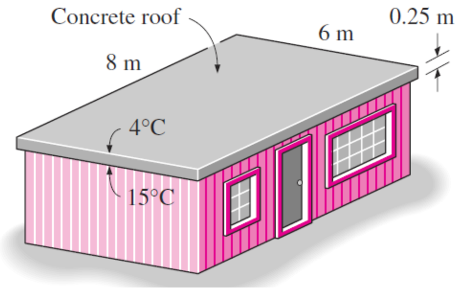
Example 1

The Cost of Heat Loss through a Roof

The roof of an electrically heated home is 6 m long, 8 m wide, and 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is k=0.8 W/m · °C. The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15°C and 4°C, respectively, for a period of 10 hours. Determine;

a) the rate of heat loss through the roof that night
b) the cost of that heat loss to the home owner if the cost of electricity is $0.08/kWh.



$$\dot{Q} = -kA\frac{dT}{dx} \text{ (Fourier's law of heat conduction)}$$

$$= -(0.8\ \mathrm{W/m\cdot °C})(6\mathrm{m}\times 8\mathrm{m})\left(\frac{15°\mathrm{C} - 4°\mathrm{C}}{0.25\mathrm{m}}\right) = 1690\ \mathrm{W} = 1.69\ \mathrm{kW}$$

$$Q = \dot{Q}\Delta t = (1.69\ \mathrm{kW})(10\ \mathrm{h}) = 16.9\ \mathrm{kWh}$$

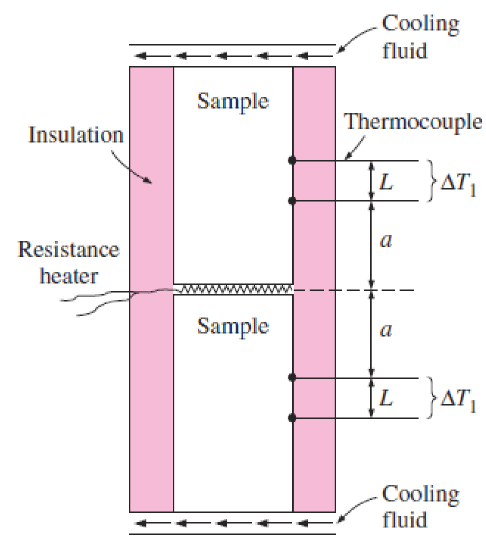
$$\text{Cost} = (\text{Amount of energy})(\text{Unit cost of energy})$$

$$= (16.9\ \mathrm{kWh})(\$0.08/\mathrm{kWh}) = \$1.35$$

Example 2

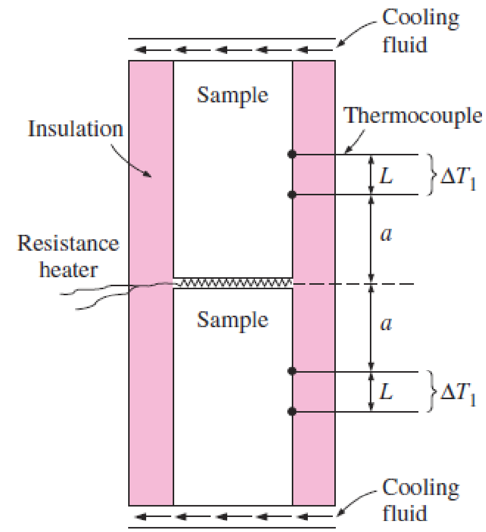
Measuring the Thermal Conductivity of a Material

A common way of measuring the thermal conductivity of a material is to sandwich an electric thermofoil heater between two identical samples of the material. The thickness of the resistance heater, including its cover, which is made of thin silicon rubber, is usually less than 0.5 mm. A circulating fluid such as tap water keeps the exposed ends of the samples at constant temperature. The lateral surfaces of the samples are well insulated to ensure that heat transfer through the samples is one-dimensional. Two thermocouples are embedded into each sample some distance L apart, and a differential thermometer reads the temperature drop T across this distance along each sample. When steady operating conditions are reached, the total rate of heat transfer through both samples becomes equal to the electric power drawn by the heater, which is determined by multiplying the electric current by the voltage.



Example 2 (cont.)

Measuring the Thermal Conductivity of a Material



In a certain experiment, cylindrical samples of diameter 5 cm and length 10 cm are used. The two thermocouples in each sample are placed 3 cm apart. After initial transients, the electric heater is observed to draw 0.4 A at 110 V, and both differential thermometers read a temperature difference of 15°C. Determine the thermal conductivity of the sample.

$$\text{Power} = \dot{W}_e = VI = (110\text{ V})(0.4\text{ A}) = 44\text{ W}$$

The rate of heat flow through each sample is

$$\dot{Q} = \frac{1}{2}\dot{W}_e = \frac{1}{2}(44\text{ W}) = 22\text{ W}$$

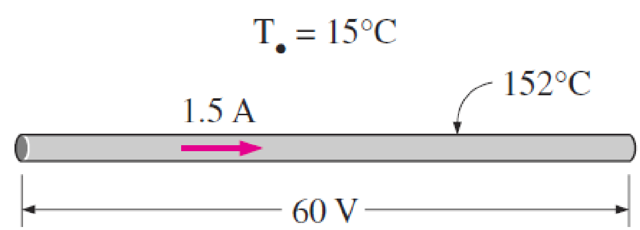
$$A = \pi\frac{D^2}{4} = \pi\frac{(0.05\text{ m})^2}{4} = 0.00196\text{ m}^2$$

$$\dot{Q} = kA\frac{\Delta T}{\Delta x} \Rightarrow k = \frac{\dot{Q}\Delta x}{A\Delta T} = \frac{(22\text{ W})(0.03\text{ m})}{(0.00196\text{ m}^2)(15°\text{C})} = 22.4\text{ W/m}\cdot°\text{C}$$

Example 3

Measuring Convection Heat Transfer Coefficient

A 2-m-long, 0.3 cm diameter electrical wire extends across a room at 15°C. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 60V and 1.5A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.



$$\dot{Q} = VI = (60\ \text{V})(1.5\ \text{A}) = 90\ \text{W}$$

$$A_s = \pi DL = \pi(0.003\ \text{m})(2\ \text{m}) = 0.01885\ \text{m}^2$$

Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q}_{conv} = hA_s\,(T_s - T_\infty)$$

$$\Rightarrow h = \frac{\dot{Q}_{conv}}{A_s\,(T_s - T_\infty)} = \frac{90\ \text{W}}{(0.01885\ \text{m}^2)(152 - 15)°\text{C}} = 34.9\ \text{W/m}^2 \cdot °\text{C}$$

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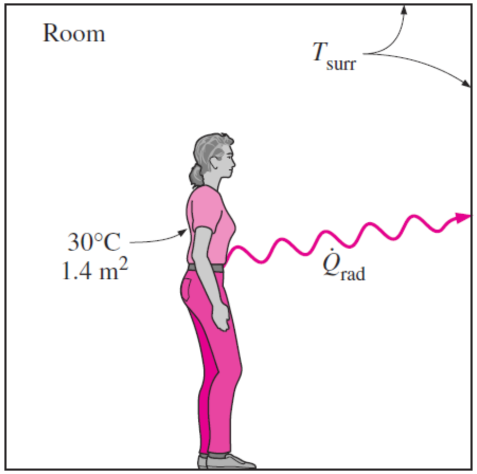
Example 4

Radiation Effect on Thermal Comfort

It is a common experience to feel "chilly" in winter and "warm" in summer in our homes even when the thermostat setting is kept the same. This is due to the so called "radiation effect" resulting from radiation heat exchange between our bodies and the surrounding surfaces of the walls and the ceiling.

Consider a person standing in a room maintained at 22°C at all times. The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 10°C in winter and 25°C in summer. Determine the rate of radiation heat transfer between this person and the surrounding surfaces if the exposed surface area and the average outer surface temperature of the person are 1.4 m^2 and 30°C, respectively. The emissivity of a person is 0.95.



$$\dot{Q}_{rad,\,winter} = \varepsilon\sigma A_s\,(T_s^4 - T_{surr,\,winter}^4)$$

$$= (0.95)(5.67\times10^{-8}\,W/m^2\cdot K^4)(1.4\,m^2)\times[(30+273)^4 - (10+273)^4]\,K^4 = 152W$$

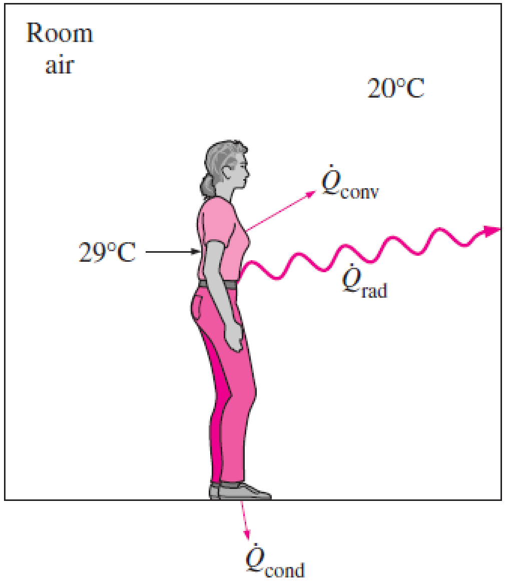
$$\dot{Q}_{rad,\,summer} = \varepsilon\sigma A_s\,(T_s^4 - T_{surr,\,summer}^4)$$

$$= (0.95)(5.67\times10^{-8}\,W/m^2\cdot K^4)(1.4\,m^2)\times[(30+273)^4 - (25+273)^4]\,K^4 = 40.9W$$

Example 5

Heat Loss from a Person

Consider a person standing in a breezy room at 20°C. Determine the total rate of heat transfer from this person if the exposed surface area and the average outer surface temperature of the person are 1.6 m^2 and 29°C, respectively, and the convection heat transfer coefficient is 6 $W/m^2 \cdot °C$. The emissivity of a person is 0.95



1. The person is completely surrounded by the interior surfaces of the room.
2. The surrounding surfaces are at the same temperature as the air in the room.
3. Heat conduction to the floor through the feet is negligible

The heat transfer between the person and the air in the room will be by convection (instead of conduction) since it is conceivable that the air in the vicinity of the skin or clothing will warm up and rise as a result of heat transfer from the body, initiating natural convection currents. It appears that the experimentally determined value for the rate of convection heat transfer in this case is 6 W per unit surface area (m^2) per unit temperature difference (K or °C) between the person and air away from the person. Thus, the rate of convection heat transfer from the person to the air in the room is;

Example 5

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$
$$= (6\ \mathrm{W/m^2 \cdot °C})(1.6\ \mathrm{m^2})(29 - 20)°\mathrm{C}$$
$$= 86.4\ \mathrm{W}$$

The person will also lose heat by radiation to the surrounding wall surfaces. We take the temperature of the surfaces of the walls, ceiling, and floor to be equal to the air temperature in this case for simplicity, but we recognize that this does not need to be the case. These surfaces may be at a higher or lower temperature than the average temperature of the room air, depending on the outdoor conditions and the structure of the walls. Considering that air does not intervene with radiation and the person is completely enclosed by the surrounding surfaces, the net rate of radiation heat transfer from the person to the surrounding walls, ceiling, and floor is

$$\dot{Q}_{rad} = \varepsilon\sigma A_s\left(T_s^4 - T_{surr}^4\right)$$
$$= (0.95)(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(1.6\ \mathrm{m^2})$$
$$\times [(29 + 273)^4 - (20 + 273)^4]\ \mathrm{K^4}$$
$$= 81.7\mathrm{W}$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = (86.4 + 81.7)\ \mathrm{W} = 168.1\mathrm{W}$$

Discussion

The heat transfer would be much higher if the person were undressed since the exposed surface temperature would be higher. Thus, an important function of the clothes is to serve as a barrier against heat transfer.

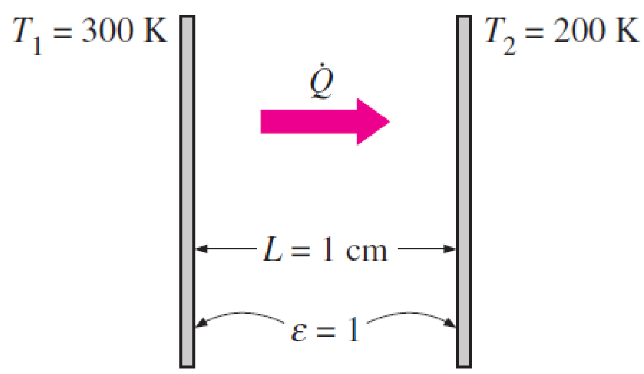
In these calculations, heat transfer through the feet to the floor by conduction, which is usually very small, is neglected. Heat transfer from the skin by perspiration, which is the dominant mode of heat transfer in hot environments, is not considered here.

Example 6

Heat Transfer between Two Isothermal Plates

Consider steady heat transfer between two large parallel plates at constant temperatures of T_1=300K and T_2=200K that are L =1cm apart. Assuming the surfaces to be black (emissivity, ε=1), determine the rate of heat transfer between the plates per unit surface area assuming the gap between the plates is filled with atmospheric air

a) evacuated
b) filled with urethane insulation,
c) filled with superinsulation that has an apparent thermal conductivity of 0.00002 W/m. °C.



Assumptions There are no natural convection currents in the air between the plates. The surfaces are black and thus ε=1.
Properties The thermal conductivity at the average temperature of 250K is k=0.0219 W/m · °C for air, 0.026 W/m · °C for urethane insulation, and 0.00002 W/m · °C for the superinsulation.

Example 6

$$\dot{Q}_{cond} = kA\frac{\Delta T}{\Delta x} = (0.0219\ \text{W/m}\cdot{}^\circ\text{C})(1\ \text{m}^2)\frac{(300 - 200)^\circ\text{C}}{0.01\ \text{m}} = 219\ \text{W}$$

$$\dot{Q}_{rad} = \varepsilon\sigma A(T_1^4 - T_2^4) = (1)(5.67\ \ 108\ \text{W/m}^2\cdot\text{K4})(1\ \text{m}^2)[(300\ \text{K})^4 - (200\ \text{K})^4] = 368\ \text{W}$$

Therefore,

$$\dot{Q}_{total} = \dot{Q}_{cond} + \dot{Q}_{rad} = 219 + 368 = 587\ \text{W}$$

The heat transfer rate in reality will be higher because of the natural convection currents that are likely to occur in the air space between the plates.

(b) When the air space between the plates is evacuated, there will be no conduction or convection, and the only heat transfer between the plates will be by radiation. Therefore,

$$\dot{Q}_{total} = 0 + \dot{Q}_{rad} = 0 + 368 = 368\ \text{W}$$

Example 6

(c) An opaque solid material placed between two plates blocks direct radiation heat transfer between the plates. Also, the thermal conductivity of an insulating material accounts for the radiation heat transfer that may be occurring through the voids in the insulating material. The rate of heat transfer through the urethane insulation is the voids in the insulating material. The rate of heat transfer through the urethane insulation is

$$\dot{Q}_{total} = \dot{Q}_{cond} = kA\frac{\Delta T}{\Delta x} = (0.026\ \mathrm{W/m\cdot{}^\circ C})(1\ \mathrm{m}^2)\frac{(300 - 200)^\circ\mathrm{C}}{0.01\mathrm{m}} = 260\ \mathrm{W}$$

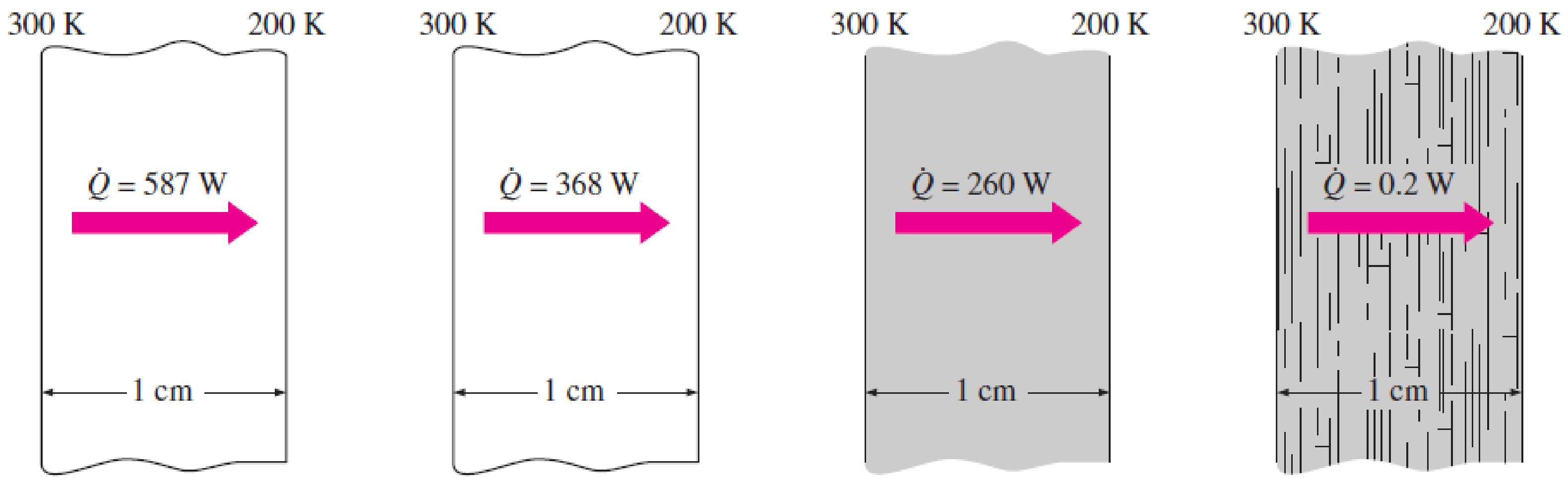
Note that heat transfer through the urethane material is less than the heat transfer through the air determined in (a), although the thermal conductivity of the insulation is higher than that of air. This is because the insulation blocks the radiation whereas air transmits it.

(d) The layers of the superinsulation prevent any direct radiation heat transfer between the plates. However, radiation heat transfer between the sheets of superinsulation does occur, and the apparent thermal conductivity of the superinsulation accounts for this effect. Therefore,

Example 6

$$\dot{Q}_{total} = kA\frac{\Delta T}{\Delta x} = (0.00002\ \text{W/m}\cdot{}^\circ\text{C})(1\ \text{m}^2)\frac{(300 - 200)^\circ\text{C}}{0.01\text{m}} = 0.2\ \text{W}$$

which is 1/1840 of the heat transfer through the vacuum.



(*a*) Air space — **Conduction, Radiation**

(*b*) Vacuum — **Radiation**

(*c*) Insulation — **Conduction**

(*d*) Superinsulation — **Conduction**