## Question 1


a) Consider a $5-\mathrm{m}$-high, $8-\mathrm{m}$-long, and $0.22-\mathrm{m}$-thick wall whose representative crosssection is as given in the figure. The thermal conductivities of various materials used, in $\mathrm{W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C}$, are $k_{A}=k_{F}=2, k_{B}=8, k_{C}=20, k_{D}=15$, and $k_{E}=35$. The left and right surfaces of the wall are maintained at uniform temperatures of $300^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$, respectively. Assuming heat transfer through the wall to be one-dimensional and the thermal contact resistance at the interfaces D-F and E-F is $0.00012 \mathrm{~m}^{2} .{ }^{\circ} \mathrm{C} / \mathrm{W}$., determine
i. the rate of heat transfer (for a $0.12-\mathrm{m}$-high $\times 1-\mathrm{m}$-deep section), Watts
ii. the rate of heat transfer through the wall, Watts
iii. the temperature at the point where the sections $B, D$, and $E$ meet, ${ }^{\circ} \mathrm{C}$ and
iv. the temperature drop across the section $F,{ }^{\circ} \mathrm{C}$.

## Solution

a)
i.


$$
\begin{aligned}
& \mathrm{R}_{1}=\mathrm{R}_{\mathrm{A}}=\left(\frac{\mathrm{L}}{\mathrm{kA}}\right)_{\mathrm{A}}=\frac{0.01}{(2)(0.12 \times 1)}=0.04^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{R}_{2}=\mathrm{R}_{4}=\mathrm{R}_{\mathrm{C}}=\left(\frac{\mathrm{L}}{\mathrm{kA}}\right)_{\mathrm{C}}=\frac{0.05}{(20)(0.04 \times 1)}=0.06^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{R}_{3}=\mathrm{R}_{\mathrm{B}}=\left(\frac{\mathrm{L}}{\mathrm{kA}}\right)_{\mathrm{B}}=\frac{0.05}{(8)(0.04 \times 1)}=0.16^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{R}_{5}=\mathrm{R}_{\mathrm{D}}=\left(\frac{\mathrm{L}}{\mathrm{kA}}\right)_{\mathrm{D}}=\frac{0.1}{(15)(0.06 \times 1)}=0.11^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{R}_{6}=\mathrm{R}_{\mathrm{E}}=\left(\frac{\mathrm{L}}{\mathrm{kA}}\right)_{\mathrm{D}}=\frac{0.1}{(35)(0.06 \times 1)}=0.05^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{R}_{7}=\mathrm{R}_{\mathrm{F}}=\left(\frac{\mathrm{L}}{\mathrm{kA}}\right)_{\mathrm{F}}=\frac{0.06}{(2)(0.12 \times 1)}=0.25^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{R}_{8}=\frac{0.00012}{0.12}=0.001^{\circ} \mathrm{C} / \mathrm{W} \\
& \frac{1}{\mathrm{R}_{\text {mid }, 1}}=\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{3}}+\frac{1}{\mathrm{R}_{4}}=\frac{1}{0.06}+\frac{1}{0.16}+\frac{1}{0.06} \Rightarrow \mathrm{R}_{\text {mid }, 1}=0.025^{\circ} \mathrm{C} / \mathrm{W} \\
& \frac{1}{\mathrm{R}_{\text {mid }, 2}}=\frac{1}{\mathrm{R}_{5}}+\frac{1}{\mathrm{R}_{6}}=\frac{1}{0.11}+\frac{1}{0.05} \Rightarrow \mathrm{R}_{\text {mid }, 2}=0.034^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{R}_{\text {total }}=\mathrm{R}_{1}+\mathrm{R}_{\text {mid }, 1}+\mathrm{R}_{\text {mid }, 2}+\mathrm{R}_{7}+\mathrm{R}_{8} \\
& =0.04+0.025+0.034+0.25+0.001=0.35^{\circ} \mathrm{C} / \mathrm{W} \\
& \mathrm{Q}_{2}=\frac{\mathrm{T}_{\infty 1}-\mathrm{T}_{\infty 2}}{\mathrm{R}_{\text {total }}}=\frac{300-100}{0.35}=571.43 \mathrm{~W}
\end{aligned}
$$

ii.

$$
\dot{Q}_{\text {total }}=\dot{Q} \frac{\text { height } \times \text { length }}{A}=(571.43) \frac{5 \times 8}{(0.12 \times 1)}=190476.67 \mathrm{~W}
$$

iii.

$$
\begin{aligned}
& \mathrm{R}_{\text {total }}=\mathrm{R}_{1}+\mathrm{R}_{\text {mid }, 1}=0.04+0.025=0.065^{\circ} \mathrm{C} / \mathrm{W} \\
& \dot{\mathrm{Q}}=\frac{\mathrm{T}_{1}-\mathrm{T}}{\mathrm{R}_{\text {total }}} \Rightarrow 571.43=\frac{300-\mathrm{T}}{0.065} \Rightarrow \mathrm{~T}=262.86^{\circ} \mathrm{C}
\end{aligned}
$$

iv.

$$
\dot{\mathrm{Q}}=\frac{\Delta \mathrm{T}}{\mathrm{R}_{\mathrm{F}}} \Rightarrow \Delta \mathrm{~T}=\dot{\mathrm{Q}} \mathrm{R}_{\mathrm{F}}=571 \times 0.25=142.75^{\circ} \mathrm{C}
$$

