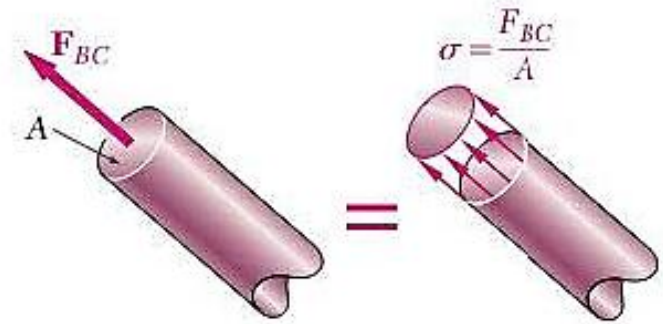
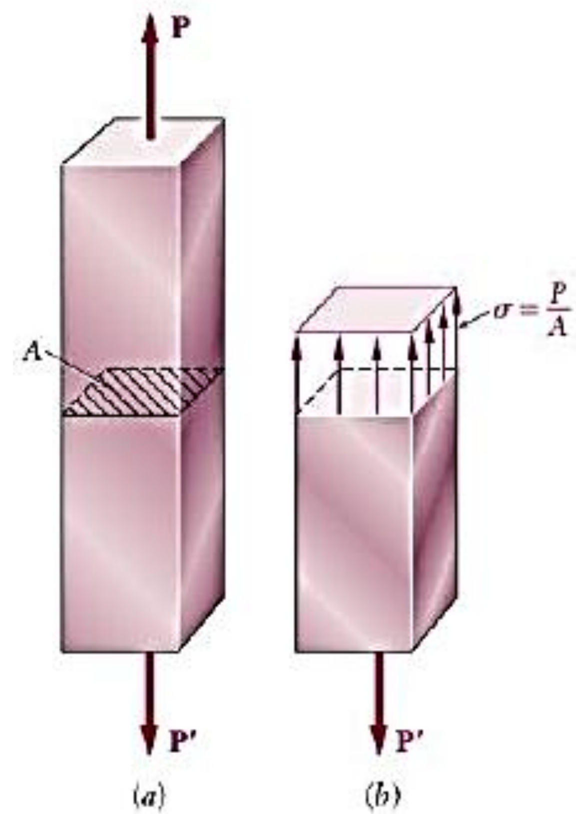
2.CONCEPT OF STRESS

While the results (force,moment) obtained from statics represent a first and necessary step in the analysis of the given structure, they do not tell us whether the given load can be safely supported.



Whether a rod, for example, will break or not under this loading depends not only upon the value found for the internal force F_{BC} ,but also upon the cross-sectional area of the rod and the material of which the rod is made.

Whether or not the rod will break under the given loading clearly depends upon the ability of the material to withstand the corresponding value F_{BC}/A of the intensity of the distributed internal forces.



CONCEPT OF STRESS

The force per unit area, or intensity of the forces distributed over a given section is called ***stress*** on that section.

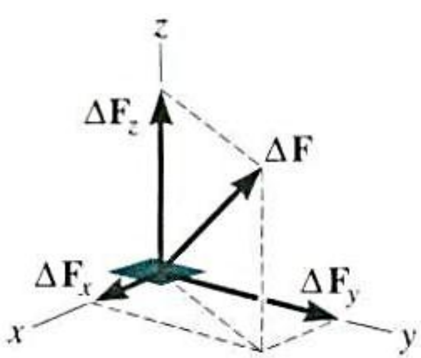
There are two main types of stresses of interest – normal stress and shear stress.

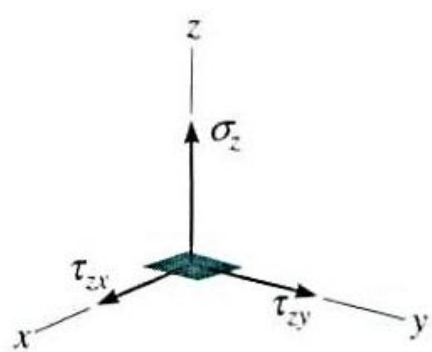
- Normal Stress The intensity of force, or force per unit area, acting normal to ΔA is defined as normal stress, σ (sigma)

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

If the normal force or stress pulls on the area element ΔA, it is referred to as *tensile stress*, whereas if it pushes on ΔA it is called *compressive stress*.

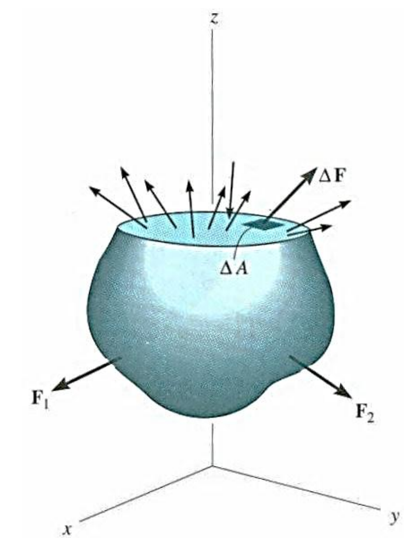
- Shear Stress The intensity of force, or force per unit area, acting tangent to ΔA is defined as shear stress, τ (tau)





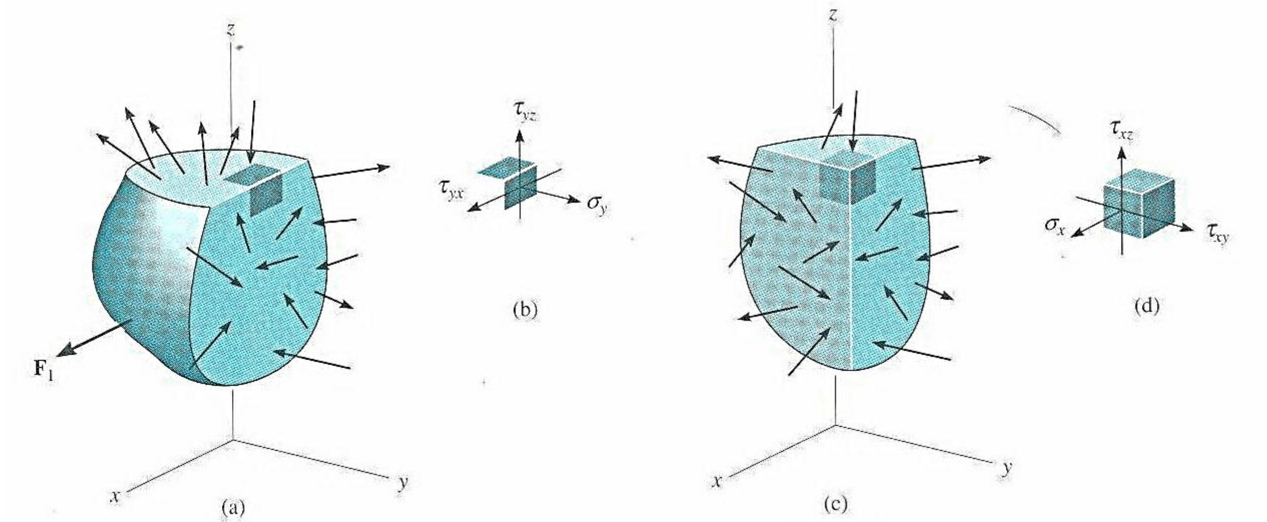
$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

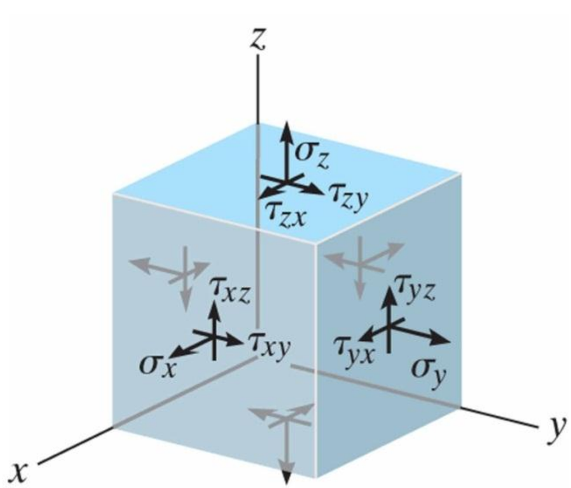
$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$



General State of Stress

If the body is sectioned by planes parallel to the *x-z* plane and the *y-z* plane we can the cut out a cubic volume element of material that represents the state of stress acting around the chosen point in the body.





General state of stress

$\sigma_x, \sigma_y, \sigma_z$ → Specifies the orientation of the area

$\tau_{xy}, \tau_{zx}, \tau_{zy}$

→ Direction lines

→ Specifies the orientation of the area

$\tau_{xy} = \tau_{yx}, \quad \tau_{zy} = \tau_{yz}, \quad \tau_{xz} = \tau_{zx}$

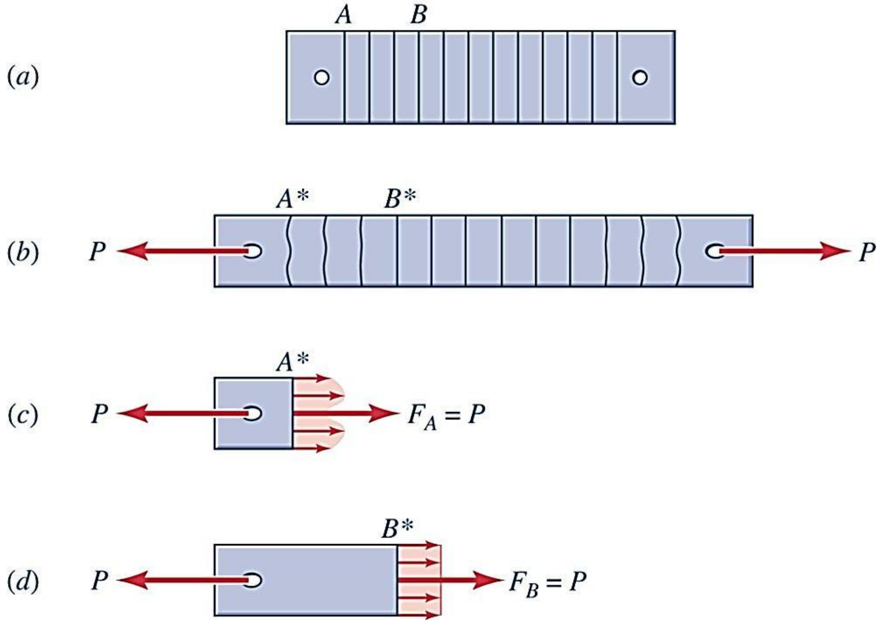
UNITS

The units of stress are units of force divided by units of area. In the U.S. Customary System of units (UCSC), stress is normally expressed in pounds (**lb**) per square inch **(psi)** or in **kips** per square inch, that is kilopounds per square inch **(ksi).**

In the International System of units (SI), stress is sprecified using basic units of force (newton) and length (meter) as newtons per meter squared (**N/m^2**). This unit, called the ***pascal*** (1 Pa=1 N/m^2) is quiate small, so in engineering work stress is normally expressed in kilopascals (1 kPa=10^3 N/m^2), megapascals (1 MPa=10^6 N/m^2), or gigapascals (1 GPa=10^9 N/m^2)

Average Normal Stress in An Axially Loaded Bar

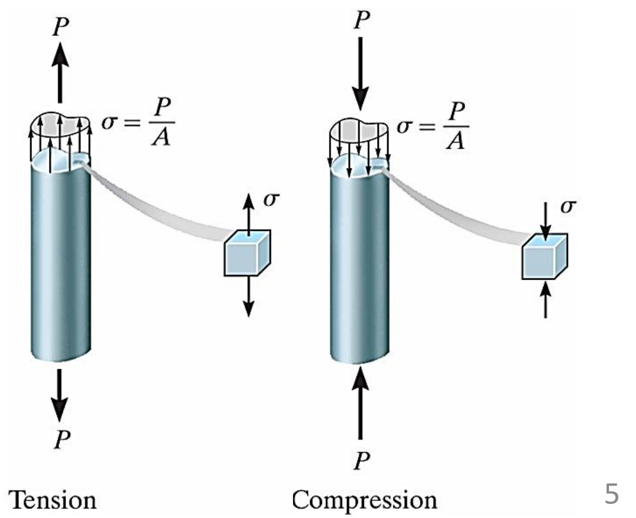
The thin arrows in figure (c) and (d) represents the distribution of force on cross section *A* and *B*. Near the ends of the bar for example at section *A*, the resultant normal force F_A, is not uniformly distributed over the cross section.



Even when the normal stress varies over the cross section, we can compute the average normal stress on the cross section by letting,

$$\sigma_{\text{avg}} = \frac{F}{A}$$

Average Normal Stress



Here,

A = cross-sectiona area of the bar where σ is determined

F or P = internal resultant normal force, which acts through the centroid of the cross-sectional area

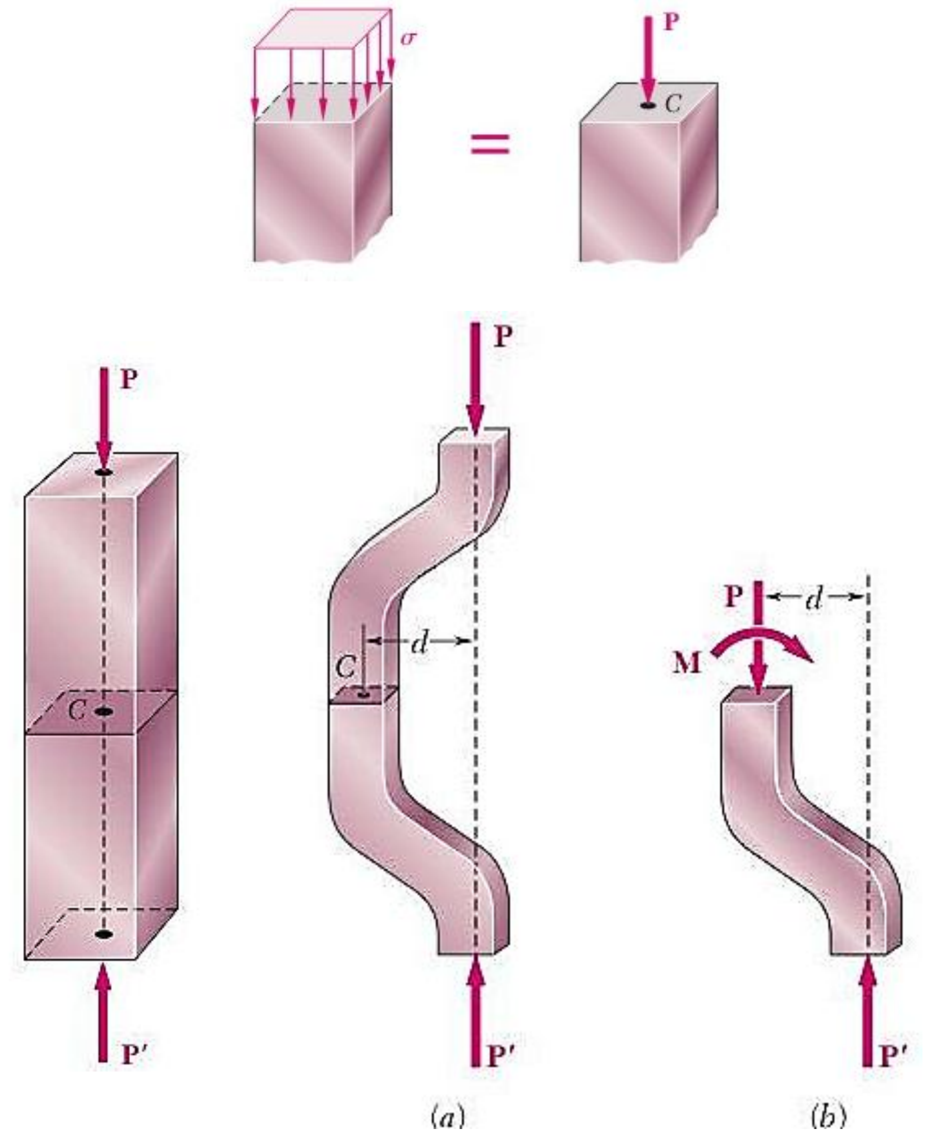
Uniform Normal Stress in An Axially Loaded Bar

A uniform distribution of stress is possible only if three conditions coexist;

- ✓ The axial force *P* acts through the centroid of the cross section. This type of loading is called centric loading. If the load is noncentric or eccentric, bending of the bar will result.

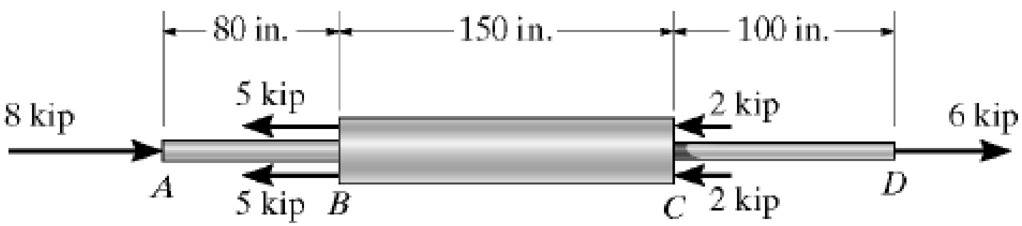
- ✓ The bar is homogeneous; that is, the bar is made of the same material throughout, with no holes, notches. Any of these give rise to stress concentrations.

- ✓ The section under consideration is remote from loaded end.



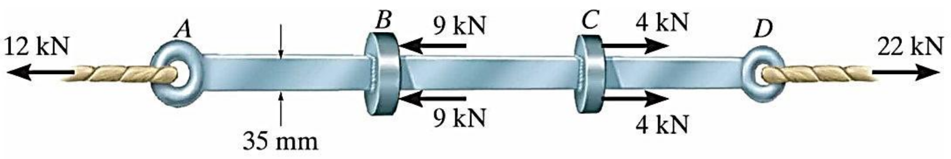
Maximum Average Normal Stress

The bar may be subjected to several external loads along its axis, or change in its cross sectional area may occur. As a result, the normal stress within the bar could be different one section to the next, and if the maximum average normal stress is to be determined then it becomes important to find the location where *P/A* is maximum.



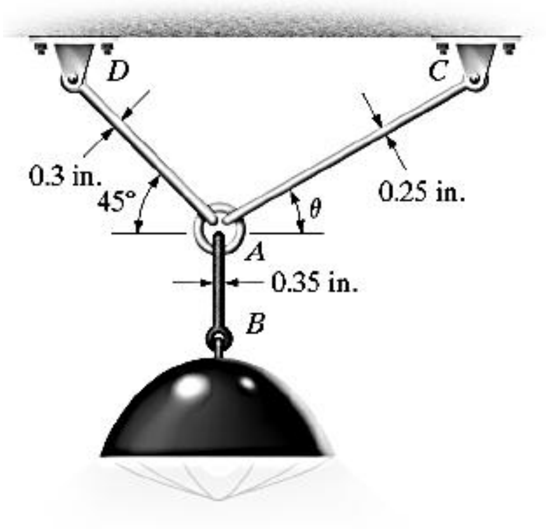
Example 2.1

The bar in Figure has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



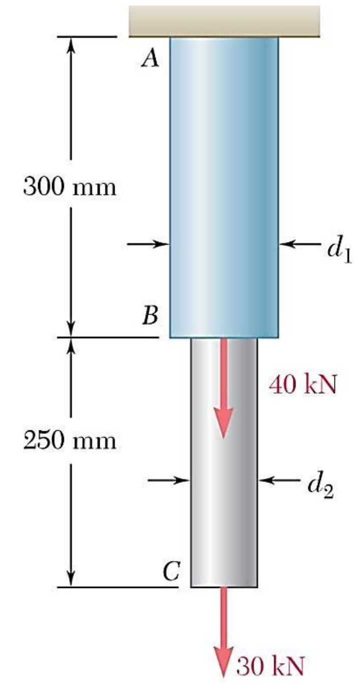
Example 2.2

Two 50-lb lamp is supported by three steel rods connected by a ring at *A*. Determine which rod is subjected to the grreater average normal stress and compute its value. Take θ =30°. The diameters of each rod is given in the figure.



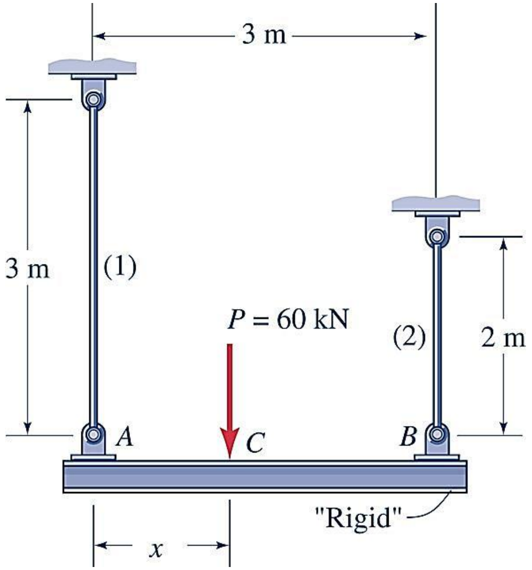
Example 2.3

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod *AB* and 150 MPa in rod *BC*, determine the smallest allowable values of d_1 and d_2.



Example 2.4

A rigid beam *AB* of total length 3 m is supported by vertical rods at its ends, and it supports a downward load at *C* of *P* = 60 kN, as shown in figure. The diameters of the steel hanger rods are d_1 = 25 mm and d_2 = 20 mm. Neglect the weight of beam *AB* and the rods. *(a)* if the load is located at *x* =1 m, what are the stresses in the hanger rods. *(b)* at what distance *x* from *A* must the load be placed such that $\sigma_1 = \sigma_2$, and what is the corresponding axial stress, in the rods?



Average Shear Stress

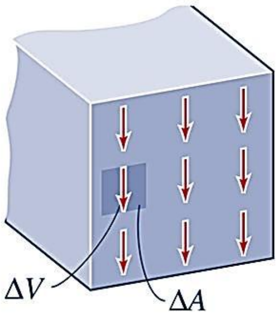
Even when the exact shear stress distribution on a surface cannot be readily determined, it is sometimes useful to calculate the average shear stress on the surface:

$$\tau_{avg} = \frac{V}{A_s}$$

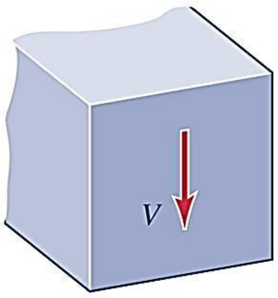
Average Shear Stress

where V is the total shear force acting on area A_s.

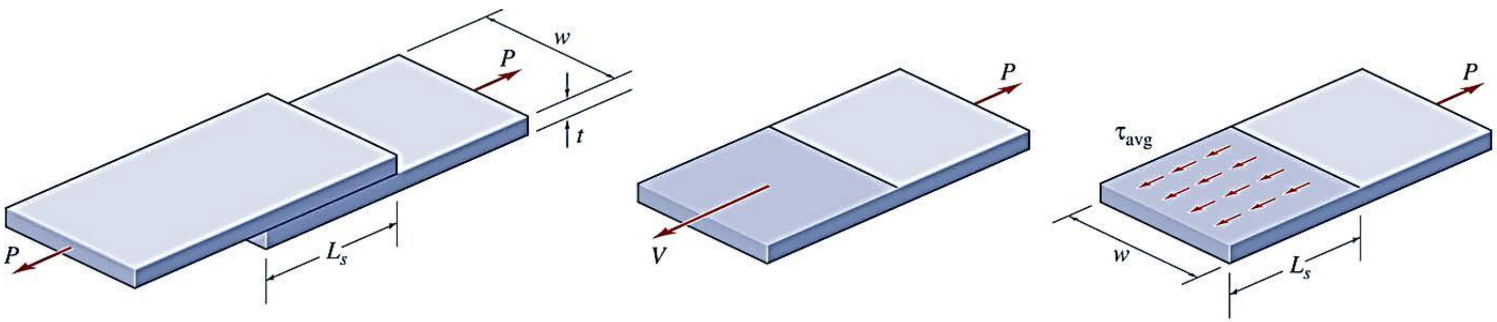
It should be noticed that the value obtained is an average value of the shearing stress over the entire section. Contrary to what we said earlier for normal stresses, the distribution of shearing stress across the section cannot be assumed uniform. As we will see later, the actual value varies from zero at the surface of the member to a maximum value that may be much larger than the average value.



(*a*) The distribution of shear force on a sectioning plane.



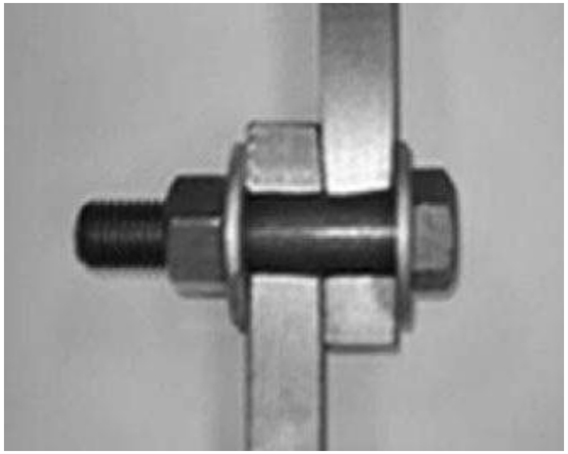
(*b*) The resultant shear force on the sectioning plane.



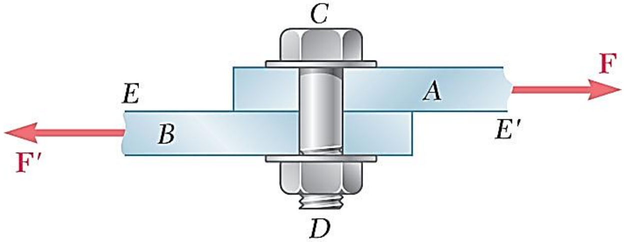
(*a*) A lap splice. (*b*) The free-body diagram. (*c*) The average-shear-stress distribution.

Average Shear Stress

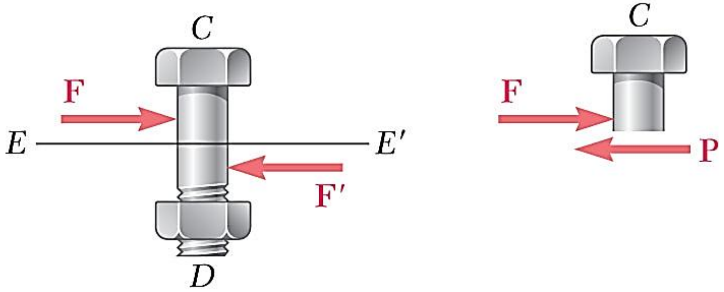
Shearing stresses are commonly found in bolts, pins and rivets used to connect various structural members and machine components. Consider the two plates *A* and *B*, which are connected by a bolt *CD*. If the plates are subjected to tension forces *F*, stresses will develop in the section of the bolt and of the portion located above the plane *EE*′. The average shearing stress in the section is obtained by dividing *F*=*P* by the area of the cross section.



$$\tau_{ave} = \frac{P}{A} = \frac{F}{A}$$



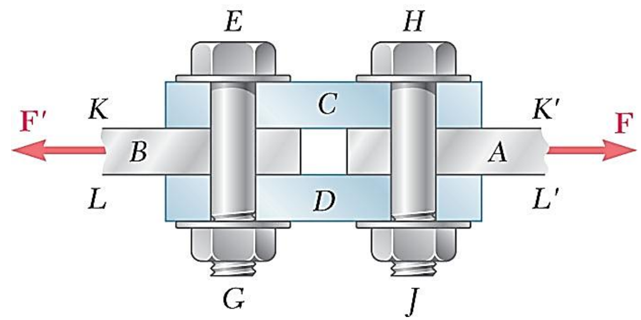
The bolt we have just considered is said to be in ***single shear***.



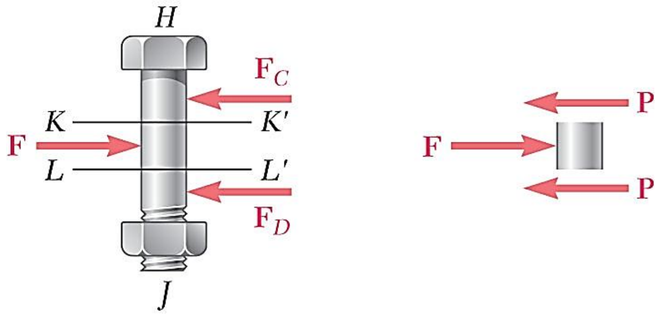
(*a*) (*b*)

Average Shear Stress

Different loading situations may arise, however. For example, if splice plates *C* and *D* are used to connect plates *A* and *B*, shear will take place in bolt *HJ* in each of two planes *KK´* and *LL´* (and similarly in bolt *EG*). The bolts are said to be in ***double shear***.

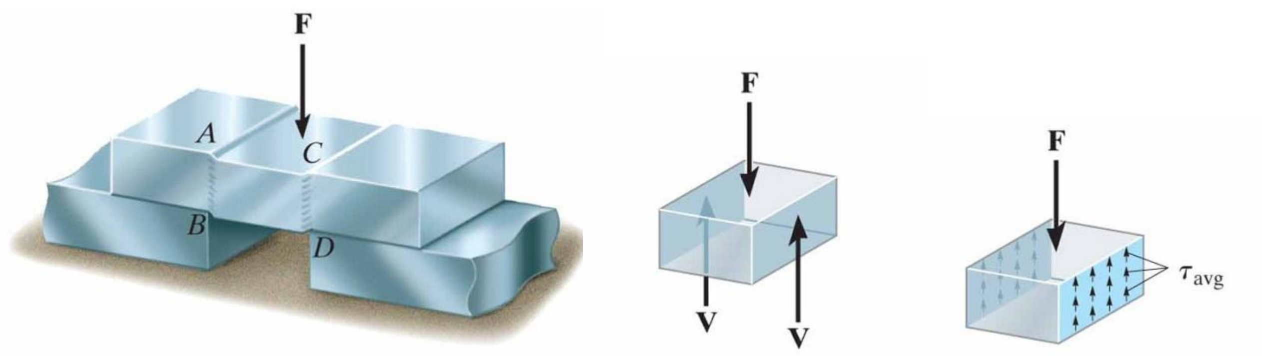


To determine the average shearing stress in each plane, we draw free-body diagrams of bolt *HJ* and of the portion of bolt located between the two planes. Observing that the shear *P* in each of the sections is *P=F/2*, we conclude that average shearing stress is



(a) (b)

$$\tau_{ave} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A}$$

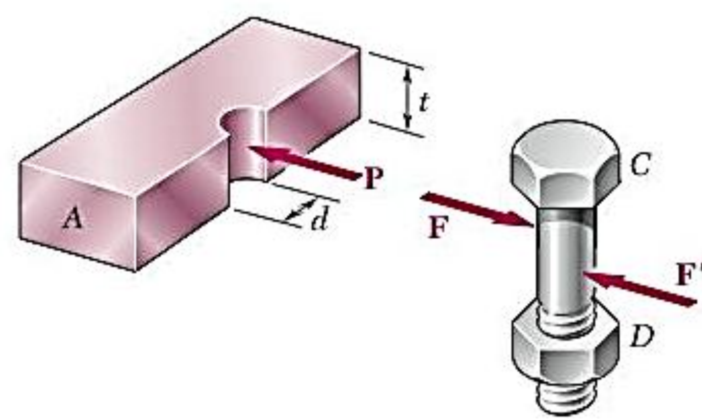


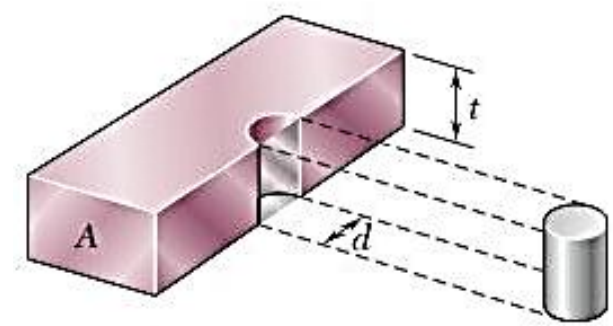
(a) (b) (c)

Bearing Stress

A normal stress that is produced by the compression of one surface against another is called ***bearing stress***. If this stress becomes large enough, it may crush or locally deform one or both of the surfaces.

Bolts, pins and rivets create stresses in the members they connect, along the bearing surface, or surface of contact. For example, in the figure, the bolts exerts on plate A a force $\boldsymbol{P}$ equal and opposite to the force $\boldsymbol{F}$ exerted by the plate on the bolt. P represents the resultant of elementary forces distributed on the inside surface of a half-cylinder of the plate. Since the distribution of these forces is quite complicated, one uses in practice an average nominal value of the stress, called the bearing stress, obtained by dividing the load by the area of the rectangle representing the projection of the bolt on the plate section.

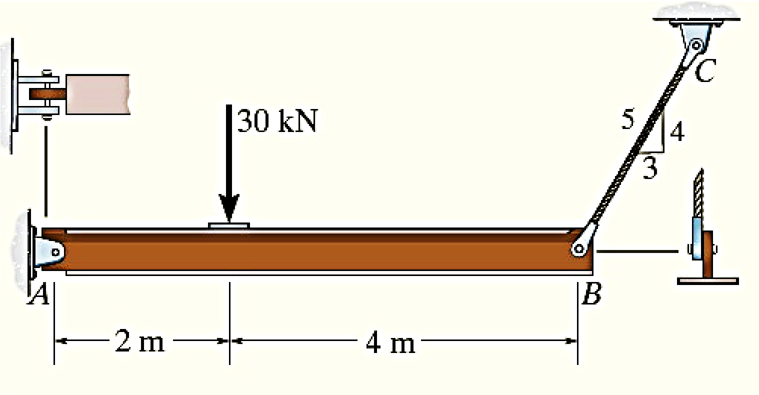


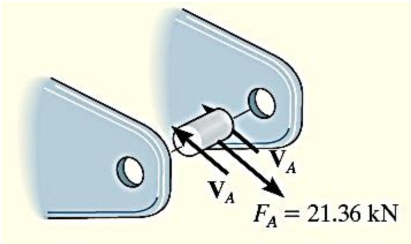


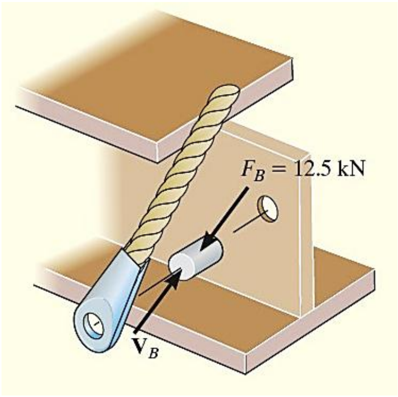
$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

Example 2.5

Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm-diameter pin at B that support the beam in the figure.

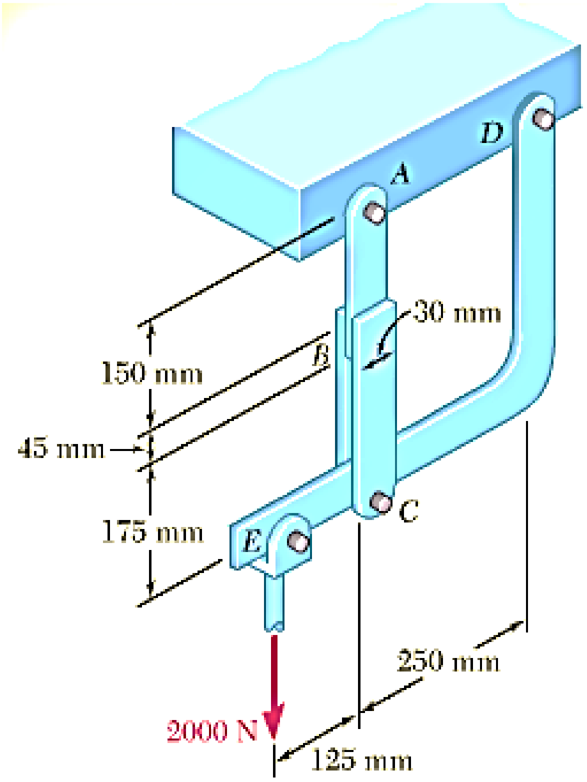






Example 2.6

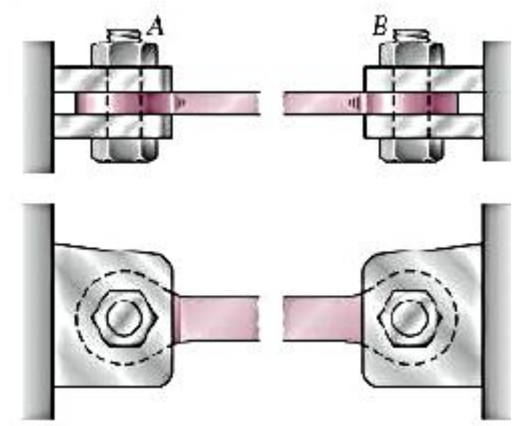
In the hanger shown, the upper portion of link *ABC* is 10 mm thick and the lower portions are each 6 mm thick. Epoxy resin is used to bond the upper and lower portions together at *B*. The pin at *A* is of 10-mm diameter while a 6-mm-diameter pin is used at *C*. Determine **(a)** the shearing stress in pin *A*. **(b)** the shearing stress in pin *C*, **(c)** the largest normal stress in link *ABC*, **(d)** the average shearing stress on the bonded surfaces at *B*, **(e)** the bearing stress in the link at *C*.

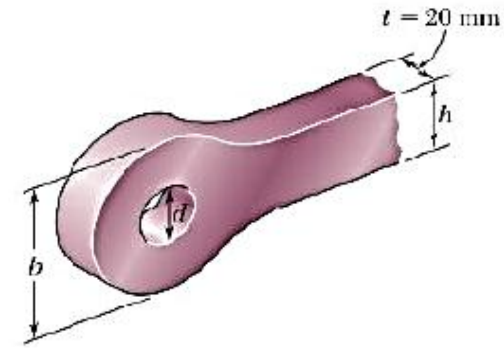


Example 2.7

The steel tie bar shown is to be designed to carry a tension force of magnitude P = 120 kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are:

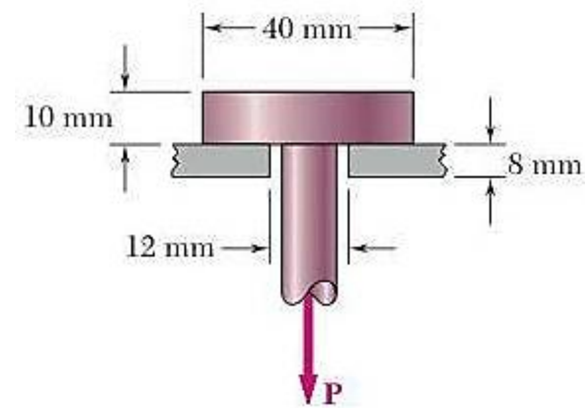
σ = 175 MPa, τ = 100 MPa, σ_b = 350 MPa. Design the tie bar by determinig the required values of **(a)** the diameter d of the bolt, **(b)** the dimension b at each end of the bar, **(c)** the dimension h of the bar.





Example 2.8

A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.



Allowable Stress and Factor of Safety

It is difficult to determine accurately the numerous factors that are involved in various aspects of analysis and design of structures. (environmental effects, vibration, corrosion, impact, human safety, variations in material properties..)

To ensure safety, it is necessary to choose an allowable stress that restricts the applied load to one that is less than the load the member can fully support.

One method of specifying the allowable load for the design or analysis of a member is to use a number called factor of safety.

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate load}}{\text{allowable load}}$$

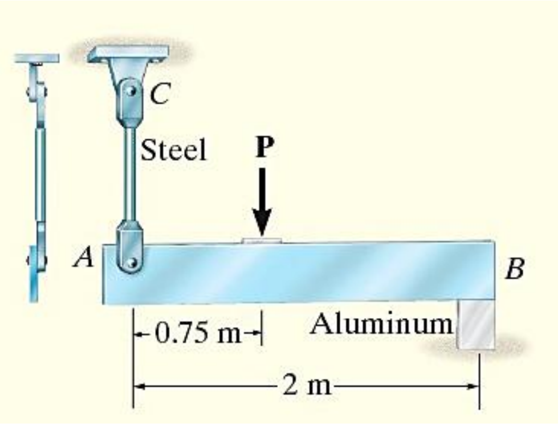
If the load applied to the member is linearly related to the stress developed within the member, it can be used as;

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

This ratio must be always greater than 1 in order to avoid the potential for failure. Specific values depend on the types of materials to be used and intended purpose of the structure or machine. For example, the F.S. Used in the design of aircraft or space-vehicle components may be close to 1 in order to reduce the weight of the vehicle. Or, in the case of a nuclear power plant, the F.S. for some of its components may be as high as 3 due to uncertainties in loading or material behavior. In many cases, the factor of safety for a specific case can be found in design codes and engineering handbooks.

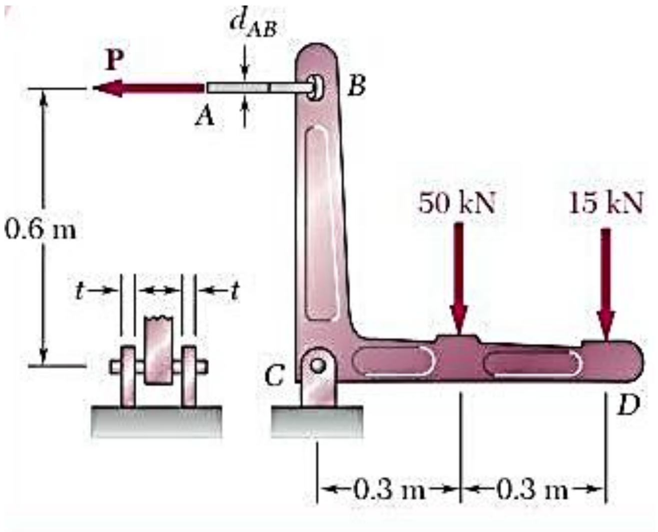
Example 2.9

The rigid bar *AB* shown in the figure is supported by a steel rod *AC* having a diameter of 20 mm and an aluminum block having a cross-sectional area of 1800 mm^2. The 18-mm-diameter pins at *A* and *C* are subjected to *single shear*. If the failure stress for the steel and aluminum is $(\sigma_{st})_{fail}$ = 680 MPa and $(\sigma_{al})_{fail}$ = 70 MPa , respectively, and the failure shear stress for each pin is τ_{fail} = 900 MPa, determine the largest load *P* that can be applied to the bar. Apply a factor of safety of F.S. = 2.



Example 2.10

Two forces are applied to the bracket *BCD* as shown. (*a*) Knowing that the control rod *AB* is to be made of a steel having an ultimate normal stress of 600 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.3. (*b*) The pin at *C* is to be made of a steel having an ultimate shearing stress of 350 MPa. Determine the diameter of the pin *C* for which the factor of safety with respect to shear will also be 3.3. (*c*) Determine the required thickness of the bracket supports at *C* knowing that the allowable bearing stress of the steel used is 300 MPa.



Example 2.11

The rigid beam BCD is attached by bolts to a control rod at B, to a hydraulic cylinder at C, and to a fixed support at D. The diameters of the bolts used are: $d_B = d_D = \frac{3}{8}$ in., $d_C = \frac{1}{2}$ in. Each bolt acts in double shear and is made from a steel for which the ultimate shearing stress is $\tau_U = 40$ ksi. The control rod AB has a diameter $d_A = \frac{7}{16}$ in. and is made of a steel for which the ultimate tensile stress is $\sigma_U = 60$ ksi. If the minimum factor of safety is to be 3.0 for the entire unit, determine the largest upward force which may be applied by the hydraulic cylinder at C.