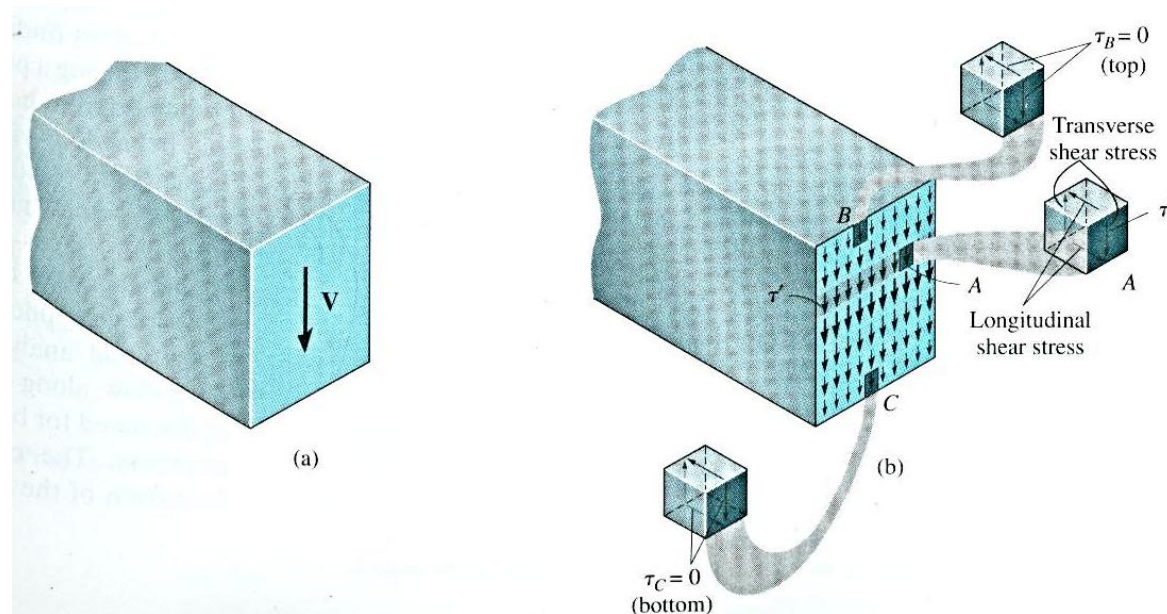


7 TRANSVERSE SHEAR

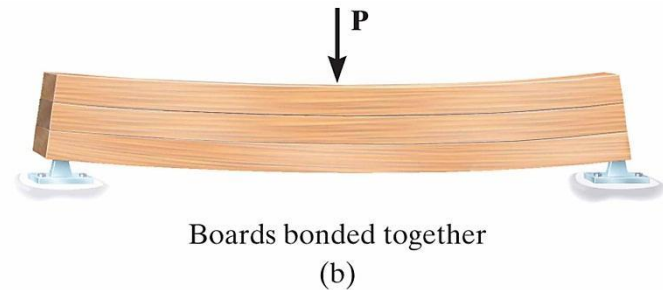
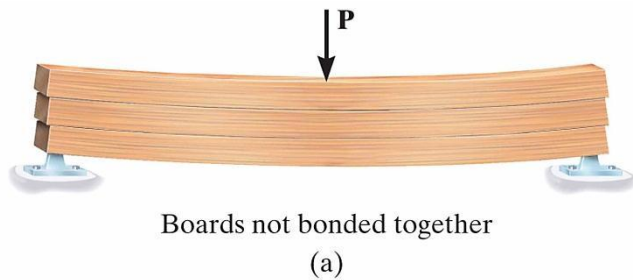
Before we develop a relationship that describes the shear-stress distribution over the cross section of a beam, we will make some preliminary remarks regarding the way shear acts within the beam. Realize that since beams are generally subjected to transverse loadings, these loadings not only cause an internal moment in the beam but also an internal shear force. This force V , shown in figure(a), is necessary for translational equilibrium, and it is the result of a **transverse shear stress** distribution that acts over the beam's cross section, figure(b).

As a result of this distribution, notice that associated **longitudinal shear stresses** will also act along longitudinal planes of the beam. For example, a typical element removed from the interior point A on the cross section is subjected to both transverse and longitudinal shear stress as shown in figure(b). Also note that the longitudinal shear stress at points B and C, located on the top and the bottom boundaries of the beam, must be zero since the top and bottom surfaces of the beam are free of any load. Consequently, the transverse shear stress at these points must also be zero.



To physically illustrate why shear stress develops on the longitudinal planes of the beam, consider the beam to be made from three boards, figure(a). If the top and bottom surfaces of each board are smooth, and the boards are not bonded together, then application of the load P will cause the boards to slide relative to one another, and so the beam will deflect as shown.

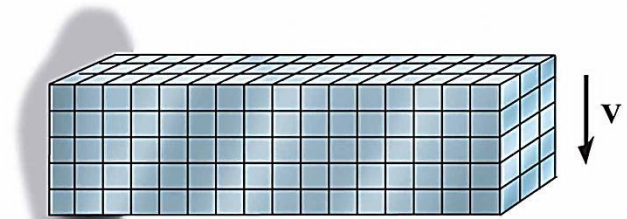
On the other hand, if the boards are bonded together, then the longitudinal shear stresses between the boards will prevent the relative sliding of the boards, and consequently the beam will act as a single unit, figure(b)



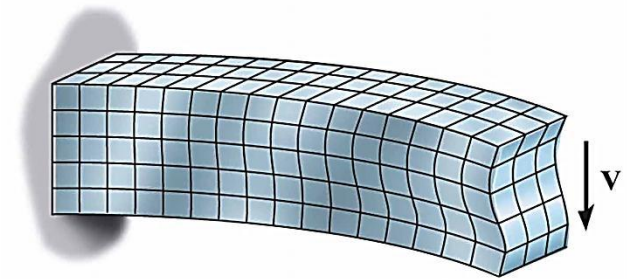
7 TRANSVERSE SHEAR

As a result of the shear stress, shear strains will be developed and these will tend to distort the cross section in a rather complex manner. For example, consider a bar made of a highly deformable material and marked with horizontal and vertical grid lines, figure(a). When the shear force is applied, it tends to deform these lines into the pattern shown in figure(b). This nonuniform shear-strain distribution will cause the cross section to warp.

Recall that in the development of the flexure formula we assumed that cross sections must remain plane and perpendicular to the longitudinal axis of the beam after deformation. Although this is violated when the beam is subjected to both bending and shear, we can generally assume the cross-sectional warping described above is small enough so that it can be neglected.



(a) Before deformation

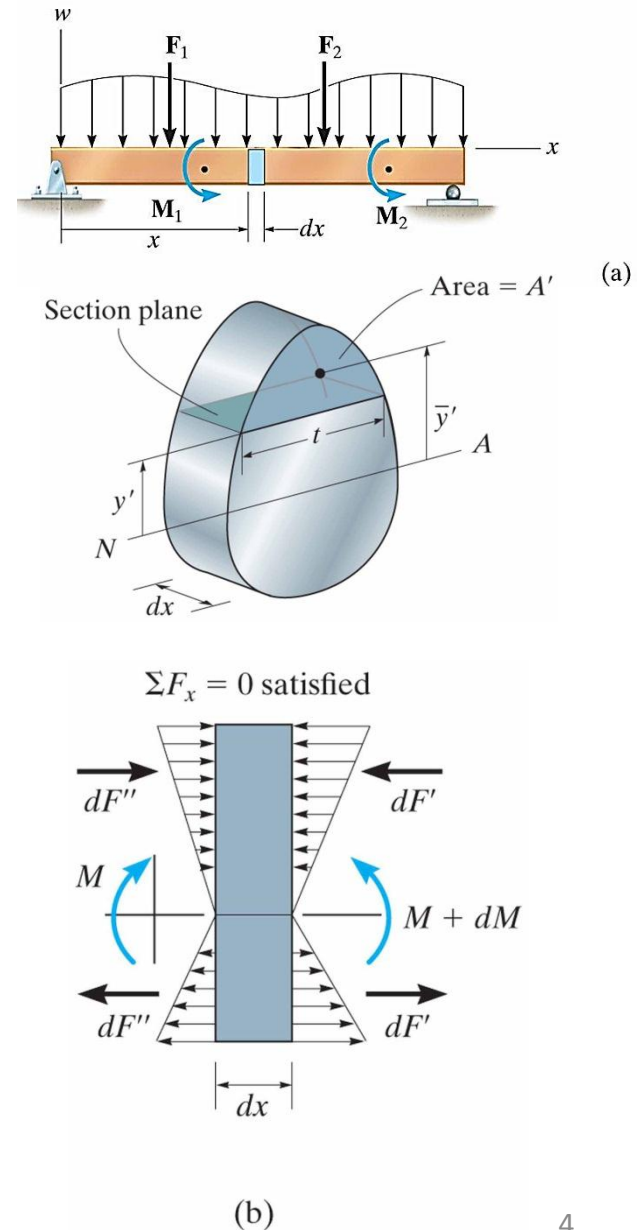


(b) After deformation

7.2 SHEAR FORMULA

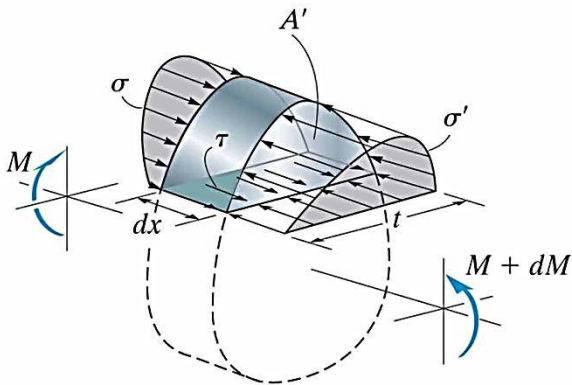
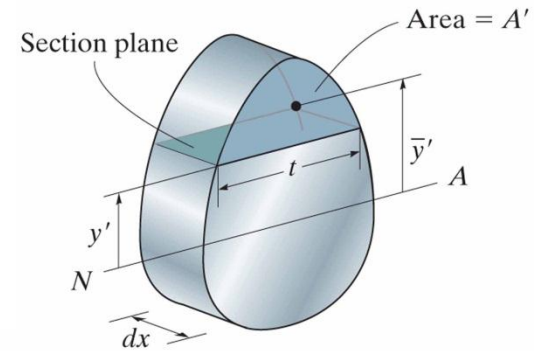
Development of a relationship between the shear-stress distribution, acting over the cross section of a beam, and the resultant shear force at the section is based on a study of the longitudinal shear stress and the results of $V=dM/dx$. To show how this relationship is established, we will consider the horizontal force equilibrium of a portion of the element taken from the beam in figure (a).

First let the element's thickness $\Delta x \rightarrow dx$, figure(a), and consider a free-body diagram of the entire element that shows only the normal-stress distribution acting on the element, figure(b). This distribution is caused by the bending moments M and $M+dM$. We have excluded the effects of V , $V+dV$, and $w(x)$ on the free-body diagram since these loadings are vertical and will therefore not be involved in a horizontal force summation. The element in figure(b) will indeed satisfy $\sum F_x = 0$ since the stress distribution on each side of the element forms only a couple moment and therefore a zero force resultant.

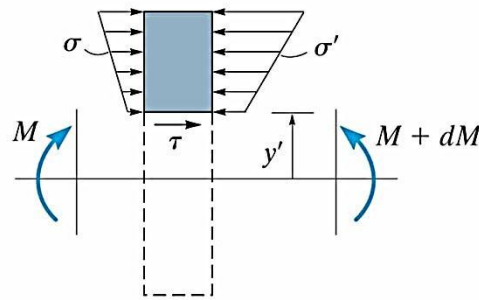


Now consider the shaded top segment of the element that has been sectioned at y' from the neutral axis, figure(a). This segment has a width t at the section, and the cross-sectional sides each have an area A' . Because the resultant moments on each side of the element differ by dM , it can be seen in figure(c) that $\sum F_x = 0$ will not be satisfied unless a longitudinal shear stress τ acts over the bottom face of the segment. In the following analysis, we will assume this shear stress is constant across the width t of the bottom face. It acts on the area $t dx$. Applying the equation of horizontal force equilibrium, and using the flexure formula, we have

$$\begin{aligned} \sum F_x = 0; \quad & \int_{A'} \sigma' dA - \int_{A'} \sigma dA - \tau(t dx) = 0 \\ \int_{A'} \left(\frac{M + dM}{I} \right) y dA - \int_{A'} \left(\frac{M}{I} \right) y dA - \tau(t dx) &= 0 \\ \left(\frac{dM}{I} \right) \int_{A'} y dA &= \tau(t dx) \end{aligned}$$



Three-dimensional view



(c)

Profile view

Solving for τ , we get

$$\tau = \frac{1}{It} \left(\frac{dM}{dx} \right) \int_{A'} y dA$$

This equation can be simplified by noting that $V = dM/dx$. Also, the integral represents the first moment of the area A' about the neutral axis. We will denote it by the symbol Q .

$$\tau = \frac{VQ}{It}$$

Here

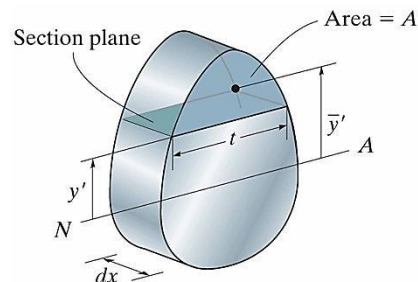
τ = the shear stress in the member at the point located a distance y' from the neutral axis, this stress is assumed to be constant and therefore averaged across the width t of the member.

V = the internal resultant shear force, determined from the method of sections and the equation of equilibrium

I = the moment of inertia of the entire cross-sectional area computed about the neutral axis.

t = the width of the member's cross-sectional area, measured at the point where τ is to be determined

$Q = \int_{A'} y dA' = \bar{y}' A'$, where A' is the top(or bottom) portion of the member's cross-sectional area, defined from the section where t is measured, and \bar{y}' is the distance to the centroid of A' , measured from the neutral axis.



$$\tau = \frac{VQ}{It}$$

The above equation is referred to as *the shear formula*. Although in the derivation we considered only the shear stresses acting on the beam's longitudinal plane, the formula applies as well for finding the transverse shear stress on the beam's cross sectional area. This, of course, is because the transverse and longitudinal shear stresses are complementary and numerically equal.

Since this equation was derived from the flexure formula, it is necessary that the material behave in a linear-elastic manner and have a modulus of elasticity that is the same in tension as it is in compression.

7.3 SHEAR STRESSES IN BEAMS

Rectangular Cross Section. Consider the beam to have a rectangular cross section of width b and height h as shown in figure(a). The distribution of the shear stress throughout the cross section can be determined by computing the shear stress at an arbitrary height y from the neutral axis, figure(b), and then plotting this function. Here the dark shaded area A' will be used for computing τ . Hence

$$Q = \bar{y}'A' = \left[y + \frac{1}{2} \left(\frac{h}{2} - y \right) \right] \left(\frac{h}{2} - y \right) b$$

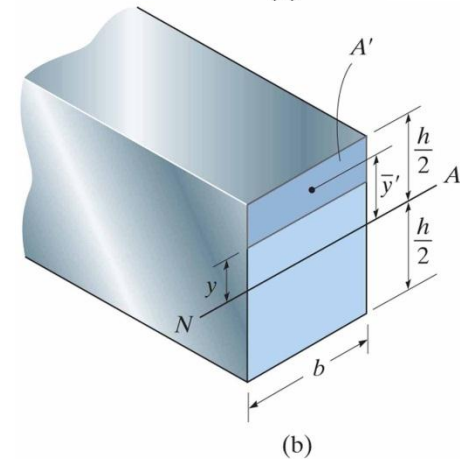
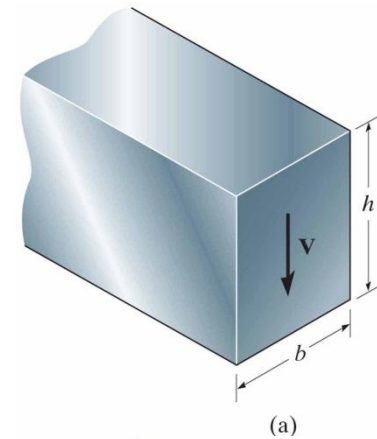
$$= \frac{1}{2} \left(\frac{h^2}{4} - y^2 \right) b$$

Applying the shear formula, we have

$$\tau = \frac{VQ}{It} = \frac{V \left(\frac{1}{2} \right) \left[\left(\frac{h^2}{4} \right) - y^2 \right] b}{\left(\frac{1}{12} b h^3 \right) b}$$

or

$$\tau = \frac{6V}{bh^3} \left(\frac{h^2}{4} - y^2 \right)$$



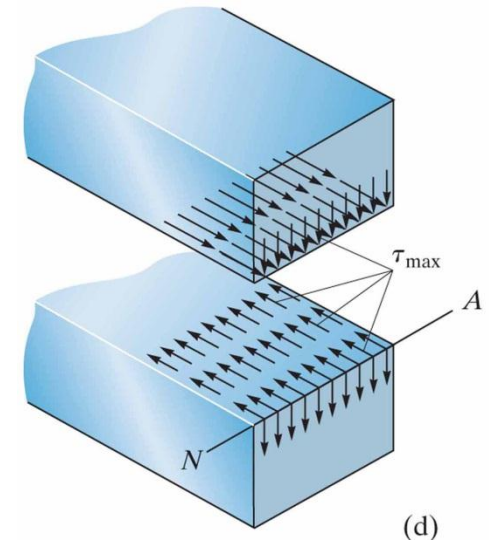
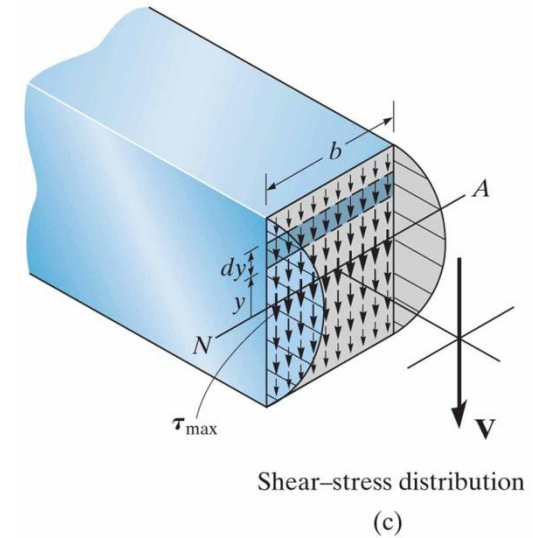
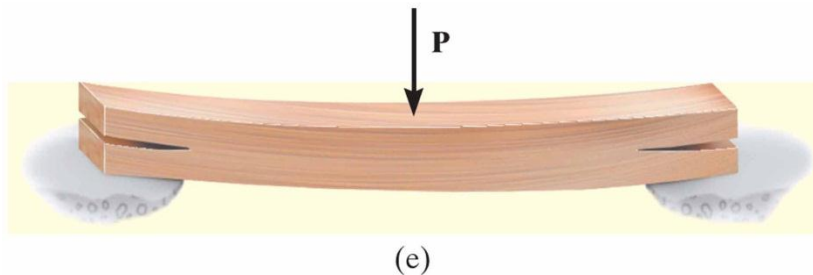
This result indicates that the shear-stress distribution over the cross section is parabolic. As shown in figure(c), the intensity varies from zero at the top and bottom, $y = \pm h/2$, to a maximum value at the neutral axis, $y = 0$. Specifically, since the area of the cross section is $A = bh$, then at $y = 0$ we have

$$\tau_{\max} = 1.5 \frac{V}{A}$$

By comparison, τ_{\max} is 50% greater than the average shear stress determined from $\tau_{\text{avg}} = V/A$.

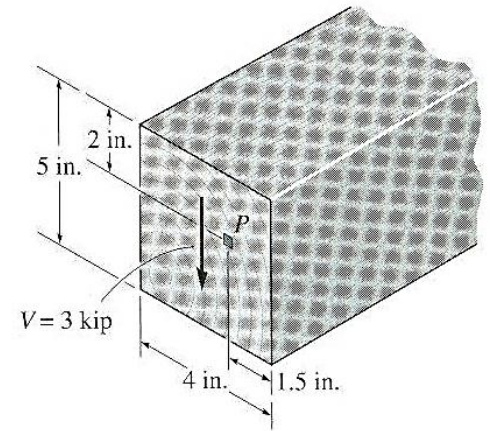
It is important to remember that for every τ acting on the cross sectional area in figure(c), there is a corresponding τ acting in the longitudinal direction along the beam. For example, if the beam is sectioned by a longitudinal plane through its neutral axis, then as noted above, the maximum shear stress acts on this plane, figure(d).

It is this stress that can cause a timber beam to fail as shown in figure(e). Here horizontal splitting of the wood starts to occur through the neutral axis at the beam's ends, since there the vertical reactions subject the beam to large shear stress and wood has a low resistance to shear along its grains, which are oriented in the longitudinal directions.



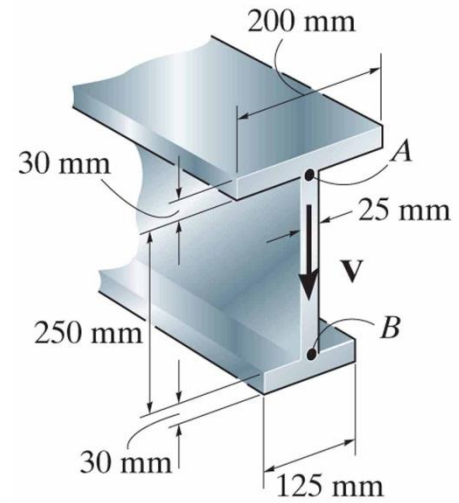
Example 7.1

The beam shown in figure is made of wood and is subjected to a resultant internal vertical shear force $V = 3$ kip. (a) determine the shear stress in the beam at point P , and (b) compute the maximum shear stress in the beam.



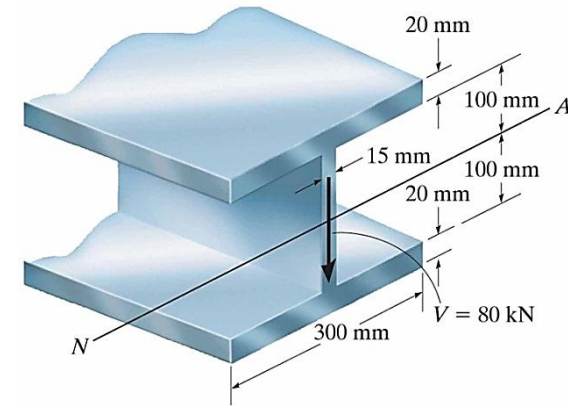
Example 7.2

If the beam is subjected to a shear of $V = 15 \text{ kN}$, determine the web's shear stress at A and B . Indicate the shear-stress components on a volume element located at these points.



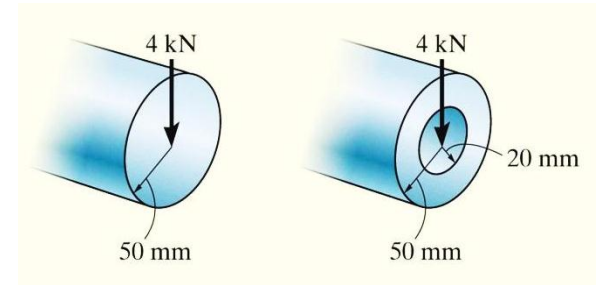
Example 7.3

A steel wide-flange beam has the dimensions shown in the figure. If it is subjected to a shear of $V = 80 \text{ kN}$, (a) plot the shear-stress distribution acting over the beam's cross-sectional area, and (b) determine the shear force resisted by the web.



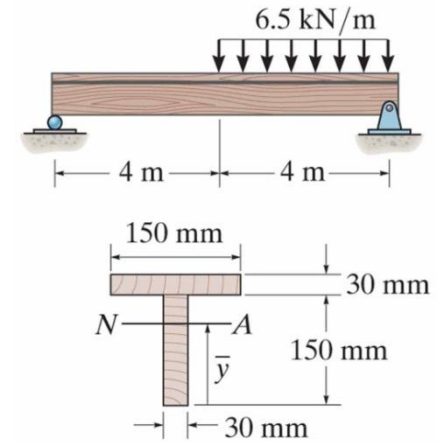
Example 7.4

The solid shaft and tube shown in the figure are subjected to the shear force of 4 kN. Determine the shear stress acting over the diameter of each cross section.



Example 7.5

The beam shown in the figure is made from two boards. Determine the maximum shear stress in the glue necessary to hold the boards together along the seam where they are joined.



Example 7.6

If the T-beam is subjected to a vertical shear of $V = 12$ kip, determine the maximum shear stress in the beam. Also compute the shear-stress jump at the flange-web junction AB . Sketch the variation of the shear-stress intensity over the entire cross section.

