# Outcome of Today's Lecture

- **After completing this lecture…**
- The students should be able to:
  - Define and apply the present worth criteria
  - Compare the two competing alternative choices using present worth
  - Apply the present worth model in cases with equal, unequal and infinite project lives
  - Compare multiple alternatives using present worth criteria

# Present Worth Analysis

- In the last two chapters we learned:
  - The idea of equivalence
  - Various compound interest factors to find the equivalent values
- Now we begin to make use of these subjects.
  - To choose among various feasible alternatives, we employ the idea of economic efficiency.
- Different techniques exist to choose among alternatives
  - Present worth analysis
  - Equivalent uniform annual analysis
  - Rate of return
  - Benefit-cost ratio

# Concepts and Assumptions

- End-of-Year Convention
  - All the series of receipts and disbursements occur at the end of the time period. If not, replace them by their equivalent values at the end of the year.

- Viewpoint of Economic Analysis Studies
  - Usually we take the point of view of an entire firm when doing an industrial economic analysis.
  - What is best for the entire firm may not be best for smaller groups in the firm. It is easy to make a bad decision if we ignore part of the problem.

- Sunk Costs
  - Events that have occurred in the past have no bearing on what we should do in the future.
  - What is important are the current and future differences between alternatives.

# Concepts and Assumptions

- Borrowed Money Viewpoint:
- There are two aspects of money to determine:
  - Financing – obtaining the money
  - Investment – spending the money
- Basic assumption is that money required to finance alternatives is considered to be borrowed at interest rate, *i*.
- Effect of Inflation
  - For the time being we assume prices are stable.
- Stability
  - The economic situation is stable.
- Determinism
  - All data of interest are known deterministically (no randomness) and can be accurately predicted.

# Present Worth Analysis

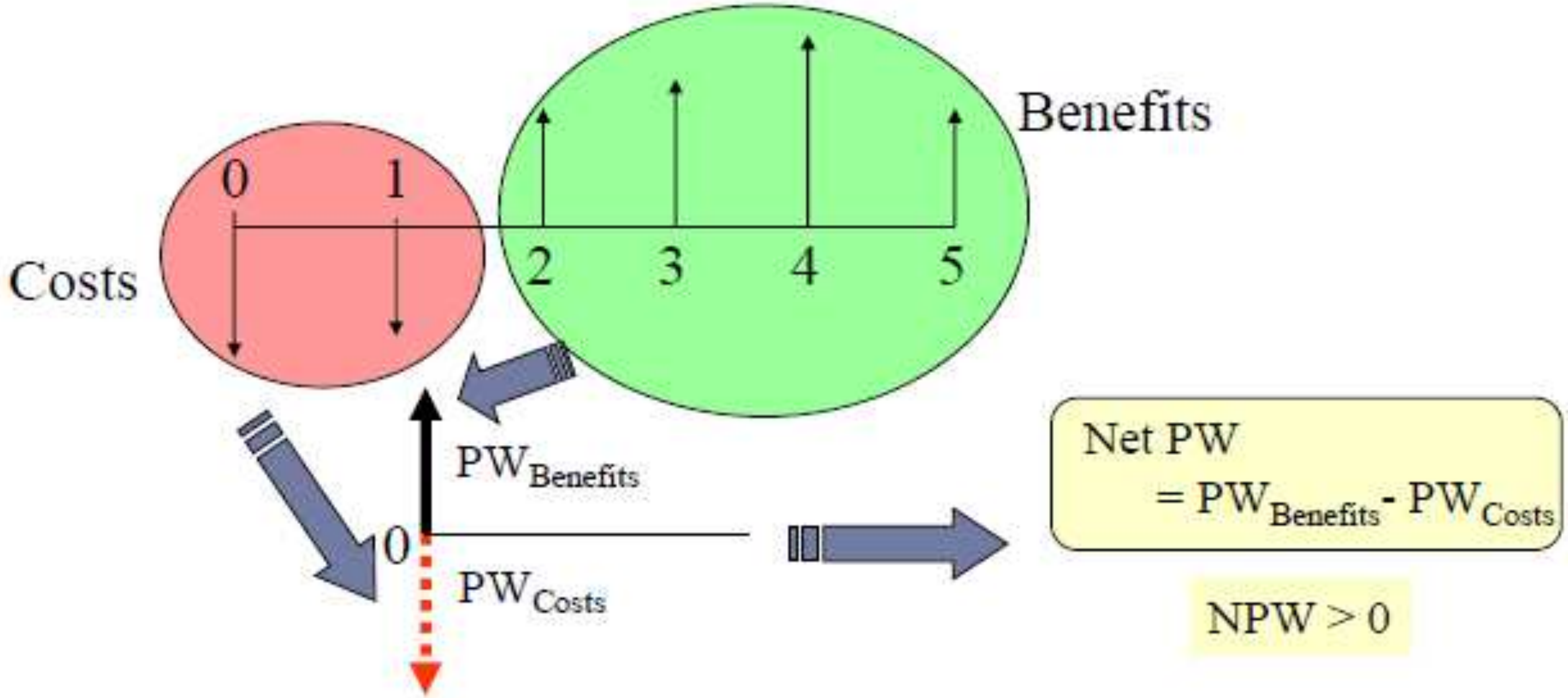
- Present worth analysis (PWA) can resolve alternatives into equivalent present consequences.
  - PWA is most frequently used to determine present value of future money receipts and disbursements.

Present Worth Analysis

| | Situation | Criterion |
|---|---|---|
| Fixed input | Amount of money or other input resources are fixed | Maximize present worth of benefits or other outputs |
| Fixed output | There is a fixed task, benefit, or other output to be accomplished | Minimize present worth of costs or other inputs |
| Neither input nor output is fixed | Neither amount of money, or other inputs, nor amount of benefits, or other output, is fixed | Maximize (present worth of benefits *minus* present worth of costs), that is, maximize net present worth |

# Present Worth Analysis

- **Principle:** Compute the equivalent net PW (PW of benefits-PW of Costs) at present (n=0) for a given interest rate of i.




- **Decision Rule:** Accept the project if the net PW is positive

# Analysis Period

- Careful consideration must be given to the time period covered by the analysis. The time period is usually called the analysis period, also called the planning horizon.
- The analysis period should be determined from the situation. The period can be:
  - Short: (2-5 years), e.g., PC manufacture
  - Intermediate length: (15-20 years) e.g., Steel manufacture
  - Indefinite length: (50 years or so) e.g., National government
- Three different analysis-period situations occur:
  - *1. Useful lives are the same for all alternatives and equal to the analysis period.*
  - *2. Useful lives are different among the alternatives, and at least one does not match the analysis period.*
  - *3. The analysis period is effectively infinite.*

# Same-Length Analysis Periods

- **Example 5-1:** A firm is considering buying device A or B. Each device can reduce costs. Each device has a useful life of 5 years, and no salvage value. Device A saves $300 a year, device B saves $400 the first year, but savings in later years decrease by $50 a year. Interest is 7%. Which device should they choose?

# Same-Length Analysis Periods

- **Example 5-2:** A municipality plans to build an aqueduct to carry water. The city can:
- a)Spend $300 million now, and enlarge the aqueduct in 25 years for $350 million more,
- b)Construct a full-size aqueduct now for $400 million.
- The analysis period is 50 years. We ignore maintenance costs. Interest is 6%. There is no salvage value.

# Same-Length Analysis Periods

- **Example 5-3:** The mailroom needs new equipment. Alternative choices are as below: Either choice will provide the same desired level of (fixed) output.

| Make | Cost | Useful Life | EOL salvage value |
|---|---|---|---|
| Choice 1 | $1,500 | 5 years | $200 |
| Choice 2 | $1,600 | 5 years | $325 |

EOL: End of Life

# Same-Length Analysis Periods

- Suppose each has a maintenance cost of C per year. Then each would have a PW of maintenance costs of: C (P/A,7%,5).

- The revised PW of the costs would be:
  - Choice 1: $1358 + C (P/A,7%,5)
  - Choice 2: $1367 + C (P/A,7%,5)

- The difference between the PW's remains the same. Unless other factors not considered above, we would still prefer Choice 1.

# Same-Length Analysis Periods

- **Example 5-4:** We must choose a weighing scale to install in a package filling operation in a plant. Either scale will allow better control of the filling operation, and result in less overfilling. Each scale has a life of 6 years. Interest is 8%.

| Alternative | Cost | Uniform annual benefit | EOL salvage value |
|---|---|---|---|
| Choice 1 | $2,000 | $450 | $100 |
| Choice 2 | $3,000 | $600 | $700 |

- We use the formula:

$$NPW = \text{PW of benefits} - \text{PW of costs}$$

# Same-Length Analysis Periods

- Choice 1:

  NPW = 450 (P/A,8%,6) + 100 (P/F,8%,6) – 2000
  = 450 (4.623) + 100 (0.6302) - 2000
  = 2080 + 63 – 2000 = $143

- Choice 2:

  NPW = 600 (P/A,8%,6) + 700 (P/F,8%,6) – 3000
  = 600 (4.623) + 7700 (0.6302) – 3000
  = 2774 + 441 – 3000 = $215

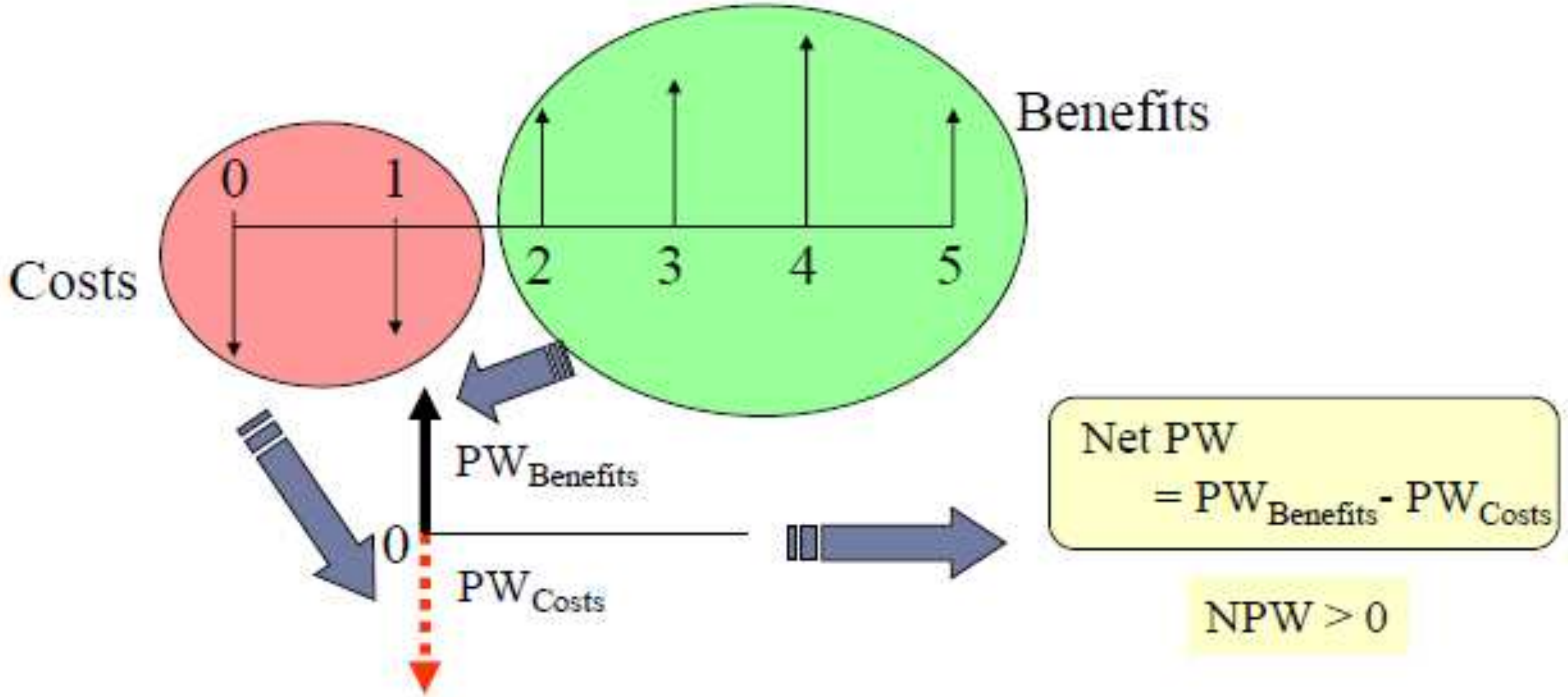
- Choice 2 looks preferable.

  - Remark. The NPV formula is of fundamental importance. It uses the fact that the PV formula is additive:

| PW (benefits – costs) = PW (benefits) – PW (costs) |
| --- |

# Present Worth Analysis

- **Principle:** Compute the equivalent net PW (PW of benefits-PW of Costs) at present (n=0) for a given interest rate of i.




- **Decision Rule:** Accept the project if the net PW is positive

# Different-Length Analysis Periods

Sometimes the useful lives of projects differ from the analysis period.

**Example:** A plant needs new equipment. Alternative choices are as follows:

| Alternative | Cost | Useful Life | EOL salvage value |
|---|---|---|---|
| Choice 1 | $1,500 | 5 years | $200 |
| Choice 2 | $1,600 | 10 years | $325 |

We no longer have a situation where either choice will provide the same desired level of (fixed) output.

Choice 1 equipment for five years is not equivalent to Choice 2 equipment for ten years.

# Different-Length Analysis Periods

- Choice 2 for 10 years:

  PW = 1600 – 325 (P/F,7%,10) = 1600 – 325 (0.5083)
  = 1600 – 165 = $1,435

- Choice 1 for 5 years:

  PW = 1500 – 200 (P/F,7%,5) = 1500 – 200 (0.7130)
  = $1,368

- We can no longer make a direct comparison:

APPLES AND ORANGES!

# Different-Length Analysis Periods

How can we solve this problem (i.e., comparing alternatives with different useful lives)?

- One possibility: Compare one Choice 2 with two Choice 1's
  - We buy a Choice 1 for $1500, use it for 5 years, get $200 salvage, buy a second Choice 1 for $1500, use it for the second 5 years, and again get $200 salvage.

- Two Choice 1's (each for 5 years, total of 10 years):

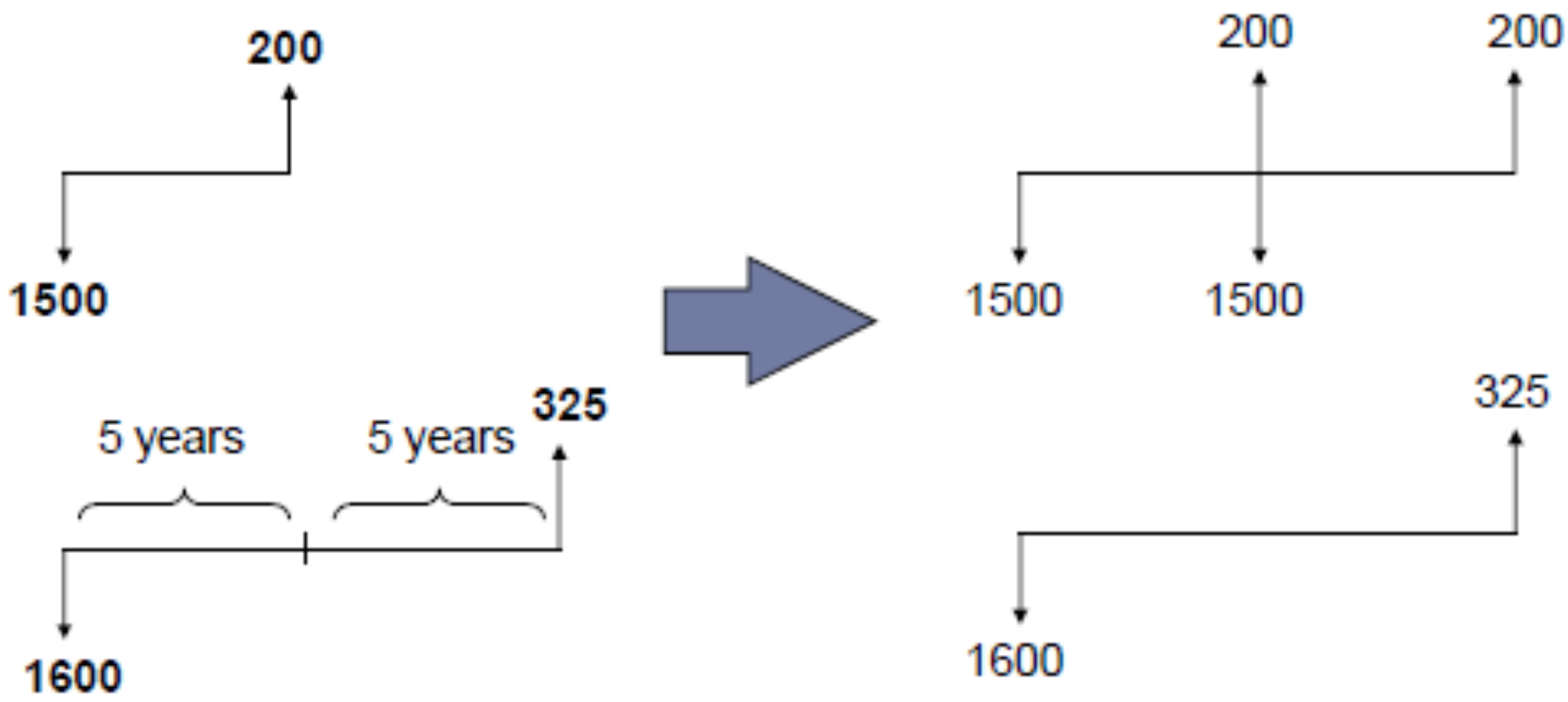
  PW = 1500 + (1500 – 200) (P/F,7%,5) – 200 (P/F,7%,10)
  = 1500 + 1300 (0.7130) – 200 (0.508)
  = 1500 + 927 – 102 = $2,325

- Choice 2 for 10 years:

  PW = 1600 – 325 (P/F,7%,10) = 1600 – 325 (0.5083)
  = 1600 – 165 = $1,435

- Now, we can clearly select Choice 2

# Different-Length Analysis Period -Repeated



# Different-Length Analysis Periods

- To compare alternatives with different analysis periods, use one of the following two methods:

- **1.Repeated: study period is equal to a the Least Common Multiplier of the lives of alternatives and assuming that economic consequences happening in initial alternative's life will repeat in all succeeding lives.**
- In previous example, we used 10 years as the analysis period (multiplier of 5 and 10)
- If one piece of equipment had a life of 7 years, and the other a life of 13 years, and we followed the same approach, we would need to use 7 (13) = 91 years. But an analysis period of 91 years is not too realistic.

- **2.Terminated: We estimate terminal values for the alternatives at some designated point prior to the end of their useful lives.**

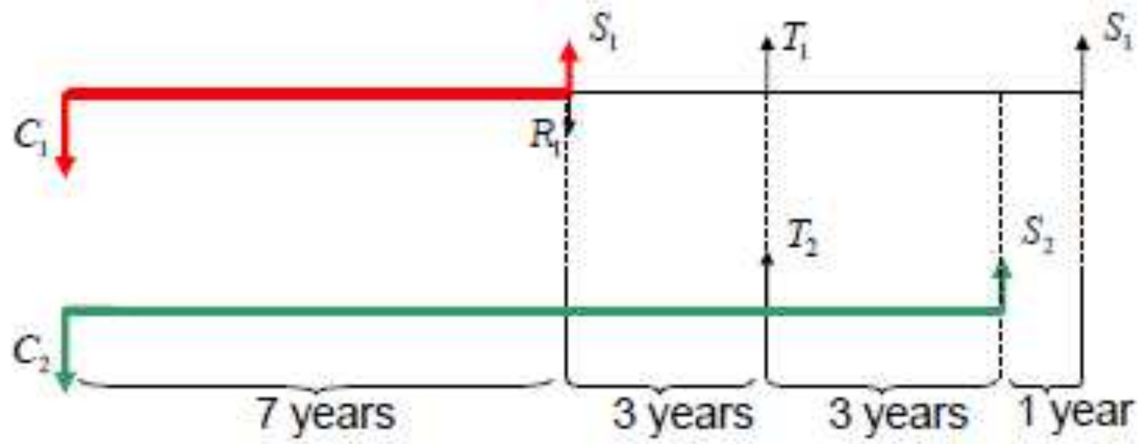
# Different-Length Analysis Periods-Terminated

**Example**

Alternative 1 (EOL=7 years):
$C_1$ = initial cost
$S_1$ = salvage value

Alternative 2 (EOL=13 years):
$C_2$ = initial cost
$S_2$ = salvage value

$R_1$ = replacement cost
$T_1$ = terminal value at end of 10th year
$T_2$ = terminal value at the end of 10th year




Present worth of costs with 10-year analysis period:

$PW_1 = C_1 + (R_1 - S_1)\ (P/F,i\%,7) - T_1\ (P/F,i\%,10)$ $PW_2 = C_2 - T_2\ (P/F,i\%,10)$

# Different-Length Analysis Periods-Terminated

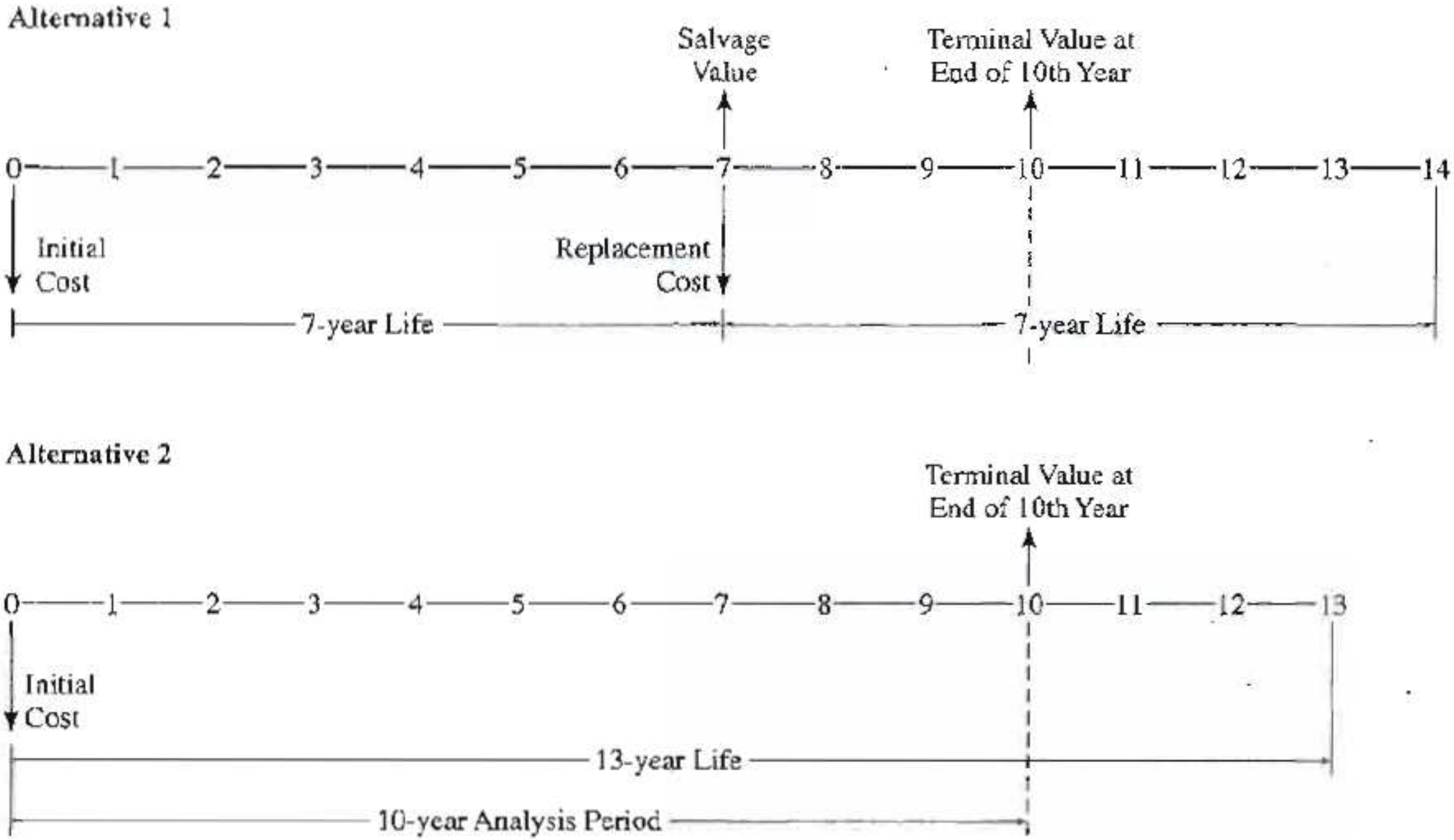
- Example: 5-5
- A diesel manufacturer is considering the two alternative production machines graphically depicted in Figure. Specific data are as follows.

| | Alt. 1 | Alt. 2 |
|---|---|---|
| Initial cost | $50,000 | $75,000 |
| Estimated salvage value at end of useful life | $10,000 | $12,000 |
| Useful life of equipment, in years | 7 | 13 |

- The manufacturer uses an interest rate of 8% and wants to use the PW method to compare these alternatives over an analysis period of 10 years.

| | Alt. 1 | Alt. 2 |
|---|---|---|
| Estimated market value, end of 10-year analysis period | $20,000 | $15,000 |

# Different-Length Analysis Periods-Terminated




Superimposing a 10-year analysis period on 7- and 13-year alternatives.

# Different-Length Analysis Periods-Terminated

$$\begin{aligned} \text{PW (Alt. 1)} &= -50{,}000 + (10{,}000 - 50{,}000)(P/F, 8\%, 7) + 20{,}000(P/F, 8\%, 10) \\ &= -50{,}000 - 40{,}000(0.5835) + 20{,}000(0.4632) \\ &= -\$64{,}076 \end{aligned}$$

$$\begin{aligned} \text{PW (Alt. 2)} &= -75{,}000 + 15{,}000(P/F, 8\%, 10) \\ &= -75{,}000 + 15{,}000(0.4632) \\ &= -\$69{,}442 \end{aligned}$$

## Infinite-Length Analysis Periods – Perpetual Projects

- Sometimes the analysis period is of indefinite length:

- The need for roads, dams, pipelines, etc. is sometimes considered permanent. This is especially true for most of civil infrastructure projects.

- The textbook refer to this situation as an infinite analysis period. Present worth analysis in this case (i.e., when the period of needed service is indefinite) is called *Capitalized Cost.*

- **Capitalized cost is the present sum of money that would need to be set aside now, at some known interest rate, to yield the funds needed to provide a service indefinitely.**

## Infinite-Length Analysis Periods –Capitalized Cost

- **Example:** A donator wants to set up a scholarship fund to provide $20,000 yearly to deserving students. The university will invest the donation, and expects it to earn 10% a year (i.e., *i=10%). How much will the donator need to donate in one lump sum so that $20,000 is available every year?*
- **Solution:**
- If person donates $200K, 10% of it is $20K:
- Year 1: *P becomes P + Pi so $200K grows in one year to $220K.*
- A scholarship of $20K is funded, so $200K remains ($220K -$20K)
- Year 2: $200K grows in one year to $220K.
- A scholarship of $20K is funded, so $220K -$20K = $200K remains and so on…
- We see that when i= 10% and A = $20K, P = A/i=$20,000/0.10 = $200,000

# Comparing Multiple Alternatives

- The previous approach generalizes to multiple alternatives. Just compute the NPW of each alternative, and then pick the one with the best NPW.

- **Example 5-8:** A contractor must build a six-miles-long tunnel. During the 5-year construction period, the contractor will need water from a nearby stream. He will construct a pipeline to carry the water to the main construction site. Various pipe diameters are being considered.

| Pipe diameter | 2" | 3" | 4" | 6" |
|---|---|---|---|---|
| Installed cost of pipeline & pump | $22,000 | $23,000 | $25,000 | $30,000 |
| Pumping Cost per hour | $1.20 | $0.65 | $0.50 | $0.40 |

  - The salvage value of the pipe and the cost to remove them may be ignored.
  - The pump will operate 2,000 hours per year.
  - The lowest interest rate at which the contractor is willing to invest money is 7%. This is called the minimum attractive rate of return (MARR).

# Comparing Multiple Alternatives

- Solution:
  - We compute the present worth of the cost for each alternative. This cost is equal to the installed cost of the pipeline and pump, plus the present worth of five years of pumping costs.
- Pumping costs:
  - 2" pipe: 1.2 (2000) (P/A,7%,5) = 1.2 (2000) (4.100) = $9,840
  - 3" pipe: 0.65 (2000) (4.100) = $5,330
  - 4" pipe: 0.50 (2000) (4.100) = $4,100
  - 6" pipe: 0.40 (2000) (4.100) = $3,280
- PW of all costs:
  - 2" pipe: $22,000 + $9,840 = $31,840
  - 3" pipe: $23,000 + $5,330 = $28,330
  - 4" pipe:$25,000 + $4,100 = $29,100
  - $6" pipe: $30,000 + $3,280 = $33,280

- Question: Which pipe size would you choose?

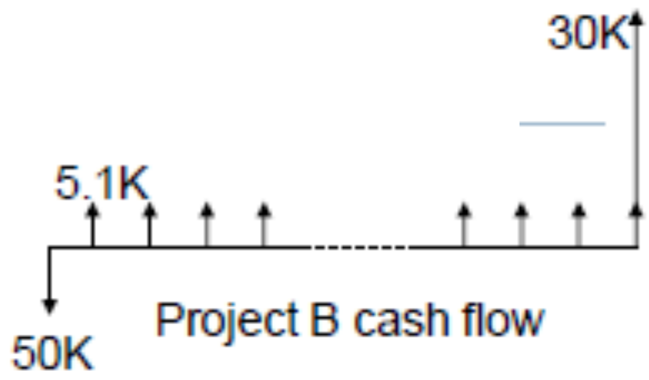
# Comparing Multiple Alternatives

- **Example 5-9:** An investor paid $8,000 to a consulting firm to analyze what to do with a parcel of land on the edge of town he bought for $30,000. The consultants suggest four alternatives.
- An investor always has the alternative to do nothing. It is not too exciting, but may be better than other choices.

| Alternative | Total Investment | Uniform net annual benefit | Terminal value, year 20 |
| --- | --- | --- | --- |
| A: Do nothing | $0 | $0 | $0 |
| B: Mini market | $50,000 | $5,100 | $30,000 |
| C: Gas station | $95,000 | $10,500 | $30,000 |
| D: Small motel | $350,000 | $36,000 | $150,000 |

# Comparing Multiple Alternatives

- **Solution:**
- We maximize the net present worth.




- Alternative A: do nothing,
- NPW = 0
- Alternative B: Mini market:
- NPW= -50,000 + 5,100 (P/A,10%,20) + 30000 (P/F,10%,20)
- = -50,000 + 5,100 (8.514) + 30000(0.1486)
- = -50,000 + 43,420 + 4,460 = -$2,120

# Comparing Multiple Alternatives

- <u>Alternative C: Gas station</u>
- NPW = -95000 + 10500 (P/A,10%,20) + 30000 (P/F,10%,20)
- = -95,000 + 10500 (8.514) + 30000(0.1486)
- = -95,000 + 9,400 + 4,460 = -$1,140

- <u>Alternative D: Small motel</u>
- NPW = -350,000 + 36000 (P/A,10%,20) + 150000 (P/F,10%,20)
- = -350,000 + 36000 (8.514) + 150000(0.1486)
- = -350,000 + 306,500 + 23,290 = -$21,210

- In this case it is best to do nothing. The $8,000 the investor spent for consulting services is a past cost, and is called a sunk cost.

# Summary

- An Alternative is a "course of action"
- Alternatives are developed from project proposals
- If there are no alternatives to consider then there really is no problem to solve!
- Mutually exclusive project proposals compete against each other (pick one and only one)
- Given a set of feasible, mutually exclusive alternatives, engineering economics is used to identify the "best" alternative