Example 8

A 5000kW gas turbine generating set operates with 2 compressor stages with intercooling between stages; the overall pressure ratio is 9/1. A HP turbine is used to drive the compressors and a LP turbine drives the generator. The temperature of the gases at the entry to the HP turbine in 650°C after the gases are reheated to 650°C after expansion in the first turbine. The exhaust gases leaving the LP turbine are passed through a heat exchanger to heat the air leaving the HP stage compressor. The compressors have equal pressure ratios and intercooling is complete between stages. The air inlet temperature to the unit is 15°C. The isentropic efficiency of each compressor stage is 0.8 and the isentropic efficiency of each turbine stage is 0.85; the HE thermal ratio is 0.75. A mechanical efficiency of 0.98 can be assumed for both of the power shaft and the compressor turbine shaft. Neglecting all pressure losses and changes in kinetic energy, calculate

- a) Cycle efficiency
- b) Work ratio
- c) Mass flow rate

For air take $C_p=1.005$ kJ/kg.K and $\gamma=1.4$ and for the gases in the combustion chamber and in the turbines and HE take $C_p=1.15$ kJ/kg.K and $\gamma=1.333$. Neglect the mass of fuel.



$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma - 1/\gamma} \text{ where } \frac{P_2}{P_1} = \sqrt{9} = 3$$

Therefore

$$T_{2s} = 288 \times 3^{0.4/1.4} = 394.20K$$

$$\eta_{\rm LC} = \left(\frac{T_{2\rm s} - T_1}{T_2 - T_1}\right) = 0.8$$

Therefore

$$T_2 = T_1 + \left(\frac{394.20 - 288}{0.8}\right) = 288 + 132.75 = 420.75K$$

 $W_{_{in,LC}} = C_{_{pa}} (T_{_2} - T_{_1}) = 1.005 \times 132.75 = 133.41 \text{kJ} / \text{kg}$

HP turbine is required to drive both compressors and to overcome mechanical friction,

$$\begin{split} W_{\text{out, HT}} &= \frac{2 \times 133.41}{0.98} = 272.27 \text{ kJ} \ / \ \text{kg} \end{split}$$
 The refore $C_{\text{pg}} \big(T_6 - T_7 \big) = 272.27 \\ 1.15 \big(923 - T_7 \big) = 272.27 \\ T_7 &= 686.24 \text{ K} \end{split}$

$$\eta_{\rm HT} = \frac{T_6 - T_7}{T_6 - T_{7\rm s}} = 0.85$$

The refore
$$923 - T_{7\rm s} = \frac{(923 - 686.24)}{0.85}$$
$$T_{7\rm s} = 644.46 \rm{K}$$

Then,

$$\frac{P_6}{P_7} = \left(\frac{T_6}{T_{7s}}\right)^{\gamma/\gamma - 1} = \left(\frac{923}{644.46}\right)^{1.333/0.333} = 4.21$$

$$\frac{P_8}{P_9} = \frac{9}{4.21} = 2.14$$
Then,
$$\frac{T_8}{P_8} = \left(\frac{P_8}{P_8}\right)^{\gamma/\gamma - 1} = (2\ 14)^{0.333/1.333} = 1\ 21$$

$$\frac{I_8}{T_{9s}} = \left(\frac{I_8}{P_9}\right) = (2.14)^{0.333/1.333} = 1.2^{-1}$$

Therefore

$$T_{9s} = \frac{923}{1.21} = 762.8K, AsT_8 = T_6$$

$$\begin{split} \eta_{\rm LT} &= \frac{T_8 - T_9}{T_8 - T_{9_8}} = 0.85 \\ \text{The refore} \\ &= \frac{923 - T_9}{923 - 762.8} = 0.85 \\ T_9 &= 786.83 \text{K} \\ \\ W_{out, \, net} &= C_{pg} \big(T_8 - T_9 \big) \times \eta_{mech} = 1.15 \big(923 - 786.83 \big) \times 0.98 = 153.46 \text{kJ} \, / \, \text{kg} \\ \text{The rmal ratio of he ate xchange re} = \frac{T_5 - T_4}{T_9 - T_4} = 0.75 \Rightarrow \text{As } T_4 = T_2 = 420.75 \text{K} \\ &= \frac{T_5 - 420.75}{786.83 - 420.75} = 0.75 \\ &= 786.83 - 420.75 \\ &= 7_5 = 695.31 \text{K} \\ \\ q_{in} &= C_{pg} \big(T_6 - T_5 \big) + C_{pg} \big(T_8 - T_7 \big) = 1.15 \big(923 - 695.31 \big) + 1.15 \big(923 - 686.24 \big) = 534.12 \text{kJ} \, / \, \text{kg} \\ \\ &= \frac{W_{out}}{q_{in}} = \frac{153.46}{534.12} = 0.2873 \\ \\ &W_{out,gross} = W_{out,HPurb} + W_{out,LPurb} = 272.27 + \frac{153.46}{0.98} = 428.86 \text{kJ} \, / \, \text{kg} \end{split}$$

Therefore

Workratio =
$$\frac{W_{\text{out, net}}}{W_{\text{out, gross}}} = \frac{153.46}{428.86} = 0.358$$

Power, P =
$$\dot{m} \times W$$
 ork ratio
 $\dot{m} = \frac{P}{W_{net}} = \frac{5000}{153.46} = 32.58 \text{kg} / \text{s}$