## Example 1

## Defrosting ice from an automobile windshield

To defrost ice accumulated on the outer surface of an automobile windshield, warm air is blown over the inner surface of the windshield. Consider an automobile windshield ( $\mathrm{kw}=1.4 \mathrm{~W} / \mathrm{m} . \mathrm{K}$ ). With an overall height of 0.5 m and thickness of 5 mm . The outside air ( 1 atm ) ambient temperature is $-20^{\circ} \mathrm{C}$ and the average air flow velocity over the outer windshield surface is $80 \mathrm{~km} / \mathrm{h}$, while the ambient temperature inside the automobile is $25^{\circ} \mathrm{C}$. Determine the value of the convection heat transfer coefficient, for the warm air blowing over the inner surface of the windshield, necessary to cause the accumulated ice to begin melting. Assume the windshield surface can be treated as aa flat plate surface.

Assumptions: the outside air pressure is 1 atm and the critical Reynolds number is $\operatorname{Re}_{\mathrm{cr}}=5 \times 10^{5}$


The properties of air at the film temperature of $\mathrm{T}_{\mathrm{f}}=\frac{\left(-20^{\circ} \mathrm{C}+0^{\circ} \mathrm{C}\right)}{2}=-10^{\circ} \mathrm{C}$ are $\mathrm{k}=0.02288 \mathrm{~W} / \mathrm{m} . \mathrm{K}, \mathrm{v}=1.252 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}$ and $\operatorname{Pr}=0.7387$ (from table)

$$
\operatorname{Re}_{\mathrm{L}}=\frac{\mathrm{VL}}{v}=\frac{(80 / 3.60 \mathrm{~m} / \mathrm{s})(0.5 \mathrm{~m})}{1.252 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}}=8.875 \times 10^{5}
$$

Since $5 \times 10^{5}<\operatorname{Re}_{\mathrm{L}}<10^{7}$ the flow is a combined laminar and turbulent flow. Using the proper relation for the Nusselt number, the average heat transfer coefficient on the outer surface of the windshield is

$$
\begin{aligned}
& \mathrm{Nu}_{\mathrm{o}}=\frac{\mathrm{h}_{0} \mathrm{~L}}{\mathrm{k}}=\left(0.037 \operatorname{Re}_{\mathrm{L}}^{0.8}-871\right) \operatorname{Pr}^{\frac{1}{3}}=\left(0.037\left(8.875 \times 10^{5}\right)^{0.8}-871\right) 0.7387^{\frac{1}{3}}=1131 \\
& \mathrm{~h}_{0}=1131 \frac{\mathrm{k}}{\mathrm{~L}}=1131 \frac{0.02288 \mathrm{~W} / \mathrm{m} \cdot \mathrm{~K}}{0.5 \mathrm{~m}}=51.75 \mathrm{~W} / \mathrm{m}^{2} . \mathrm{K}
\end{aligned}
$$

From the energy balance the heat transfer through the windshield thickness can be written as;

$$
\frac{\mathrm{T}_{\infty, \mathrm{o}}-\mathrm{T}_{\mathrm{s}, \mathrm{o}}}{1 / \mathrm{h}_{\mathrm{o}}}=\frac{\mathrm{T}_{\mathrm{s}, \mathrm{o}}-\mathrm{T}_{\infty, \mathrm{i}}}{\mathrm{t} / \mathrm{k}_{\mathrm{w}}+1 / \mathrm{h}_{\mathrm{i}}}
$$

For the ice to begin melting, the outer surface temperature of the windshield $\left(\mathrm{T}_{\mathrm{s}, \mathrm{o}}\right)$ should be at least $0^{\circ} \mathrm{C}$. Then the convection heat transfer coefficient for the warm air blowing over the inner surface of the wind shield must be;

$$
\mathrm{h}_{\mathrm{i}}=\left(\frac{1}{\mathrm{~h}} \frac{\mathrm{~T}_{\mathrm{s}, \mathrm{o}}-\mathrm{T}_{\infty, \mathrm{i}}}{\mathrm{~T}_{\infty, 0}-\mathrm{T}_{\mathrm{s}, \mathrm{o}}}-\frac{\mathrm{t}}{\mathrm{k}_{\mathrm{w}}}\right)^{-1}=\left(\frac{1}{51.75 \mathrm{~W} / \mathrm{m}^{2} \cdot \mathrm{~K}} \frac{(0-25)^{\circ} \mathrm{C}}{(-20-0)^{\circ} \mathrm{C}}-\frac{0.005 \mathrm{~m}}{1.4 \mathrm{~W} / \mathrm{m} \cdot \mathrm{~K}}\right)^{-1}=48.6 \mathrm{~W} / \mathrm{m}^{2} \cdot \mathrm{~K}
$$

In practical situation the ambient temperature and the convective heat transfer coefficient outside the automobile vary with weather conditions and the automobile speed. Therefore, the convection heat transfer coefficient of the warm air necessary to melt the ice should be varied as well. This is done by adjusting the warm air flow rate and temperature.

## (ㅇ)UTM <br> Example 2

## Cooling of Plastic Sheets by Forced Air

The forming section of a plastics plant puts out a continuous sheet of plastic that is 1.2 m wide and 0.1 cm thick at a velocity of $9 \mathrm{~m} / \mathrm{min}$. The temperature of the plastic sheet is $95^{\circ} \mathrm{C}$ when it is exposed to the surrounding air, and a $0.6-\mathrm{m}$-long section of the plastic sheet is subjected to air flow at $25^{\circ} \mathrm{C}$ at a velocity of 3 $\mathrm{m} / \mathrm{s}$ on both sides along its surfaces normal to the direction of motion of the sheet. Determine
(a) the rate of heat transfer from the plastic sheet to air by forced convection and radiation
(b) the temperature of the plastic sheet at the end of the cooling section. Take the density, specific heat, and emissivity of the plastic sheet to be $\varrho=1200 \mathrm{~kg} / \mathrm{m}^{3}$, $C_{p}=1.7 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$, and $\varepsilon=0.9$.


## (0) UTM

Critical Reynolds number is $\operatorname{Re}_{\mathrm{cr}}=5 \times 10^{5}$. The local atmospheric pressure is 1 atm and the surrounding surfaces are at the temperature of the room air. The properties of air at the film temperature of $\mathrm{T}_{\mathrm{f}}=\left(\mathrm{T}_{\mathrm{s}}+\mathrm{T}_{\infty}\right) / 2=(95+25) / 2=60^{\circ} \mathrm{C}$ and 1 atm pressure are (given in table)
$\mathrm{k}=0.02808 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C}$
$\operatorname{Pr}=0.7202$
$v=1.896 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}$

We expect the temperature of the plastic sheet to drop somewhat as it flows through the $0.6-\mathrm{m}$ - long cooling section, but at this point we do not know the magnitude of that drop. Therefore, we assume the plastic sheet to be isothermal at $95^{\circ} \mathrm{C}$ to get started. We will repeat the calculations if necessary to account for the temperature drop of the plastic sheet. Noting that $\mathrm{L}=1.2 \mathrm{~m}$, the Reynolds number at the end of the air flow across the plastic sheet is

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$\operatorname{Re}_{\mathrm{L}}=\frac{\mathrm{VL}}{v}=\frac{(3 \mathrm{~m} / \mathrm{s})(1.2 \mathrm{~m})}{1.896 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}}=1.899 \times 10^{5}$
which is less than the critical Reynolds number. Thus, we have laminar flow over the entire sheet, and the Nusselt number is determined from the laminar flow relations for a flat plate to be
$\mathrm{Nu}=\frac{\mathrm{hL}}{\mathrm{k}}=\left(0.664 \operatorname{Re}_{\mathrm{L}}^{0.5}\right) \operatorname{Pr}^{\frac{1}{3}}=\left(0.664\left(1.899 \times 10^{5}\right)^{0.5}\right) 0.7202^{\frac{1}{3}}=259.3$
Then,
$\mathrm{h}=\frac{\mathrm{k}}{\mathrm{L}} \mathrm{Nu}=\frac{0.02808 \mathrm{~W} / \mathrm{m} .{ }^{\circ} \mathrm{C}}{1.2 \mathrm{~m}}(259.3)=6.07 \mathrm{~W} / \mathrm{m}^{2} .{ }^{\circ} \mathrm{C}$
$\mathrm{A}_{\mathrm{s}}=(0.6 \mathrm{~m})(1.2 \mathrm{~m})(2$ sides $)=1.44 \mathrm{~m}^{2}$
$\dot{\mathrm{Q}}_{\text {conv }}=\mathrm{hA}_{\mathrm{s}}\left(\mathrm{T}_{\mathrm{s}}-\mathrm{T}_{\infty}\right)=\left(6.07 \mathrm{~W} / \mathrm{m}^{2} .{ }^{\circ} \mathrm{C}\right)\left(1.44 \mathrm{~m}^{2}\right)(95-25)^{\circ} \mathrm{C}=612 \mathrm{~W}$
$\dot{\mathrm{Q}}_{\text {rad }}=\varepsilon \sigma \mathrm{A}_{\mathrm{s}}\left(\mathrm{T}_{\mathrm{s}}{ }^{4}-\mathrm{T}_{\text {surr }}{ }^{4}\right)=(0.9)\left(5.67 \times 10^{-8} \mathrm{~W} / \mathrm{m}^{2} . \mathrm{K}^{4}\right)\left(1.44 \mathrm{~m}^{2}\right)\left(368^{4}-298^{4}\right) \mathrm{K}=768 \mathrm{~W}$
Therefore, the rate of cooling of the plastic sheet by combined convection and radiation is
$\dot{\mathrm{Q}}_{\text {total }}=\dot{\mathrm{Q}}_{\text {conv }}+\dot{\mathrm{Q}}_{\mathrm{rad}}=612 \mathrm{~W}+768 \mathrm{~W}=1380 \mathrm{~W}$

## Example 3

Heat Loss from a Steam Pipe in Windy Air
A long $10-\mathrm{cm}$-diameter steam pipe whose external surface temperature is $110^{\circ} \mathrm{C}$ passes through some open area that is not protected against the winds. Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and $10^{\circ} \mathrm{C}$ and the wind is blowing across the pipe at a velocity of
 $8 \mathrm{~m} / \mathrm{s}$.
The properties of air at the film temperature of $\mathrm{T}_{\mathrm{f}}=\frac{\left(110^{\circ} \mathrm{C}+10^{\circ} \mathrm{C}\right)}{2}=60^{\circ} \mathrm{C}$ and 1 atm pressure are (from table)

$$
\begin{aligned}
& \mathrm{k}=0.02808 \mathrm{~W} / \mathrm{m} .{ }^{\circ} \mathrm{C} \\
& \mathrm{v}=1.896 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s} \\
& \operatorname{Pr}=0.7202 \\
& \mathrm{Re}_{\mathrm{L}}=\frac{\mathrm{VL}}{\mathrm{v}}=\frac{(8 \mathrm{~m} / \mathrm{s})(0.1 \mathrm{~m})}{1.896 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}}=4.219 \times 10^{4}
\end{aligned}
$$

## (C) UTM

The Nusselt number is determined from

$$
\begin{aligned}
& \mathrm{Nu}=\frac{\mathrm{hD}}{\mathrm{k}}=0.3+\frac{0.62 \operatorname{Re}^{\frac{1}{2}} \operatorname{Pr}^{\frac{1}{3}}}{\left[1+(0.4 / \operatorname{Pr})^{\frac{2}{3}}\right]^{\frac{1}{4}}}\left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}} \\
& =0.3+\frac{0.62\left(4.219 \times 10^{4}\right)^{\frac{1}{2}}(0.7202)^{\frac{1}{3}}}{\left[1+(0.4 / 0.7202)^{\frac{2}{3}}\right]^{\frac{1}{4}}}\left[1+\left(\frac{4.219 \times 10^{4}}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}=124
\end{aligned}
$$

Then,

$$
\mathrm{h}=\frac{\mathrm{k}}{\mathrm{D}} \mathrm{Nu}=\frac{0.02808 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C}}{0.1 \mathrm{~m}}(124)=34.8 \mathrm{~W} / \mathrm{m}^{2} .{ }^{\circ} \mathrm{C}
$$

Then the rate of heat transfer from the pipe per unit of its length becomes
$\mathrm{A}_{\mathrm{s}}=\mathrm{pL}=\pi \mathrm{DL}=\pi(0.1 \mathrm{~m})(1 \mathrm{~m})=0.314 \mathrm{~m}^{2}$
$\dot{\mathrm{Q}}_{\text {conv }}=\mathrm{hA}_{s}\left(\mathrm{~T}_{\mathrm{s}}-\mathrm{T}_{\infty}\right)=\left(34.8 \mathrm{~W} / \mathrm{m}^{2} .{ }^{\circ} \mathrm{C}\right)\left(0.314 \mathrm{~m}^{2}\right)(110-10)^{\circ} \mathrm{C}=1093 \mathrm{~W}$
The rate of heat loss from the entire pipe can be obtained by multiplying the
value above by the length of the pipe in $m$.

## Example 4

## Cooling of a Steel Ball by Forced Air

A 25-cm-diameter stainless steel ball ( $8055 \mathrm{~kg} / \mathrm{m}^{3}, C_{p}$ $480 \mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}$ ) is removed from the oven at a uniform temperature of $300^{\circ} \mathrm{C}$. The ball is then subjected to the flow of air at 1 atm pressure and $25^{\circ} \mathrm{C}$ with a velocity of $3 \mathrm{~m} / \mathrm{s}$. The surface temperature of the ball eventually
 drops to $200^{\circ} \mathrm{C}$. Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.

## Assumptions

- Radiation effects are negligible.
- The outer surface temperature of the ball is uniform at all times.
- The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of $(300+200) / 2=250^{\circ} \mathrm{C}$ in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

The dynamic viscosity of air at the average surface temperature is
$\mu_{\mathrm{s}}=\mu_{\circledast 250^{\circ} \mathrm{C}}=2.76 \times 10^{5} \mathrm{~kg} / \mathrm{m} \cdot \mathrm{s}$. The properties of air at the free - stream temperature of $25^{\circ} \mathrm{C}$ and 1 atm are

$$
\begin{gathered}
\mathrm{k}=0.02551 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C} \\
\mathrm{v}=1.849 \times 10^{-5} \mathrm{~kg} / \mathrm{m} \cdot \mathrm{~s} \\
v=1.562 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s} \\
\operatorname{Pr}=0.7296 \\
\operatorname{Re}_{\mathrm{L}}=\frac{\mathrm{VL}}{v}=\frac{(3 \mathrm{~m} / \mathrm{s})(0.25 \mathrm{~m})}{1.562 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}}=4.802 \times 10^{4}
\end{gathered}
$$

The Nusselt number is

$$
\begin{aligned}
& \mathrm{Nu}=\frac{\mathrm{hD}}{\mathrm{k}}=2+\left[0.4 \operatorname{Re}^{1 / 2}+0.06 \operatorname{Re}^{2 / 3}\right] \operatorname{Pr}^{0.4}\left(\frac{\mu_{\infty}}{\mu_{\mathrm{s}}}\right)^{1 / 4} \\
& =2+\left[0.4\left(4.802 \times 10^{4}\right)^{1 / 2}+0.06\left(4.802 \times 10^{4}\right)^{2 / 3}\right](0.7296)^{0.4}\left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{1 / 4}=135
\end{aligned}
$$

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Then the average convection heat transfer coefficient becomes
$\mathrm{h}=\frac{\mathrm{k}}{\mathrm{D}} \mathrm{Nu}=\frac{0.02551 \mathrm{~W} / \mathrm{m} .{ }^{\circ} \mathrm{C}}{0.25 \mathrm{~m}}(135)=13.8 \mathrm{~W} / \mathrm{m}^{2} .{ }^{\circ} \mathrm{C}$

In order to estimate the time of cooling of the ball from $300^{\circ} \mathrm{C}$ to $200^{\circ} \mathrm{C}$, we determine the average rate of heat transfer from Newton's law of cooling by using the average surface temperature. That is,

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{s}}=\pi \mathrm{D}_{2}=(0.25 \mathrm{~m})^{2}=0.1963 \mathrm{~m}^{2} \\
& \dot{\mathrm{Q}}_{\mathrm{ave}}=\mathrm{hA}_{\mathrm{s}}\left(\mathrm{~T}_{\mathrm{s}, \text { ave }}-\mathrm{T}_{\infty}\right)=\left(13.8 \mathrm{~W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right)\left(0.1963 \mathrm{~m}^{2}\right)(250-25)^{\circ} \mathrm{C}=610 \mathrm{~W}
\end{aligned}
$$

Next we determine the total heat transferred from the ball, which is simply the change in the energy of the ball as it cools from $300^{\circ} \mathrm{C}$ to $200^{\circ} \mathrm{C}$ :

$$
\begin{aligned}
& \mathrm{m}=\rho \mathrm{V}=\rho \frac{1}{6} \pi \mathrm{D}^{3}=\left(8055 \mathrm{~kg} / \mathrm{m}^{3}\right)(0.25 \mathrm{~m})^{3}=65.9 \mathrm{~kg} \\
& \mathrm{Q}_{\text {total }}=\mathrm{mC}_{\mathrm{p}}\left(\mathrm{~T}_{2}-\mathrm{T}_{1}\right)=(65.9 \mathrm{~kg})\left(480 \mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)(300-200)^{\circ} \mathrm{C}=3,163,000 \mathrm{~J}
\end{aligned}
$$

## (ㅈ)UTM

In this calculation, we assumed that the entire ball is at $200^{\circ} \mathrm{C}$, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$
\Delta t \approx \frac{\mathrm{Q}}{\dot{\mathrm{Q}}_{\text {ave }}} \approx \frac{3,163,000 \mathrm{~J}}{610 \mathrm{~J} / \mathrm{s}}=5185 \mathrm{~s} 1 \mathrm{~h} 26 \mathrm{~min}
$$

The time of cooling could also be determined more accurately using the transient temperature charts or relations introduced previously. But the simplifying assumptions we made above can be justified if all we need is a ballpark value. It will be naive to expect the time of cooling to be exactly 1 h 26 min, but, using our engineering judgment, it is realistic to expect the time of cooling to be somewhere between one and two hours.

## (0) UTM <br> Example 5

## Heating of water in a tube by steam

Water enters a 2.5 cm internal diameter thin cooper tube of a heat exchanger at $15{ }^{\circ} \mathrm{C}$ at a rate of $0.3 \mathrm{~kg} / \mathrm{s}$ and is heated by steam condensing outside at $120^{\circ} \mathrm{C}$. If the average heat transfer coefficient is $800 \mathrm{~W} / \mathrm{m}^{2} .{ }^{\circ} \mathrm{C}$, determine the length of the tube required in order to
 heat the water to $115{ }^{\circ} \mathrm{C}$.

The conduction resistance of a cooper tube is negligible so that the inner surface temperature of the tube is equal to the condensation temperature of steam. The specific heat of water at the bulk mean temperature of $(15+115) / 2=65{ }^{\circ} \mathrm{C}$ is $4187 \mathrm{~J} / \mathrm{kg} .{ }^{\circ} \mathrm{C}$. The heat of condensation of steam at $120^{\circ} \mathrm{C}$ is $2203 \mathrm{~kJ} / \mathrm{kg}$
$\dot{\mathrm{Q}}=\dot{\mathrm{mc}}_{\mathrm{p}}\left(\mathrm{T}_{\mathrm{e}}-\mathrm{T}_{\mathrm{i}}\right)=(0.3 \mathrm{~kg} / \mathrm{s})\left(4.187 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)(115-15)^{\circ} \mathrm{C}=125.6 \mathrm{~kW}$

The logarithmic mean temperature

$$
\begin{aligned}
& \Delta \mathrm{T}_{\mathrm{e}}=\mathrm{T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{e}}=120^{\circ} \mathrm{C}-115^{\circ} \mathrm{C}=5^{\circ} \mathrm{C} \\
& \Delta \mathrm{~T}_{\mathrm{i}}=\mathrm{T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{i}}=120^{\circ} \mathrm{C}-15^{\circ} \mathrm{C}=105^{\circ} \mathrm{C} \\
& \Delta \mathrm{~T}_{\ln }=\frac{\Delta \mathrm{T}_{\mathrm{e}}-\Delta \mathrm{T}_{\mathrm{i}}}{\ln \left(\Delta \mathrm{~T}_{\mathrm{e}} / \Delta \mathrm{T}_{\mathrm{i}}\right)}=\frac{5-105}{\ln (5 / 105)}=32.85^{\circ} \mathrm{C}
\end{aligned}
$$

The heat transfer surface area is

$$
\dot{\mathrm{Q}}_{\mathrm{ave}}=\mathrm{hA}_{\mathrm{s}} \Delta \mathrm{~T}_{\mathrm{ln}} \rightarrow \mathrm{~A}_{\mathrm{s}}=\frac{\dot{\mathrm{Q}}_{\mathrm{ave}}}{\mathrm{~h} \Delta \mathrm{~T}_{\mathrm{ln}}}=\frac{125.6 \mathrm{~kW}}{0.8 \mathrm{~kW} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\left(32.85^{\circ} \mathrm{C}\right)}=4.78 \mathrm{~m}^{2}
$$

Then required tube length becomes

$$
\mathrm{A}_{\mathrm{s}}=\pi \mathrm{DL} \rightarrow \mathrm{~L}=\frac{\mathrm{A}_{\mathrm{s}}}{\pi \mathrm{D}}=\frac{4.78 \mathrm{~m}^{2}}{\pi(0.025 \mathrm{~m})}=61 \mathrm{~m}
$$

The bulk mean temperature of water during this heating process is $65^{\circ} \mathrm{C}$ and thus the arithmetic mean temperature difference is $\Delta \mathrm{T}_{\mathrm{am}}=120-65=55^{\circ} \mathrm{C}$. Using $\Delta \mathrm{T}_{\mathrm{am}}$ instead of $\Delta \mathrm{T}_{\text {ln }}$ would give $\mathrm{L}=36 \mathrm{~m}$ which is grossly in error. This shows the importance of using the logarithmic mean temperature in calculation.

## (0) UTM <br> Example 6

## Flow of Oil in a Pipeline through a Lake

Consider the flow of oil at $20^{\circ} \mathrm{C}$ in a 30 - cm -diameter pipeline at an average velocity of $2 \mathrm{~m} / \mathrm{s}$. A 200-m-long section of the pipeline passes through icy waters of a lake at $0^{\circ} \mathrm{C}$. Measurements indicate that the surface temperature of the pipe is very nearly $0^{\circ} \mathrm{C}$. Disregarding the thermal resistance of the pipe material, determine
a) the temperature of the oil when the pipe
b) leaves the lake
c) the rate of heat transfer from the oil

d) the pumping power required to overcome the pressure losses and to maintain the flow of the oil in the pipe

Assumptions The thermal resistance of the pipe is negligible. The inner surfaces of the pipeline are smooth. The flow is hydrodynamically developed when the pipeline reaches the lake.

## (0) UTM

We do not know the exit temperature of the oil, and thus we cannot determine the bulk mean temperature, which is the temperature at which the properties of oil are to be evaluated. The mean temperature of the oil at the inlet is $20^{\circ} \mathrm{C}$, and we expect this temperature to drop somewhat as a result of heat loss to the icy waters of the lake. We evaluate the properties of the oil at the inlet temperature, but we will repeat the calculations, if necessary, using properties at the evaluated bulk mean temperature. At $20^{\circ} \mathrm{C}$ we read

$$
\begin{gathered}
\rho=888 \mathrm{~kg} / \mathrm{m}^{3} \\
\mathrm{k}=0.145 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C} \\
v=901 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s} \\
\operatorname{Pr}=10400 \\
\mathrm{C}_{\mathrm{p}}=1880 \mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C} \\
\operatorname{Re}_{\mathrm{L}}=\frac{\mathrm{V}_{\mathrm{m}} \mathrm{D}_{\mathrm{h}}}{v}=\frac{(2 \mathrm{~m} / \mathrm{s})(0.3 \mathrm{~m})}{901 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}}=666
\end{gathered}
$$

Note that this Nusselt number is considerably higher than the fully developed value of 3.66. Then,

$$
\begin{aligned}
& \mathrm{h}=\frac{\mathrm{k}}{\mathrm{D}} \mathrm{Nu}=\frac{0.145 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C}}{0.3 \mathrm{~m}}(37.3)=18.0 \mathrm{~W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C} \\
& \text { Also, } \\
& \mathrm{A}_{\mathrm{s}}=\mathrm{pL}=\pi \mathrm{DL}=\pi(0.3 \mathrm{~m})(200 \mathrm{~m})=188.5 \mathrm{~m}^{2} \\
& \dot{\mathrm{~m}}=\rho \mathrm{A}_{\mathrm{c}} \mathrm{~V}_{\mathrm{m}}=\left(888 \mathrm{~kg} / \mathrm{m}^{3}\right)\left[\frac{1}{4} \pi(0.3 \mathrm{~m})^{2}\right](2 \mathrm{~m} / \mathrm{s})=125.5 \mathrm{~kg} / \mathrm{s}
\end{aligned}
$$

Next we determine the exit temperature of oil from

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{e}}=\mathrm{T}_{\mathrm{s}}-\left(\mathrm{T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{i}}\right) \exp \left(-\mathrm{hA}_{\mathrm{s}} / \dot{\mathrm{mc}}_{\mathrm{p}}\right) \\
& =0^{\circ} \mathrm{C}-\left[(0-20)^{\circ} \mathrm{C}\right] \exp \left[-\frac{\left(18.0 \mathrm{~W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right)\left(188.5 \mathrm{~m}^{2}\right)}{(125.5 \mathrm{~kg} / \mathrm{s})\left(1880 \mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)}\right]=19.71^{\circ} \mathrm{C}
\end{aligned}
$$

Thus, the mean temperature of oil drops by a mere $0.29^{\circ} \mathrm{C}$ as it crosses the lake. This makes the bulk mean oil temperature $19.86^{\circ} \mathrm{C}$, which is practically identical to the inlet temperature of $20^{\circ} \mathrm{C}$. Therefore, we do not need to reevaluate the properties.

The logarithmic mean temperature difference and the rate of heat loss from the oil are;

$$
\begin{aligned}
& \Delta \mathrm{T}_{\mathrm{ln}}=\frac{\mathrm{T}_{\mathrm{i}}-\mathrm{T}_{\mathrm{e}}}{\ln \left(\frac{\mathrm{~T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{e}}}{\mathrm{~T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{i}}}\right)}=\frac{20-19.71}{\ln \left(\frac{0-19.71}{0-20}\right)}=-19.85^{\circ} \mathrm{C} \\
& \dot{\mathrm{Q}}_{\text {ave }}=\mathrm{hA}_{\mathrm{s}} \Delta \mathrm{~T}_{\mathrm{ln}}=\left(18.0 \mathrm{~kW} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right)\left(188.5 \mathrm{~m}^{2}\right)\left(-19.85^{\circ} \mathrm{C}\right)=-6.74 \times 10^{4}
\end{aligned}
$$

Therefore, the oil will lose heat at a rate of 67.4 kW as it flows through the pipe in the icy waters of the lake. Note that $\Delta T_{\text {ln }}$ is identical to the arithmetic mean temperature in this case, since $\Delta T_{i} \approx \Delta T_{e}$.

The laminar flow of oil is hydrodynamically developed. Therefore, the friction factor can be determined from

$$
\mathrm{f}=\frac{64}{\mathrm{Re}}=\frac{64}{666}=0.0961
$$

Then the pressure drop in the pipe and the required pumping power become

$$
\begin{aligned}
& \Delta \mathrm{P}=\mathrm{f} \frac{\mathrm{~L}}{\mathrm{D}} \frac{\rho \mathrm{~V}_{\mathrm{m}}^{2}}{2}=0.0961 \frac{200 \mathrm{~m}}{0.3 \mathrm{~m}} \frac{\left(888 \mathrm{~kg} / \mathrm{m}^{3}\right)(2 \mathrm{~m} / \mathrm{s})^{2}}{2}=1.14 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2} \\
& \dot{\mathrm{~W}}_{\text {pump }}=\frac{\dot{\mathrm{m}} \Delta \mathrm{P}}{\rho}=\frac{(125.5 \mathrm{~kg} / \mathrm{s})\left(1.14 \times 10^{5} \mathrm{~N} / \mathrm{m}^{2}\right)}{888 \mathrm{~kg} / \mathrm{m}^{3}}=16.1 \mathrm{~kW}
\end{aligned}
$$

We will need a 16.1-kW pump just to overcome the friction in the pipe as the oil flows in the 200-m-long pipe through the lake. If the pipe is long, a more optimal design to reduce pump work would be to increase the pipe diameter D, since pump work is inversely proportional to D5. Optimal pipe sizing requires an economic life-cycle cost analysis

## Example 7

## Heating of Water by Resistance Heaters in a Tube

Water is to be heated from $15^{\circ} \mathrm{C}$ to $65^{\circ} \mathrm{C}$ as it flows through a 3 -cm-internaldiameter $5-\mathrm{m}$-long tube. The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate
 of $10 \mathrm{~L} / \mathrm{min}$, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at the exit.

The properties of water at the bulk mean temperature of $T_{b}=\left(T_{i}+T_{e}\right) / 2=(15+$ $65) / 2=40^{\circ} \mathrm{C}$.

$$
\begin{aligned}
& \rho=992.1 \mathrm{~kg} / \mathrm{m}^{3} \\
& \mathrm{k}=0.631 \mathrm{~W} / \mathrm{m} .{ }^{\circ} \mathrm{C} \\
& v=\mu / \rho=0.658 \times 10^{6} \mathrm{~m}^{2} / \mathrm{s} \\
& \operatorname{Pr}=4.32 \\
& \mathrm{C}_{\mathrm{p}}=4179 \mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}
\end{aligned}
$$

The cross sectional and heat transfer surface areas are

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{c}}=\frac{1}{4} \pi \mathrm{D}^{2}=\frac{1}{4} \pi(0.03 \mathrm{~m})^{2}=7.069 \times 10^{4} \mathrm{~m}^{2} \\
& \mathrm{~A}_{\mathrm{s}}=\mathrm{pL}=\pi \mathrm{DL}=\pi(0.03 \mathrm{~m})(5 \mathrm{~m})=0.471 \mathrm{~m}^{2}
\end{aligned}
$$

The volume flow rate of water is given as $\dot{V}=10 \mathrm{~L} / \mathrm{min}=0.01 \mathrm{~m}^{3} / \mathrm{min}$. Then the mass flow rate becomes

$$
\dot{\mathrm{m}}=\rho \dot{\mathrm{V}}=\left(992.1 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(0.01 \mathrm{~m}^{3} / \mathrm{min}\right)=9.921 \mathrm{~kg} / \mathrm{min}=0.1654 \mathrm{~kg} / \mathrm{s}
$$

## (0) UTM

To heat the water at this mass flow rate from $15^{\circ} \mathrm{C}$ to $65^{\circ} \mathrm{C}$, heat must be supplied to the water at a rate of
$\dot{\mathrm{Q}}=\dot{\mathrm{m}} \mathrm{C}_{\mathrm{p}}\left(\mathrm{T}_{\mathrm{e}}-\mathrm{T}_{\mathrm{i}}\right)=(0.1654 \mathrm{~kg} / \mathrm{s})\left(4.179 \mathrm{~kJ} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)(65-15)^{\circ} \mathrm{C}=34.6 \mathrm{~kJ} / \mathrm{s}=34.6 \mathrm{~kW}$

All of this energy must come from the resistance heater. Therefore, the power rating of the heater must be 34.6 kW .
The surface temperature Ts of the tube at any location can be determined from
$\dot{\mathrm{q}}_{\mathrm{s}}=\mathrm{h}\left(\mathrm{T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{m}}\right) \rightarrow \mathrm{T}_{\mathrm{s}}=\mathrm{T}_{\mathrm{m}}+\frac{\dot{\mathrm{q}}_{\mathrm{s}}}{\mathrm{h}}$
where $h$ is the heat transfer coefficient and Tm is the mean temperature of the fluid at that location. The surface heat flux is constant in this case, and its value can be determined from
$\dot{\mathrm{q}}_{\mathrm{s}}=\frac{\dot{\mathrm{Q}}}{\mathrm{A}_{\mathrm{c}}}=\frac{34.6 \mathrm{~kW}}{0.471 \mathrm{~m}^{2}}=73.46 \mathrm{~kW} / \mathrm{m}^{2}$

## (0) UTM

To determine the heat transfer coefficient, we first need to find the mean velocity of water and the Reynolds number :
$\mathrm{V}_{\mathrm{m}}=\frac{\dot{\mathrm{V}}}{\mathrm{A}_{\mathrm{c}}}=\frac{0.010 \mathrm{~m}^{3} / \mathrm{min}}{7.069 \times 10^{-4} \mathrm{~m}^{2}}=14.15 \mathrm{~m} / \mathrm{min}=0.236 \mathrm{~m} / \mathrm{s}$
$\operatorname{Re}=\frac{\mathrm{V}_{\mathrm{m}} \mathrm{D}}{v}=\frac{(0.236 \mathrm{~m} / \mathrm{s})(0.03 \mathrm{~m})}{0.658 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}}=10,760$
which is greater than 10,000 . Therefore, the flow is turbulent and the entry length is roughly
$\mathrm{L}_{\mathrm{h}} \approx \mathrm{L}_{\mathrm{t}} \approx 10 \mathrm{D}=10 \times 0.03=0.3 \mathrm{~m}$
which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe and determine the Nusselt number from
$\mathrm{Nu}=\frac{\mathrm{hD}}{\mathrm{k}}=0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{0.4}=0.023(10,760)^{0.8}(4.34)^{0.4}=69.5$

Then;

$$
\mathrm{h}=\frac{\mathrm{k}}{\mathrm{D}} \mathrm{Nu}=\frac{0.631 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C}}{0.03 \mathrm{~m}}(69.5)=1462 \mathrm{~W} / \mathrm{m}^{2}{ }^{\circ} \mathrm{C}
$$

and the surface temperature of the pipe at the exit becomes

$$
\mathrm{T}_{\mathrm{s}}=\mathrm{T}_{\mathrm{m}}+\frac{\dot{\mathrm{q}}_{\mathrm{s}}}{\mathrm{~h}}=65^{\circ} \mathrm{C}+\frac{73,460 \mathrm{~W} / \mathrm{m}^{2}}{1462 \mathrm{~W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}}=115^{\circ} \mathrm{C}
$$

## Discussion

Note that the inner surface temperature of the pipe will be $50^{\circ} \mathrm{C}$ higher than the mean water temperature at the pipe exit. This temperature difference of $50^{\circ} \mathrm{C}$ between the water and the surface will remain constant throughout the fully developed flow region.

## (ㅇ)UTM <br> Example 8

## Heat Loss from the Ducts of a Heating System

Hot air at atmospheric pressure and $80^{\circ} \mathrm{C}$ enters an $8-$ m -long uninsulated square duct of cross section 0.2 m 0.2 m that passes through the attic of a house at a rate of $0.15 \mathrm{~m}^{3} / \mathrm{s}$. The duct is observed to be nearly isothermal at $60^{\circ} \mathrm{C}$. Determine the exit temperature of the air and the rate of heat loss from the duct to the
 attic space.

We do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk mean temperature of air, which is the temperature at which the properties are to be determined. The temperature of air at the inlet is $80^{\circ} \mathrm{C}$ and we expect this temperature to drop somewhat as a result of heat loss through the duct whose surface is at $60^{\circ} \mathrm{C}$. At $80^{\circ} \mathrm{C}$ and 1 atm we read;

$$
\begin{array}{ll}
\rho=0.99994 \mathrm{~kg} / \mathrm{m}^{3} & \mathrm{k}=0.02953 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C} \\
v=\mu / \rho=2.097 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s} & \operatorname{Pr}=0.7154 \\
\mathrm{c}_{\mathrm{p}}=1008 \mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C} &
\end{array}
$$

The characteristic length (which is the hydraulic diameter), the mean velocity, and the Reynolds number in this case are
$\mathrm{D}_{\mathrm{h}}=\frac{4 \mathrm{~A}_{\mathrm{c}}}{\mathrm{p}}=\frac{4 \mathrm{a}^{2}}{4 \mathrm{a}}=\mathrm{a}=0.2 \mathrm{~m}$
$\mathrm{V}_{\mathrm{m}}=\frac{\dot{\mathrm{V}}}{\mathrm{A}_{\mathrm{c}}}=\frac{0.15 \mathrm{~m}^{3} / \mathrm{min}}{(0.2 \mathrm{~m})^{2}}=3.75 \mathrm{~m} / \mathrm{s}$
$\operatorname{Re}=\frac{\mathrm{V}_{\mathrm{m}} \mathrm{D}}{v}=\frac{(3.75 \mathrm{~m} / \mathrm{s})(0.2 \mathrm{~m})}{2.097 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}}=35,765$
which is greater than 10,000 . Therefore, the flow is turbulent and the entry length is roughly
$\mathrm{L}_{\mathrm{h}} \approx \mathrm{L}_{\mathrm{t}} \approx 10 \mathrm{D}=10 \times 0.2 \mathrm{~m}=2 \mathrm{~m}$
which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct and determine the Nusselt number from
$\mathrm{Nu}=\frac{\mathrm{hD}}{\mathrm{h}} \mathrm{k}=0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{0.3}=0.023(35,765)^{0.8}(0.7154)^{0.3}=91.4$

Then, $\mathrm{h}=\frac{\mathrm{k}}{\mathrm{D}_{\mathrm{h}}} \mathrm{Nu}=\frac{0.02953 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C}}{0.2 \mathrm{~m}}(91.4)=13.5 \mathrm{~W} / \mathrm{m}^{2} .{ }^{\circ} \mathrm{C}$
Also,

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{s}}=\mathrm{pL}=4 \mathrm{aL}=4 \times(0.2 \mathrm{~m})(8 \mathrm{~m})=6.4 \mathrm{~m}^{2} \\
& \dot{\mathrm{~m}}=\rho \dot{\mathrm{V}}=\left(1.009 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(0.15 \mathrm{~m}^{3} / \mathrm{s}\right)=0.151 \mathrm{~kg} / \mathrm{s}
\end{aligned}
$$

Next we determine the exit temperature of oil from

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{e}}=\mathrm{T}_{\mathrm{s}}-\left(\mathrm{T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{i}}\right) \exp \left(-\mathrm{hA}_{\mathrm{s}} / \dot{\mathrm{mc}}_{\mathrm{p}}\right) \\
& =60^{\circ} \mathrm{C}-\left[(60-80)^{\circ} \mathrm{C}\right] \exp \left[-\frac{\left(13.5 \mathrm{~W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right)\left(6.4 \mathrm{~m}^{2}\right)}{(0.151 \mathrm{~kg} / \mathrm{s})\left(1008 \mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)}\right]=71.3^{\circ} \mathrm{C}
\end{aligned}
$$

The logarithmic mean temperature difference and the rate of heat loss from the air become

$$
\begin{aligned}
\Delta \mathrm{T}_{\mathrm{ln}}= & \frac{\mathrm{T}_{\mathrm{i}}-\mathrm{T}_{\mathrm{e}}}{\ln \frac{\mathrm{~T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{e}}}{\mathrm{~T}_{\mathrm{s}}-\mathrm{T}_{\mathrm{i}}}}=\frac{80-71.3_{\mathrm{e}}}{\ln \frac{60-71.3}{60-80}}=-15.2^{\circ} \mathrm{C} \\
& \quad \dot{\mathrm{Q}}=\mathrm{hA}_{\mathrm{s}} \Delta \mathrm{~T}_{\mathrm{ln}}=\left(13.5 \mathrm{~W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right)\left(6.4 \mathrm{~m}^{2}\right)\left(-15.2^{\circ} \mathrm{C}\right)=-1313 \mathrm{~W}
\end{aligned}
$$

Therefore, air will lose heat at a rate of 1313 W as it flows through the duct in the attic.

## Discussion

The average fluid temperature is $(80+71.3) / 2=75.7^{\circ} \mathrm{C}$, which is sufficiently close to $80^{\circ} \mathrm{C}$ at which we evaluated the properties of air. Therefore, it is not necessary to re-evaluate the properties at this temperature and to repeat the calculations.

