8.COMBINED LOADING

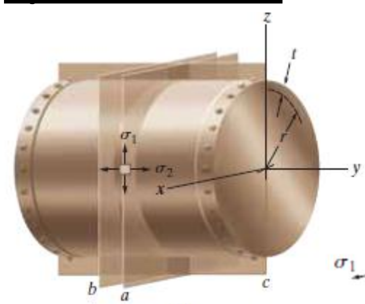
8.1. Thin-Walled Pressure Vessels

Cylindrical or spherical vessels are commonly used in industry to serve as boilers or tanks. When under pressure, the material of which they are made is subjected to loading from all directions. Although this is the case, the vessel can be analyzed in a simple manner provided it has a thin wall. In general, thin wall refers to a vessel having an inner-radius-to wall-thickness ratio of 10 or more ($r/t \geq 10$). Specifically, when $r/t = 10$ the results of a thin-wall analysis will predict a stress that is approximately 4% less than the actual maximum stress in the vessel. For larger r/t ratios this error will be even smaller.

Figure: 08-01-UN
Cylindrical pressure vessels, such as this gas tank, have semi-spherical end caps rather than flat ones in order to reduce the stress in the tank.

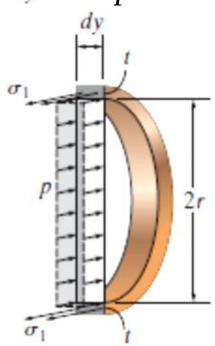
Provided the vessel wall is thin, the stress distribution throughout its thickness wll not vary significantly, and so we will assume that it is uniform or constant.

<u>Cylindrical Vessels</u>



(a)

Consider the cylindrical vessel in Figure(a), subjected to pressure p that developed within the vessel by a contained gas. Due to this loading, a small element of the vessel that is sufficiently removed from the ends and oriented as shown in figure(b), is subjected to normal stresses σ_1 in the *circumferential* or *hoop* direction and σ_2 in the *longitudinal* or *axial direction.*



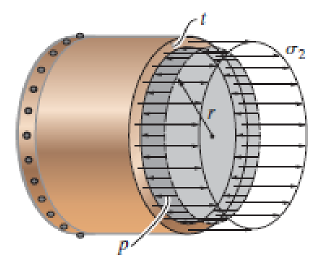
(b)

$$\sum F_x = 0; \quad 2[\sigma_1(t\,dy)] - p(2r\,dy) = 0$$

$$\sigma_1 = \frac{p\,r}{t}$$

$$\sum F_y = 0; \qquad \sigma_2\,(2\pi r t) - p(\pi r^2) = 0$$

$$\sigma_2 = \frac{p\,r}{2t}$$



(c)

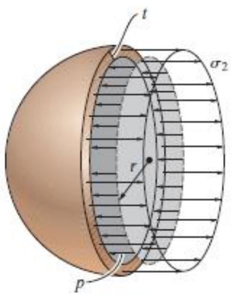
t : the thickness of the wall

r : the inner radius of the cylinder

p : the internal gauge pressure developed by the contained gas.

Spherical Vessels

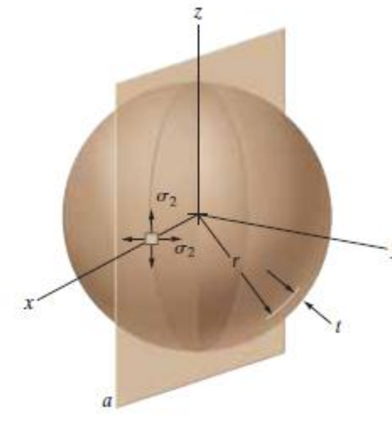
We can analyze a spherical pressure vessel in a similar manner. To do this, consider the vessel to have a wall thickness *t*, inner radius *r*, and subjected to internal gauge pressure *p*, Figure(a). If the vessel is sectioned in half, the resulting free-body diagram is shown figure(b).



(b)

$$\sum F_y = 0; \qquad \sigma_2\,(2\pi r t) - p(\pi r^2) = 0$$

$$\sigma_2 = \frac{p\,r}{2t}$$

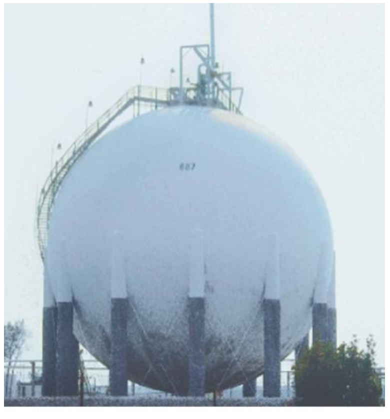


(a)

This is the same result as that obtained for the longitudinal stress in the cylindrical pressure vessel. Furthermore, from the analysis, this stress will be the same regardless of the orientation of the hemispheric free-body diagram. Consequently, a small element of the material is subjected to the state of stress shown in Figure(a)

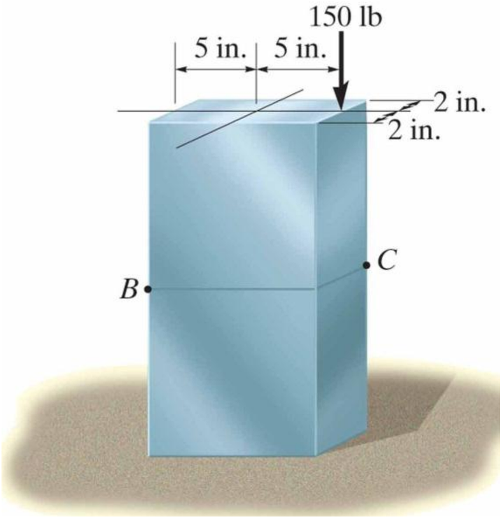
Example 8.1(Hibbeler)

A cylindrical pressure vessel has an inner diameter of 1.2 m and a thickness of 12 mm. Determine the maximum internal pressure it can sustain so that neither its circumferential nor its longitudinal stress component exceeds 140 MPa. Under the same conditions, what is the maximum internal pressure that a similar-size spherical vessel can sustain?



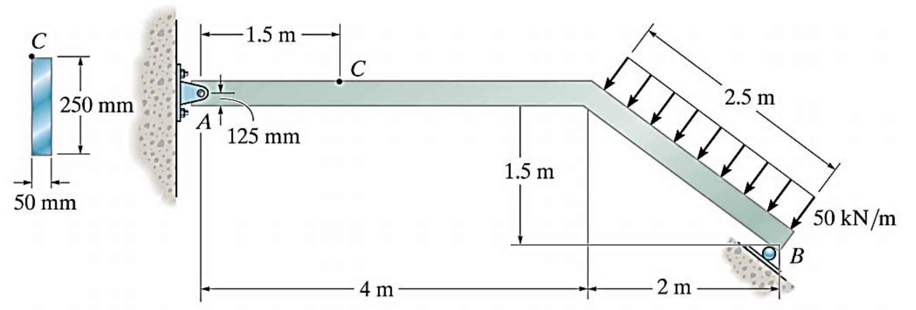
Example 8.2(hibbeler)

A force of 150 lb is applied to the edge of the member shown in the figure. Neglect the weight of the member and determine the state of stress at points *B* and *C*.



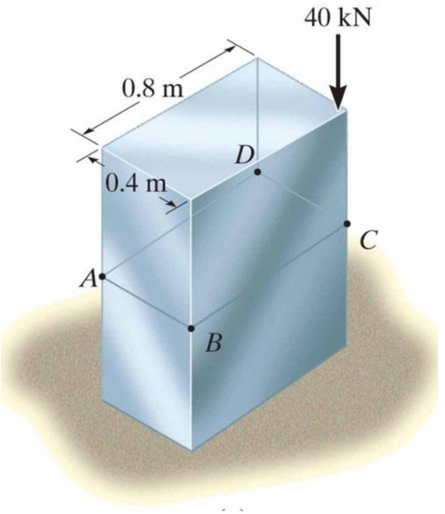
Example 8.3(Hibbeler)

The member shown in the the figure has a rectangular cross section. Determine the state of stress that the loading produces at point *C*.



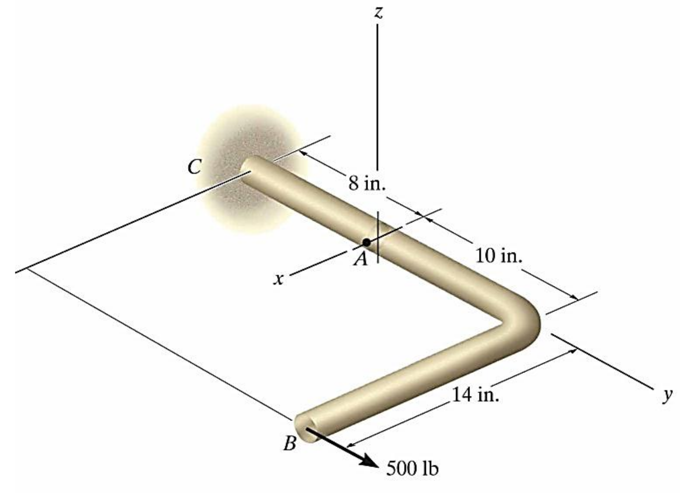
Example 8.4(Hibbeler)

The rectangular block of negligible weight in the figure is subjected to a vertical force of 40 kN, which is applied to its corner. Determine the largest normal stress acting on a section through *ABCD*.



Example 8.5(Hibbeler)

The solid rod shown in the figure has a radius of 0.75 in. If it is subjected to the force of 500 lb, determine the state of stress at point *A*.



Example 8.6(Hibbeler)

The solid rod shown in the figure has a radius of 0.75 in. If it is subjected to the force of 800 lb, determine the state of stress at point *A*.