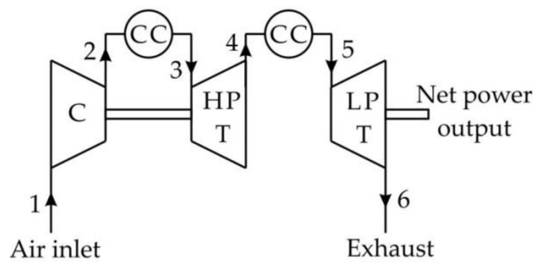
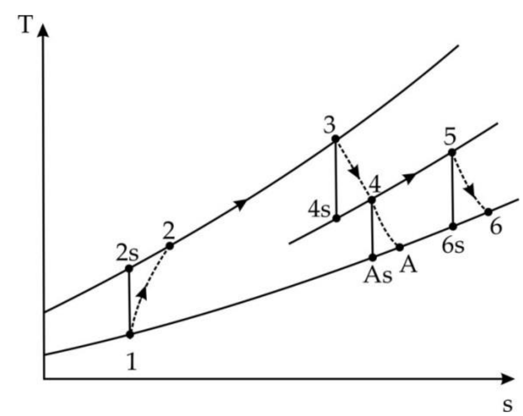
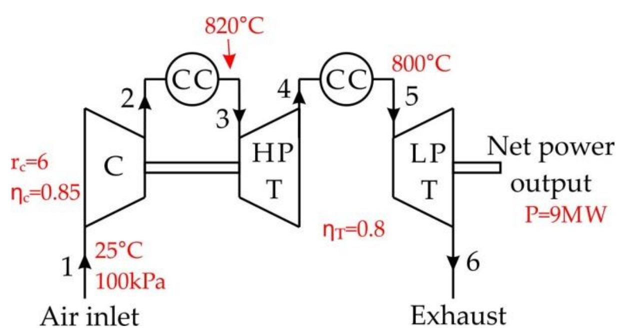
A gas turbine power plant operates with two-stage expansion and a reheat process. The compressor is driven by the high-pressure turbine while the low-pressure turbine delivers 9 MW of power to drive an electric generator. Air enters the compressor at 100 kPa and 25°C and the pressure ratio of the compressor is 6. The gases temperature at the entry to the high-pressure turbine is 820°C while the temperature of gases leaving the reheater is 800°C. The isentropic efficiency of the compressor is 0.85 and the isentropic efficiency of each turbine is 0.8. Sketch the schematic diagram of the plant and the cycle on a T-s diagram and calculate,

i) the mass flow rate of the combustion gases, kg/s
ii) the total heat input, kW, and
iii) the thermal efficiency, %.

For the compression process take $c_P = 1.005$ kJ/kgK and $\gamma = 1.40$ and for the combustion and expansion process take $c_P = 1.15$ kJ/kgK and $\gamma = 1.33$.







The entire work output of HPT is consumed by the compressor. The work delivered by LPT is used to run the electric generator.

Solution

$$T_{2s}=T_1 r_c^{\frac{\alpha-1}{\alpha}}=(25+273)\times 6^{\frac{1.4-1}{1.4}}=298\times 6^{\frac{1.4-1}{1.4}}=497.20\text{K}$$

$$T_2=T_1+\frac{(T_{2s}-T_1)}{\eta_c}=(25+273)+\frac{(497.20-(25+273))}{0.85}=532.352\text{K}$$

$$\dot{W}_c=\dot{m}C_p(T_2-T_1)=\dot{m}C_p(532.352-(25+273))=234.35\dot{m}\times 1.005=235.5218\dot{m}$$

$$\dot{W}_{HPT}=\dot{m}C_p(T_3-T_4)=\dot{W}_c$$
$$\dot{m}C_p((820+273)-T_4)=235.5218\dot{m}$$
$$\dot{m}\times 1.15\times(1093-T_4)=235.5218\dot{m}C_p$$
$$T_4=888.198\text{K}$$

$$\eta=\frac{T_3\text{ - }T_4}{T_3\text{ - }T_{4s}}=\frac{(820+273)\text{ - }888.198}{(820+273)\text{ - }T_4 s}=\frac{204.802}{(820+273)\text{ - }T_4 s}=0.8$$
$$T_{4s}=836.9975\text{K}$$

$$T_{4s}=\frac{T_3}{r_{HPT}^{\frac{\alpha-1}{\alpha}}}\Rightarrow 836.9975=\frac{820+273}{r_{HPT}^{0.2857}}\Rightarrow r_{HPT}^{0.481}=1.3058\Rightarrow r_{HPT}=\frac{P_3}{P_4}=2.932$$

$$P_3=100\text{kPa}\times 6=600\text{kPa}, \text{therefore}\frac{P_3}{P_4}=2.932\quad\Rightarrow\frac{600\text{kPa}}{P_4}=2.932\quad\Rightarrow P_4=204.6385\text{kPa}$$

$$r_{LPT}=\frac{P_4}{P_6}=\frac{204.6385\text{kPa}}{100\text{kPA}}=2.04638$$

$$T_{6s}=\frac{T_5}{r_{LPT}^{\frac{\alpha-1}{\alpha}}}=\frac{800+273}{2.04638^{0.2481}}=898.334\text{K}$$

$$\eta=\frac{T_5\text{ - }T_6}{T_5\text{ - }T_{6s}}\Rightarrow T_6=(800+273)-((800+273)-898.334)\times 0.8=933.267\text{K}$$

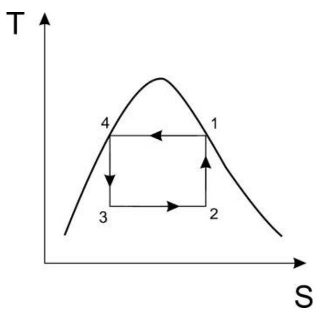
$$\dot{W}_{LPT}=\dot{m}C_p(T_5-T_6)=9000\text{kW}\Rightarrow\dot{m}\times 1.15((800+273)-933.267)=9000\text{kW}$$
$$\dot{m}=56.01\text{kg}/\text{s}$$

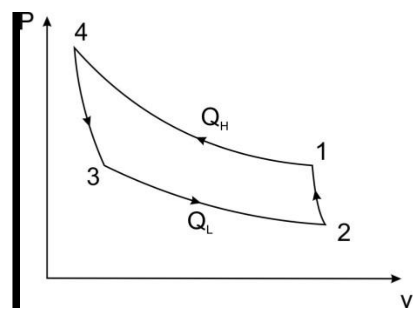
$$\dot{Q} = \dot{m}C_p(T_3 - T_2) + \dot{m}C_p(T_5 - T_4)$$
$$= 51.65 \times 1.005[(1093 - 532.352) + (1073 - 888.198)]$$
$$= 56.01 \times 857.27$$
$$= 48015.69 kW$$

$$\eta = \frac{\dot{W}}{\dot{Q}} = \frac{9000}{48015.69} = 0.1874 or 18.74\%$$

Q2

a) T-s and P-h diagrams of Carnot cycle



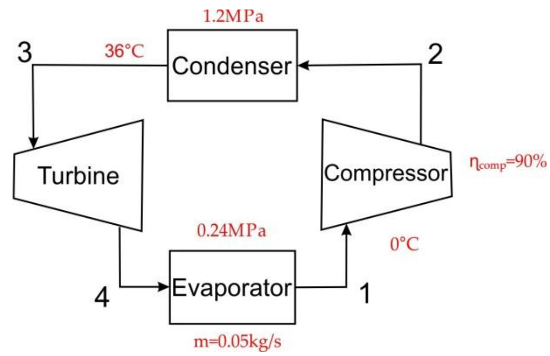


The Carnot cycle is not a realistic model for steam power plants because limiting the heat

1. transfer processes to two-phase systems to maintain isothermal conditions severely limits the maximum temperature that can be used in the cycle
2. the turbine will have to handle steam with a high moisture content which causes erosion
3. it is not practical to design a compressor that will handle two phases

b) Refrigeration cycle

Re frigerant 134a



From table,

$P_1 = 0.24MPa@\text{saturated vapour}$

$h_1 = h_{g@0.24MPa} = 247.28kJ/kg$

$s_1 = s_{g@0.24MPa} = 0.75664kJ/kg \cdot K$

$P_2 = 1.2MPa$

$s_{2s} = s_1 = 0.93458kJ/kg \cdot K$

$h_{2s} = h_{1.2MPa} = 280.852kJ/kg$

$T_{2s} = 52.27°C$

$P_3 = 1.2MPa$

$h_3 = 117.77kJ/kg$

$T_3 = 46.29°C$

$h_4 \cong h_3 = 117.77kJ/kg(\text{throttling})$

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.05(247.28 - 117.77) = 6.4755kW$$

$$\eta_{comp} = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{comp}} \Rightarrow h_2 = 247.28 + \frac{280.852 - 247.28}{0.9} = 284.58kJ/kg$$

$$\dot{Q}_{H,\text{rejection to environment}} = \dot{m}(h_2 - h_3) = 0.05(284.58 - 117.77) = 8.34kW$$

$$\dot{W}_{in} = (h_{2s} - h_1)/\eta_{comp} = \dot{m}\left[(h_{2s} - h_1)/\eta_{comp}\right] = 0.05 \times [(280.852 - 247.28)/0.9] = 1.8651kW$$

$$COP = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{6.4755kW}{1.8651kW} = 3.4719$$

Q3

a) **Forced convection**

In forced convection, the fluid is forced to move by external means such as a fan, pump, or the wind. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect that manifests itself as the rise of the warmer fluid and the fall of the cooler fluid. The convection caused by winds is natural convection for the earth, but it is forced convection for bodies subjected to the winds since for the body it makes no difference whether the air motion is caused by a fan or by the winds.

The convection heat transfer coefficient will usually be higher in forced convection since heat transfer coefficient depends on the fluid velocity, and forced convection involves higher fluid velocities.

b) An electrical cable

$$\text{radius}, r = 1.94\text{mm}$$
1atm air
$$T_{air} = 30°C$$
$$v = 50\text{m/s}$$
$$T_{air} = 50°C$$

$$T_f = \frac{50+30}{2} = 40°C$$
From the table A - 22;

$$Pr = 0.7255$$
$$\rho = 1.127$$
$$k = 0.02662\,W/m \cdot K$$
$$\upsilon = 1.702 \times 10^{-5}\,m^2/s$$
$$\mu = 1.918 \times 10^{-5}\,kg/m \cdot s$$

$$Re = \frac{vd}{\upsilon} = \frac{(50)\left(2 \times 1.94 \times 10^{-3}\right)}{1.702 \times 10^{-5}} = 11398.35 = \frac{\rho v d}{\mu} = \frac{1.127(50)\left(2 \times 1.94 \times 10^{-3}\right)}{1.918 \times 10^{-5}} = 11399.27$$

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5} \quad \text{for } Re < 20000$$

$$\frac{hD}{k} = 0.3 + \frac{0.62(11398.35)^{1/2}(0.7255)^{1/3}}{\left[1+(0.4/0.7255)^{2/3}\right]^{1/4}}\left[1+\left(\frac{11398.35}{282000}\right)^{5/8}\right]^{4/5} = 58.164$$

$$h = \frac{58.164 \times k}{D} = \frac{58.164 \times k}{D} = \frac{58.164 \times 0.02662}{\left(2 \times 1.94 \times 10^{-3}\right)} = 399.053\,W/m^2C$$

Rate of heat transfer per unit length,

$$A_s = \pi D L = \pi\left(2 \times 1.94 \times 10^{-3}\right)1m = 0.012189\,m^2$$
$$\dot{Q} = hA_s(T_s - T_\infty) = 399.053 \times 0.12189 \times (50-30) = 97.2811\,W/m$$

Q4

a) **Grashof number**

The Grashof number represents the ratio of the buoyancy force to the viscous force acting on a fluid.

The inertial forces in Reynolds number is replaced by the buoyancy forces in Grashof number.

b) Circular plate

$$D = 600\times10^{-3}\,\mathrm{m}$$
$$T_{room} = 30°C$$
$$T_{s_1} = 90°C$$
$$T_{s_2} = \text{insulated}$$

$$T_f = \frac{(T_s + T_\infty)}{2} = \frac{(90+30)}{2} = 60°C$$

From table

$$k = 0.02808\,\mathrm{W/m\cdot K}$$
$$Pr = 0.7202$$
$$v = 1.896\times10^{-5}\,\mathrm{m^2/s}$$
$$\beta = \frac{1}{T_f} = \frac{1}{60+273} = \frac{1}{333}$$

a) Horizontal with hot surface facing up

$$L_c = \frac{A_s}{p} = \frac{\pi\left(\frac{D}{2}\right)^2}{2\pi\left(\frac{D}{2}\right)} == \frac{\pi\left(150\times10^{-3}\right)^2}{2\pi\left(150\times10^{-3}\right)} = 0.075\,\mathrm{m}$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L_c^3}{v^2}Pr = \frac{(9.81)\left(\frac{1}{333}\right)(90-30)(0.075)^3}{\left(1.896\times10^{-5}\right)^2}\times0.7202 = 2.07436\times10^6$$

$$Nu = 0.54Ra_L^{1/4} = 0.54\left(2.07436\times10^6\right)^{1/4} = 20.49$$

$$h = \frac{k}{L_c}Nu = \frac{0.02808}{0.075}\times20.49 = 7.673\,\mathrm{W/m°C}$$

$$Q = hA_s(T_s - T_\infty) = 7.673\left(\pi\left(300\times10^{-3}\right)^2\right)(90-30) = 130.164\,\mathrm{W}$$

b) Vertical

$L_c = 0.075\text{m}$

$Ra_L = 2.07436 \times 10^6$

$Nu = 0.59 Ra_L{}^{1/4} = 0.59\left(2.07436 \times 10^6\right)^{1/4} = 22.39$

$h = \dfrac{k}{L_c} Nu = \dfrac{0.02808}{0.075} \times 22.39 = 8.3832 W/m°C$

$Q = hA_s\left(T_s - T_\infty\right) = 8.3832\left(\pi\left(300 \times 10^{-3}\right)^2\right)(90 - 30) = 142.217W$