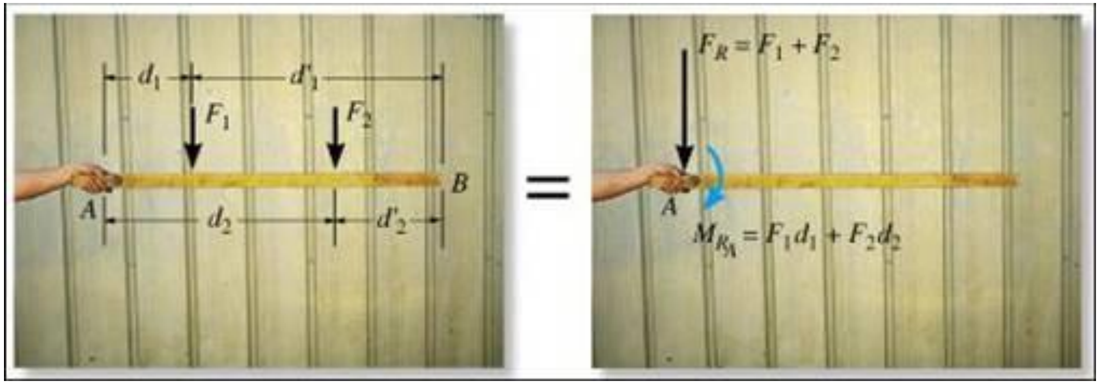
SIMPLIFICATION OF FORCE AND COUPLE SYSTEMS & THEIR FURTHER SIMPLIFICATION

Today's Objectives:

Students will be able to:

a) Determine the effect of moving a force.

b) Find an equivalent force-couple system for a system of forces and couples.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Equivalent Systems
- System Reduction
- Example Problems
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. A general system of forces and couple moments acting on a rigid body can be reduced to a ___ .

   A) single force

   B) single moment

   C) single force and two moments

   D) single force and a single moment

2. The original force and couple system and an equivalent force-couple system have the same ______ effect on a body.

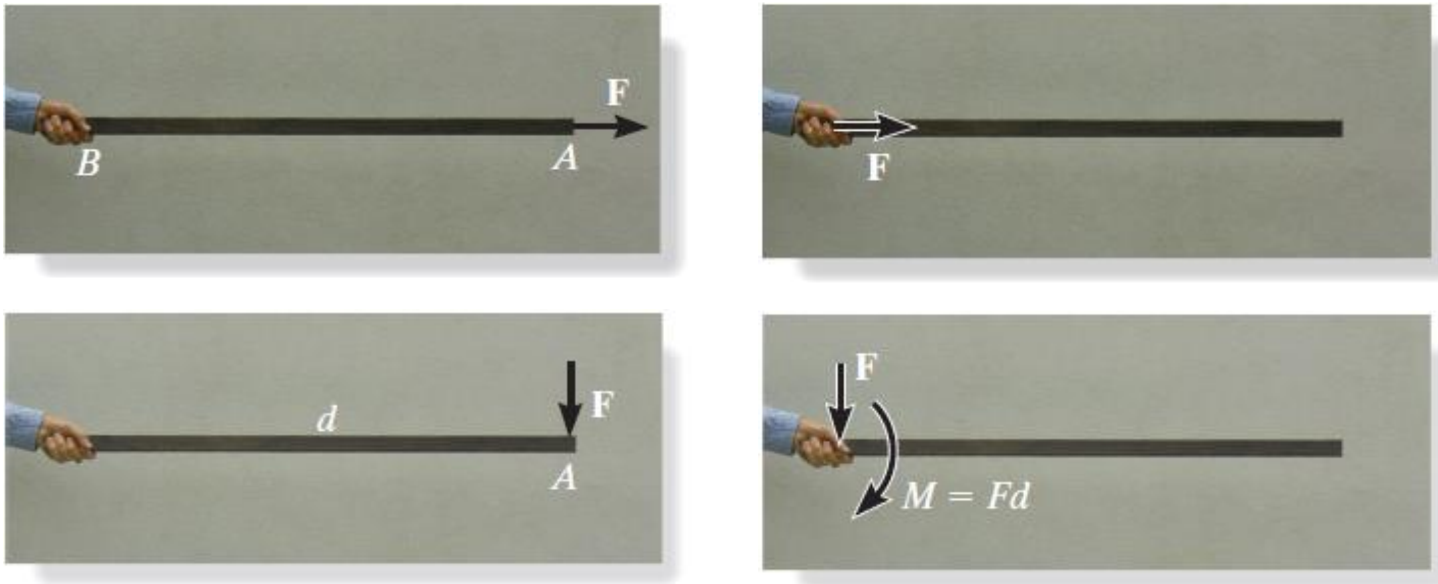
   A) internal

   B) external

   C) internal and external

   D) microscopic

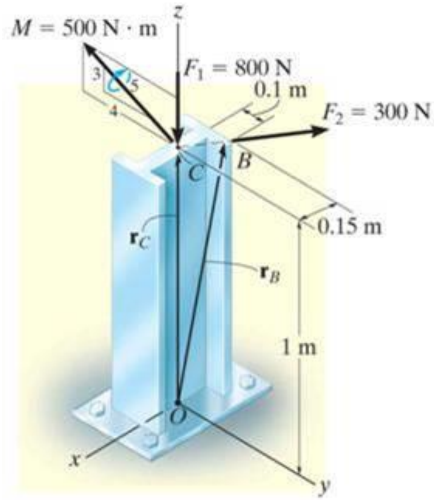
APPLICATIONS



What are the resultant effects on the person's hand when the force is applied in these four different ways?

Why is understanding these differences important when designing various load-bearing structures?

APPLICATIONS (continued)



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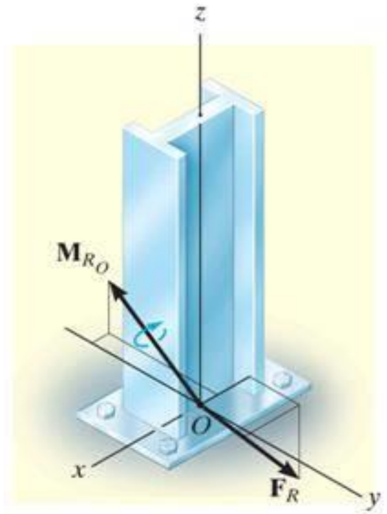
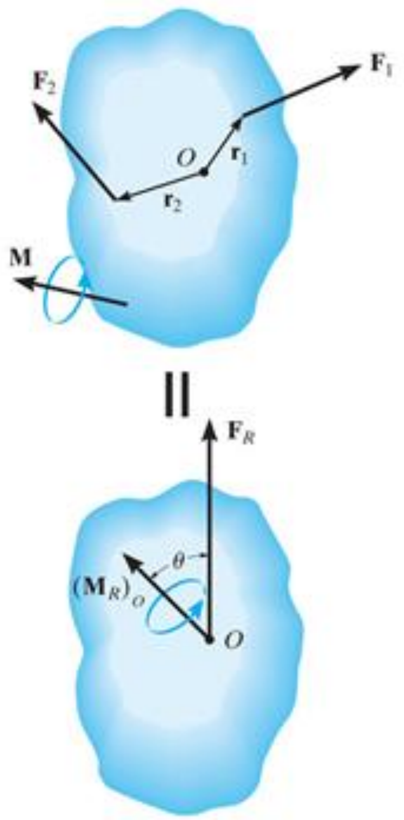


Several forces and a couple moment are acting on this vertical section of an I-beam.

For the process of designing the I-beam, it would be very helpful if you could replace the various forces and moment just one force and one couple moment at point O with the same external effect? How will you do that?

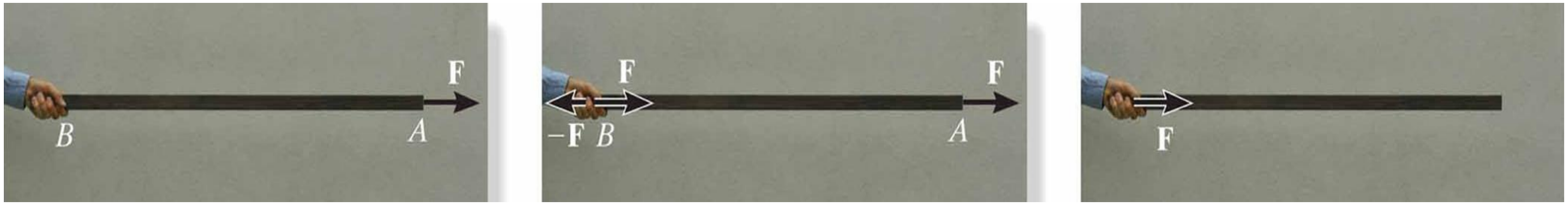
SIMPLIFICATION OF FORCE AND COUPLE SYSTEM (Section 4.7)



When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect.

The two force and couple systems are called equivalent systems since they have the same external effect on the body.

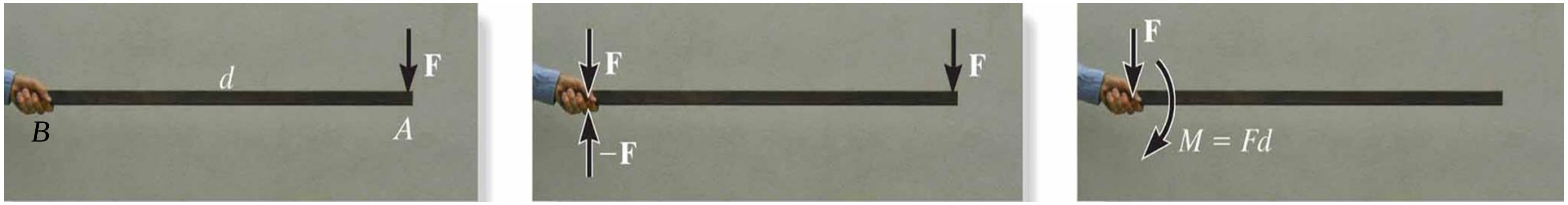
MOVING A FORCE ON ITS LINE OF ACTION



Moving a force from A to B, when both points are on the vector's line of action, does not change the external effect.

Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).

MOVING A FORCE OFF OF ITS LINE OF ACTION

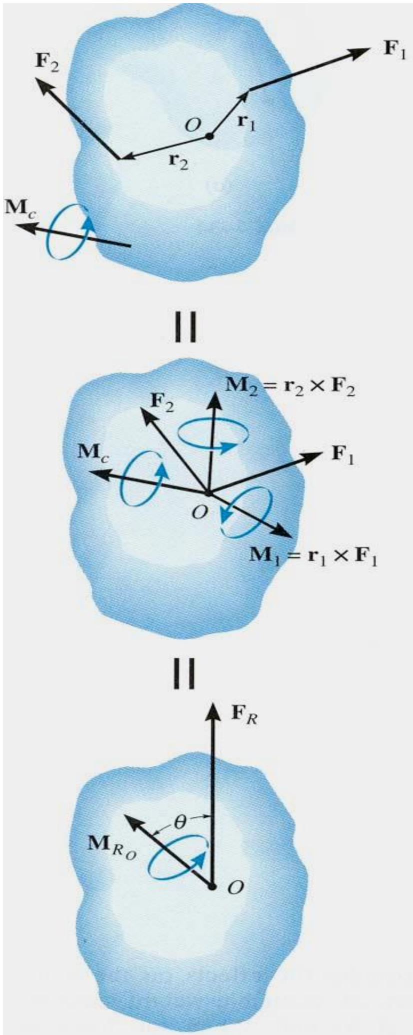


When a force is moved, but not along its line of action, there is a change in its external effect!

Essentially, moving a force from point A to B (as shown above) requires creating an additional couple moment. So moving a force means you have to “add” a new couple.

Since this new couple moment is a “free” vector, it can be applied at any point on the body.

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM



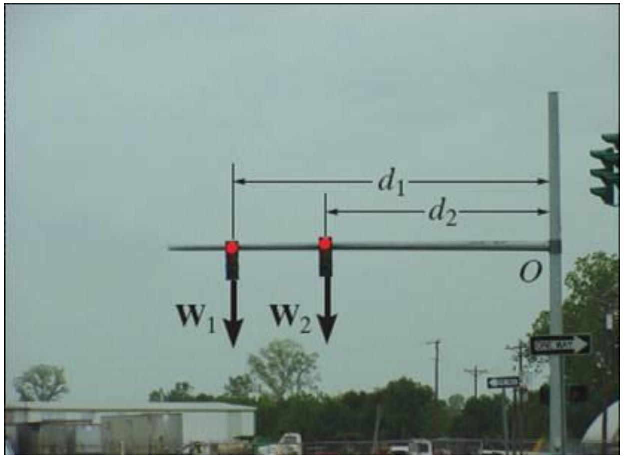
When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

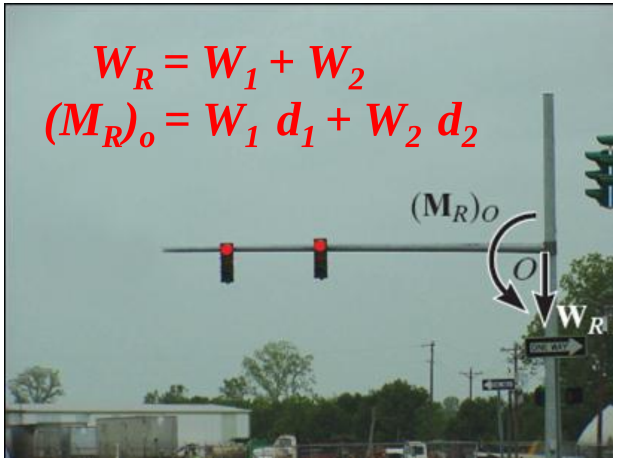
Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM (continued)





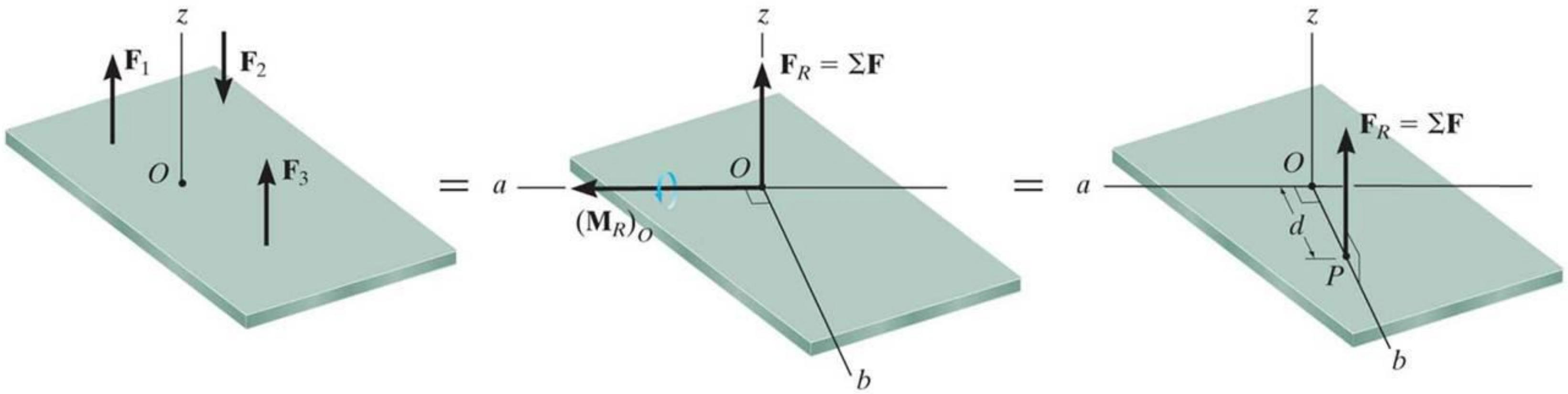
If the force system lies in the x-y plane (a 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$

$$F_{R_y} = \Sigma F_y$$

$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

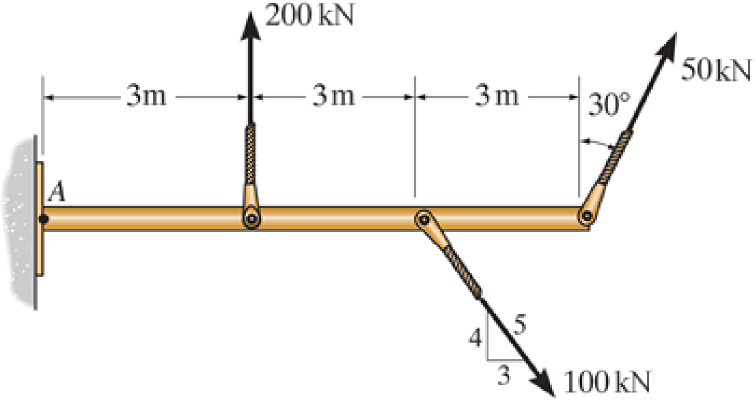
FURTHER SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM (Section 4.8)



If F_R and M_{RO} are perpendicular to each other, then the system can be further reduced to a single force, F_R , by simply moving F_R from O to P.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

EXAMPLE I



Given: A 2-D force system with geometry as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location measured from A.

Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force component to A.

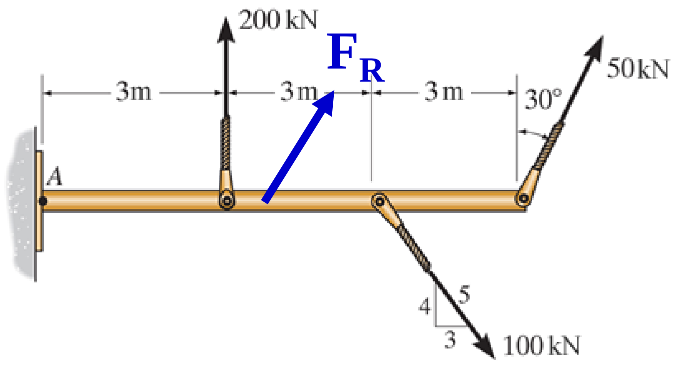
3) Shift F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE I (continued)

$+\rightarrow \Sigma F_{Rx} = 50(\sin 30) + 100(3/5)$
$= 85 \text{ kN}$

$+\uparrow \Sigma F_{Ry} = 200 + 50(\cos 30) - 100(4/5)$
$= 163.3 \text{ kN}$

$+\circlearrowleft M_{RA} = 200\,(3) + 50\,(\cos 30)\,(9)$
$- 100\,(4/5)\,6 =$ 509.7 kN·m ↺



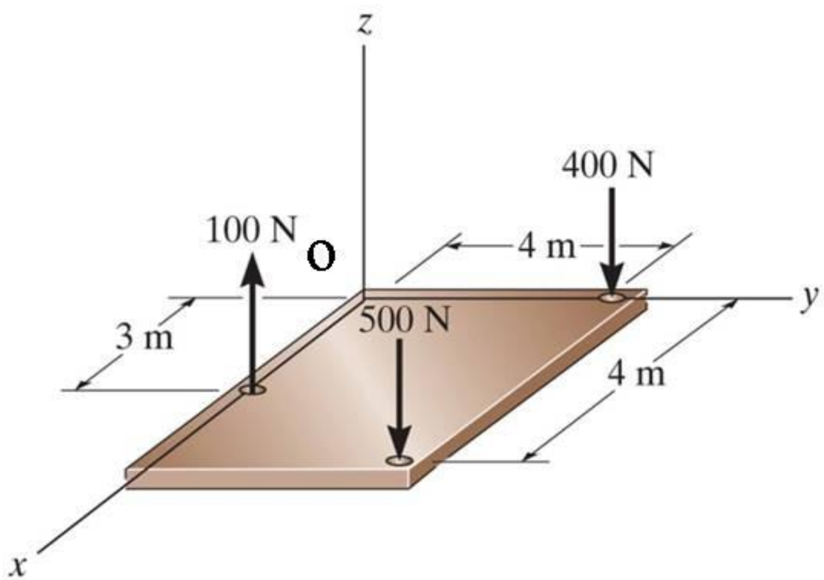
$F_R = (85^2 + 163.3^2)^{1/2} =$ 184 kN

$\angle\ \theta = \tan^{-1}(163.3/85) =$ 62.5°

The equivalent single force F_R can be located at a distance d measured from A.

$d = M_{RA}/F_{Ry} = 509.7 / 163.3 =$ 3.12 m

EXAMPLE II



Given: The slab is subjected to three parallel forces.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x, y) of the single equivalent resultant force.

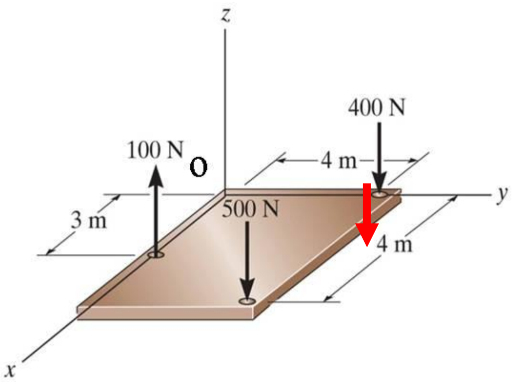
Plan:

1) Find $\boldsymbol{F}_{RO} = \sum \boldsymbol{F}_i = F_{Rzo}\, \boldsymbol{k}$

2) Find $\boldsymbol{M}_{RO} = \sum (\boldsymbol{r}_i \times \boldsymbol{F}_i) = M_{RxO}\, \boldsymbol{i} + M_{RyO}\, \boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO} / F_{RzO}$ and $y = M_{RxO} / F_{RzO}$

EXAMPLE II (continued)



$\boldsymbol{F}_{RO} = \{100\,\boldsymbol{k} - 500\,\boldsymbol{k} - 400\,\boldsymbol{k}\} = -\,800\,\boldsymbol{k}$ N

$\boldsymbol{M}_{RO} = (3\,\boldsymbol{i}) \times (100\,\boldsymbol{k}) + (4\,\boldsymbol{i} + 4\,\boldsymbol{j}) \times (-500\,\boldsymbol{k})$

$+ (4\,\boldsymbol{j}) \times (-400\,\boldsymbol{k})$

$= \{-300\,\boldsymbol{j} + 2000\,\boldsymbol{j} - 2000\,\boldsymbol{i} - 1600\,\boldsymbol{i}\}$

$= \{\,\underline{-\,3600\,\boldsymbol{i}} + \underline{1700\,\boldsymbol{j}}\,\}\underline{\text{N}\cdot\text{m}}$

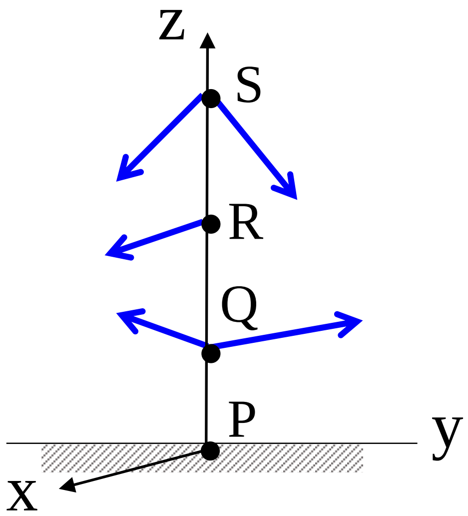
The location of the single equivalent resultant force is given as,

$x = -\,M_{Ryo} / F_{Rzo} = (-1700) / (-800) = \underline{2.13\text{ m}}$

$y = M_{Rxo} / F_{Rzo} = (-3600) / (-800) = \underline{4.5\text{ m}}$

CONCEPT QUIZ

1. The forces on the pole can be reduced to a single force and a single moment at point ____ .



A) P B) Q C) R

D) S E) Any of these points.

2. Consider two couples acting on a body. The simplest possible equivalent system at any arbitrary point on the body will have

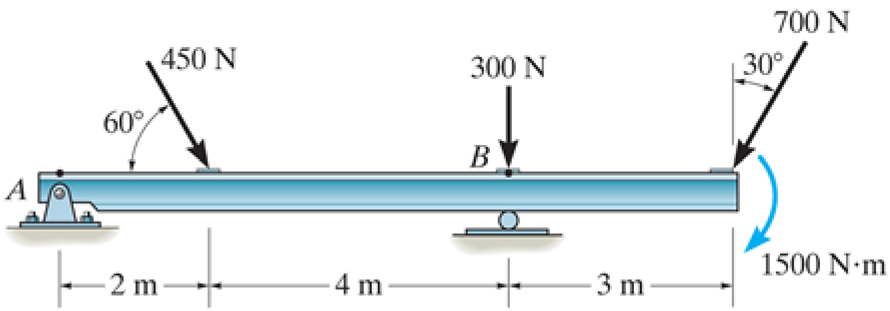
A) One force and one couple moment.

B) One force.

C) One couple moment.

D) Two couple moments.

GROUP PROBLEM SOLVING I



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A.

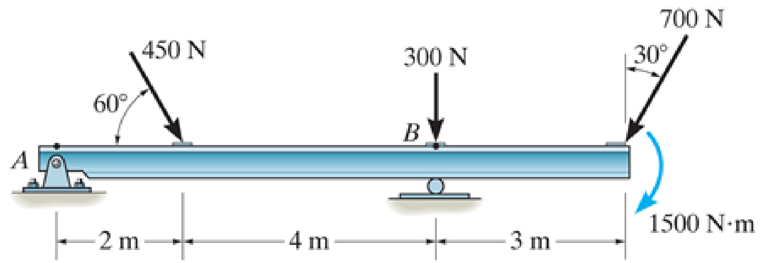
Plan:

1) Sum all the x and y components of the two forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A and add them to the 1500 N·m free moment to find the resultant M_{RA} .

GROUP PROBLEM SOLVING I (continued)

Summing the force components:



$+\rightarrow \Sigma F_x = 450\,(\cos 60) - 700\,(\sin 30)$

$= -125 \text{ N}$

$+\uparrow \Sigma F_y = -450\,(\sin 60) - 300 - 700\,(\cos 30)$

$= -1296 \text{ N}$

Now find the magnitude and direction of the resultant.

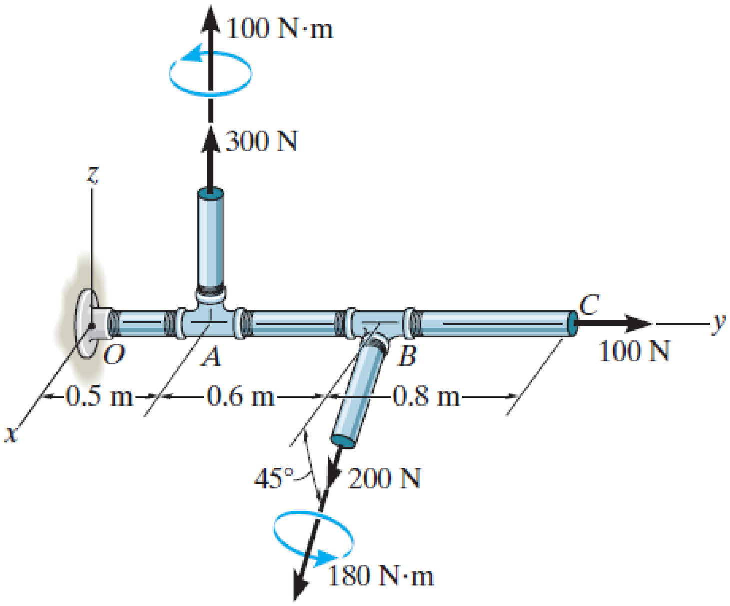
$F_{RA} = (125^2 + 1296^2)^{1/2} = \underline{1302 \text{ N}}$ and $\theta = \tan^{-1}(1296/125)$

$= \underline{84.5°}$

$+\circlearrowleft M_{RA} = 450\,(\sin 60)\,(2) + 300\,(6) + 700\,(\cos 30)\,(9) + 1500$

$= \underline{9535 \text{ N·m}} \circlearrowleft$

GROUP PROBLEM SOLVING II



Given: Forces and couple moments are applied to the pipe.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $\boldsymbol{F}_{RO} = \Sigma\, \boldsymbol{F}_i = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$

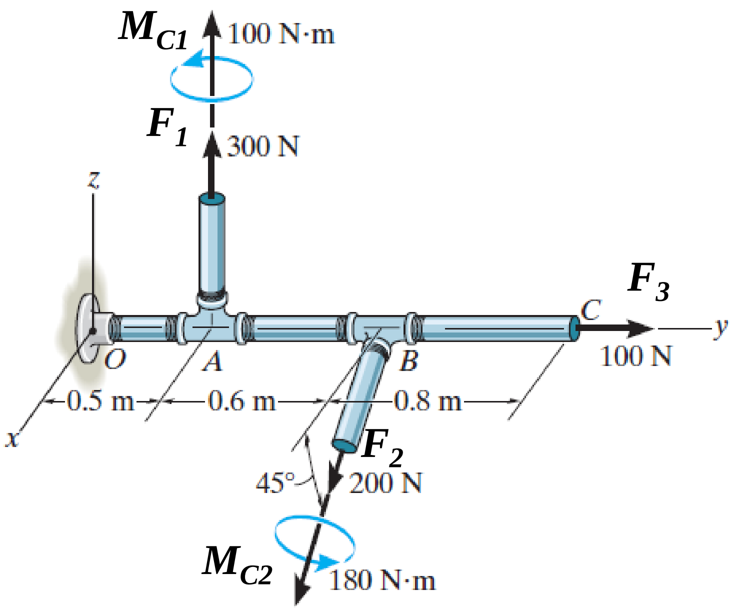
b) Find $\boldsymbol{M}_{RO} = \Sigma\, \boldsymbol{M}_C + \Sigma\, (\boldsymbol{r}_i \times \boldsymbol{F}_i)$

where,

$\boldsymbol{M}_C$ are any free couple moments.

$\boldsymbol{r}_i$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F}_i$.

GROUP PROBLEM SOLVING II (continued)



$\boldsymbol{F_1} = \{300\ \boldsymbol{k}\}$ N

$\boldsymbol{F_2} = 200\{\cos 45°\ \boldsymbol{i} - \sin 45°\ \boldsymbol{k}\}$ N

$= \{141.4\ \boldsymbol{i} - 141.4\ \boldsymbol{k}\}$ N

$\boldsymbol{F_3} = \{100\ \boldsymbol{j}\}$ N

$\boldsymbol{r_1} = \{0.5\ \boldsymbol{i}\}$ m, $\boldsymbol{r_2} = \{1.1\ \boldsymbol{i}\}$ m,

$\boldsymbol{r_3} = \{1.9\ \boldsymbol{i}\}$ m

Free couple moments are:

$\boldsymbol{M_{C1}} = \{100\ \boldsymbol{k}\}$ N·m

$\boldsymbol{M_{C2}} = 180\{\cos 45°\ \boldsymbol{i} - \sin 45°\boldsymbol{k}\}$N·m

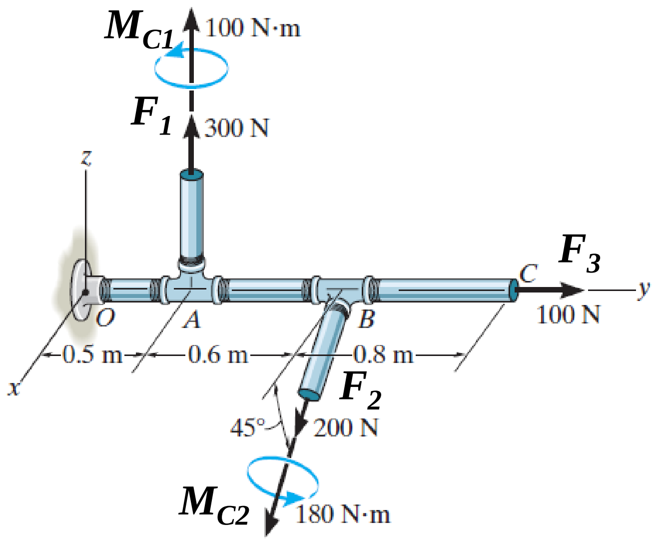
$= \{127.3\ \boldsymbol{i} - 127.3\boldsymbol{k}\}$N·m

GROUP PROBLEM SOLVING II (continued)

Resultant force and couple moment at point O:

$\boldsymbol{F}_{RO} = \Sigma\, \boldsymbol{F}_i = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$

$= \{300\, \boldsymbol{k}\} + \{141.4\, \boldsymbol{i} - 141.4\, \boldsymbol{k}\} + \{100\, \boldsymbol{j}\}$



$\boldsymbol{F}_{RO} = \{\underline{141}\, \boldsymbol{i} + \underline{100}\, \boldsymbol{j} + \underline{159}\, \boldsymbol{k}\}\ \underline{\text{N}}$

$\boldsymbol{M}_{RO} = \Sigma\, \boldsymbol{M}_C + \Sigma\, (\boldsymbol{r}_i \times \boldsymbol{F}_i)$

$\boldsymbol{M}_{RO} = \{100\, \boldsymbol{k}\} + \{127.3\, \boldsymbol{i} - 127.3\boldsymbol{k}\}$

$$+ \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1.1 & 0 \\ 141.4 & 0 & -141.4 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1.9 & 0 \\ 0 & 100 & 0 \end{vmatrix}$$

$\boldsymbol{M}_{RO} = \{\underline{122}\, \boldsymbol{i}\ \underline{-183}\, \boldsymbol{k}\}\ \underline{\text{N·m}}$

ATTENTION QUIZ

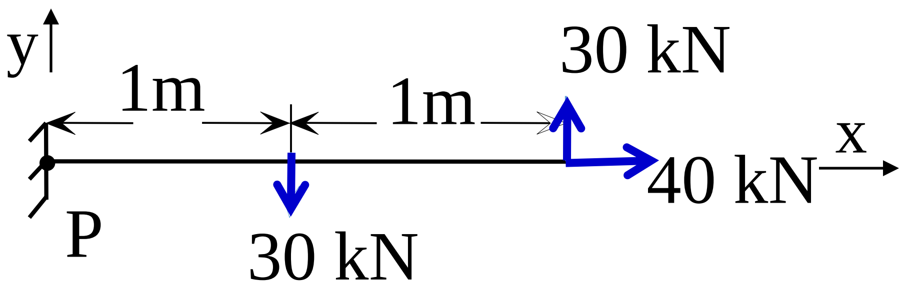
1. For this force system, the equivalent system at P is ____________ .

A) F_{RP} = 40 kN (along +x-dir.) and M_{RP} = +60 kN · m

B) F_{RP} = 0 kN and M_{RP} = +30 kN · m

C) F_{RP} = 30 kN (along +y-dir.) and M_{RP} = -30 kN · m

D) F_{RP} = 40 kN (along +x-dir.) and M_{RP} = +30 kN · m



ATTENTION QUIZ

2. Consider three couples acting on a body. Equivalent systems will be _______ at different points on the body.

A) Different when located

B) The same even when located

C) Zero when located

D) None of the above.

End of the Lecture

Let Learning Continue