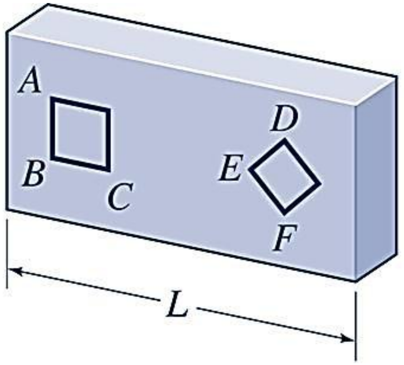
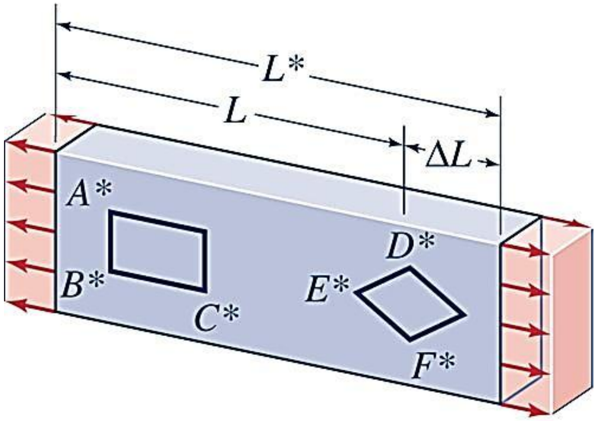
3.STRAIN

When a solid body is subjected to external loading and/or temperature changes, it deforms; that is, changes occur in the size and/or the shape of the body. The general term ***deformation*** includes both changes of lengths and changes of angles. They may be either highly visible or practically unnoticeable.

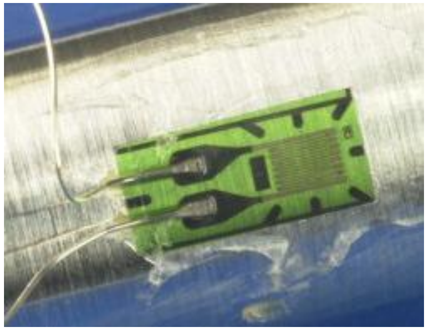
In order to describe the deformation by changes in length of line segments and the changes in the angles between them, we will develop the concept of strain. Measurements of strain are actually made by experiments, and once the strains are obtained, it will be shown later how they may be related to the applied loads, or stresses, acting within the body.



(*a*) The undeformed bar.



(*b*) The deformed bar.



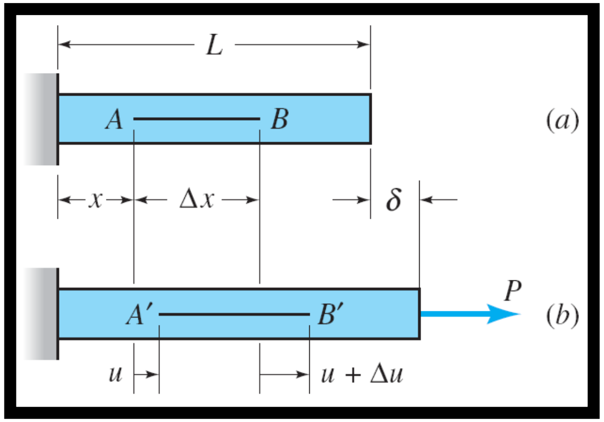
- Normal Strain

The elongation or contraction of a line segment per unit of length is referred to as *normal strain*. Denoting the average normal strain by ε_{avg}, we write,

$$\epsilon_{avg} = \delta / L$$

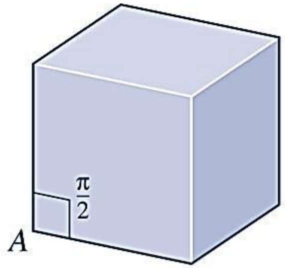
If the bar stretches, the strain is positive and is called *tensile strain*. A shortening of the bar results in a negative value of strain and is referred to as *compressive strain*.

Notice that normal strain is a dimensionless quantity, since it is a ratio of two lengths. Although this is the case, it is common practice to state it in terms of ratio of length units (m/m for SI system, in./in. for U.S. customary system). Ordinarily, for most engineering applications strain will be very small, so measurements of strain are in micrometers per meter (μm/m)



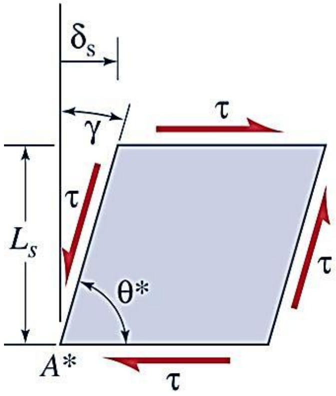
- Shear Strain

The change in angle that occurs between two line segments that were originally perpendicular to one another is referred to as *shear strain*. This angle is denoted by γ (gamma) and is measured in radians (rad).



(*a*) Original (undeformed) element.

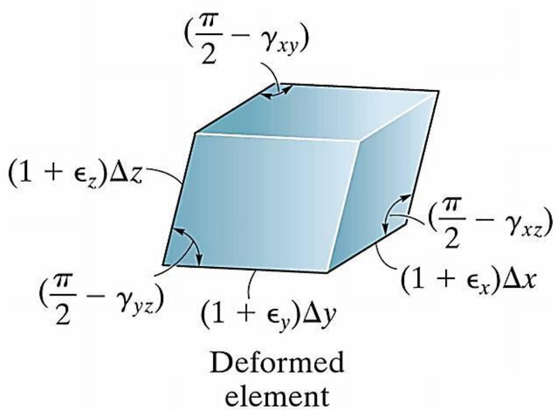
$$\gamma = \frac{\pi}{2} - \theta^*$$



(*b*) Pure shear deformation.

Not that if θ^* is smaller than $\pi/2$ the shear strain is positive, whereas if θ^* is larger than $\pi/2$ the shear strain is negative.

In particular, notice that the **normal strains cause a change in volume** of the rectangular element, wheras the **shear strains cause a change in its shape.**

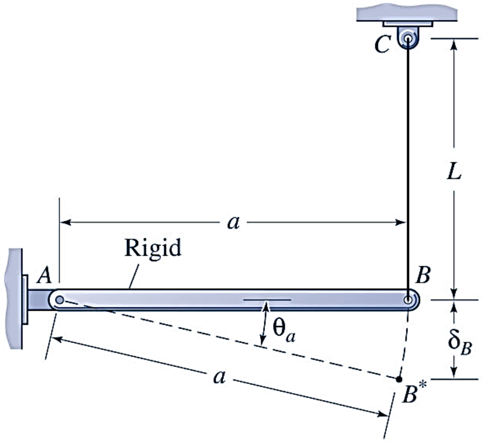


Small Strain Analysis

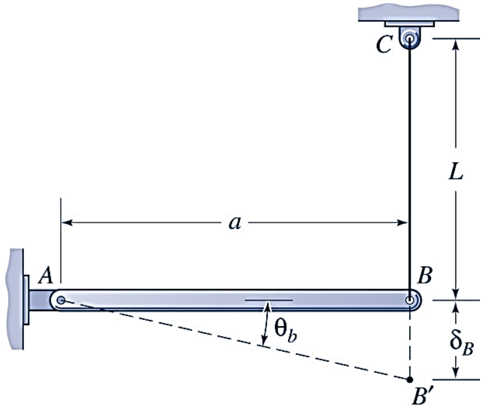
Most engineering design involves applications for which only small deformations are allowed.

Consider an extensible rod *BC* that pinned at *B* to a rigid beam and let point *B* move downward by a small distance, δ_B, where $\delta_B \ll L$ and $\delta_B \ll a$. Since *AB* is assumed to be rigid, point *B* actually moves in a circular path around a center at *A*, as shown in figure(*a*). Figure(*b*) illustrates a simplifying assumption: point *B* can be assumed to move vertically downward to point *B*′, rather than a circular path shown in figure(*a*).

$$\theta_b = \frac{\delta_B}{a}$$



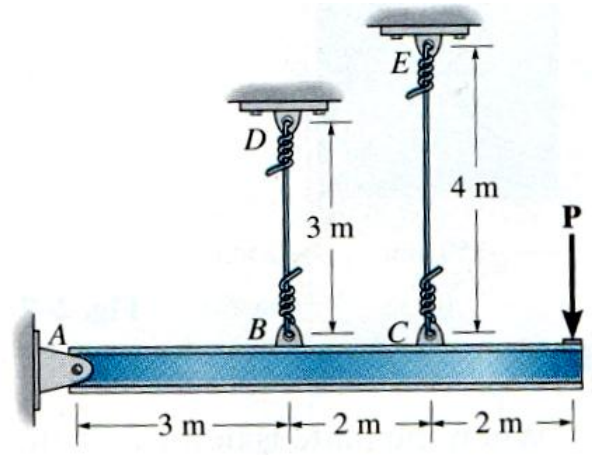
(*a*) Actual displacement of *B*.



(*b*) Displacement of *B* approximated to be vertical.

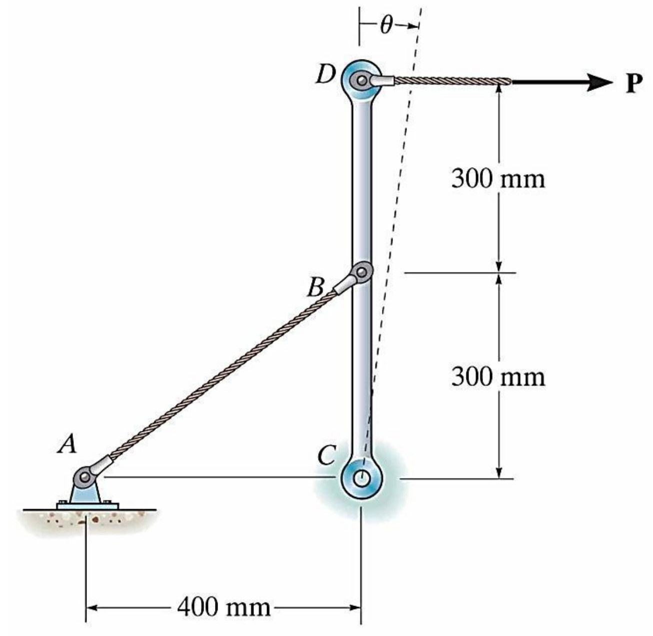
Example 3.1

The rigid beam is supported by a pin at A and wires BD and CE. If the maximum allowable normal strain in each wire is $\epsilon_{max} = 0.002mm/mm$, determine the maximum vertical displacement of the load P.



Example 3.2

Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point *D*. Originally the cable is unstretched.



Example 3.3

The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners *A* and *B* if the plastic distorts as shown by the dashed lines.