) software, plot the water temperature in the container as a function of the heating time as the time varies from 30 min to 10 h, and discuss the results.

9–48 A room is to be heated by a coal-burning stove, which is a cylindrical cavity with an outer diameter of 32 cm and a height of 70 cm. The rate of heat loss from the room is estimated to be 1.2 kW when the air temperature in the room is maintained constant at 24°C. The emissivity of the stove surface is 0.85 and the average temperature of the surrounding

wall surfaces is 17°C. Determine the surface temperature of the stove. Neglect the transfer from the bottom surface and take the heat transfer coefficient at the top surface to be the same as that on the side surface.

The heating value of the coal is 30,000 kJ/kg, and the combustion efficiency is 65 percent. Determine the amount of coal burned a day if the stove operates 14 h a day.

9–49 The water in a 40-L tank is to be heated from 15°C to 45°C by a 6-cm-diameter spherical heater whose surface temperature is maintained at 85°C. Determine how long the heater should be kept on.

Natural Convection from Finned Surfaces and PCBs

9–50C Why are finned surfaces frequently used in practice? Why are the finned surfaces referred to as heat sinks in the electronics industry?

9–51C Why are heat sinks with closely packed fins not suitable for natural convection heat transfer, although they increase the heat transfer surface area more?

9–52C Consider a heat sink with optimum fin spacing. Explain how heat transfer from this heat sink will be affected by (*a*) removing some of the fins on the heat sink and (*b*) doubling the number of fins on the heat sink by reducing the fin spacing. The base area of the heat sink remains unchanged at all times.

9–53 Aluminum heat sinks of rectangular profile are commonly used to cool electronic components. Consider a 7.62-cm-long and 9.68-cm-wide commercially available heat sink whose cross section and dimensions are as shown in Figure P9–53. The heat sink is oriented vertically and is used to cool a power transistor that can dissipate up to 125 W of power. The back surface of the heat sink is insulated. The surfaces of the heat sink are untreated, and thus they have a low emissivity (under 0.1). Therefore, radiation heat transfer from the heat sink can be neglected. During an experiment conducted in room air at 22°C, the base temperature of the heat sink was measured to be 120°C when the power dissipation of the transistor was 15 W. Assuming the entire heat sink to be at the base temperature, determine the average natural convection heat transfer coefficient for this case. *Answer:* 7.1 W/m2 · °C

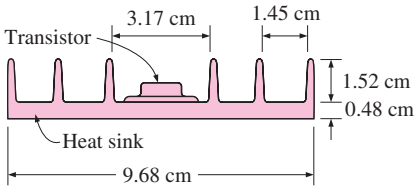


FIGURE P9–53

9–54 Reconsider the heat sink in Problem 9–53. In order to enhance heat transfer, a shroud (a thin rectangular metal plate) whose surface area is equal to the base area of the heat sink is placed very close to the tips of the fins such that the interfin spaces are converted into rectangular channels. The base temperature of the heat sink in this case was measured to be 108°C. Noting that the shroud loses heat to the ambient air from both sides, determine the average natural convection heat transfer coefficient in this shrouded case. (For complete details, see Çengel and Zing, Ref. 9).

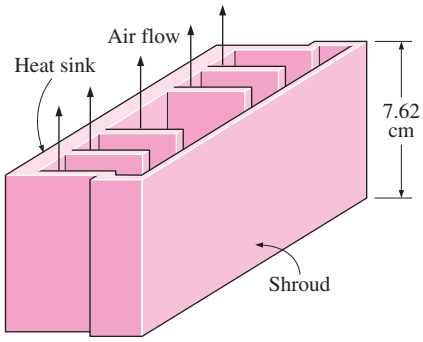


FIGURE P9–54

9–55E A 6-in.-wide and 8-in.-high vertical hot surface in 78°F air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.08 in. thick and 8 in. long in the vertical direction and have a height of 1.2 in. from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 180°F.

9–56E Reconsider Problem 9–55E. Using EES (or other) software, investigate the effect of the length of the fins in the vertical direction on the optimum fin spacing and the rate of heat transfer by natural convection. Let the fin length vary from 2 in. to 10 in. Plot the optimum fin spacing and the rate of convection heat transfer as a function of the fin length, and discuss the results.

9–57 A 12.1-cm-wide and 18-cm-high vertical hot surface in 25°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction. Determine the optimum fin height and the rate of heat transfer by natural convection from the heat sink if the base temperature is 65°C.

Natural Convection inside Enclosures

9–58C The upper and lower compartments of a well-insulated container are separated by two parallel sheets of glass with an air space between them. One of the compartments is to be filled with a hot fluid and the other with a cold fluid. If it is desired that heat transfer between the two compartments be minimal, would you recommend putting the hot fluid into the upper or the lower compartment of the container? Why?

9–59C Someone claims that the air space in a double-pane window enhances the heat transfer from a house because of the natural convection currents that occur in the air space and

recommends that the double-pane window be replaced by a single sheet of glass whose thickness is equal to the sum of the thicknesses of the two glasses of the double-pane window to save energy. Do you agree with this claim?

9–60C Consider a double-pane window consisting of two glass sheets separated by a 1-cm-wide air space. Someone suggests inserting a thin vinyl sheet in the middle of the two glasses to form two 0.5-cm-wide compartments in the window in order to reduce natural convection heat transfer through the window. From a heat transfer point of view, would you be in favor of this idea to reduce heat losses through the window?

9–61C What does the effective conductivity of an enclosure represent? How is the ratio of the effective conductivity to thermal conductivity related to the Nusselt number?

9–62 Show that the thermal resistance of a rectangular enclosure can be expressed as $R = \delta/(Ak\,\mathrm{Nu})$, where k is the thermal conductivity of the fluid in the enclosure.

9–63E A vertical 4-ft-high and 6-ft-wide double-pane window consists of two sheets of glass separated by a 1-in. air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 65°F and 40°F, determine the rate of heat transfer through the window by (*a*) natural convection and (*b*) radiation. Also, determine the *R*-value of insulation of this window such that multiplying the inverse of the *R*-value by the surface area and the temperature difference gives the total rate of heat transfer through the window. The effective emissivity for use in radiation calculations between two large parallel glass plates can be taken to be 0.82.

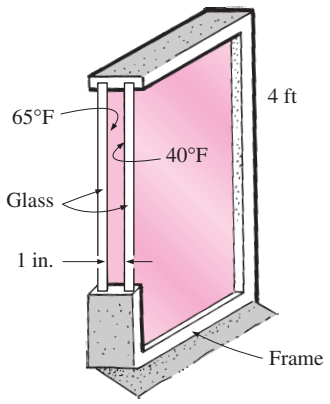


FIGURE P9–63E

9–64E Reconsider Problem 9–63E. Using EES (or other) software, investigate the effect of the air gap thickness on the rates of heat transfer by natural convection and radiation, and the *R*-value of insulation. Let the air gap thickness vary from 0.2 in. to 2.0 in. Plot the rates of heat transfer by natural convection and radiation, and the *R*-value of insulation as a function of the air gap thickness, and discuss the results.

9–65 Two concentric spheres of diameters 15 cm and 25 cm are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_1 = 350$ K and $T_2 = 275$ K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

9–66 Reconsider Problem 9–65. Using EES (or other) software, plot the rate of natural convection heat transfer as a function of the hot surface temperature of the sphere as the temperature varies from 300 K to 500 K, and discuss the results.

9–67 Flat-plate solar collectors are often tilted up toward the sun in order to intercept a greater amount of direct solar radiation. The tilt angle from the horizontal also affects the rate of heat loss from the collector. Consider a 2-m-high and 3-m-wide solar collector that is tilted at an angle θ from the horizontal. The back side of the absorber is heavily insulated. The absorber plate and the glass cover, which are spaced 2.5 cm from each other, are maintained at temperatures of 80°C and 40°C, respectively. Determine the rate of heat loss from the absorber plate by natural convection for θ = 0°, 20°, and 90°.

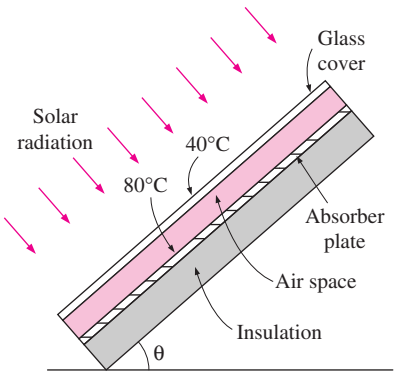


FIGURE P9–67

9–68 A simple solar collector is built by placing a 5-cm-diameter clear plastic tube around a garden hose whose outer diameter is 1.6 cm. The hose is painted black to maximize solar absorption, and some plastic rings are used to keep the spacing between the hose and the clear plastic cover constant. During a clear day, the temperature of the hose is measured to be 65°C,

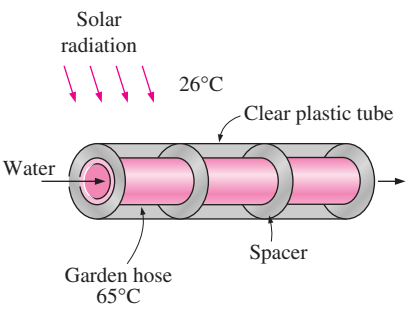


FIGURE P9–68

while the ambient air temperature is 26°C. Determine the rate of heat loss from the water in the hose per meter of its length by natural convection. Also, discuss how the performance of this solar collector can be improved. *Answer:* 8.2 W

9–69 Reconsider Problem 9–68. Using EES (or other) software, plot the rate of heat loss from the water by natural convection as a function of the ambient air temperature as the temperature varies from 4˚C to 40˚C, and discuss the results.

9–70 A vertical 1.3-m-high, 2.8-m-wide double-pane window consists of two layers of glass separated by a 2.2-cm air gap at atmospheric pressure. The room temperature is 26˚C while the inner glass temperature is 18˚C. Disregarding radiation heat transfer, determine the temperature of the outer glass layer and the rate of heat loss through the window by natural convection.

9–71 Consider two concentric horizontal cylinders of diameters 55 cm and 65 cm, and length 125 cm. The surfaces of the inner and outer cylinders are maintained at 46˚C and 74˚C, respectively. Determine the rate of heat transfer between the cylinders by natural convection if the annular space is filled with (*a*) water and (*b*) air.

Combined Natural and Forced Convection

9–72C When is natural convection negligible and when is it not negligible in forced convection heat transfer?

9–73C Under what conditions does natural convection enhance forced convection, and under what conditions does it hurt forced convection?

9–74C When neither natural nor forced convection is negligible, is it correct to calculate each independently and add them to determine the total convection heat transfer?

9–75 Consider a 5-m-long vertical plate at 85°C in air at 30°C. Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible. *Answer:* 9.04 m/s

9–76 Reconsider Problem 9–75. Using EES (or other) software, plot the forced motion velocity above which natural convection heat transfer is negligible as a function of the plate temperature as the temperature varies from 50˚C to 150˚C, and discuss the results.

9–77 Consider a 5-m-long vertical plate at 60°C in water at 25°C. Determine the forced motion velocity above which natural convection heat transfer from this plate is negligible. Take $\beta = 0.0004\ K^{-1}$ for water.

9–78 In a production facility, thin square plates 2 m × 2 m in size coming out of the oven at 270°C are cooled by blowing ambient air at 30°C horizontally parallel to their surfaces. Determine the air velocity above which the natural convection effects on heat transfer are less than 10 percent and thus are negligible.

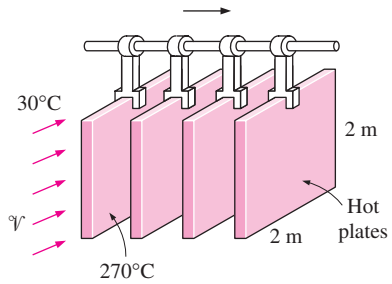


FIGURE P9–78

9–79 A 12-cm-high and 20-cm-wide circuit board houses 100 closely spaced logic chips on its surface, each dissipating 0.05 W. The board is cooled by a fan that blows air over the hot surface of the board at 35°C at a velocity of 0.5 m/s. The heat transfer from the back surface of the board is negligible. Determine the average temperature on the surface of the circuit board assuming the air flows vertically upwards along the 12-cm-long side by (*a*) ignoring natural convection and (*b*) considering the contribution of natural convection. Disregard any heat transfer by radiation.

Special Topic: Heat Transfer through Windows

9–80C Why are the windows considered in three regions when analyzing heat transfer through them? Name those regions and explain how the overall *U*-value of the window is determined when the heat transfer coefficients for all three regions are known.

9–81C Consider three similar double-pane windows with air gap widths of 5, 10, and 20 mm. For which case will the heat transfer through the window will be a minimum?

9–82C In an ordinary double-pane window, about half of the heat transfer is by radiation. Describe a practical way of reducing the radiation component of heat transfer.

9–83C Consider a double-pane window whose air space width is 20 mm. Now a thin polyester film is used to divide the air space into two 10-mm-wide layers. How will the film affect (*a*) convection and (*b*) radiation heat transfer through the window?

9–84C Consider a double-pane window whose air space is flashed and filled with argon gas. How will replacing the air in the gap by argon affect (*a*) convection and (*b*) radiation heat transfer through the window?

9–85C Is the heat transfer rate through the glazing of a double-pane window higher at the center or edge section of the glass area? Explain.

9–86C How do the relative magnitudes of *U*-factors of windows with aluminum, wood, and vinyl frames compare? Assume the windows are identical except for the frames.

9–87 Determine the *U*-factor for the center-of-glass section of a double-pane window with a 13-mm air space for winter

design conditions. The glazings are made of clear glass having an emissivity of 0.84. Take the average air space temperature at design conditions to be 10°C and the temperature difference across the air space to be 15°C.

9–88 A double-door wood-framed window with glass glazing and metal spacers is being considered for an opening that is 1.2 m high and 1.8 m wide in the wall of a house maintained at 20°C. Determine the rate of heat loss through the window and the inner surface temperature of the window glass facing the room when the outdoor air temperature is −8°C if the window is selected to be (*a*) 3-mm single glazing, (*b*) double glazing with an air space of 13 mm, and (*c*) low-e-coated triple glazing with an air space of 13 mm.

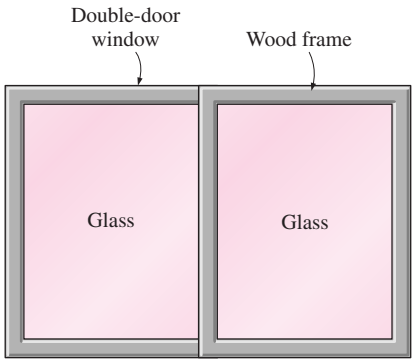


FIGURE P9–88

9–89 Determine the overall *U*-factor for a double-door-type wood-framed double-pane window with 13-mm air space and metal spacers, and compare your result to the value listed in Table 9–6. The overall dimensions of the window are 2.00 m × 2.40 m, and the dimensions of each glazing are 1.92 m × 1.14 m.

9–90 Consider a house in Atlanta, Georgia, that is maintained at 22°C and has a total of 20 m^2 of window area. The windows are double-door-type with wood frames and metal spacers. The glazing consists of two layers of glass with 12.7 mm of air space with one of the inner surfaces coated with reflective film. The winter average temperature of Atlanta is 11.3°C. Determine the average rate of heat loss through the windows in winter.
Answer: 456 W

9–91E Consider an ordinary house with *R*-13 walls (walls that have an *R*-value of 13 h · ft^2 · °F/Btu). Compare this to the *R*-value of the common double-door windows that are double pane with $\frac{1}{4}$ in. of air space and have aluminum frames. If the windows occupy only 20 percent of the wall area, determine if more heat is lost through the windows or through the remaining 80 percent of the wall area. Disregard infiltration losses.

9–92 The overall *U*-factor of a fixed wood-framed window with double glazing is given by the manufacturer to be U = 2.76 W/m^2 · °C under the conditions of still air inside and winds of 12 km/h outside. What will the *U*-factor be when the wind velocity outside is doubled? *Answer:* 2.88 W/m^2 · °C

9–93 The owner of an older house in Wichita, Kansas, is considering replacing the existing double-door type wood-framed single-pane windows with vinyl-framed double-pane windows with an air space of 6.4 mm. The new windows are of double-door type with metal spacers. The house is maintained at 22°C at all times, but heating is needed only when the outdoor temperature drops below 18°C because of the internal heat gain from people, lights, appliances, and the sun. The average winter temperature of Wichita is 7.1°C, and the house is heated by electric resistance heaters. If the unit cost of electricity is \$0.07/kWh and the total window area of the house is 12 m^2, determine how much money the new windows will save the home owner per month in winter.

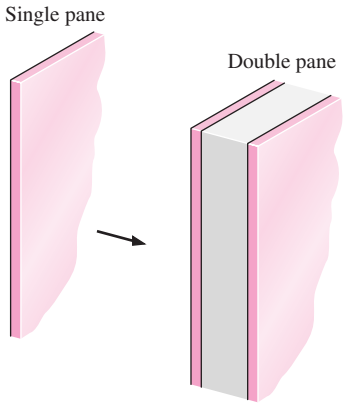


FIGURE P9–93

Review Problems

9–94E A 0.1-W small cylindrical resistor mounted on a lower part of a vertical circuit board is 0.3 in. long and has a diameter of 0.2 in. The view of the resistor is largely blocked by another circuit board facing it, and the heat transfer through the connecting wires is negligible. The air is free to flow through the large parallel flow passages between the boards as a result of natural convection currents. If the air temperature at the vicinity of the resistor is 120°F, determine the approximate surface temperature of the resistor. *Answer:* 212°F

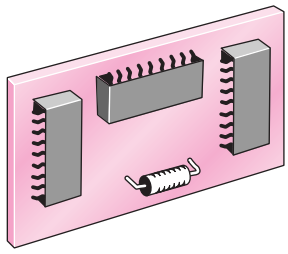


FIGURE P9–94E

9–95 An ice chest whose outer dimensions are 30 cm × 40 cm × 40 cm is made of 3-cm-thick styrofoam (k = 0.033 W/m · °C). Initially, the chest is filled with 30 kg of ice at 0°C, and the inner surface temperature of the ice chest can be taken to be 0°C at all times. The heat of fusion of water at 0°C is 333.7 kJ/kg, and the surrounding ambient air is at 20°C. Disregarding any heat transfer from the 40 cm × 40 cm base of the ice chest, determine how long it will take for the ice in the chest to melt completely if the ice chest is subjected to (*a*) calm air and (*b*) winds at 50 km/h. Assume the heat transfer coefficient on the front, back, and top surfaces to be the same as that on the side surfaces.

9–96 An electronic box that consumes 180 W of power is cooled by a fan blowing air into the box enclosure. The dimensions of the electronic box are 15 cm × 50 cm × 50 cm, and all surfaces of the box are exposed to the ambient except the base surface. Temperature measurements indicate that the box is at an average temperature of 32°C when the ambient temperature and the temperature of the surrounding walls are 25°C. If the emissivity of the outer surface of the box is 0.85, determine the fraction of the heat lost from the outer surfaces of the electronic box.

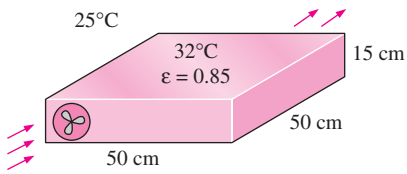


FIGURE P9–96

9–97 A 6-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel (k = 15 W/m · °C) is used to store iced water at 0°C in a room at 20°C. The walls of the room are also at 20°C. The outer surface of the tank is black (emissivity ε = 1), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. Assuming the entire steel tank to be at 0°C and thus the thermal resistance of the tank to be negligible, determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period.

Answers: (*a*) 15.4 kW, (*b*) 3988 kg

9–98 Consider a 1.2-m-high and 2-m-wide double-pane window consisting of two 3-mm-thick layers of glass (k = 0.78 W/m · °C) separated by a 3-cm-wide air space. Determine the steady rate of heat transfer through this window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is 0°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be h_1 = 10 W/m² · °C and h_2 = 25 W/m² · °C and disregard any heat transfer by radiation.

9–99 An electric resistance space heater is designed such that it resembles a rectangular box 50 cm high, 80 cm long, and 15 cm wide filled with 45 kg of oil. The heater is to be placed against a wall, and thus heat transfer from its back surface is negligible for safety considerations. The surface temperature of the heater is not to exceed 45°C in a room at 25°C. Disregarding heat transfer from the bottom and top surfaces of the heater in anticipation that the top surface will be used as a shelf, determine the power rating of the heater in W. Take the emissivity of the outer surface of the heater to be 0.8 and the average temperature of the ceiling and wall surfaces to be the same as the room air temperature.

Also, determine how long it will take for the heater to reach steady operation when it is first turned on (i.e., for the oil temperature to rise from 25°C to 45°C). State your assumptions in the calculations.

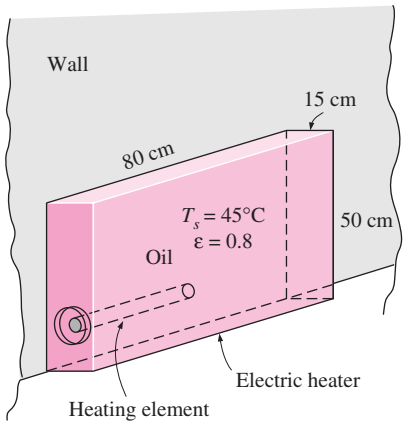


FIGURE P9–99

9–100 Skylights or "roof windows" are commonly used in homes and manufacturing facilities since they let natural light in during day time and thus reduce the lighting costs. However, they offer little resistance to heat transfer, and large amounts of energy are lost through them in winter unless they are equipped with a motorized insulating cover that can be used in cold weather and at nights to reduce heat losses. Consider a 1-m-wide and 2.5-m-long horizontal skylight on the roof of a house that is kept at 20°C. The glazing of the skylight is made of a single layer of 0.5-cm-thick

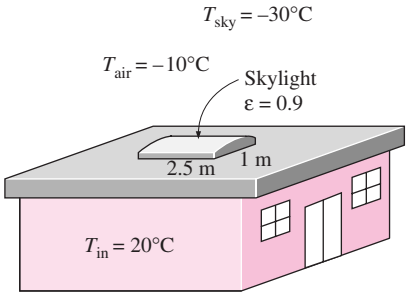


FIGURE P9–100

glass (k = 0.78 W/m · °C and ε = 0.9). Determine the rate of heat loss through the skylight when the air temperature outside is −10°C and the effective sky temperature is −30°C. Compare your result with the rate of heat loss through an equivalent surface area of the roof that has a common R-5.34 construction in SI units (i.e., a thickness–to–effective-thermal-conductivity ratio of 5.34 m² · °C/W).

9–101 A solar collector consists of a horizontal copper tube of outer diameter 5 cm enclosed in a concentric thin glass tube of 9 cm diameter. Water is heated as it flows through the tube, and the annular space between the copper and glass tube is filled with air at 1 atm pressure. During a clear day, the temperatures of the tube surface and the glass cover are measured to be 60°C and 32°C, respectively. Determine the rate of heat loss from the collector by natural convection per meter length of the tube. *Answer:* 17.4 W

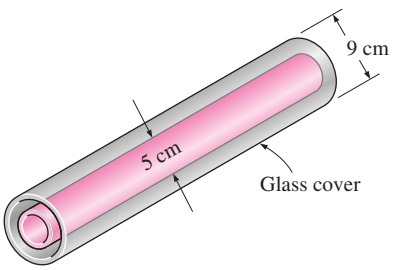


FIGURE P9–101

9–102 A solar collector consists of a horizontal aluminum tube of outer diameter 4 cm enclosed in a concentric thin glass tube of 7 cm diameter. Water is heated as it flows through the aluminum tube, and the annular space between the aluminum and glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 20 W per meter length, and the temperature of the ambient air outside is 30°C. Approximating the surfaces of the tube and the glass cover as being black (emissivity ε = 1) in radiation calculations and taking the effective sky temperature to be 20°C, determine the temperature of the aluminum tube when equilibrium is established (i.e., when the net heat loss from the tube by convection and radiation equals the amount of solar energy absorbed by the tube).

9–103E The components of an electronic system dissipating 180 W are located in a 4-ft-long horizontal duct whose cross-section is 6 in. × 6 in. The components in the duct are cooled by forced air, which enters at 85°F at a rate of 22 cfm and leaves at 100°F. The surfaces of the sheet metal duct are not painted, and thus radiation heat transfer from the outer surfaces is negligible. If the ambient air temperature is 80°F, determine (*a*) the heat transfer from the outer surfaces of the duct to the ambient air by natural convection and (*b*) the average temperature of the duct.

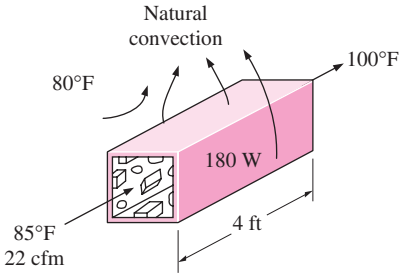


FIGURE P9–103E

9–104E Repeat Problem 9–103E for a circular horizontal duct of diameter 4 in.

9–105E Repeat Problem 9–103E assuming the fan fails and thus the entire heat generated inside the duct must be rejected to the ambient air by natural convection through the outer surfaces of the duct.

9–106 Consider a cold aluminum canned drink that is initially at a uniform temperature of 5°C. The can is 12.5 cm high and has a diameter of 6 cm. The emissivity of the outer surface of the can is 0.6. Disregarding any heat transfer from the bottom surface of the can, determine how long it will take for the average temperature of the drink to rise to 7°C if the surrounding air and surfaces are at 25°C. *Answer:* 12.1 min

9–107 Consider a 2-m-high electric hot water heater that has a diameter of 40 cm and maintains the hot water at 60°C. The tank is located in a small room at 20°C whose walls and the ceiling are at about the same temperature. The tank is placed in a 46-cm-diameter sheet metal shell of negligible thickness, and the space between the tank and the shell is filled with foam insulation. The average temperature and emissivity of the outer surface of the shell are 40°C and 0.7, respectively. The price of

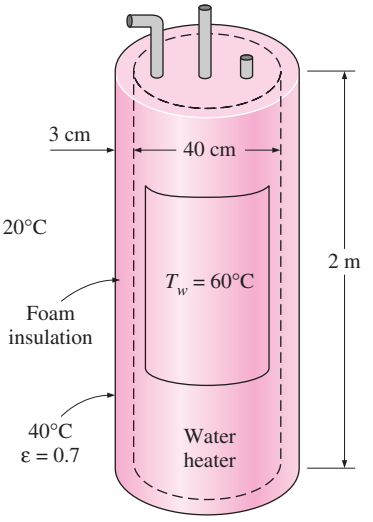


FIGURE P9–107

electricity is \$0.08/kWh. Hot water tank insulation kits large enough to wrap the entire tank are available on the market for about \$30. If such an insulation is installed on this water tank by the home owner himself, how long will it take for this additional insulation to pay for itself? Disregard any heat loss from the top and bottom surfaces, and assume the insulation to reduce the heat losses by 80 percent.

9–108 During a plant visit, it was observed that a 1.5-m-high and 1-m-wide section of the vertical front section of a natural gas furnace wall was too hot to touch. The temperature measurements on the surface revealed that the average temperature of the exposed hot surface was 110°C, while the temperature of the surrounding air was 25°C. The surface appeared to be oxidized, and its emissivity can be taken to be 0.7. Taking the temperature of the surrounding surfaces to be 25°C also, determine the rate of heat loss from this furnace.

The furnace has an efficiency of 79 percent, and the plant pays \$0.75 per therm of natural gas. If the plant operates 10 h a day, 310 days a year, and thus 3100 h a year, determine the annual cost of the heat loss from this vertical hot surface on the front section of the furnace wall.

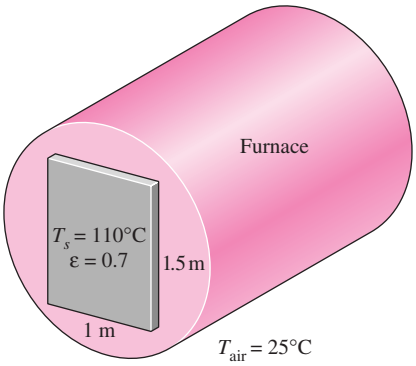


FIGURE P9–108

9–109 A group of 25 power transistors, dissipating 1.5 W each, are to be cooled by attaching them to a black-anodized square aluminum plate and mounting the plate on the wall of a room at 30°C. The emissivity of the transistor and the plate surfaces is 0.9. Assuming the heat transfer from the back side of the plate to be negligible and the temperature of the surrounding surfaces to be the same as the air temperature of the room, determine the size of the plate if the average surface temperature of the plate is not to exceed 50°C. *Answer:* 43 cm 3 43 cm

9–110 Repeat Problem 9–109 assuming the plate to be positioned horizontally with (*a*) transistors facing up and (*b*) transistors facing down.

9–111E Hot water is flowing at an average velocity of 4 ft/s through a cast iron pipe (k = 30 Btu/h · ft · °F) whose inner and outer diameters are 1.0 in. and 1.2 in., respectively. The pipe passes through a 50-ft-long section of a basement whose temperature is 60°F. The emissivity of the outer surface of the pipe

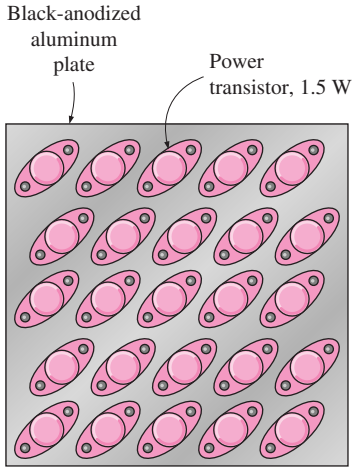


FIGURE P9–109

is 0.5, and the walls of the basement are also at about 60°F. If the inlet temperature of the water is 150°F and the heat transfer coefficient on the inner surface of the pipe is 30 Btu/h · ft^2 · °F, determine the temperature drop of water as it passes through the basement.

9–112 Consider a flat-plate solar collector placed horizontally on the flat roof of a house. The collector is 1.5 m wide and 6 m long, and the average temperature of the exposed surface of the collector is 42°C. Determine the rate of heat loss from the collector by natural convection during a calm day when the ambient air temperature is 15°C. Also, determine the heat loss by radiation by taking the emissivity of the collector surface to be 0.9 and the effective sky temperature to be −30°C. *Answers:* 1295 W, 2921 W

9–113 Solar radiation is incident on the glass cover of a solar collector at a rate of 650 W/m^2. The glass transmits 88 percent of the incident radiation and has an emissivity of 0.90. The hot water needs of a family in summer can be met completely by a

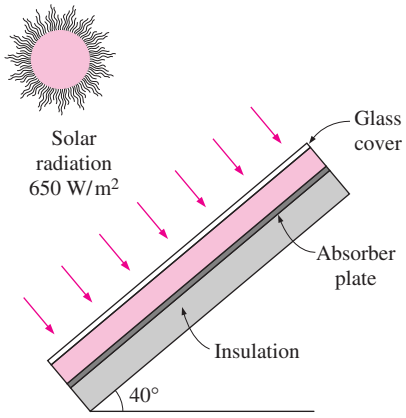


FIGURE P9–113

collector 1.5 m high and 2 m wide, and tilted 40° from the horizontal. The temperature of the glass cover is measured to be 40°C on a calm day when the surrounding air temperature is 20°C. The effective sky temperature for radiation exchange between the glass cover and the open sky is −40°C. Water enters the tubes attached to the absorber plate at a rate of 1 kg/min. Assuming the back surface of the absorber plate to be heavily insulated and the only heat loss occurs through the glass cover, determine (*a*) the total rate of heat loss from the collector, (*b*) the collector efficiency, which is the ratio of the amount of heat transferred to the water to the solar energy incident on the collector, and (*c*) the temperature rise of water as it flows through the collector.

Design and Essay Problems

9–114 Write a computer program to evaluate the variation of temperature with time of thin square metal plates that are removed from an oven at a specified temperature and placed vertically in a large room. The thickness, the size, the initial temperature, the emissivity, and the thermophysical properties of the plate as well as the room temperature are to be specified by the user. The program should evaluate the temperature of the plate at specified intervals and tabulate the results against time. The computer should list the assumptions made during calculations before printing the results.

For each step or time interval, assume the surface temperature to be constant and evaluate the heat loss during that time interval and the temperature drop of the plate as a result of this heat loss. This gives the temperature of the plate at the end of a time interval, which is to serve as the initial temperature of the plate for the beginning of the next time interval.

Try your program for 0.2-cm-thick vertical copper plates of 40 cm × 40 cm in size initially at 300°C cooled in a room at 25°C. Take the surface emissivity to be 0.9. Use a time interval of 1 s in calculations, but print the results at 10-s intervals for a total cooling period of 15 min.

9–115 Write a computer program to optimize the spacing between the two glasses of a double-pane window. Assume the spacing is filled with dry air at atmospheric pressure. The program should evaluate the recommended practical value of the spacing to minimize the heat losses and list it when the size of the window (the height and the width) and the temperatures of the two glasses are specified.

9–116 Contact a manufacturer of aluminum heat sinks and obtain their product catalog for cooling electronic components by natural convection and radiation. Write an essay on how to select a suitable heat sink for an electronic component when its maximum power dissipation and maximum allowable surface temperature are specified.

9–117 The top surfaces of practically all flat-plate solar collectors are covered with glass in order to reduce the heat losses from the absorber plate underneath. Although the glass cover reflects or absorbs about 15 percent of the incident solar radiation, it saves much more from the potential heat losses from the absorber plate, and thus it is considered to be an essential part of a well-designed solar collector. Inspired by the energy efficiency of double-pane windows, someone proposes to use double glazing on solar collectors instead of a single glass. Investigate if this is a good idea for the town in which you live. Use local weather data and base your conclusion on heat transfer analysis and economic considerations.