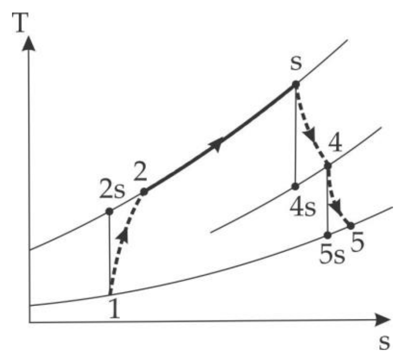
Example 7

A gas-turbine unit takes in air at 17°C and 1.01bar and the pressure ratio is 8/1. The compressor is driven by HP turbine and LP turbine drives a separate power shaft. The isentropic efficiencies of the compressor and the HP and LP turbines are 0.8, 0.85 and 0.83 respectively. Calculate the pressure and temperature of the gases entering the power turbine, the net power developed by the unit kg/s mass flow rate, the work ratio and the cycle efficiency of the unit. The maximum cycle temperature is 650°C. For the compression process take Cp = 1.005kJ/kg K and γ=1.4; for the combustion process and for the expansion process take Cp = 1.15kJ/kg K and γ=1.333. Neglect the mass of fuel.



From the equation for an isentropic process

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1}\right)^{(\gamma-1)/\gamma}$$

Therefore,

$$T_{2s} = 290 \times (8)^{(0.4)/1.4} = 525.32\text{K}$$

Then,

$$\eta_{comp} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{525.32 - 290}{T_2 - 290} = 0.8$$

$$T_2 = 584.15\text{K}$$

$$\text{Compressor work input} = C_{pa}(T_2 - T_1) = C_{pa}(584.15 - 290)$$

$$= 1.005 \times 294.15 = 295.62\,\text{kJ/kg}$$

Now the work output from the HP turbine must be sufficient to drive the compressor,

$$\text{Turbine work output from HP turbine} = C_p(T_3 - T_4) = 295.62\ \text{kJ/kg}$$

$$(T_3 - T_4) = \frac{295.62}{1.15} = 257.06\text{K}$$

Therefore,

$$T_4 = T_3 - 257.06 = 923 - 257.06 = 665.94\text{K}$$

Then,

$$\eta_{th} \text{ for HP turbine} = \frac{T_3 - T_4}{T_3 - T_{4s}} = \frac{923 - 665.94}{923 - T_{4s}} = 0.85$$

$$T_{4s} = 620.57\text{K}$$

Similarly for turbine

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4}\right)^{(\gamma-1)/\gamma}$$

or

$$\frac{P_3}{P_{4s}} = \left(\frac{T_3}{T_{4s}}\right)^{\gamma/(\gamma-1)} = \left(\frac{923}{620.57}\right)^{1.333/(0.333)} = 4.9$$

$$P_{4s} = \frac{8 \times 1.01}{4.9} = 1.65\text{bar}$$

Hence, the pressure and temperature at entry to the LP turbine are 1.65bar and 665.94K (392.94°C).

To find the power output it is now necessary to evaluate T_5. The pressure ratio, (p_4/p_5), is given by $(p_4/p_3)\times(p_3/p_5)$, is given by

$$\frac{P_{4s}}{P_{5s}} = \frac{P_{4s}}{P_3} \times \frac{P_{2s}}{P_1} \left(\text{Since } P_2 = P_3 \text{ and } P_5 = P_1\right)$$

Therefore,

$$\frac{P_{4s}}{P_{5s}} = \frac{8}{4.9} = 1.63$$

Then,

$$\frac{T_4}{T_{5s}} = \left(\frac{P_{4s}}{P_{5s}}\right)^{(\gamma-1)/\gamma} = 1.63^{0.333/1.333} = 1.13$$

Therefore,

$$T_{5s} = \frac{665.94}{1.13} = 589.18\text{K}$$

Next,

$$\eta_{\text{turb}} \text{ for the LP turbine} = \frac{T_4 - T_5}{T_4 - T_{5s}} = 0.83$$

$$\frac{T_4 - T_5}{665.94 - 589.18} = 0.83$$

$$T_4 - T_5 = 63.71$$

Then,

$$\text{Work output from LP turbine} = C_{pg}(T_4 - T_5) = 1.15(63.71) = 73.27 \text{ kJ/kg}$$

Hence,

$$\text{Net power output} = 73.27 \times 1 = 73.27\text{kW}$$

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross work output}} = \frac{73.27}{73.27 + 295.62} = 0.1986$$

and

$$\text{Heat supplied} = C_{pg}(T_3 - T_2) = 1.15(923 - 584.15) = 389.68 \text{ kJ/kg}$$

$$\eta_{\text{cycle}} = \frac{\text{Net work output}}{\text{Heat supplied}} = \frac{73.27}{389.68} = 0.188$$