## Engineering Economics

Annual Cash Flow Analysis

## Annual Cash Flow Analysis

- Concepts of Annual Cash Flow Analysis
- Comparing Alternatives using Annual Cash Flow Analysis:
- Same-Length Analysis Period
- Different-Length Analysis Periods
- Infinite-Length Analysis Period
- Other Analysis Periods


## Techniques for Cash Flow Analysis

- Present Worth Analysis:
- $\mathrm{PW}_{\mathrm{A}}=-\mathrm{R}_{\mathrm{A}}+\mathrm{A}_{\mathrm{A}}(\mathrm{P} / \mathrm{A}, \mathrm{i}, \mathrm{n})+\mathrm{S}_{\mathrm{A}}(\mathrm{P} / \mathrm{F}, \mathrm{i}, \mathrm{n})$
- $\mathrm{PW}_{\mathrm{B}}=-\mathrm{R}_{\mathrm{B}}+\mathrm{R}_{\mathrm{B}}(\mathrm{P} / \mathrm{A}, \mathrm{i}, \mathrm{n})+\mathrm{S}_{\mathrm{B}}(\mathrm{P} / \mathrm{F}, \mathrm{i}, \mathrm{n})$
- If $\mathrm{PW}_{\mathrm{A}}>\mathrm{PW}_{\mathrm{B}}=>$ Choose A ,
b otherwise => choose B.
- Annual Cash Flow Analysis:

- EUAB: Equivalent Uniform Annual Benefit
- $\mathrm{EUAB}_{\mathrm{A}}=\mathrm{A}_{\mathrm{A}}+\mathrm{S}_{\mathrm{A}}(\mathrm{A} / \mathrm{F}, \mathrm{i}, \mathrm{n}) ; \mathrm{EUAB}_{\mathrm{B}}=\mathrm{A}_{\mathrm{B}}+\mathrm{S}_{\mathrm{B}}(\mathrm{A} / \mathrm{F}, \mathrm{i}, \mathrm{n})$
- (EUAB-EUAC) $A_{A}=A+S_{A}(A / F, i, n)-R_{A}(A / P, i, n)$
- (EUAB-EUAC) ${ }_{B}=\mathrm{B}+\mathrm{S}_{\mathrm{B}}(\mathrm{A} / \mathrm{F}, \mathrm{i}, \mathrm{n})-\mathrm{R}_{\mathrm{B}}(\mathrm{A} / \mathrm{P}, \mathrm{i}, \mathrm{n})$

- If $(E U A B-E U A C)_{A}>(E U A B-E U A C)_{B}$
b => Choose A, otherwise => choose B.


## Problem 6-1

- Compute the value of C for the following diagram, based on " $10 \%$ interest rate.



## Problem 6-8

- As shown in the cash flow diagram, there is an annual disbursement of money that varies from year to year from $\$ 100$ to $\$ 300$ in a fixed pattern that repeats forever. If interest is $10 \%$, compute the value of $A$, also continuing forever, that is equivalent to the fluctuating disbursements.



## Problem 6-8



Pattern repeats infinitely

There is a repeating series:; $100-200-300-200$. Solving this series for A gives us the A for the infinite series.

## Problem 6-8



$$
\begin{aligned}
A= & \$ 100 \\
=\$ 100 & +[\$ 100(\mathrm{P} / \mathrm{F}, 10 \%, 2)+\$ 200(\mathrm{P} / \mathrm{F}, 10 \%, 3)+\$ 100(\mathrm{P} / \mathrm{F}, 10 \%, 4)](\mathrm{A} / \mathrm{P}, \mathrm{I} 0 \%, 4) \\
& =\$ 100+[\$ 301.20](0.3155) \\
& =\$ 195.03
\end{aligned}
$$

## Annual Cash Flow Analysis

- The basic idea is to convert all cash flows to a series of EUAW (equivalent uniform annual worth):

Net EUAW = EUAB -EUAC

- EUAC: Equivalent Uniform Annual Cost
- EUAB: Equivalent Uniform Annual Benefit
- An expenditure increases EUAC and a receipt of money decreases EUAC.
- To convert a PW of a cost to EUAC, use:

$$
\text { EUAC = (PW of cost) }(\mathrm{A} / \mathrm{P}, \mathrm{i} \%, \mathrm{n})
$$

- Where there is salvage value?

$$
A=F(A / F, i \%, n)
$$

- A salvage value will reduce EUAC and increase EUAB
- When there is an arithmetic gradient, use the (A/G, i\%, n) factor.
- If there are irregular cash flows, try to first find PW of these flows; then, EUAC may be calculated from this PW.
- Criteria for selection of an alternative:
- Maximize Net EUAW (EUAB -EUAC)
- Minimize EUAC OR Maximize EUAB


## Analysis Period Equal to Alternative Lives

- We have an ideal situation (rarely the case in 'real-life' ):
- Study period $=$ life-cycle of any of the alternatives
- Example 6-6: In addition to the do-nothing alternative, three alternatives are being considered for improving the operation of an assembly line. Each of the alternatives has a 10 -years life and a scrap value equal to $10 \%$ of its original cost. If interest is $8 \%$, which alternative should be adopted.

| Plan | A | B | C |
| ---: | :---: | :---: | :---: |
| Installed cost of equipment | $\$ 15,000$ | $\$ 25,000$ | $\$ 33,000$ |
| Material and labor savings per year | $\$ 14,000$ | $\$ 9,000$ | $\$ 14,000$ |
| Annual operating expense | $\$ 8,000$ | $\$ 6,000$ | $\$ 6,000$ |
| End-of-useful life scrap value | $\$ 1,500$ | $\$ 2,500$ | $\$ 3,300$ |

## Analysis Period Equal to Alternative Lives

Plan A


Plan B


## Analysis Period Equal to Alternative Lives

| Plan | A | B | C | Do <br> Nothing |
| :---: | :---: | :---: | :---: | :---: |
| EUAB |  |  |  |  |
| Material and labor savings per year | \$14,000 | \$9,000 | \$14,000 | \$0 |
| Scrap value (A/F, $8 \%, 10$ ) | \$104 | \$172 | \$228 | 0 |
| Total EUAB | \$14,104 | \$9,172 | \$14,228 | \$0 |
| EUAC |  |  |  |  |
| Installed cost (A/P,8\%,10) | \$2,235 | \$3,725 | \$4,927 | 0 |
| Annual operating expenses | \$8,000 | \$6,000 | \$6,000 | 0 |
| Total EUAC | \$10,235 | \$9,725 | \$10,927 | 0 |
| EUAB - EUAC | \% \$3,869** | -\$553 | \$3,311 | \$0 |
| $\begin{aligned} & (\mathrm{A} / \mathrm{F}, 8 \%, 10)=0.0690 \\ & (\mathrm{~A} / \mathrm{P}, 8 \%, 10)=0.1490 \end{aligned}$ |  | Choose P | an A |  |

## Problem 6-32

Two possible routes for a power line are under study. Data on the routes are as follows:

|  | Around <br> the Lake | Under <br> the Lake |
| :--- | :--- | :--- |
| Length | 15 km | 5 km |
| First cost | $\$ 5000 / \mathrm{km}$ | $\$ 25,000 / \mathrm{km}$ |
| Maintenance | $\$ 200 / \mathrm{km} / \mathrm{yr}$ | $\$ 400 / \mathrm{km} / \mathrm{yr}$ |
| Useful life, in years | 15 | 15 |
| Salvage value | $\$ 3000 / \mathrm{km}$ | $\$ 5000 / \mathrm{km}$ |
| Yearly power loss | $\$ 500 / \mathrm{km}$ | $\$ 500 / \mathrm{km}$ |
| Aonual property taxes | $2 \%$ of first cost | $2 \%$ of first cost |

If $7 \%$ interest is used, should the power line be routed around the lake or under the lake? (Answer: Around the lake.)

## Problem 6-32

Length
First cost
Maintenance
Useful life, in years
Salvage value
Yearly power loss
Annual property taxes
Around Under
the Lake the Lake
$15 \mathrm{~km} \quad 5 \mathrm{~km}$
$\$ 5000 / \mathrm{kmo} \quad \$ 25,000 / \mathrm{km}$
$\$ 200 / \mathrm{km} / \mathrm{yr} \quad \$ 400 / \mathrm{km} / \mathrm{yr}$
15
$\$ 3000 / \mathrm{km} \quad \$ 5000 / \mathrm{km}$
$\$ 500 / \mathrm{km} \quad \$ 500 / \mathrm{km}$
$2 \%$ of first cost $2 \%$ of first cost

First Cost
Maintenance
Annual Power Loss
Property Taxes
Salvage Value
Useful Life

## Around the Lake

 \$75,000\$3,000/yr
\$7,500/yr
\$1,500/yr
\$45,000
15 years

Under the Lake
\$125,000
\$2,000/yr
\$2,500/yr
\$2,500/yr
\$25,000
15 years

## Problem 6-32



Around the Lake
EUAC $=\$ 75,000(A / P, 7 \%, 15)+\$ 12,000-\$ 45,000(A / F, 7 \%, 15)$

$$
\begin{aligned}
& =\$ 75,000(0.1098)+\$ 12,000-\$ 45,000(0.0398) \\
& =\$ 18,444
\end{aligned}
$$

Under the Lake
EUAC $=\$ 125,000(A / P, 7 \%, 15)+\$ 7,000-\$ 25,000(A / F, 7 \%, 15)$
$=\$ 125,000(0.1098)+\$ 7,000-\$ 25,000(0.0398)$
= \$19,730
Go around the lake.

