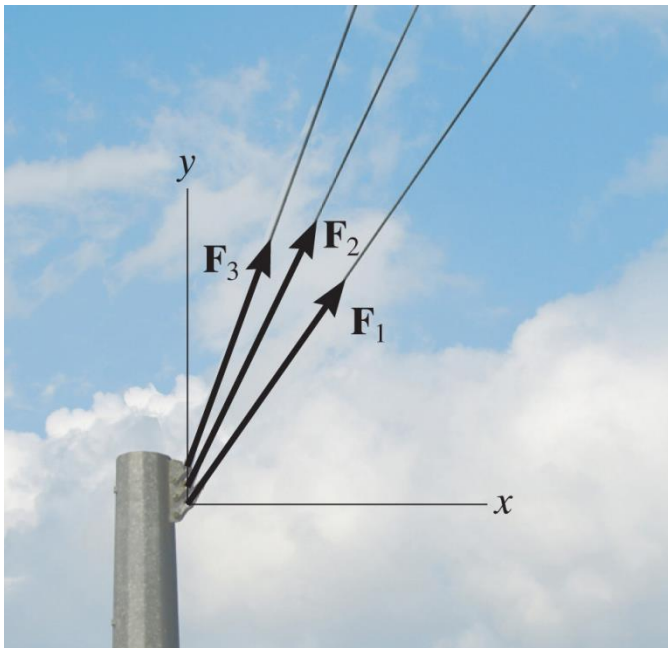


# FORCE VECTORS, VECTOR OPERATIONS & ADDITION COPLANAR FORCES

## Today's Objective:

Students will be able to :

- Resolve a 2-D vector into components.
- Add 2-D vectors using Cartesian vector notations.



## In-Class activities:

- Check Homework
- Reading Quiz
- Application of Adding Forces
- Parallelogram Law
- Resolution of a Vector Using Cartesian Vector Notation (CVN)
- Addition Using CVN
- Example Problem
- Concept Quiz
- Group Problem
- Attention Quiz

## READING QUIZ

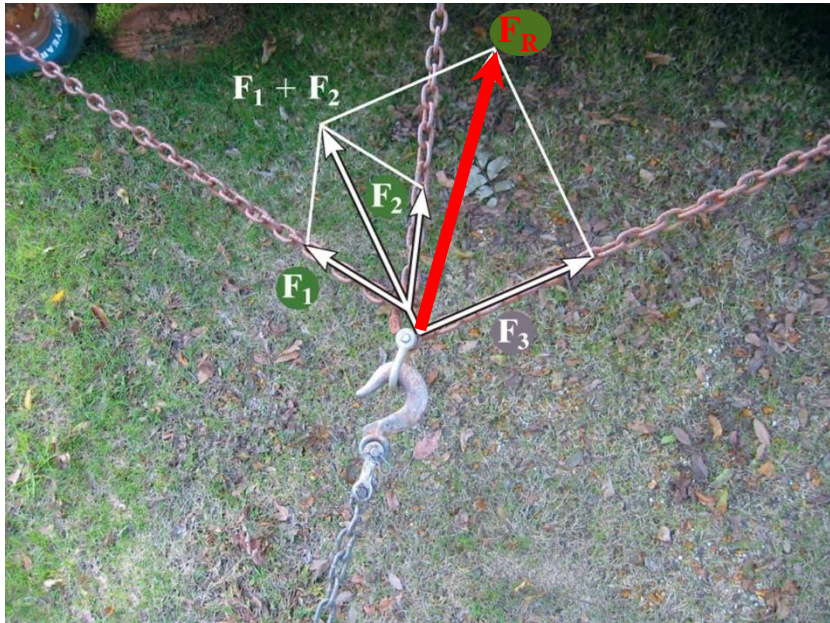
1. Which one of the following is a scalar quantity?

- A) Force      B) Position  
C) Mass      D) Velocity

2. For vector addition, you have to use \_\_\_\_\_ law.

- A) Newton's Second  
B) the arithmetic  
C) Pascal's  
D) the parallelogram

# APPLICATION OF VECTOR ADDITION



There are three concurrent forces acting on the hook due to the chains.

We need to decide if the hook will fail (bend or break).

To do this, we need to know the resultant or total force acting on the hook as a result of the three chains.

# SCALARS AND VECTORS

## (Section 2.1)

### Scalars

### *Vectors*

Examples:

Mass, Volume

Force, Velocity

Characteristics:

It has a magnitude  
(positive or negative)

It has a magnitude  
**and** direction

Addition rule:

Simple arithmetic

Parallelogram law

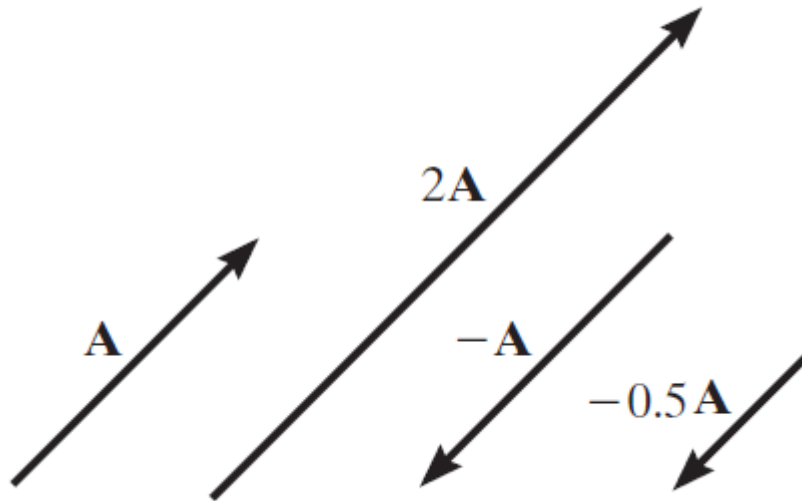
Special Notation:

None

Bold font, a line, an  
arrow or a “carrot”

In these PowerPoint presentations, a vector quantity is represented *like this* (in **bold**, *italics*, and **red**).

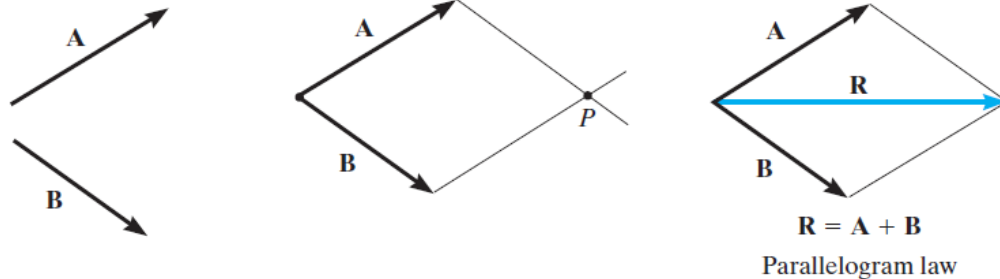
## VECTOR OPERATIONS (Section 2.2)



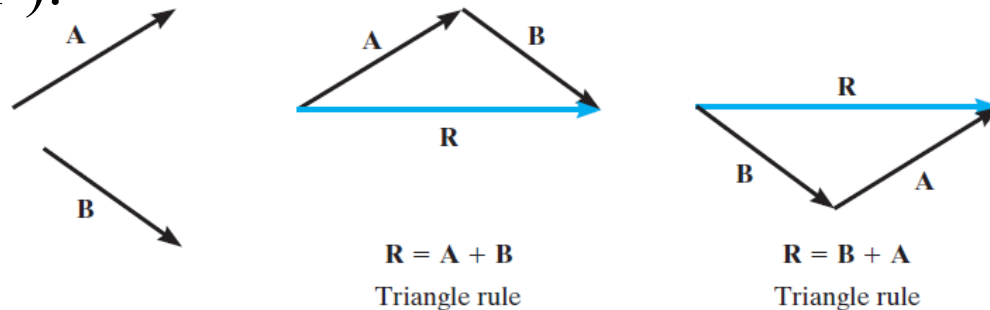
Scalar Multiplication  
and Division

# VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

Parallelogram Law:



Triangle method  
(always 'tip to tail'):

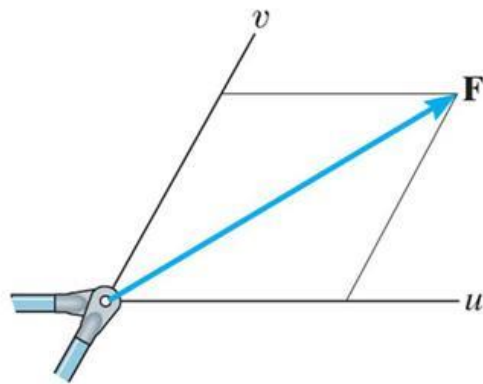


How do you subtract a vector?

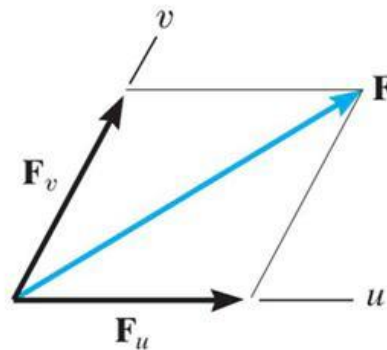
How can you add more than two concurrent vectors graphically?

# RESOLUTION OF A VECTOR

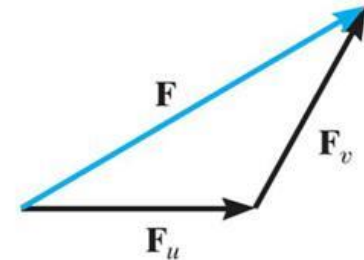
“Resolution” of a vector is breaking up a vector into components.



(a)



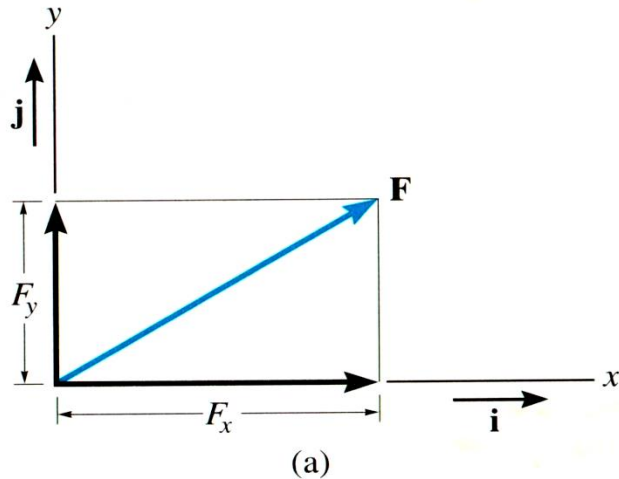
(b)



(c)

It is kind of like using the parallelogram law in reverse.

# ADDITION OF A SYSTEM OF COPLANAR FORCES (Section 2.4)

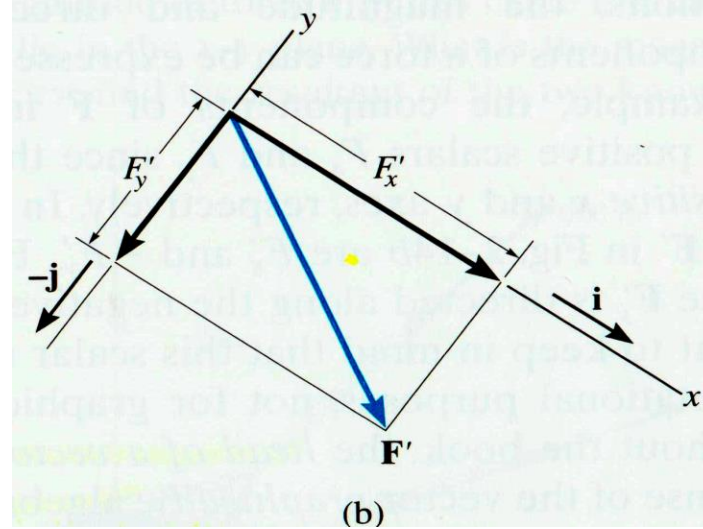
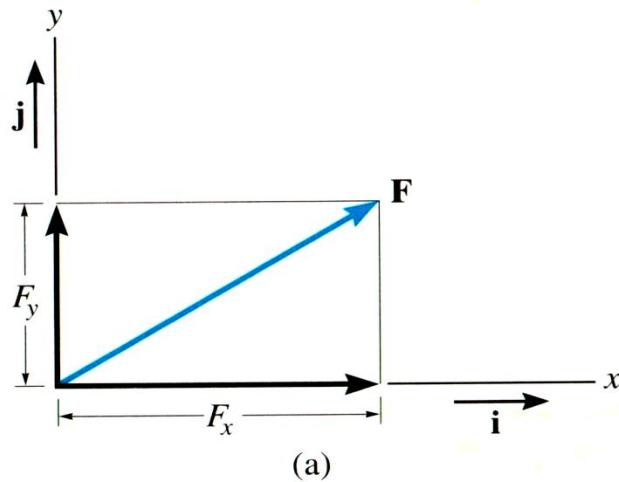


- We ‘resolve’ vectors into components using the x and y-axis coordinate system.
  - Each component of the vector is shown as a magnitude and a direction.
- 
- The directions are based on the x and y axes. We use the “unit vectors”  $\mathbf{i}$  and  $\mathbf{j}$  to designate the x and y-axes.



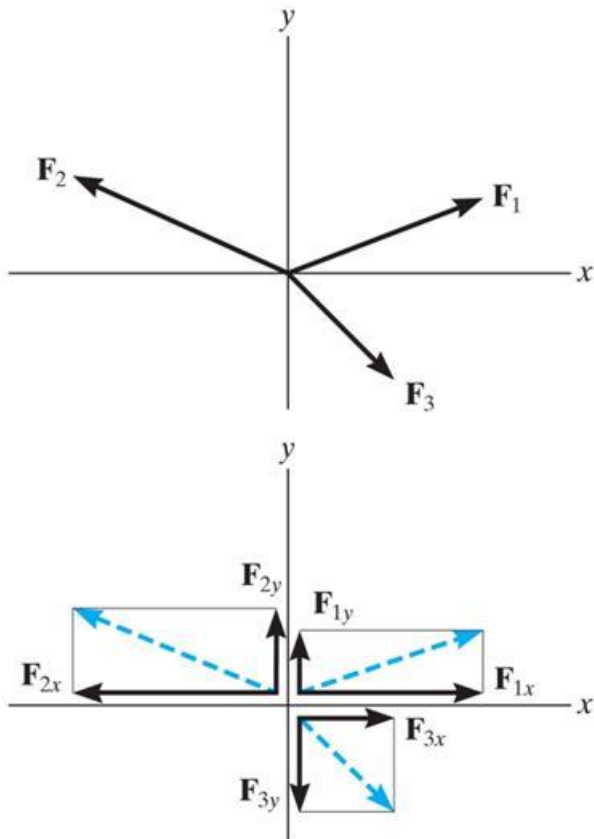
For example,

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad \text{or} \quad \mathbf{F}' = F'_x \mathbf{i} + (-F'_y) \mathbf{j}$$



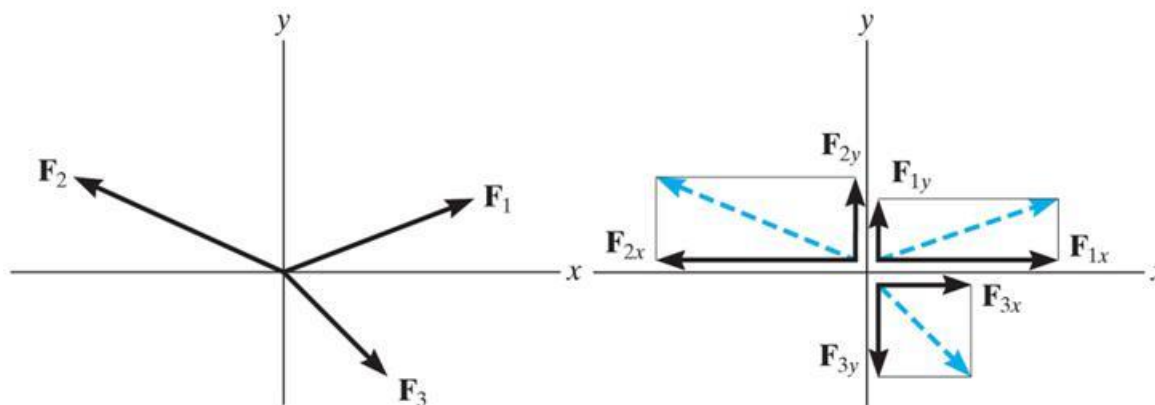
The x and y-axis are always perpendicular to each other. Together, they can be “set” at any inclination.

# ADDITION OF SEVERAL VECTORS



- Step 1 is to resolve each force into its components.
- Step 2 is to add all the x-components together, followed by adding all the y-components together. These two totals are the x and y-components of the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

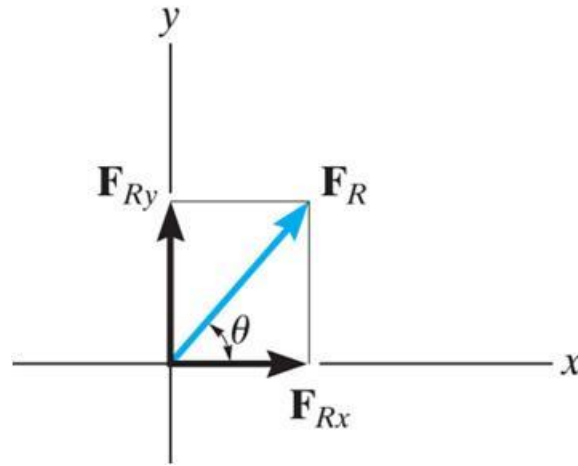
An example of the process:



Break the three vectors into components, then add them.

$$\begin{aligned} \mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x} \mathbf{i} + F_{1y} \mathbf{j} - F_{2x} \mathbf{i} + F_{2y} \mathbf{j} + F_{3x} \mathbf{i} - F_{3y} \mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x}) \mathbf{i} + (F_{1y} + F_{2y} - F_{3y}) \mathbf{j} \\ &= (F_{Rx}) \mathbf{i} + (F_{Ry}) \mathbf{j} \end{aligned}$$

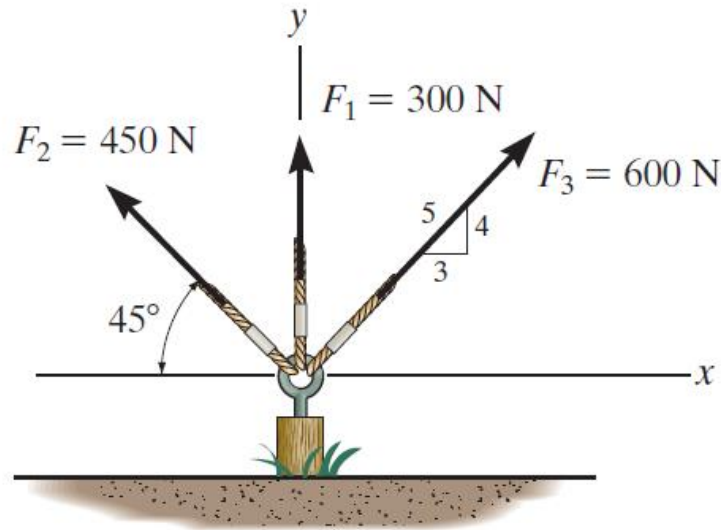
**You can also represent a 2-D vector with a magnitude and angle.**



$$\theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

## EXAMPLE I



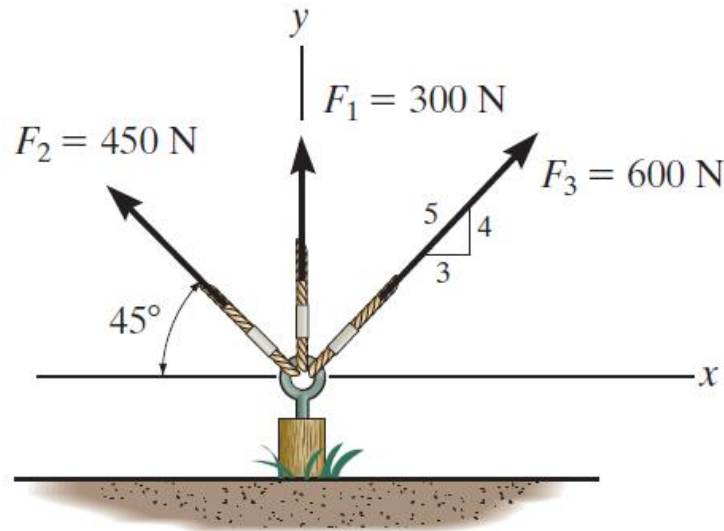
**Given:** Three concurrent forces acting on a tent post.

**Find:** The magnitude and angle of the resultant force.

### Plan:

- Resolve** the forces into their x-y components.
- Add** the respective **components** to get the resultant vector.
- Find **magnitude** and **angle** from the resultant components.

## EXAMPLE I (continued)



$$\mathbf{F}_1 = \{ 0 \mathbf{i} + 300 \mathbf{j} \} \text{ N}$$

$$\begin{aligned} \mathbf{F}_2 &= \{ -450 \cos(45^\circ) \mathbf{i} + 450 \sin(45^\circ) \mathbf{j} \} \text{ N} \\ &= \{ -318.2 \mathbf{i} + 318.2 \mathbf{j} \} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_3 &= \{ (3/5) 600 \mathbf{i} + (4/5) 600 \mathbf{j} \} \text{ N} \\ &= \{ 360 \mathbf{i} + 480 \mathbf{j} \} \text{ N} \end{aligned}$$

## EXAMPLE I (continued)

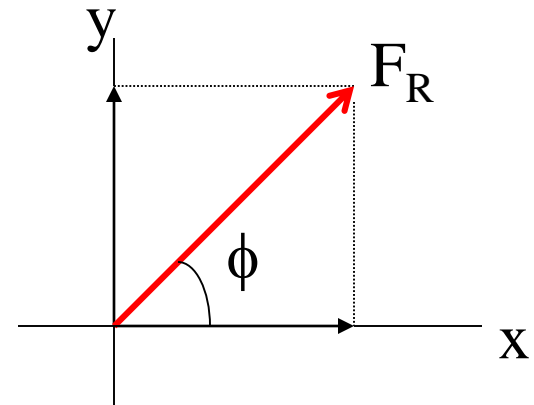
Summing up all the  $i$  and  $j$  components respectively, we get,

$$\begin{aligned} \mathbf{F}_R &= \{ (0 - 318.2 + 360) \mathbf{i} + (300 + 318.2 + 480) \mathbf{j} \} \text{ N} \\ &= \{ 41.80 \mathbf{i} + 1098 \mathbf{j} \} \text{ N} \end{aligned}$$

Using magnitude and direction:

$$F_R = ((41.80)^2 + (1098)^2)^{1/2} = \underline{1099 \text{ N}}$$

$$\phi = \tan^{-1}(1098/41.80) = \underline{87.8^\circ}$$

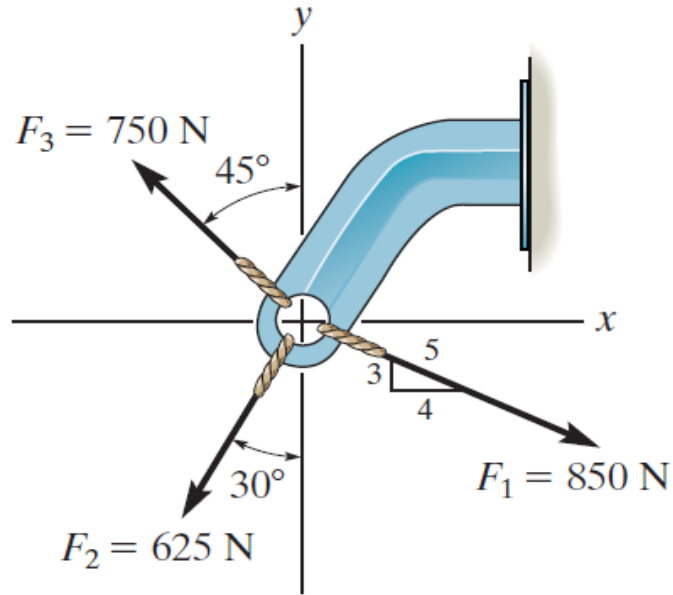


## CONCEPT QUIZ

1. Can you resolve a 2-D vector along two directions, which are not at  $90^\circ$  to each other?
  - A) Yes, but not uniquely.
  - B) No.
  - C) Yes, uniquely.
  
2. Can you resolve a 2-D vector along three directions (say at  $0^\circ$ ,  $60^\circ$ , and  $120^\circ$ )?
  - A) Yes, but not uniquely.
  - B) No.
  - C) Yes, uniquely.



# GROUP PROBLEM SOLVING



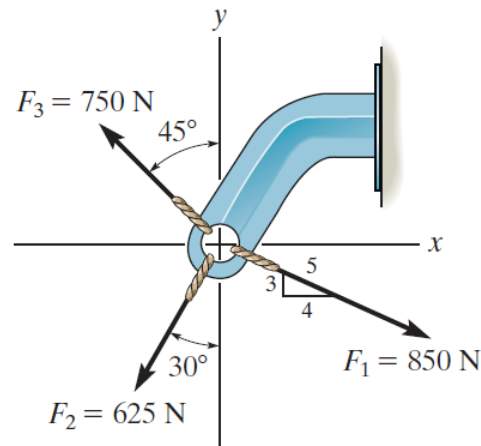
**Given:** Three concurrent forces acting on a bracket.

**Find:** The magnitude and angle of the resultant force. Show the resultant in a sketch.

## Plan:

- Resolve the forces into their x and y-components.
- Add the respective components to get the resultant vector.
- Find magnitude and angle from the resultant components.

## GROUP PROBLEM SOLVING (continued)



$$\begin{aligned} \mathbf{F}_1 &= \{ 850 (4/5) \mathbf{i} - 850 (3/5) \mathbf{j} \} \text{ N} \\ &= \{ 680 \mathbf{i} - 510 \mathbf{j} \} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_2 &= \{ -625 \sin(30^\circ) \mathbf{i} - 625 \cos(30^\circ) \mathbf{j} \} \text{ N} \\ &= \{ -312.5 \mathbf{i} - 541.3 \mathbf{j} \} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_3 &= \{ -750 \sin(45^\circ) \mathbf{i} + 750 \cos(45^\circ) \mathbf{j} \} \text{ N} \\ &= \{ -530.3 \mathbf{i} + 530.3 \mathbf{j} \} \text{ N} \end{aligned}$$

## GROUP PROBLEM SOLVING (continued)

Summing all the  $i$  and  $j$  components, respectively, we get,

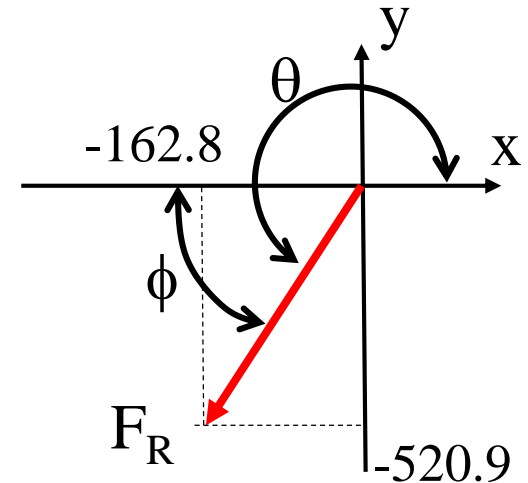
$$\begin{aligned} \mathbf{F}_R &= \{ (680 - 312.5 - 530.3) \mathbf{i} + (-510 - 541.3 + 530.3) \mathbf{j} \} \text{N} \\ &= \{ -162.8 \mathbf{i} - 520.9 \mathbf{j} \} \text{N} \end{aligned}$$

Now find the magnitude and angle,

$$F_R = ((-162.8)^2 + (-520.9)^2)^{1/2} = \underline{546 \text{ N}}$$

$$\phi = \tan^{-1}(520.9 / 162.8) = \underline{72.6^\circ}$$

From the positive x-axis,  $\theta = 253^\circ$



## ATTENTION QUIZ

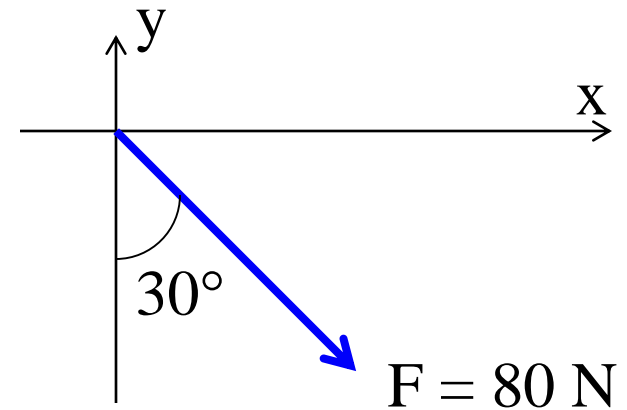
1. Resolve  $F$  along x and y axes and write it in vector form.  $F = \{ \underline{\hspace{2cm}} \}$  N

A)  $80 \cos (30^\circ) \mathbf{i} - 80 \sin (30^\circ) \mathbf{j}$

B)  $80 \sin (30^\circ) \mathbf{i} + 80 \cos (30^\circ) \mathbf{j}$

C)  $80 \sin (30^\circ) \mathbf{i} - 80 \cos (30^\circ) \mathbf{j}$

D)  $80 \cos (30^\circ) \mathbf{i} + 80 \sin (30^\circ) \mathbf{j}$



2. Determine the magnitude of the resultant ( $F_1 + F_2$ ) force in N when  $F_1 = \{ 10 \mathbf{i} + 20 \mathbf{j} \}$  N and  $F_2 = \{ 20 \mathbf{i} + 20 \mathbf{j} \}$  N .

A) 30 N

B) 40 N

C) 50 N

D) 60 N

E) 70 N

***End of the Lecture***

***Let Learning Continue***