Heat Loss from Hot Water Pipes

A 6-m-long section of an 8-cm-diameter horizontal hot water pipe passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.



The properties of air at the film temperature of $T_f = (T_s + T_{\infty})/2 = (70 + 20)/2 = 45^{\circ}C$ and 1 atm are k = 0.02699 W/m °C Pr = 0.7241

 $v = 1.749 \times 10^{-5} \text{ m}^2/\text{s}$

$$\beta = \frac{1}{T_f} = \frac{1}{318K}$$





The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.08$ m. Then the Rayleigh number becomes

$$Ra_{D} = \frac{g\beta(T_{s} - T_{\infty})D^{3}}{v^{2}}Pr = \frac{9.81 \times (1/318)(70 - 20) \times 0.08^{3}}{(1.749 \times 10^{-5})^{2}}(0.7241) = 1.869 \times 10^{6}$$

The natural convection Nusselt number

$$Nu = \left\{ 0.6 + \frac{0.387 \text{ Ra}_{D}^{1/6}}{\left[1 + (0.559/\text{Pr})^{9/16}\right]^{8/27}} \right\}^{2} = \left\{ 0.6 + \frac{0.387(1869 \times 10^{6})^{1/6}}{\left[1 + (0.559/0.7241)^{9/16}\right]^{8/27}} \right\}^{2} = 17.40$$

Then,

$$h = \frac{k}{D} Nu = \frac{0.02699 \text{ W/m} \ ^{\circ}\text{C}}{0.08 \text{ m}} (17.40) = 5.869 \text{ W/m} \ ^{\circ}\text{C}$$
$$A_{s} = \pi DL = \pi (0.08 \text{ m})(6 \text{ m}) = 1.508 \text{ m}^{2}$$
and
$$\dot{Q} = hA_{s} (T_{s} - T_{\infty}) = (5.869 \text{ W/m}^{2} \cdot ^{\circ}\text{C})(1.508 \text{ m}^{2})(70 - 20)^{\circ}\text{C} = 443 \text{ W}$$

Therefore, the pipe will lose heat to the air in the room at a rate of 443 W by natural convection.

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Cooling of a Plate in Different Orientations

Consider a 0.6-m x 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated. Determine the rate of heat transfer from the plate by natural convection if the plate is (a) vertical, (b) horizontal with hot surface facing up, and (c) horizontal with hot surface facing down.

The properties of air at the film temperature of $T_f = (T_s + T_{\infty})/2 = (90 + 30)/2 = 60^{\circ}C$ and 1 atm are k = 0.02808 W/m °C

Pr = 0.7202



$$\beta = \frac{1}{T_{\rm f}} = \frac{1}{333K}$$





(c) Hot surface facing down





$$Ra_{D} = \frac{g\beta(T_{s} - T_{\infty})L^{3}}{v^{2}}Pr = \frac{9.81 \times (1/333)(90 - 30) \times 0.6^{3}}{(1.896 \times 10^{-5})^{2}}(0.722) = 7.656 \times 10^{8}$$

The natural convection Nusselt number

$$Nu = \left\{ 0.825 + \frac{0.387 \text{ Ra}_{D}^{1/6}}{\left[1 + (0.492/\text{Pr})^{9/16}\right]^{8/27}} \right\}^{2} = \left\{ 0.825 + \frac{0.387(7.656 \times 10^{8})^{1/6}}{\left[1 + (0.492/0.7202)^{9/16}\right]^{8/27}} \right\}^{2} = 113.4$$

Then,

$$h = \frac{k}{D} Nu = \frac{0.02808 \text{ W/m} \ ^{\circ}\text{C}}{0.6 \text{ m}} (113.4) = 5.306 \text{ W/m} \ ^{\circ}\text{C}$$
$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$
and

$$\dot{Q} = hA_s(T_s - T_{\infty}) = (5.306 \text{ W/m}^2 \cdot {}^{\circ}\text{C})(0.6 \text{ m}^2)(90 - 30){}^{\circ}\text{C} = 115 \text{ W}$$



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(b) Horizontal with hot surface facing up. The characteristic length and the Rayleigh number in this case are

$$\begin{split} & L_{c} = \frac{A_{s}}{p} = \frac{L^{2}}{4L} = \frac{L}{4} = \frac{0.6}{4} = 0.15m\\ & Ra_{L} = \frac{g\beta(T_{s} - T_{\infty})L^{3}}{v^{2}}Pr = \frac{9.81 \times (1/333) \times (90 - 30) \times 0.15^{3}}{(1.896 \times 10^{-5})^{2}}(0.7202) = 1.196 \times 10^{7}\\ & The natural convection Nusselt number\\ & Nu = 0.54 Ra_{L}^{1/4} = 0.54(1.196 \times 10^{7})^{1/4} = 31.76\\ & Then, \end{split}$$

$$h = \frac{k}{L_c} Nu = \frac{0.0280 \text{ W/m} \ ^{\circ}\text{C}}{0.15 \text{ m}} (31.76) = 5.946 \text{ W/m} \ ^{\circ}\text{C}$$
$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$
and

$$\dot{Q} = hA_s(T_s - T_{\infty}) = (5.946 \text{ W/m}^2 \cdot {}^{\circ}\text{C})(0.36 \text{ m}^2)(90 - 30){}^{\circ}\text{C} = 128 \text{ W}$$



(c) Horizontal with hot surface facing down. The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (b).

Nu = 0.27 Ra_L^{1/4} = 0.27(1.196 × 10⁷)^{1/4} = 15.86
h =
$$\frac{k}{L_c}$$
 Nu = $\frac{0.02808 \text{ W/m} \ ^{\circ}\text{C}}{0.15 \text{ m}}$ (15.86) = 2.973 W/m $\ ^{\circ}\text{C}$
and
 $\dot{Q} = hA_s(T_s - T_{\infty}) = (2.973 \text{ W/m}^2 \cdot ^{\circ}\text{C})(0.36 \text{ m}^2)(90 - 30)^{\circ}\text{C} = 64.2 \text{ W}$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is "trapped" under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer.



Optimum Fin Spacing of a Heat Sink

A 12-cm-wide and 18-cm-high vertical hot surface in 30°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.

The properties of air at the film temperature of $T_f = (T_s + T_{\infty})/2 = (80 + 30)/2 = 55^{\circ}C$ and 1 atm are k = 0.02772 W/m °C Pr = 0.7215

$$v = 1.846 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\beta = \frac{1}{T_{_f}} = \frac{1}{328K}$$





$$Ra_{D} = \frac{g\beta(T_{s} - T_{\infty})L^{3}}{v^{2}}Pr = \frac{9.81 \times (1/328)(80 - 30) \times 0.18^{3}}{(1.846 \times 10^{-5})^{2}}(0.7215) = 1.846 \times 10^{7}$$

The optimum fin spacing is

$$S_{opt} = 2.714 \frac{L}{Ra_{L}^{0.25}} = 2.714 \frac{0.8 \text{ m}}{(1.846 \times 10^7)^{0.25}} = 7.45 \times 10^{-3} \text{ m} = 7.45 \text{ mm}$$

which is about seven times the thickness of the fins. Therefore, the assumption of negligible fin thickness in this case is acceptable. The number of fins and the heat transfer coefficient for this optimum fin spacing case are,

$$n = \frac{W}{S+t} = \frac{0.12 \text{ m}}{(0.00745 \ 0.0001) \text{ m}} \approx 15 \text{ fins}$$

The convection coefficient for this optimum in spacing case is,

$$h = Nu_{opt} \frac{k}{S_{opt}} = 1.307 \frac{0.02772 \text{ W/m} \ ^{\text{o}}\text{C}}{0.00745 \text{ m}} = 0.2012 \text{W/m} \ ^{\text{o}}\text{C}$$

Then the rate of natural convection heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_{\infty}) = h(2nLH)(T_s - T_{\infty})$$

= $(0.2012 W/m^2 \cdot {}^{\circ}C)[2 \times 15(0.18 m)(0.024 m)](80 - 30){}^{\circ}C = 1.30 W$







Heat Loss through a Double-Pane Window

The vertical 0.8-m-high, 2-m-wide double-pane window consists of two sheets of glass separated by a 2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 12°C and 2°C, determine the rate of heat transfer through the window.

The properties of air at the film temperature of $T_f = (T_s + T_{\infty})/2 = (12 + 2)/2 = 7^{\circ}C$ and 1 atm are k = 0.02416 W/m °C

Pr = 0.7344

$$v = 1.399 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\beta = \frac{1}{T_{\rm f}} = \frac{1}{280K}$$





$$Ra_{L} = \frac{g\beta(T_{s} - T_{\infty})L^{3}}{v^{2}}Pr = \frac{9.81 \times (1/280) \times (12 - 2) \times 0.02^{3}}{(1.399 \times 10^{-5})^{2}}(0.7344) = 1.051 \times 10^{4}$$

The aspect ratio of the geometry is H/L = 0.8/0.02 = 40. Then the Nusselt number in this case

Nu = 0.42 Ra_L^{1/4} Pr ^{0.012}
$$\left(\frac{H}{L}\right)^{-0.3} = 0.42 \times (1.051 \times 10^4)^{1/4} Pr ^{0.012} \left(\frac{0.8}{0.02}\right)^{-0.3} = 1.401$$

Then

Then,

$$A_s = H \times W = (0.8 \text{ m})(2 \text{ m}) = 1.6 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_1 - T_2) = kNuAs \frac{(T_1 - T_2)}{L}$$

= (0.02416 W/m² · °C)(1.401 m²)(1.6 m²) $\frac{(12 - 2)^{\circ}C}{0.02m} = 27.1 W$



Heat Transfer through a Spherical Enclosure

The two concentric spheres of diameters $D_i = 20$ cm and $D_o = 30$ cm shown are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_i = 320$ K and $T_o = 280$ K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.



The properties of air at the film temperature of $T_{ave} = (T_i + T_o)/2 = (320 + 280)/2 = 300^{\circ}K = 270^{\circ}C$ and 1 atm are k = 0.02566 W/m °C

Pr = 0.729

$$v = 1.580 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\beta = \frac{1}{T_{ave}} = \frac{1}{300K}$$



We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

 $L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05 m$

The Rayleigh number is

 $Ra_{L} = \frac{g\beta(T_{i} - T_{o})L^{3}}{v^{2}}Pr = \frac{9.81 \times (1/300) \times (320 - 280) \times 0.05^{3}}{(1.58 \times 10^{-5})^{2}}(0.729) = 4.776 \times 10^{5}$

The effective thermal conductivity is

$$F_{sph} = \frac{L_{c}}{(D_{i}D_{o})^{4}(D_{i}^{-7/5} + D_{o}^{-7/5})^{5}} = \frac{0.05}{(0.2 \times 0.3)^{4}(0.2^{-7/5} + 0.3^{-7/5})^{5}} = 0.005229$$

Then,

$$k = 0.74 k \left(\frac{Pr}{0.861 + Pr}\right)^{1/4} \left(F_{sph} Ra_{L}\right)^{1/4}$$
$$= 0.74 (0.02566 \text{ W/m}^{\circ}\text{C}) \left(\frac{0.729}{0.861 + 0.729}\right)^{1/4} (0.005229 \times 4.776 \times 10^{5})^{1/4} = 0.1104 \text{ W/m} ^{\circ}\text{C}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{eff} \pi \frac{D_i D_o}{L_c} (T_i - T_o) = (0.1104) \pi \frac{0.2 \times 0.3}{0.05} (320 - 280)^\circ C = 16.7 \text{ W}$$



Heating Water in a Tube by Solar Energy

A solar collector consists of a horizontal aluminum tube having an outer diameter of 0.0508 enclosed in a concentric thin glass tube of 0.1016-diameter. Water is heated as it flows through the tube, and the annular space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 8.792 Watt, and the temperature of the ambient air outside is 21.11°C. Disregarding any heat loss by radiation, determine the temperature of the aluminum tube when steady operation is established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).





The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 43°C, and use properties at an anticipated average temperature of (21.11 + 43.33)/2 = 32.22°C (Table A - 15E), k = 0.02604 W/m °K

Pr = 0.729

$$v = 1.6289 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\beta = \frac{1}{T_{ave}} = \frac{1}{(273 + 32.22)K}$$

We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{tube-glass} = \dot{Q}_{glass-ambient} = \dot{Q}_{solar gain} = 8.792W \text{ (per m of tube)}$$

The heat transfer surface area of the glass cover is $A_o = A_{glass} (\pi D_o L) = \pi (0.1016/0.3048 \text{ m})(0.0254 \text{ m}) = 0.0266 \text{ m}^2 \text{ (per m of tube)}$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial - and - error approach. Assuming the glass cover temperature to be 43.33°C, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$Ra_{L} = \frac{g\beta(T_{s} - T_{\infty})D_{o}^{3}}{v^{2}}Pr = \frac{9.81 \times (1/305.22) \times (43.33 - 21.11)^{\circ}C \times 0.05^{3}}{(1.6289 \times 10^{-5} \text{ m}^{2}/\text{s})^{2}}(0.729) = 2.453 \times 10^{5}$$



$$Ra_{L} = \frac{g\beta(T_{s} - T_{\infty})D_{o}^{3}}{v^{2}}Pr = \frac{9.81 \times (1/305.22K) \times (316.33 - 294.11)K \times (0.1016)^{3}}{(1.6289 \times 10^{-5} \text{ m}^{2}/\text{s})^{2}} (0.729)$$
$$= 2.0579 \times 10^{6}$$

$$Nu = \left\{ 0.6 + \frac{0.387 \operatorname{Ra}_{D}^{1/6}}{\left[1 + (0.559/\operatorname{Pr})^{9/16}\right]^{8/27}} \right\}^{2} = \left\{ 0.6 + \frac{0.387 \left(2.0579 \times 10^{6}\right)^{1/6}}{\left[1 + (0.559/0.729)^{9/16}\right]^{8/27}} \right\}^{2}$$

$$k = 0.02604 \text{ W/m °K}$$

$$Pr = 0.729$$

$$v = 1.6289 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\beta = \frac{1}{T_{ave}} = \frac{1}{(273 + 32.22)\text{K}}$$
taksiaplagi!!!

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