Engineering Economics

Benefit Cost Analysis

Techniques for Cash Flow Analysis

- Present Worth Analysis
- Annual Cash Flow Analysis
- Rate of Return Analysis
- Incremental Analysis
- Other Techniques:
  - Future Worth Analysis
  - Benefit-Cost Ratio Analysis
  - Payback Period Analysis

Benefit-Cost Analysis

- The Benefit-cost analysis is commonly used to evaluate public (government) projects.

- Benefits of a nonmonetary nature need to be quantified in dollar terms as much as possible and factored into the analysis.

- A broad range of project users distinct from the sponsor should be considered—benefits and costs to all these users can (and should) be taken into account.

Benefit-Cost Ratio Criterion

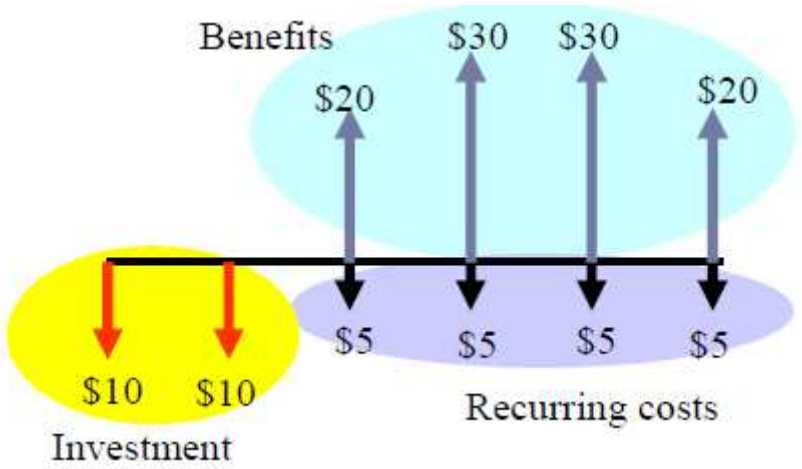
- The Net B/C ratio expresses the net benefit expected per dollar invested.

$$\text{Benefit - Cost Ratio (B/C)} = \frac{\text{Equivalent Net Benefits}}{\text{Equivalent Net Costs}} = \frac{\text{PW of Benefits}}{\text{PW of Costs}} = \frac{\text{EUAB}}{\text{EUAC}}$$

- **Decision Rule: If B/C ratio > 1, the project can be justified/acceptable.**

Example 1: Benefit-Cost Ratio Computation

- Compute Benefit-Cost Ratio for given CFD:



- Solution:
- Benefit, B = $20 (P/F,10%,2) + $30 (P/F,10%,3) + $30 (P/F,10%,4) + $20 (P/F,10%,5) = $71.98
- Cost, C = $10 + $10 (P/F,10%,1) + $5 (P/A,10%,4) (P/F,10%,1) = $25
- Benefit-cost ratio=B/C = 72/25 = 2.88 => Accept the project

Example 2: B/C Ratio Analysis

- Each of the five mutually exclusive alternatives presented below will last for 20 years and has no salvage value. MARR = 6%.

	A	B	C	D	E	F
PWC	$4,000	$2,000	$6,000	$1,000	$9,000	$10,000
PWB	$7,330	$4,700	$8,730	$1,340	$9,000	$9,500
B/C	1.83	2.35	1.46	1.34	1.00	0.95
NPV	$3,330	$2,700	$1,730	$340	0	-$500

- The steps are the same as in incremental ROR, except that the criterion is now B/C, and the cutoff is 1 instead of the MARR:

- 1. Be sure you identify all alternatives.
- 2. (Optional) Compute the B/C ratio for each alternative. Discard any with a B/C < 1. (We can discard F).
- 3. Arrange the remaining alternatives in ascending order of investment.

Example 2: Cost Ratio Analysis

	D	B	A	C	E	F
Cost	$1000	$2000	$4000	$6000	$9000	$10000
PWB	$1340	$4700	$7330	$8730	$9000	$9500
B/C	1.34	2.35	1.83	1.46	1.00	0.95
NPV	340	2700	3330	1730	0	-500

4. Comparing $\Delta B/\Delta C$ with 1 for consecutive alternatives select the best alternative.

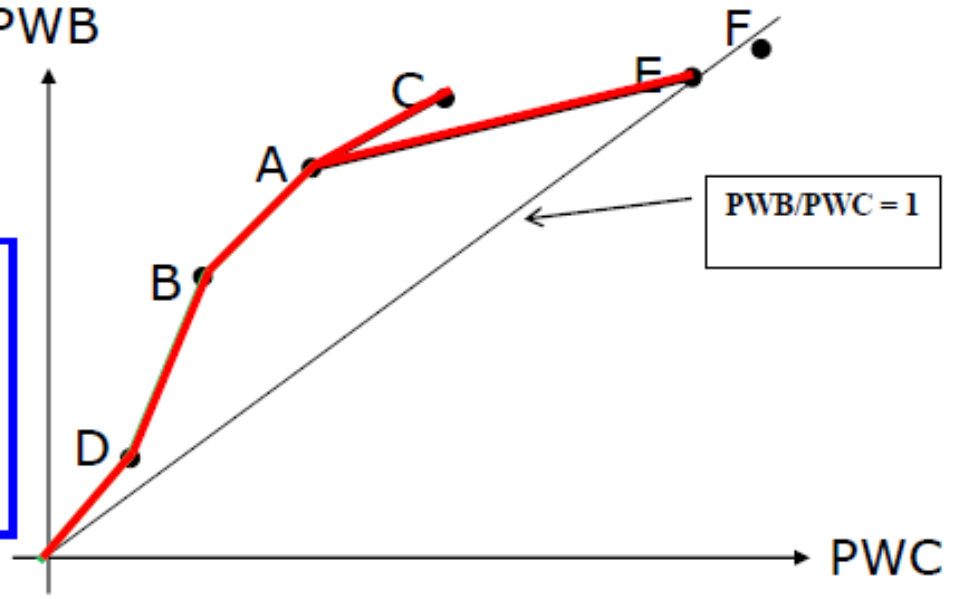
	B-D	A-B	C - A	E-A
Incremental Cost	$1000	$2000	$2000	$5000
Incremental Benefit	$3360	$2630	$1400	$1670
Incr.B/Incr. C	3.36	1.32	0.76	0.33

- Thus, for the example, the increments B-D and A-B are attractive. We prefer B to D, and we prefer A to B. Increment C-A is not attractive, as $\Delta B/\Delta C = 0.76 < 1$. Comparing A to E, again A is best. Finally A is the best project.

Example 2: Cost Ratio Analysis

- ✓ A, B, C, and D are above the 45-degree line; their B/C ratio is > 1.
- ✓ F is below the line: B/C ratio is < 1.
- ✓ We can discard F if we wish.

Examine each separable increment of investment.
$\Delta B/\Delta C < 1 \Rightarrow$ increment is not attractive
$\Delta B/\Delta C \geq 1 \Rightarrow$ increment is desirable.



Begin with D & B: $\Delta B/\Delta C > 1$. B "wins".
Next consider A: $\Delta B/\Delta C > 1$. A "wins".
C: $\Delta B/\Delta C < 1$; discard C.
E: $\Delta B/\Delta C < 1$; discard E.
F was discarded earlier
Conclude A is best.
Note: Alt. B has the highest B/C ratio

	B-D	A-B	C - A	E-A
Incremental Cost	$1000	$2000	$2000	$5000
Incremental Benefit	$3360	$2630	$1400	$1670
Incr.B/Incr. C	3.36	1.32	0.76	0.33

Example 3: Cost Ratio Analysis

- You are deciding between three alternatives and you need to pick the best one. The lifetimes of all machines is 20 years. Assuming a 5% interest rate, which machine should you select?
- Use B/C ratio to make your decision.

	Alternative A	Alternative B	Alternative C
Benefits			
Taxes	$7,000 per year	$3,000 per year	$8,000 per year
Salvage Value	$30,000	$15,000	$25,000
Costs			
First Cost	$45,000 (present)	$25,000 (present)	$65,000 (present)
Operating Expenses	$1,500 per year	$2,500 per year	$1000 per year
Maintenance Costs	$2,000 per year	$3,000 per year	$1500 per year
Lifetime	20 years	20 years	20 years

Example 3: Benefit-Cost Ratio Analysis

- **Solution:**
- Analysis of Alternative A
- B/C ratio for Alt A = Benefits/Cost
- = [7,000 (P/A, 5%, 20) + 30,000 (P/F, 5%, 20)] / [45,000 + (1,500+2000) (P/A, 5%, 20)]
- = 98,542/88,617 = 1.1199 > 1 (Good)
- Analysis of Alternative B
- B/C ratio for Alt B = Benefits/Cost
- = [3000 (P/A, 5%, 20) + 15,000 (P/F, 5%, 20)] / [25,000 + (2,500+3000) (P/A, 5%, 20)]
- = 43,040 / 93,542 = 0.4601 < 1 (Bad, Not good)
- If we do the same for Alternative C we get a B/C ratio of 1.135, which is > 1 (Good)

Example 3: Cost Ratio Analysis

	Alternative A	Alternative B	Alternative C	(C - A)
Benefits				
Taxes	$7,000	$3,000	$8,000	$1,000
Salvage Value	$30,000	$15,000	$25,000	($5,000)
Costs				
First Cost	$45,000	$25,000	$65,000	$20,000
Operating Expenses	$1,500	$2,500	$1,000	($500)
Maintenance Costs	$2,000	$3,000	$1,500	($500)
Lifetime	20 years	20 years	20 years	
Benefits	$98,542.4000	$43,040.1000	$109,120.1000	$10,577.7000
Costs	$88,617.70	$93,542.10	$96,155.50	$7,537.80
B/C Ratio	1.111994556	0.46011475	1.134829521	1.403287431
Decision	Good, Compare	Not Good, Eliminate	Good, Compare	Good, prefer C

Example 3: Cost Ratio Analysis

- Note that the benefits and costs are obtained from the previous analysis (we made the analysis in terms of Present Worth).

- For example, for Alternative A:
- Benefits = 7,000 (P/A, 5%, 20) + 30,000 (P/F, 5%, 20)= $98,542
- Costs = 45,000 + (1,500+2000) (P/A, 5%, 20) = $88,617

- In this case, since Incremental B/C of (C-A) = 1.40 we prefer Alternative C over Alternative A.

- Since we have no more alternatives we decide that Alternative C is the best one.

Other Analysis Techniques

- **Future worth analysis** is equivalent to present worth analysis. There are many situations where we want to know what a future situation will be, if we take some particular course of action now. This is called future worth analysis.

- **Payback period** is an approximate analysis method. For example, if a $1000 investment today generates $500 annually in savings, we say its payback period is 1000/500 = 2 years.

- **Sensitivity analysis** identifies how sensitive economic conclusions are to the values of the data, and allows making decisions for an entire range of the data.

- **Breakeven analysis** is closely related to sensitivity analysis, and determines conditions when two alternatives are equivalent (as well as when each is better than the other). It can be viewed as a type of sensitivity analysis.

9-29

- A project will cost $50,000. The benefits at the end of the first year are estimated to be $10,000, increasing at a 10% uniform rate in subsequent years. Using an 8-year analysis period and a 10% interest rate, compute the benefit-cost ratio.

- Geometric gradient at a 10% uniform rate.
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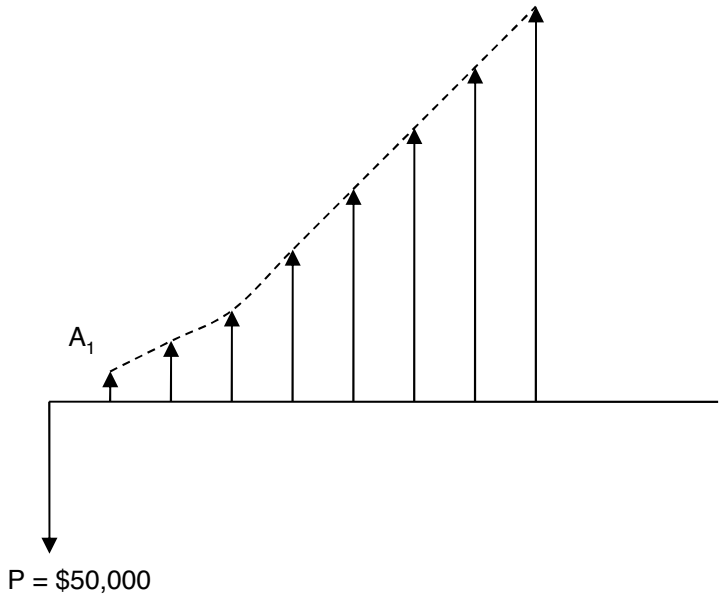
- A_1 = $10,000
- i = 10%
- g = 10%
- n = 8 yrs
- Where i = g: $P = A_1 n (1 + i)^{-1}$



- B/C = PW of Benefits/PW of Cost = [$10,000 (8) $(1 + 0.10)^{-1}$]/$50,000
- B/C = 1.45

9-30

Each of the three alternatives shown has a 5-year useful life. If the MARR is 10%, which alternative should be selected? Solve the problem by benefit–cost ratio analysis.

	A	*B*	*C*
Cost	$600.0	$500.0	$200.0
Uniform annual benefit	158.3	138.7	58.3

- $B/C_{OF\ A}$ = \$158.3/[\$600 (A/P, 10%, 5)] = 1.00
- $B/C_{OF\ B}$ = \$138.7/[\$500 (A/P, 10%, 5)] = 1.05
- $B/C_{OF\ C}$ = \$58.3/[\$200 (A/P, 10%, 5)] = 1.11

- All alternatives have a B/C ratio > 1.00. Proceed with incremental analysis.

9-30

	B- C	A- B
Cost	$300	$100
Uniform Annual Benefit	$80.4	$19.6

Incremental Analysis

$B/C_{OF\ B\text{-}C}$ = $80.4/[$300 (A/P, 10%, 5)] = 1.02

Desirable increment. Reject C.

$B/C_{OF\ A\text{-}B}$ = $19.6/[$100 (A/P, 10%, 5)] = 0.74

Undesirable increment. Reject A.

Conclusion: Select B.

9-31

- Consider three alternatives, each with a 10-year useful life. If the MARR is 10%, which alternative should be selected? Solve the problem by benefit-cost ratio analysis.

	A	*B*	*C*
Cost	$800	$300	$150
Uniform annual benefit	142	60	33.5

B/C ratio

B/C_A = ($142 (P/A, 10%, 10))/$800 = 1.09
B/C_B = ($60 (P/A, 10%, 10))/$300 = 1.23
B/C_C = ($33.5 (P/A, 10%, 10))/$150 = 1.37

9-31

- **Incremental Analysis**
- B- C Increment
- Δ Cost $_{(B- C)}$ = \$150
- Δ UAB= \$26.5
- $\Delta B/\Delta C$= (\$26.5 (P/A, 10%, 10))/\$150 = 1.09
- This is a desirable increment. Reject C.

- A- B Increment
- Δ Cost $_{(A- B)}$ = \$500
- Δ UAB =\$82
- $\Delta B/\Delta C$ = (\$82 (P/A, 10%, 10))/\$500 = 1.01
- This is a desirable increment. Reject B.

Conclusion: Select A.

9-33

Using benefit–cost ratio analysis, determine which one of the three mutually exclusive alternatives should be selected.

	A	*B*	*C*
First cost	$560	$340	$120
Uniform annual benefit	140	100	40
Salvage value	40	0	0

B/C ratio

Alternative A: B/C = [$140 (P/A, 10%, 6)]/[$560 - $40 (P/F, 10%, 6)]
= [$140 (4.355)]/($560 - $40 (0.5645)]
= 1.13

Alternative B: B/C = [$100 (P/A, 10%, 6)]/$340
= 1.28

Alternative C: B/C = [$40 (P/A, 10%, 6)]/$120
= 1.45

9-33

	B- C	A- B
Δ First Cost	$220	$220
Δ Uniform Annual Benefit	$60	$40
Δ Salvage Value	$0	$40
Compute ΔB/ΔC value	1.19	0.88

Incremental Analysis

B- C ΔB/ΔC = [$60 (P/A, 10%, 6)]/$220 = 1.19

B- C is a desirable increment.

A- B ΔB/ΔC = [$40 (P/A, 10%, 6)/[$220 - $40 (P/F, 10%, 6)] = 0.88

A- B is an undesirable increment.

Conclusion: Choose B.

9-33

- **NPW Solution**

- NPW_A = \$140 (P/A, 10%, 6) + \$40 (P/F, 10%, 6) - \$560
- = \$140 (4.355) + \$40 (0.5645) - \$560
- = +\$72.28

- NPW_B = \$100 (P/A, 10%, 6) - \$340
- = +\$95.50

- NPW_C = \$40 (P/A, 10%, 6) - \$120
- = +\$54.20

Select B

9-33

Rate of Return Solution

	B- C	A- B
Δ Cost	$220	$220
Δ Uniform Annual Benefit	$60	$40
Δ Salvage Value	$0	$40
Computed Δ ROR	16.2%	6.6%
Decision	> 10% Accept B. Reject C.	< 10% Reject A.

Select B.