Engineering Economics

Rate of Return Analysis

Outcome of Today's Lecture

- **After completing this lecture…**
- The students should be able to:
- Evaluate project cash flows with the internal rate of return measure
- Plot a project's present worth against the interest rate
- Use an incremental rate of return analysis to evaluate competing alternatives

Techniques for Cash Flow Analysis

- Present Worth Analysis
- Annual Cash Flow Analysis
- Rate of Return Analysis
- Incremental Analysis
- Other Techniques:
  - Future Worth Analysis
  - Benefit-Cost Ratio Analysis
  - Payback Period Analysis

Rate of Return Analysis

- Internal Rate of Return
- Calculating Rate of Return
- Rate of Return Analysis
- Incremental Cash Flow Analysis

Internal Rate of Return (IRR) Lender's Viewpoint

Year	Cash flow
0	-5000
1	+1252
2	+1252
3	+1252
4	+1252
5	+1252

1. We know that the PW of five payments of $1,252 are equivalent to $5,000 when interest rate is 8%.

2. At the end of 5 years, the payments exactly repaid the $5,000 debt with interest rate 8%. *We say the lender received 8% rate of return.*

- The interest rate on the balance of a loan such that the unpaid loan balance equals zero when the final payment is made

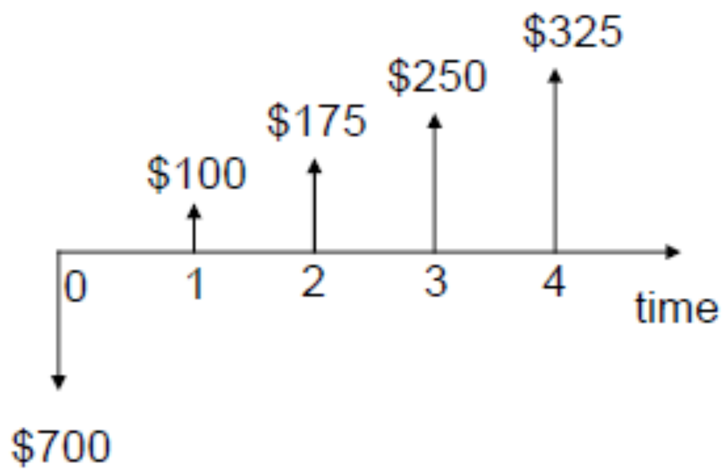
Internal Rate of Return (IRR)

- Simple Definition:
- Given a cash flow stream, rate of return (a.k.a. IRR) is the interest rate *i* at which the benefits are equivalent to the costs:*

  - NPW=0
  - PW of benefits - PW of costs = 0
  - PW of benefits = PW of costs
  - PW of benefits / PW of costs = 1
  - EUAB -EUAC = 0

Internal Rate of Return (IRR)

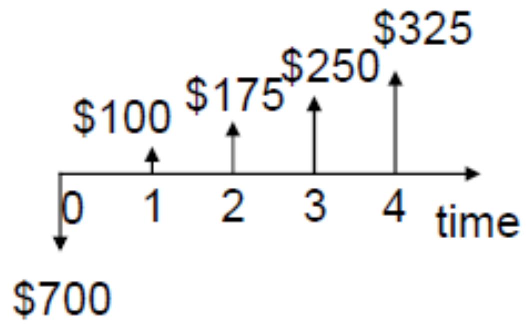
- Suppose you have the following cash flow stream. You invest $700, and then receive $100, $175, $250, and $325 at the end of years 1, 2, 3 and 4 respectively. What is the IRR for your investment?



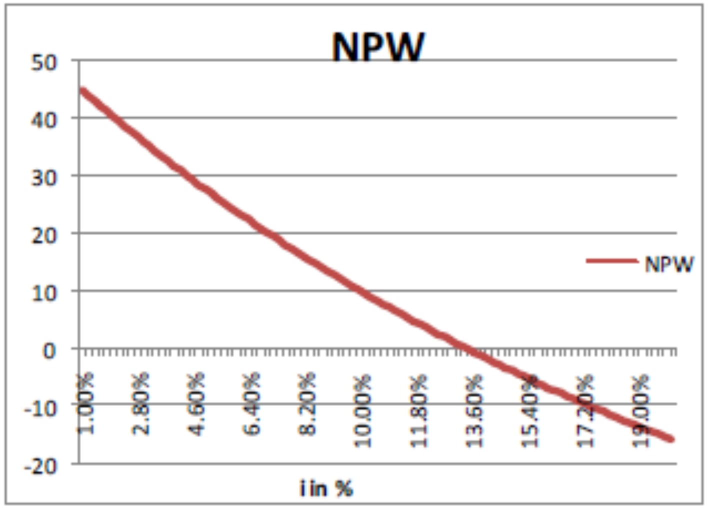
- $700 = 100/(1+i) + 175/(1+i)^2 + 250/(1+i)^3 + 325/(1+i)^4$
- Solving for *i* >>> *It turns out that i* = 6.09 %*

Calculating Internal Rate of Return

- Ways to find the IRR:
- 1. Compound Interest Tables (you may need to use interpolation)
- 2. Trial-and-error
- 3. Numerically (Excel's IRR function, MATLAB, or other root finding methods)
- 4. Graphically
- If you have a CFS with an investment *(-P) followed by benefits (non negative) from the investment:*
- The graph of NPW versus *i will have the same general form.*
- It will decrease at a decreasing rate and have a value 0 at some unique value *i**.
- Where the graph has a value 0 defines the IRR.

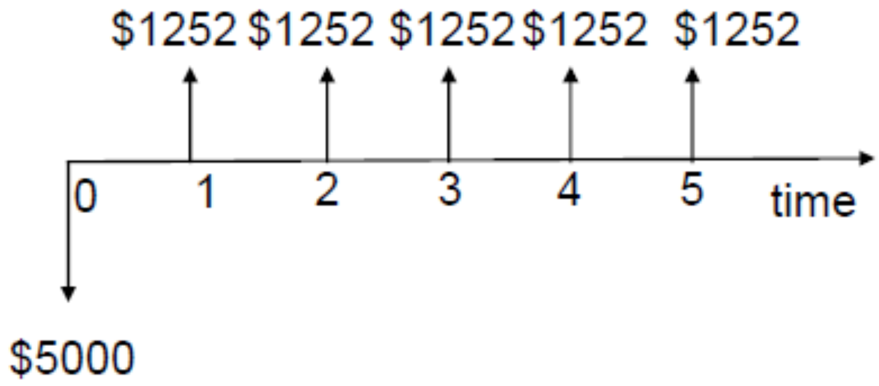


$$NPW = -700 + 100/(1+i) + 175/(1+i)^2 + 250/(1+i)^3 + 325/(1+i)^4$$



Example 1: Solution Using Interest Tables

- Given the following CFD, find *i**



- PWB/PWC = 1
- 1252(P/A,i,5)/5000 = 1
- (P/A,i,5) = 5000/1252 = 3.993
- From Compound Interest Tables:

i=8%

Interest rate	(P/A,i,5)
7%	4.100
8%	3.993
9%	3.890

Example 2: Solution Using Interest Tables

- An investment resulted in the following cash flow. Compute the rate of return.

Year	Cash Flow
0	−$700
1	+100
2	+175
3	+250
4	+325

$$\text{EUAB} - \text{EUAC} = 0$$

$$100 + 75(A/G, i, 4) - 700(A/P, i, 4) = 0$$

Solve the equation by trial and error

At $i = 5\%$,

$$\text{EUAB} - \text{EUAC} = 208 - 197 = +11$$

At $i = 8\%$,

$$\text{EUAB} - \text{EUAC} = 205 - 211 = -6$$

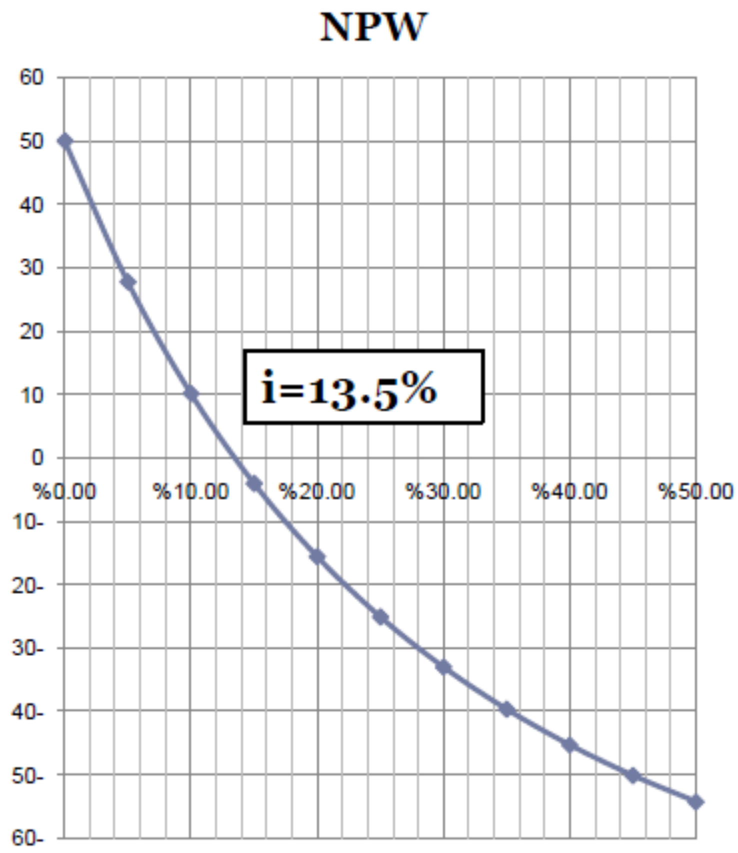
$i = 7\%$:

$$\text{EUAB} - \text{EUAC} = 0$$

Example 3: Graphical Solution

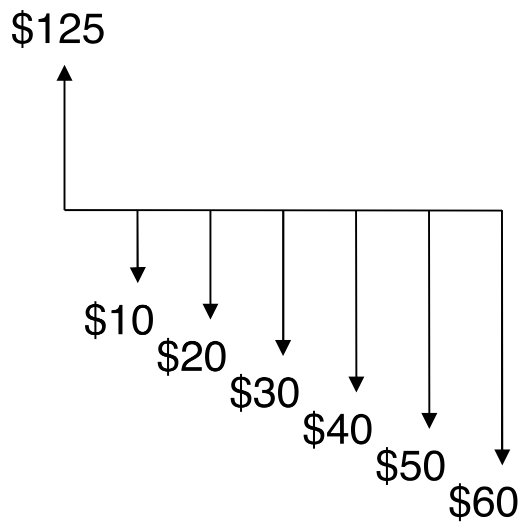
- Given the following CFS, find i*

Year	Cash
0	-100
1	20
2	30
3	20
4	40
5	40



- PW of costs = PW of benefits
- $100=20/(1+i)+30/(1+i)^2+20/(1+i)^3+40/(1+i)^4+40/(1+i)^5$
- $NPW=-100+20/(1+i)+30/(1+i)^2+20/(1+i)^3+40/(1+i)^4+40/(1+i)^5$

Problem: 7-1



- $125 = $10 (P/A, i%, 6) + $10 (P/G, i%, 6)
- **LHS=RHS**

  at 12%, RHS= $10 (4.111) + $10 (8.930) = $130.4

  at 15%, RHS= $10 (3.784) + $10 (7.937) = $117.2

- i* = 12% + (3%) ((130.4 – 125).(130.4-117.2)) = 13.23%

Problem 7-8

Year	Cash Flow
0	−$400
1	0
2	+200
3	+150
4	+100
5	+50

PWC=$400

PWB = [$200 (P/A, i%, 4) - $50 (P/G, i%, 4)] (P/F, i%, 1)

PWC=PWB

- **Try i = 7%**
- PWB=[$200 (3.387) - $50 (4.795)] (0.9346) = 409.03
- **Try i = 8%**
- PWB=[$200 (3.312) - $50 (4.650)] (0.9259) = $398.08
- i* = 7% + (1%) [($409.03 - $400)/($409.03 - $398.04)]
- = 7.82%

Problem 7-10

Solve the following cash flow for the rate of return to within an $^1/_2$%.

Year	Cash Flow
0	–$500
1	–100
2	+300
3	+300
4	+400
5	+500

PWC=$500 + $100 (P/F, i%, 1)

PWB= $300 (P/A, i%, 2) (P/F, i%, 1) + $400 (P/F, i%, 4) + $500 (P/F, i%, 5)

PWC-PWB=0

Problem 7-10

- **Try i = 30%**
- PWC=$500 + $100 (0.7692)= $576.92
- PWB=$300 (1.361) (0.7692) + $400 (0.6501) + $500 (0.2693)= $588.75
- PWC-PWB = 11.83
- **Try i = 35%**
- PWC=$500 + $100 (0.7407)= $574.07
- PWB=$300 (1.289) (0.7407) + $400 (0.3011) + $500 (0.2230) = $518.37
- PWC-PWB= 55.70

- Rate of Return, i* = 30% + (5%) [11.83/55.70)= 31.06%
- Exact Answer: 30.81%

Rate of Return (RoR) Analysis

- Example statements about a project:
  - The net present worth of the project is $32,000
  - The equivalent uniform annual benefit is $2,800
  - The project will produce a 23% rate of return

- The third statement is perhaps most widely understood.
- Rate of return analysis is probably the most frequently used analysis technique in industry.
- Its major advantage is that it provides a figure of merit that is readily understood.

Rate of Return (RoR) Analysis

- Rate of return analysis has another advantage: With NPW or EUAB one must choose an interest rate for using in the calculations.
- This choice may possibly be difficult or controversial.

- With RoR analysis no (exterior) interest rate is introduced into the calculations.
- Instead, we compute a RoR from the CFS.

- Warning: Relying only on RoR is not always a good idea.

Rate of Return (RoR) Analysis

- **Example:** Which of the following two investment options would you select?

- Option 1:
- Invest $2,000 today. At the end of years 1, 2, and 3 get $100, $100, and $500 profit; at the end of year 4, you get $2,200.

- Option 2:
- Invest $2,000 today. At the end of years 1, 2, and 3 get $100, $100, and $100 profit; at the end of year 4, you get $2,000.

Rate of Return (RoR) Analysis

- Find out the implicit interest rate you would be receiving; that is, solve for the interest rate in which the PW of benefits are equal to your payments $2,000.

- Option 1:
- $2000 = 100/(1+i)^1 + 100/(1+i)^2 + 500/(1+i)^3 + 2200/(1+i)^4$
- IRR: i= 10.78%

- Option 2:
- $2000 = 100/(1+i)^1 + 100/(1+i)^2 + 100/(1+i)^3 + 2000/(1+i)^4$
- IRR: i= 3.82%

Which deal would you prefer?

The Minimum Attractive Rate of Return (MARR)

- The MARR is a minimum return the company will accept on the money it invests
- The MARR is usually calculated by financial analysts in the company and provided to those who evaluate projects
- It is the same as the interest rate used for Present Worth and Annual Worth analysis.

Incremental Cash Flow Analysis (ΔCFS)

- Suppose you must choose between projects A or B.
- We can rewrite the CFS for B as B = A + (B –A).
- In this representation B has two CFS components:
  - 1. the same CFS as A, and
  - 2. the incremental component (B –A).

- B is preferred to A when the IRR on the difference (B–A) exceeds the MARR.
- Thus, to choose one between B and A, IRR analysis is done by computing the IRR on the incremental investment (B-A) between the projects.

Incremental Cash Flow Analysis (ΔCFS)

- Steps to conduct ΔCFS on two CFS's:
- 1. Number them CFS1and CFS2, with CFS1 having the largest initial (year 0) cost (in absolute value)
- 2. Compute ΔCFS = CFS1–CFS2 (It's year 0 entry must be negative)
- 3. Find the IRR for ΔCFS, say ΔIRR
- 4. If ΔIRR ≥ MARR, choose CFS1; if not, choose CFS2

- Example: There are two cash flows: (-20,28) and (-10,15) and MARR = 6%.
  - 1.CFS1= (-20,28), CFS2= (-10,15)
  - 2. ΔCFS = CFS1-CFS2=(-10,13)
  - 3. ΔIRR = 30%.
  - 4. ΔIRR > MARR => we choose CFS1= (-20,28)

Incremental Cash Flow Analysis (ΔCFS)

- In summary, we compute the CFS for the difference between the projects by subtracting the cash flow for the lower investment-cost project (A) from that of the higher investment-cost project (B).

- Then, the decision rule is as follows:
  - IF $\Delta IRR_{B-A} > MARR$, select B
  - IF $\Delta IRR_{B-A} = MARR$, select either A or B
  - IF $\Delta IRR_{B-A} < MARR$, select A
  - Here, B-A is an investment increment.

Why We Use ΔIRR in IRR analysis?

Years	A	B	B-A	
0	-10	-20	-10	MARR=6%
1	15	28	13	

	A	B	B-A		
IRR	50%	40%		Select A	
$\Delta IRR_{B\text{-}A}$			30%	$MARR < \Delta IRR_{B\text{-}A}$	Select B
NPV	3.92	6.05		Select B	

- Although the rate of return of A is higher than B, B got $8 return from the $20 investment and A only got $5 return from $10 investment.
- Project B: you put $20 in project B to get a return $8.
- Project A: you put $10 in project A (and $10 in your pocket) to get a return $5.
- From this example, we know that we can't evaluate two projects by comparing the IRRs of the projects. Instead, we use ΔIRR and MARR to make the decision.

Problem 7-47

Two alternatives are as follows:

Year	*A*	*B*
0	−$2000	−$2800
1	+800	+1100
2	+800	+1100
3	+800	+1100

If 5% is considered the minimum attractive rate of return, which alternative should be selected?

Problem 7-47

Year	A	B	(B- A)
0	-$2,000	-$2,800	-$800
1- 3	+$800	+$1,100	+$300
Computed ROR	9.7%	8.7%	6.1%

The rate of return on the increment (B - A) exceeds the Minimum Attractive Rate of Return (MARR), therefore the higher cost alternative B should be selected.

Problem 7-51

Consider two mutually exclusive alternatives:

Year	*X*	*Y*
0	−$5000	−$5000
1	−3000	+2000
2	+4000	+2000
3	+4000	+2000
4	+4000	+2000

If the MARR is 8%, which alternative should be selected?

Problem 7-51

Year	X	Y	X- Y
0	-$5,000	-$5,000	$0
1	-$3,000	+$2,000	-$5,000
2	+$4,000	+$2,000	+$2,000
3	+$4,000	+$2,000	+$2,000
4	+$4,000	+$2,000	+$2,000
Computed ROR	16.9%	21.9%	9.7%

Since X- Y difference between alternatives is desirable, select Alternative X.

Summary

- RoR analysis is often used but not always well understood by practitioners
- RoR can be computationally difficult manually; a spreadsheet model helps reduce solution time
- If an exact RoR is not necessary, use the PW or AW methods
- Use incremental analysis when using IRR