

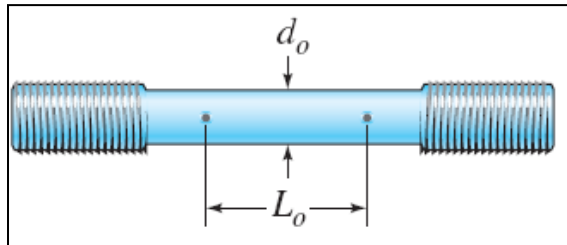
4.MECHANICAL PROPERTIES OF MATERIALS

The diagram representing the relation between stress and strain in a given material is an important characteristic of the material. To obtain the stress-strain diagram of the material, one usually conducts a *tensile test* on a specimen of the material such as metals, ceramics, polymers and composites.

The values of normal stress and extensional strain that are used in plotting a *conventional stress-strain diagram* are the engineering stress and engineering strain.

$$\sigma = \frac{P}{A_0}, \quad \epsilon = \frac{L^* - L_0}{L_0}$$

A_0 is the specimen's initial cross-sectional area. We can determine the nominal or engineering stress by dividing the applied load P by the specimen's original cross-sectional area. Likewise, the nominal or engineering strain is found directly from the strain gauge reading, or by dividing the change in the specimen's gauge length, δ , by the specimen's original gauge length L_0 .

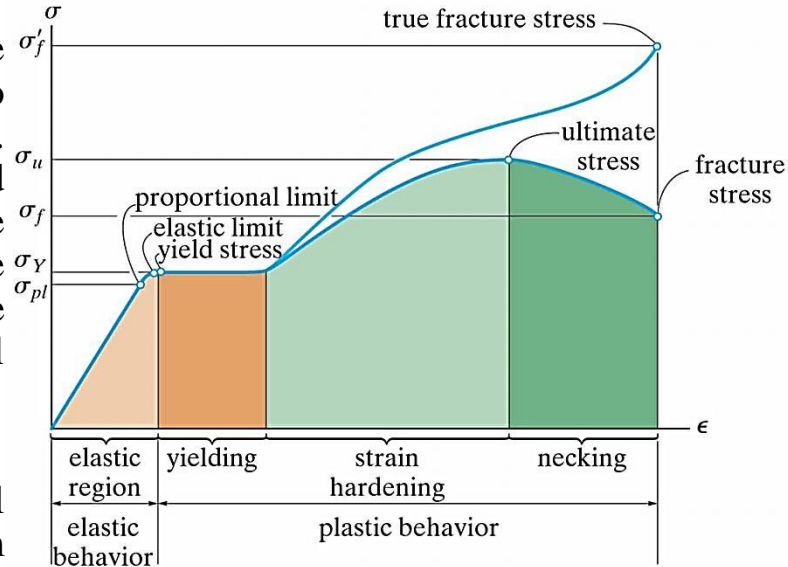


It is possible to distinguish some common characteristics among the stress-strain diagrams of various groups of materials and to divide materials into two broad categories on the basis of these characteristics, namely, *ductile* materials and the *brittle* materials.

Stress-strain diagram for ductile materials.

Elastic Behavior. Here the curve is actually a straight line throughout most of this region, so that the stress is proportional to the strain. The material in this region is said to be *linear elastic*. The upper stress limit to this linear relationship is called **proportional limit**, σ_{PL} . If the stress exceeds this limit, the curve tends to bend and flatten out as shown. This continues until the stress reaches the **elastic limit**. Upon reaching this point, if the load is removed the specimen will still return back to its original shape.

Yielding. A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to deform permanently. This behavior is called **yielding**. The stress that causes yielding is called the **yield stress** or **yield point**, σ_Y , and the deformation that occurs is called **plastic deformation**. Although not shown in the diagram, for low-carbon steels or those that are hot rolled, the yield point is often distinguished by two values. The **upper yield point** occurs first, followed by a sudden decrease in load-carrying capacity to a **lower yield point**. Notice that once the yield point is reached, then the specimen will continue to elongate without any increase in load. When the material is in this state, it is often referred to as being **perfectly plastic**.



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

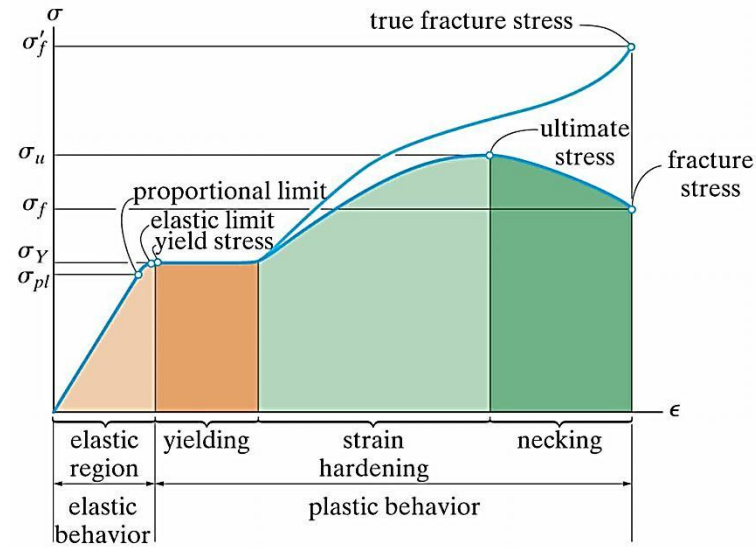
$$E = \frac{\Delta\sigma}{\Delta\epsilon}, \quad \sigma < \sigma_{PL}$$

Young's Modulus

Strain Hardening. When yielding has ended, an increase in load can be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the *ultimate stress*, σ_u . The rise in the curve in this manner is called *strain hardening*.

Necking. Up to the ultimate stress, as the specimen elongates, its cross-sectional area will decrease. This decrease is fairly uniform over the specimen's entire gauge length; however, just after, at the ultimate stress, the cross-sectional area will begin to decrease in a localized region of the specimen. As a result, a constriction or neck tends to form in this region as the specimen elongates further. Here the stress-strain diagram tends to curve downward until the specimen breaks at the *fracture stress*, σ_f .

True stress-strain diagram. Instead of always using the original length and cross-sectional area to calculate the stress and strain, we could have used the actual area and length at the instant the load is measured. Note that the conventional and true diagrams are practically coincident when the strain is small. Although the diagrams are different, most engineering design is done so that the material support a stress within the elastic range.



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

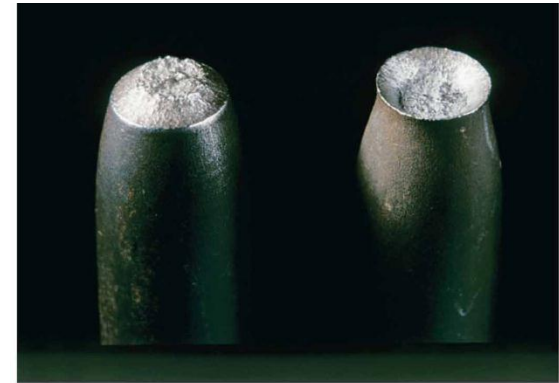


Figure: 03-03-UN
This steel specimen clearly shows the necking that occurred just before the specimen failed. This resulted in the formation of a "cup-cone" shape at the fracture location, which is characteristic of ductile materials.



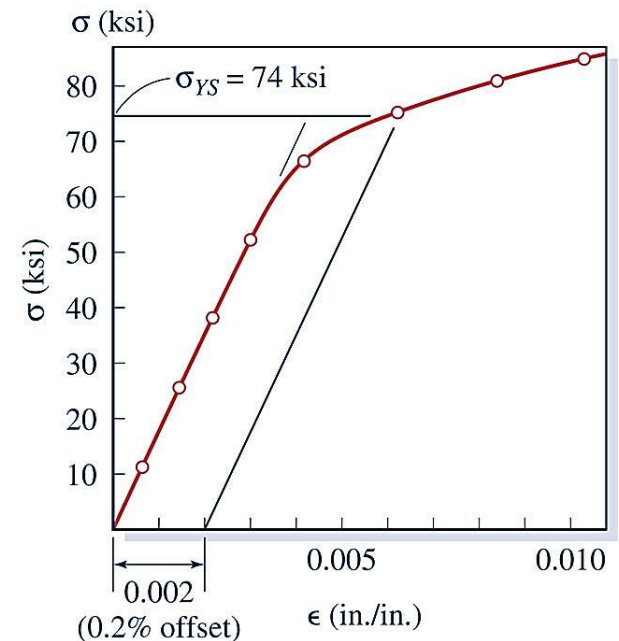
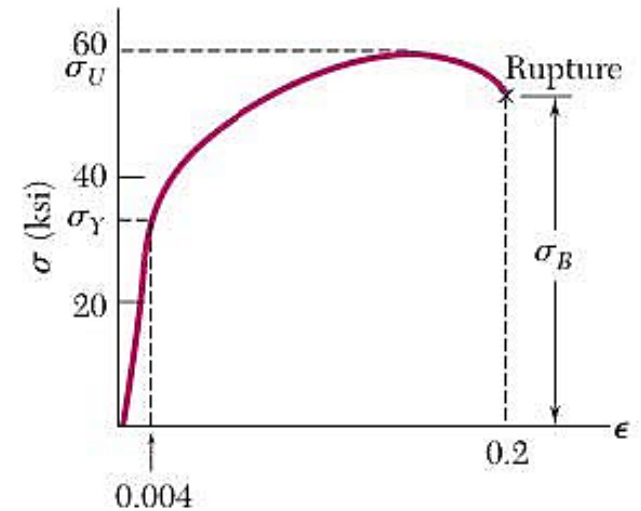
Necking

Failure of a ductile material

Off-set method for some ductile materials

For materials like aluminum that have no clearly defined yield point, a stress value called the offset yield stress, is used as a yield-point stress.

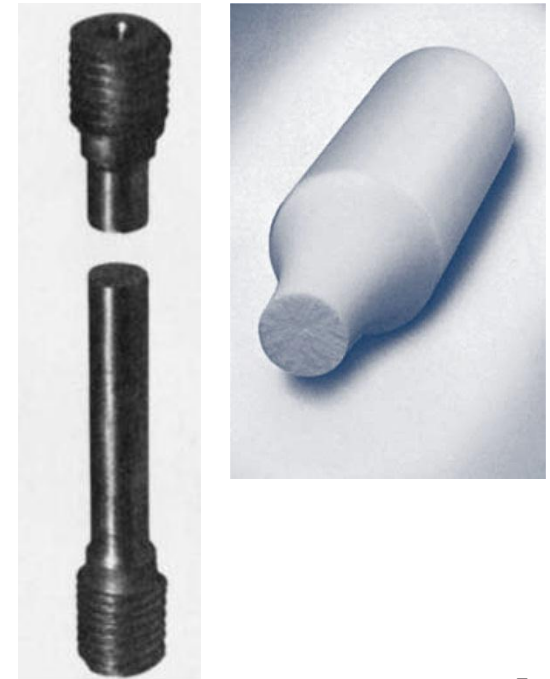
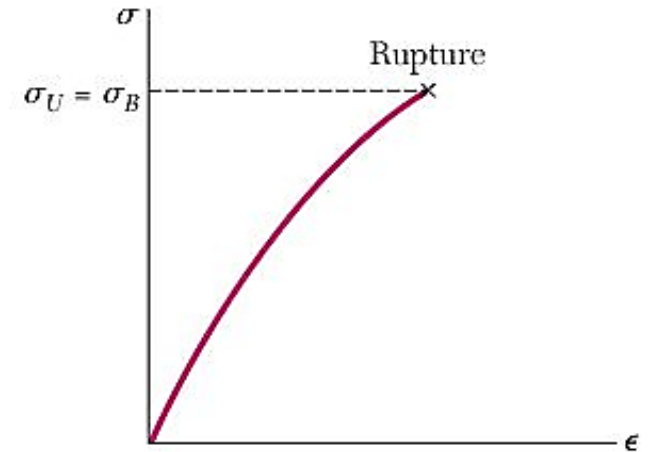
As illustrated in figure, the offset yield stress is determined by first drawing a straight line that fits the data in initial portion of stress-strain diagram. A second line is then drawn parallel to the original line but offset by a specified amount of strain. The intersection of this second line with the stress-strain curve determine the offset yield stress. In figure the commonly used offset value of 0.002 is illustrated.



Stress-strain diagram for brittle materials.

Brittle materials, which comprise cast iron, glass and stone are characterized by the fact that rupture occurs without any noticeable prior change in the rate of elongation. Thus, for brittle materials, there is no difference between the ultimate strength and the breaking strength.

Also, the strain at the time of rupture is much smaller for brittle than for ductile materials. From the figures, we note the absence of any necking of the specimen in the case of brittle material, and observe that rupture occurs along a surface perpendicular to the load.



Hooke's Law and Poisson's Ratio .

Most engineering structures are designed to undergo relatively small deformations, involving only the straight-line portion of the corresponding stress-strain diagram. For that initial portion of the diagram the stress is directly proportional to the strain and we can write,

$$\sigma = E \epsilon$$

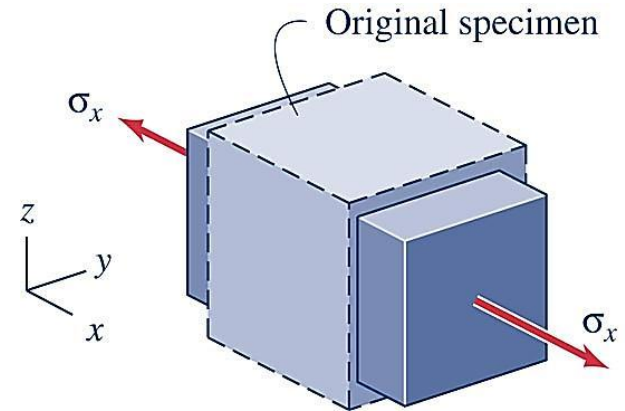
This relation is known as *Hooke's law*.

Associated with the elongation of a member in axial tension, there is a transverse contraction. The transverse contraction during a tensile test is related to the longitudinal elongation by

$$\epsilon_{\text{transv}} = -\nu \epsilon_{\text{longit}}$$

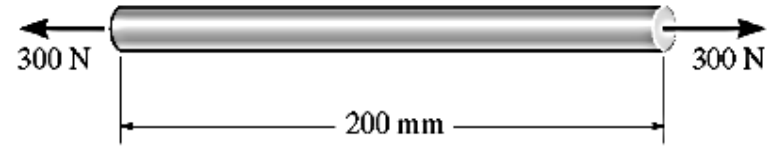
Where ν (greek symbol nu) is Poisson's ratio. Poisson's ratio is dimensionless, with typical values in the 0.25-0.35 range. For orientation of axes in figure the transverse strains are related to the longitudinal strain by

$$\epsilon_y = \epsilon_z = -\nu \epsilon_x$$



Example 4.1

The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



5. AXIAL LOAD

- A structural member having a straight longitudinal axis is said to undergo **axial deformation** if, when loads are applied to the member or it is subjected to temperature change: (1) the axis of the member remains straight, and (2) cross sections of the member remain plane, remain perpendicular to the axis, and do not rotate about the axis as the member deforms.

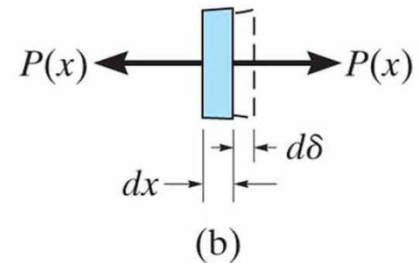
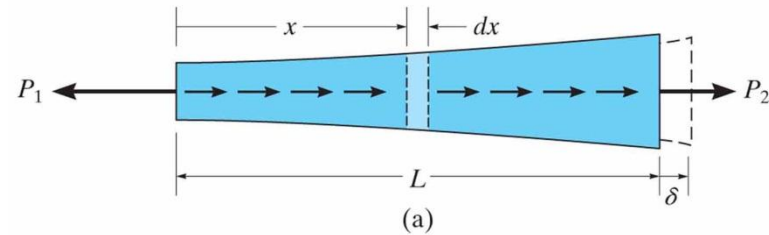
ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

Using the method of sections, a differential element of length dx and cross-sectional area $A(x)$ is isolated from the bar at the arbitrary position x . The resultant internal axial force $P(x)$ will deform the element into the shape indicated by dashed outline. The stress and strain in the element are

$$\sigma = \frac{P(x)}{A(x)} \qquad \epsilon = \frac{d\delta}{dx}$$

Using the Hooke's law

$$\sigma = E \epsilon \rightarrow \frac{P(x)}{A(x)} = E \left(\frac{d\delta}{dx} \right) \rightarrow d\delta = \frac{P(x)dx}{A(x)E}$$



For the entire length L of the bar, we must integrate this expression to find the required end displacement. This yields

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

Where

δ : displacement of one point on the bar relative to another point

L : distance between the points

$P(x)$: internal axial force at the section, located a distance x from one end

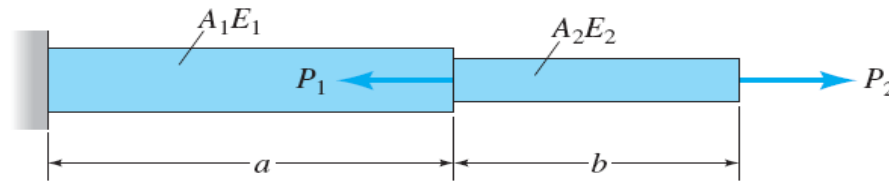
$A(x)$: cross sectional area of the bar, expressed as a function of x

E : modulus of elasticity for the material

Constant Load and Cross-Sectional Area. In many cases the bar will have a constant cross-sectional area A ; and the material will be homogeneous, so E is constant. Furthermore, if a constant external force is applied at each end, then the internal force P throughout the length of the bar is also constant.

$$\delta = \frac{PL}{AE}$$

If the bar is subjected to several different axial forces, or the cross-sectional area or modulus of elasticity changes abruptly from one region of the bar to the next, the above equation can be applied to each segment of the bar where these quantities are all constant. The displacement of one end of the bar with respect to the other is then found from the vector addition of the end displacements of each segment.



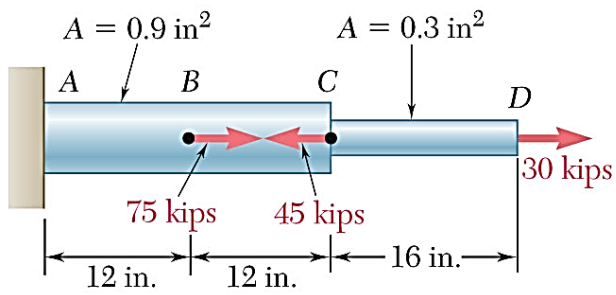
$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$

Sign Convention. In order to apply this equation we must develop a sign convention for the internal force and the displacement of one end of the bar with respect to the other end. We will consider both the force and displacement to be positive if they cause tension and elongation, respectively, whereas a negative force and displacement will cause compression and contraction, respectively.



Example 5.1(Beer&Johnston)

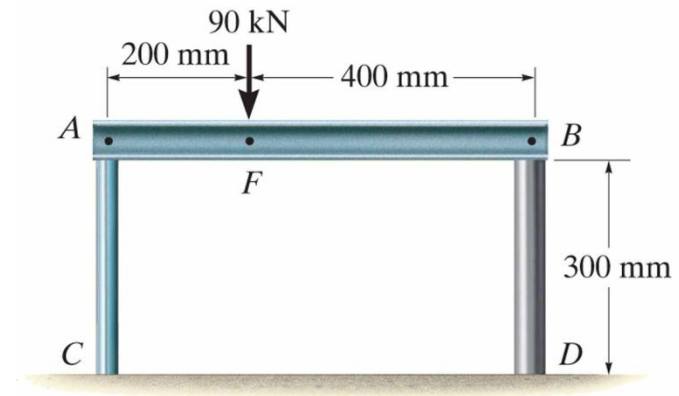
Determine the deformation of the steel rod shown in figure under the given loads. $E = 29(10^6) psi$



Example 5.2

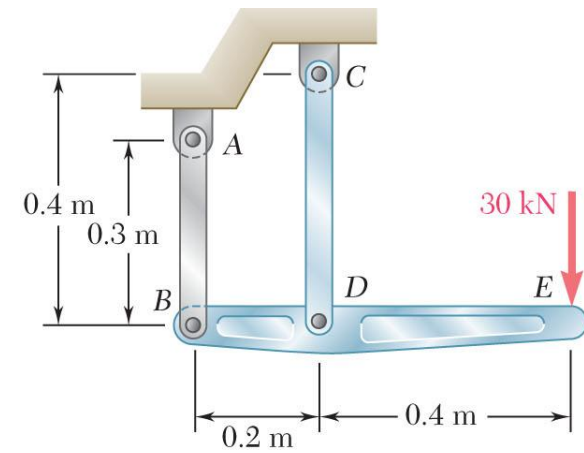
Rigid beam AB rests on the two short posts shown in the figure. AC is made of steel and has a diameter of 20 mm, and BD is made of aluminum and has a diameter of 40 mm. Determine the displacement of point F on AB if a vertical load of 90 kN is applied over this point. Take

$$E_{st} = 200 \text{ GPa}, E_{al} = 70 \text{ GPa}$$



Example 5.3 (Beer & Johnston)

The rigid bar BDE is supported by two links AB and CD . Link AB is made of aluminum ($E = 70 \text{ GPa}$) and has a cross-sectional area of 500 mm^2 ; link CD is made of steel ($E = 200 \text{ GPa}$) and has a cross-sectional area of 600 mm^2 . For the 30-kN force shown, determine the deflection (a) of B , (b) of D , (c) of E .



Example 5.4

The assembly consists of three titanium (Ti-6Al-4V) rods and a rigid bar AC . The cross-sectional area of each rod is given in the figure. If a force of 6 kip is applied to the ring F , determine the angle of tilt of bar AC .

$$E = 17.4 (10^3) \text{ ksi}$$

