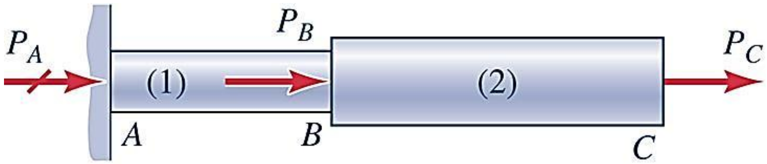
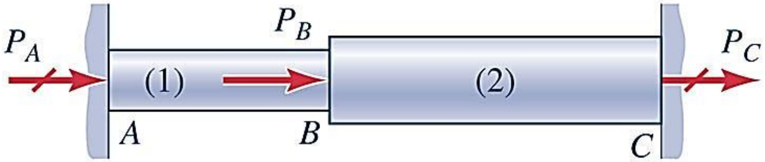
STATICALLY INDETERMINATE AXIAL LOADED STRUCTURES

The figure shows two structures, each consisting of two collinear elements. Acting on the structure in figure (a) are two known forces, P_B and P_C, and one reaction, P_A. The reaction at the left end and the axial force in each of the two elements of the structure in figure (a) can be determined from statics alone, that is by drawing free-body diagrams and solving equilibrium equations. The values of these forces are independent of the materials involved and other member properties. Structures of this type are called **statically determinate structures**.



(a) A statically determinate structure.

On the other hand, both ends of the structure in Figure *(b)* are attached to rigid walls, so there are two unknown reactions P_A and P_C, but only one known load P_B. Since there is only one equilibrium equation, summation of forces in the axial direction, it is not possible to determine both reactions from equilibrium alone. To determine the reactions and element forces for this case it is necessary to consider the deformation of the elements, and this involves member sizes and materials. Such structures are classified as **statically indeterminate**.



(b) A statically indeterminate structure.

$$P_A + P_B + P_C = 0$$

For the figure shown, the force equilibrium equation;

$$\sum F = 0; \quad R_A + R_B - P = 0$$

In this case the bar is called *statically indeterminate*, since the equilibrium equation is not sufficent to determine the reactions. In order to establish an additional equation needed for solution, it is necessary to consider the geometry of the deformation. Specifically, an equation that specifies the conditions for displacement is referred to as a ***compatibility*** or ***kinematic condition***. A suitable compatibility condition would require the relative displacement of one end of the bar with respect to the other end to be equal zero, since the end supports are fixed:

$$\delta_{A/B} = 0$$

This equation can be expressed in terms of the applied loads by using a load-displacement relationship, which depends on the material behavior.

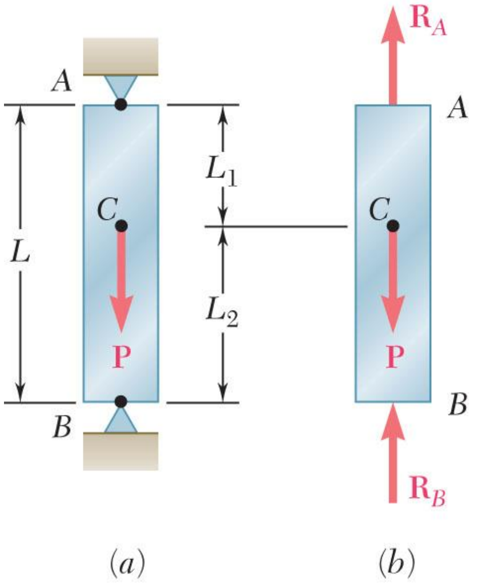
$$\delta_{A/B} = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} = 0$$

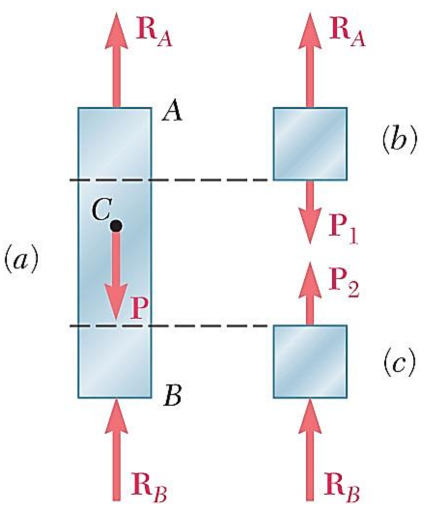
Realizing that the internal force in segment AC is $P_1 = R_A$ and in segment CB the internal force is $P_2 = -R_B$, (AE, constant)

$$R_A L_1 - R_B L_2 = 0$$

and using the equilibrium equation

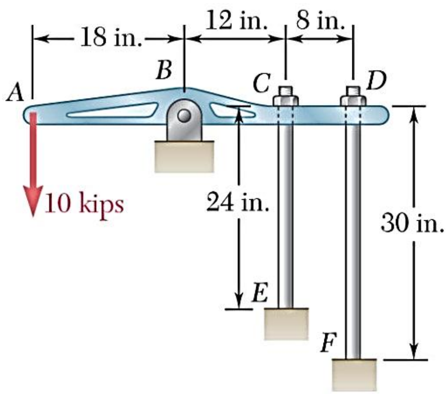
$$R_A = P\frac{L_2}{L}, \qquad R_B = P\frac{L_1}{L}$$

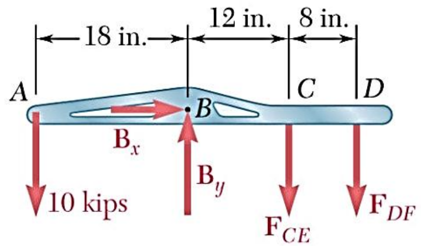


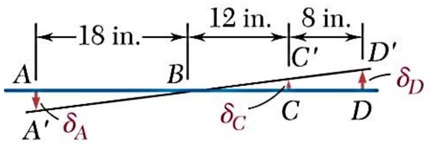


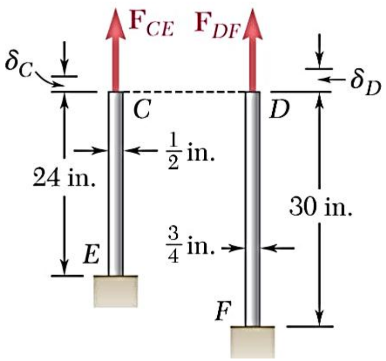
Example 5.4 (Beer&Johnston)

The ½ in.-diameter rod *CE* and the 3/4-in.-diameter rod *DF* are attached to the rigid bar *ABCD* as shown. Knowing that the rods are made of aluminum and using $E = 10.6(10)^6$ psi, determine (*a*) the force in each rod caused by the loading shown, (*b*) the corresponding deflection of point *A*.



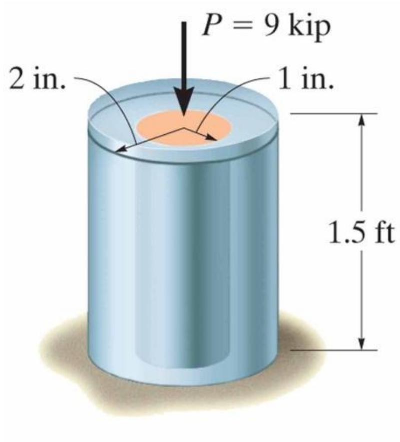




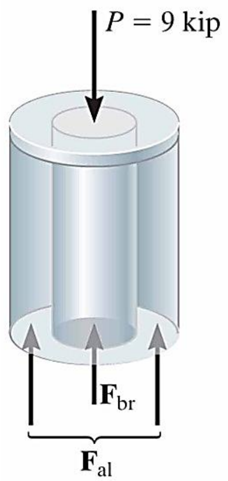


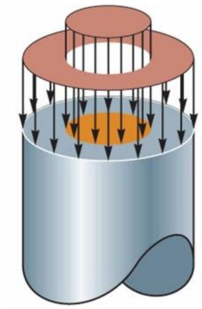
Example 5.5 (Hibbeler)

The aluminum post shown in Figure(a) is reinforced with a brass core. If this assembly supports an axial compressive load of P = 9 kip, applied to the rigid cap, determine the average normal stress in the aluminum and the brass. Take E_{al} =10(10^3) ksi and E_{br} = 15(10^3) ksi.



(a)





THE FORCE METHOD OF ANALYSIS FOR AXIALLY LOADED STRUCTURES (SUPERPOSITION METHOD)

It is also possible to solve statically indeterminate problems by writing the compatibility equation using the superposition of the forces acting on the free-body diagram. This method of solution is often referred to as the flexibility or force method of analysis.

We will first choose any one of the two supports as 'redundant' and temporarily remove its effect on the bar. The word redundant, as used here indicates that the support is not needed to hold the bar in stable equilibrium, so that when it is removed, the bar becomes statically determinate. Here we will choose the support at *B* as redundant. By using the principle of superposition, the bar having its original loading on it is equivalent to the bar subjected only to the external load $\boldsymbol{P}$ plus the bar subjected only to the external load $\boldsymbol{F_B}$.

If the load $\boldsymbol{P}$ causes *B* to be displaced downward by an amount δ_P, the reaction $\boldsymbol{F_B}$ must be capable of displacing the end *B* of the bar upward by an amount δ_B, such that no displacement occurs at *B* when the two loadings are superimposed. Thus

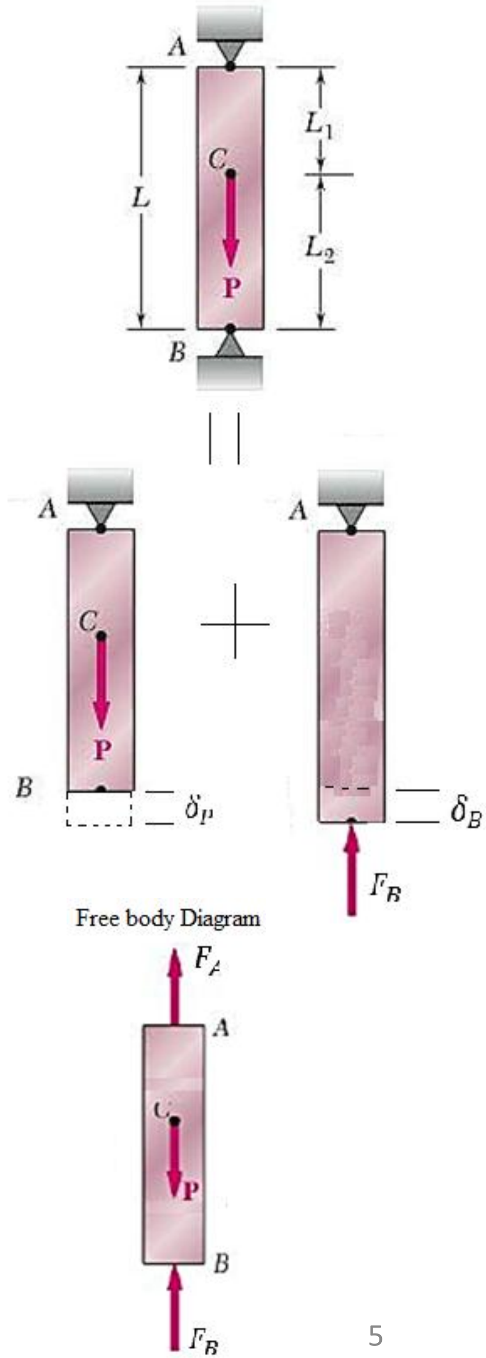
$$0 = \delta_P - \delta_B$$

Applying the load-displacement relationship to each case, we have $\delta_P = PL_{AC}/AE$ and $\delta_B = F_B L/AE$. Then

$$F_B = PL_{AC}/L$$

From the free-body diagram of the bar, the reaction at A can be determined from the equation of equilibrium

$$F_B + F_A - P = 0$$

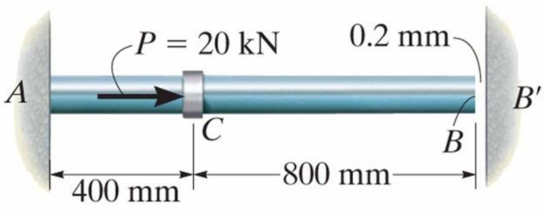


Superposition principle procedure

1. One of the unknown reactions is designated as *redundant* and released from the member by removing the support.
2. The remaining member, which is rendered statically determinate, is loaded by the actual load (P) and the redundant (R_B) itself. Note that the redundant is considered to be an *unknown load*.
3. The expressions for the displacements due to these loads are obtained and substituted into the equation of geometric compatibility to calculate the redundant reaction. The other unknown reaction is found by applying statics.

Example 5.6 (Hibbeler)

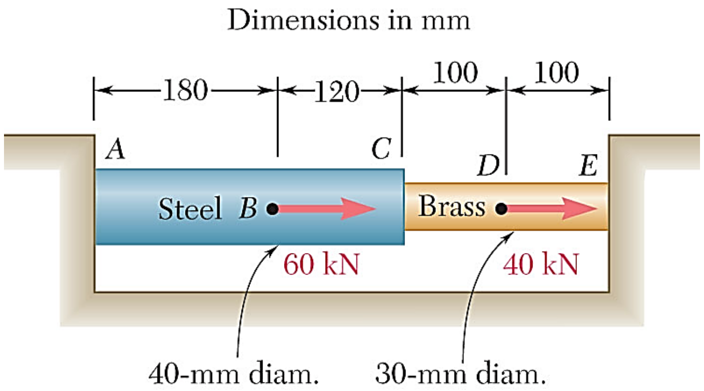
The steel rod shown in figure has a diameter of 10 mm. It is fixed to the wall at *A*, and before it is loaded, there is a gap of 0.2 mm between the wall at *B'* and the rod. Determine the reactions at *A* and *B'* if the rod is subjected to an axial force of $P = 20$ kN as shown. Neglect the size of the collar at *C*. Take $E_{st} = 200$ GPa.



(a)

Example 5.7 (Beer&Johnston)

Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that E_{st} = 200 GPa and E_{br} = 105 GPa, determine (a) the reactions at A and E, (b) the deflection of point C.



THERMAL STRESS

A change in temperature can cause a material to change its dimensions. If the temperature increases, generally a material expands, whereas if the temperature decreases the material will contract. Ordinarilly this expansion or contraction is linearly related to the temperature increase or decrease that occurs. If this is the case, and the material is homogeneous and isotropic., it has been found from experiment that the deformation of a member having length L can be calculated using the formula

$$\delta_T = \alpha \Delta T L$$

Where;

α : a property of the material, referred to as the linear coefficient of thermal expansion. The units measure strain per degree of temperature. They are 1/°F in the Foot-Pound-Second system, and 1/°C or 1/°K in the SI system

ΔT : the algebraic change in temperature of the member

L : the original length of the member

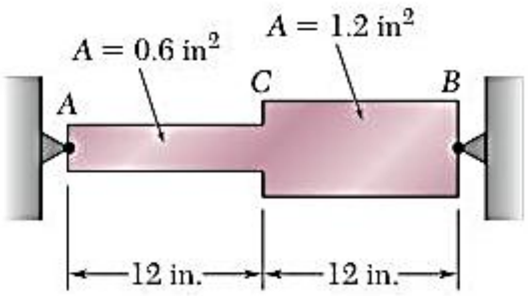
δ_T : the algebraic chamge in length of the member

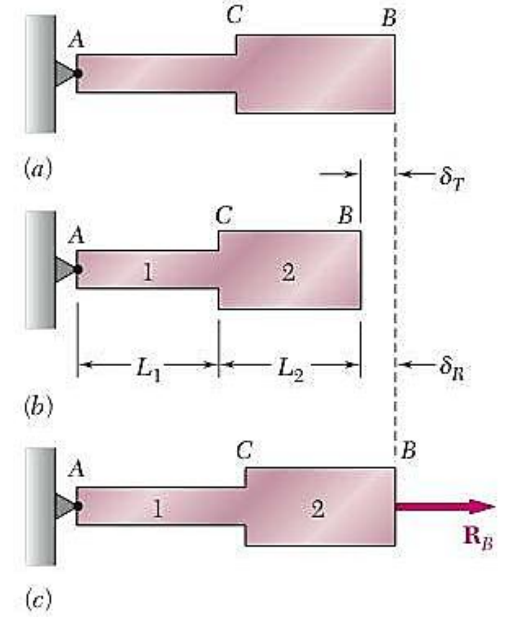
If the change in temperature varies throughout the length of the member, i.e. ΔT = ΔT (x), or if α varies along the length, then

$$\delta_T = \int \alpha \Delta T dx$$

Example 5.8(Beer&Johnston)

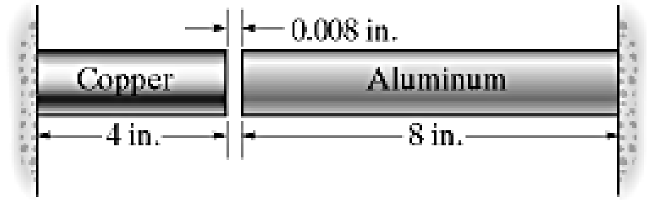
Determine the values of the stress in portions *AC* and *CB* of the steel bar shown in the figure when the temperature of the bar is – 50 °F, knowing that a close fit exists at both of the rigid supports when the temperature is 75 °F. Use the values $E = 20\ (10^6)$ psi and $\alpha = 6.5\ (10^{-6})/^{\circ}\text{F}$ for steel.





Example 5.9(Hibbeler)

The two circular rod segments, one of aluminum and the other of copper, are fixed to the rigid walls such that there is a gap of 0.008 in. between them when T_1 = 60 ^{0}F. Each rod has a diameter of 1.25 in., α_{al} = 13(10^{-6})/^{0}F, E_{al} = 10(10^3) ksi, α_{cu} = 9.4(10^{-6})/^{0}F, E_{cu} = 18(10^3) ksi. Determine the average normal stress in each rod if T_2 = 300 ^{0}F, and also calculate the new length of the aluminum segment.



Example 5.10(Hibbeler)

The rigid bar is fixed to the top of the three posts made of A-36 steel and 2014-T6 aluminum. The posts each have a length of 250 mm when no load is applied to the bar, and the temperature is $T_1 = 20^\circ$ C. Determine the force supported by each post if the bar is subjected to a uniform distributed load of 150 kN/m and the temperature is raised to $T_2 = 20^\circ$ C.