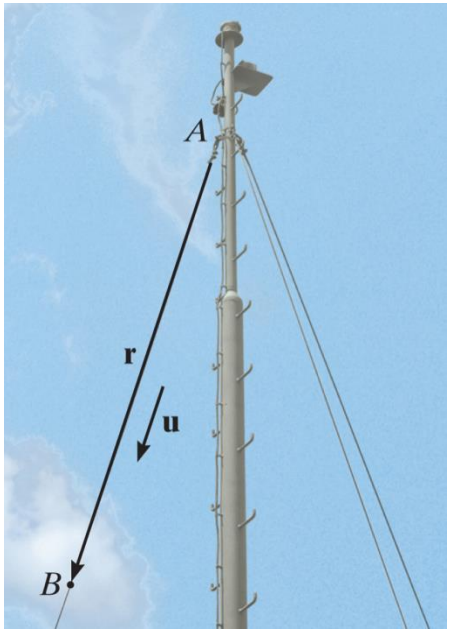
POSITION VECTORS & FORCE VECTORS

Today's Objectives:

Students will be able to :

a) Represent a position vector in Cartesian coordinate form, from given geometry.

b) Represent a force vector directed along a line.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications / Relevance
- Write Position Vectors
- Write a Force Vector along a line
- Example Problem
- Concept Quiz
- Group Problem
- Attention Quiz

READING QUIZ

1. The position vector $\boldsymbol{r_{PQ}}$ is obtained by

   A) Coordinates of Q minus coordinates of the origin

   B) Coordinates of P minus coordinates of Q

   C) Coordinates of Q minus coordinates of P

   D) Coordinates of the origin minus coordinates of P

2. A force of magnitude F, directed along a unit vector $\boldsymbol{U}$, is given by $\boldsymbol{F}$ = _____ .

   A) F ($\boldsymbol{U}$)

   B) $\boldsymbol{U}$ / F

   C) F / $\boldsymbol{U}$

   D) F + $\boldsymbol{U}$

   E) F – $\boldsymbol{U}$

APPLICATIONS

This ship's mooring line, connected to the bow, can be represented as a Cartesian vector.

What are the forces in the mooring line and how do we find their directions?

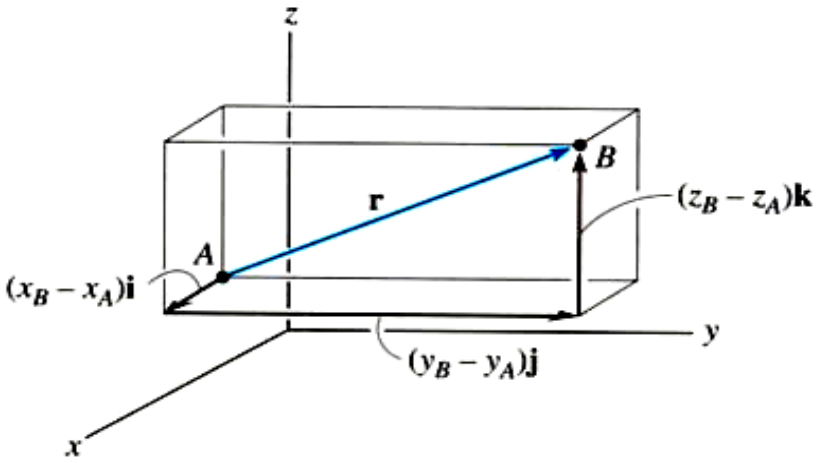
Why would we want to know these things?

APPLICATIONS (continued)

This awning is held up by three chains. What are the forces in the chains and how do we find their directions? Why would we want to know these things?

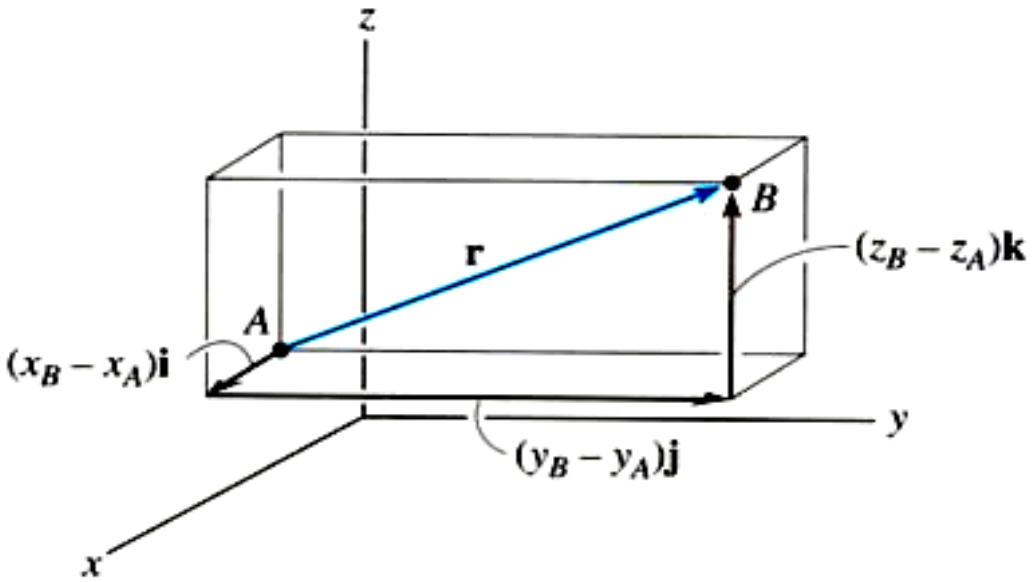
POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.



Consider two points, A and B, in 3-D space.
Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.

POSITION VECTOR (continued)

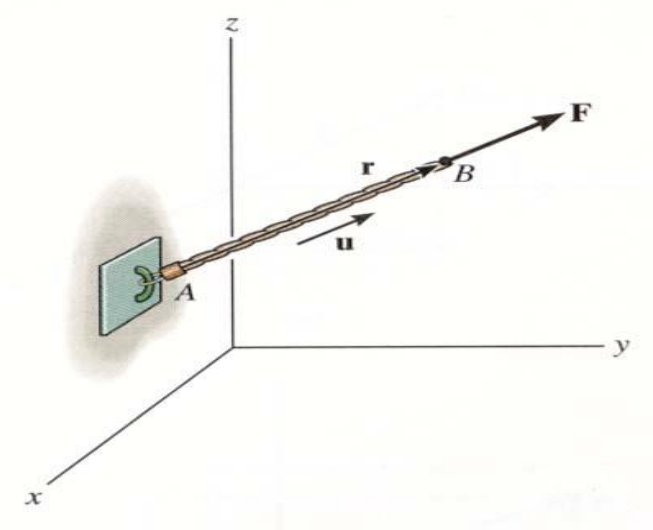


The position vector directed from A to B, $\boldsymbol{r_{AB}}$, is defined as

$$\boldsymbol{r_{AB}} = \{(X_B - X_A)\,\boldsymbol{i} + (Y_B - Y_A)\,\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}\text{m}$$

Please note that B is the ending point and A is the starting point. ALWAYS subtract the "tail" coordinates from the "tip" coordinates!

FORCE VECTOR DIRECTED ALONG A LINE (Section 2.8)



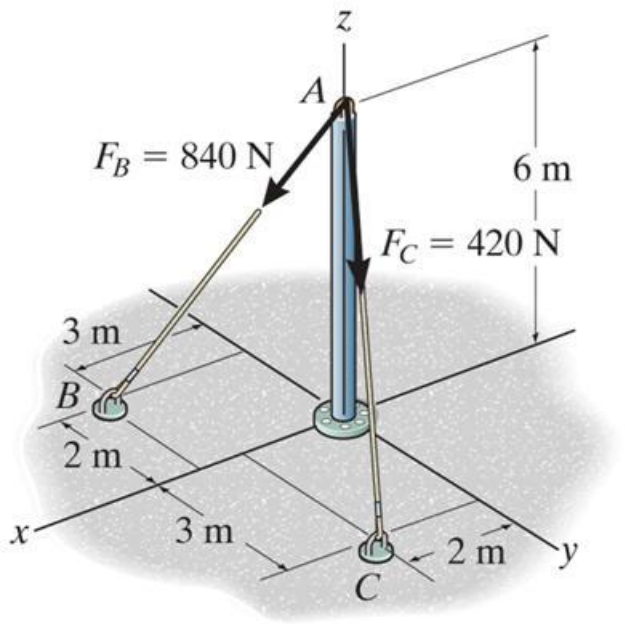
If a force is directed along a line, then we can represent the force vector in Cartesian coordinates by using a unit vector and the force's magnitude. So we need to:

a) Find the position vector, $\boldsymbol{r_{AB}}$, along two points on that line.

b) Find the unit vector describing the line's direction, $\boldsymbol{u_{AB}} = (\boldsymbol{r_{AB}}/r_{AB})$.

c) Multiply the unit vector by the magnitude of the force, $\boldsymbol{F} = F\ \boldsymbol{u_{AB}}$.

EXAMPLE



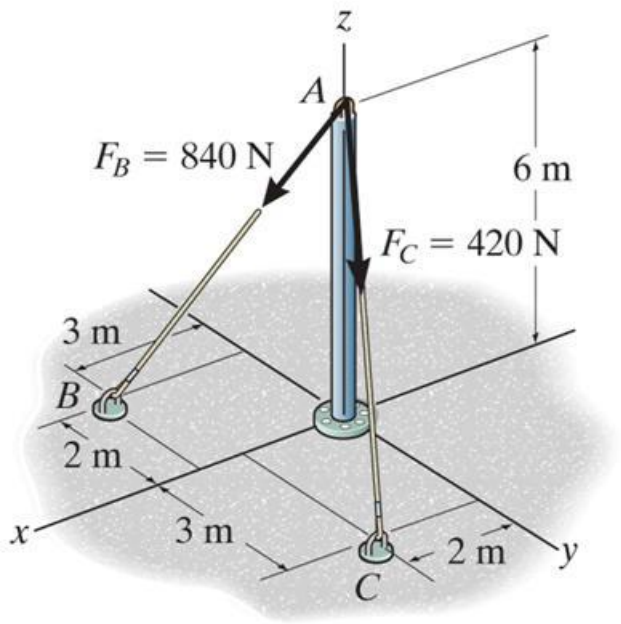
Given: The 420 N force along the cable AC.

Find: The force $\boldsymbol{F_{AC}}$ in the Cartesian vector form.

Plan:

1. Find the position vector $\boldsymbol{r_{AC}}$ and its unit vector $\boldsymbol{u_{AC}}$.
2. Obtain the force vector as $\boldsymbol{F_{AC}}$ = 420 N $\boldsymbol{u_{AC}}$.

EXAMPLE (continued)



As per the figure, when relating A to C, we will have to go 2 m in the x-direction, 3 m in the y-direction, and -6 m in the z-direction. Hence,

$\boldsymbol{r}_{AC} = \{2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m.

(We can also find $\boldsymbol{r}_{AC}$ by subtracting the coordinates of A from the coordinates of C.)

$r_{AC} = \{2^2 + 3^2 + (-6)^2\}^{1/2} = 7$ m

Now $\boldsymbol{u}_{AC} = \boldsymbol{r}_{AC}/r_{AC}$ and $\boldsymbol{F}_{AC} = 420\,\boldsymbol{u}_{AC} = 420\,(\boldsymbol{r}_{AC}/r_{AC})$

So $\boldsymbol{F}_{AC} = 420\{(2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 6\,\boldsymbol{k})/7\}$ N

$= \{120\,\boldsymbol{i} + 180\,\boldsymbol{j} - 360\,\boldsymbol{k}\}$ N

CONCEPT QUIZ

1. **P** and **Q** are two points in a 3-D space. How are the position vectors $\boldsymbol{r}_{PQ}$ and $\boldsymbol{r}_{QP}$ related?

   A) $\boldsymbol{r}_{PQ} = \boldsymbol{r}_{QP}$ B) $\boldsymbol{r}_{PQ} = -\boldsymbol{r}_{QP}$

   C) $\boldsymbol{r}_{PQ} = 1/\boldsymbol{r}_{QP}$ D) $\boldsymbol{r}_{PQ} = 2\,\boldsymbol{r}_{QP}$

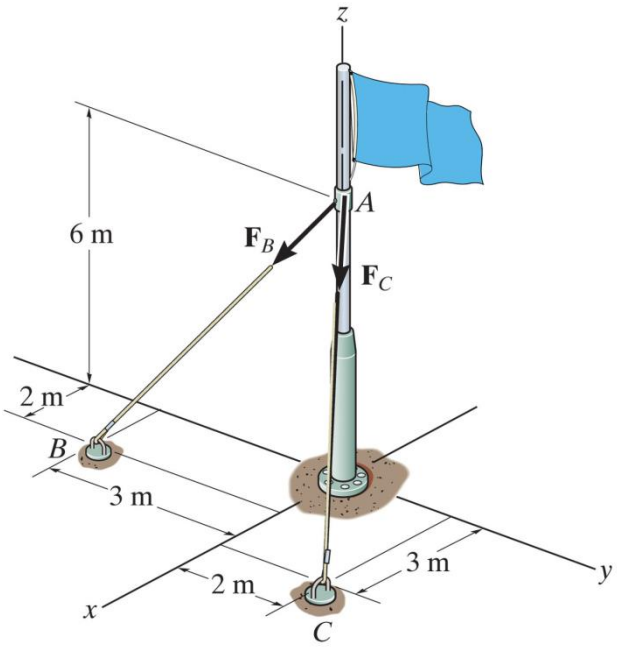
2. If $\boldsymbol{F}$ and $\boldsymbol{r}$ are force and position vectors, respectively, in SI units, what are the units of the expression (r * ($\boldsymbol{F}$ / F)) ?

   A) Newton B) Dimensionless

   C) Meter D) Newton - Meter

   E) The expression is algebraically illegal.

GROUP PROBLEM SOLVING



Given: Two forces are acting on a flag pole as shown in the figure. F_B = 560 N and F_C = 700 N

Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

1) Find the forces along AB and AC in the Cartesian vector form.
2) Add the two forces to get the resultant force, $\boldsymbol{F_R}$.
3) Determine the magnitude and the coordinate angles of $\boldsymbol{F_R}$.

GROUP PROBLEM SOLVING (continued)

$\boldsymbol{r}_{AB} = \{2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m

$\boldsymbol{r}_{AC} = \{3\,\boldsymbol{i} + 2\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m

$r_{AB} = \{2^2 + (-3)^2 + (-6)^2\}^{1/2} = 7$ m

$r_{AC} = \{3^2 + 2^2 + (-6)^2\}^{1/2} = 7$ m

$\boldsymbol{F}_{AB} = 560\,(\boldsymbol{r}_{AB} / r_{AB})$ N

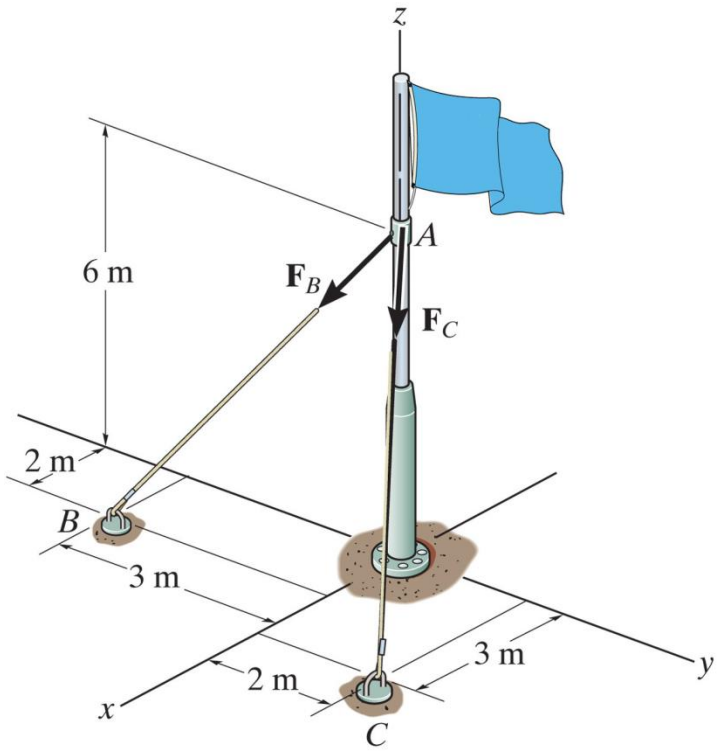
$\boldsymbol{F}_{AB} = 560\,(2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 6\,\boldsymbol{k}) / 7$ N

$\boldsymbol{F}_{AB} = (160\,\boldsymbol{i} - 240\,\boldsymbol{j} - 480\,\boldsymbol{k})$ N

$\boldsymbol{F}_{AC} = 700\,(\boldsymbol{r}_{AC} / r_{AC})$ N

$\boldsymbol{F}_{AC} = 700\,(3\,\boldsymbol{i} + 2\,\boldsymbol{j} - 6\,\boldsymbol{k}) / 7$ N

$\boldsymbol{F}_{AC} = \{300\,\boldsymbol{i} + 200\,\boldsymbol{j} - 600\,\boldsymbol{k}\}$ N

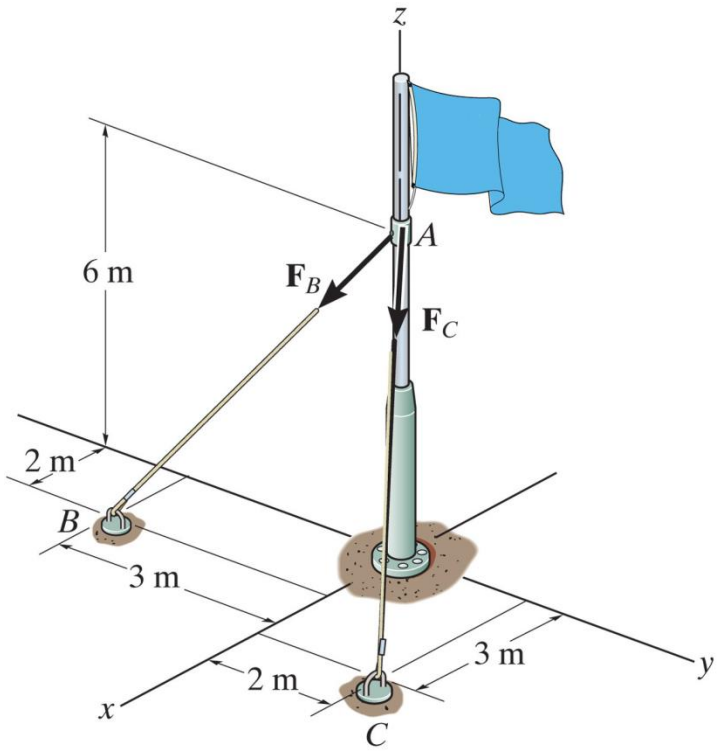


GROUP PROBLEM SOLVING (continued)

$$\boldsymbol{F}_R = \boldsymbol{F}_{AB} + \boldsymbol{F}_{AC}$$
$$= \{460\,\boldsymbol{i} - 40\,\boldsymbol{j} - 1080\,\boldsymbol{k}\}\ \text{N}$$

$$F_R = \{460^2 + (-40)^2 + (-1080)^2\}^{1/2}$$
$$= 1174.6\ \text{N}$$
$$F_R = \underline{1175\ \text{N}}$$

$$\alpha = \cos^{-1}(460/1175) = \underline{66.9°}$$
$$\beta = \cos^{-1}(-40/1175) = \underline{92.0°}$$
$$\gamma = \cos^{-1}(-1080/1175) = \underline{157°}$$



ATTENTION QUIZ

1. Two points in 3–D space have coordinates of P (1, 2, 3) and Q (4, 5, 6) meters. The position vector $\boldsymbol{r}_{QP}$ is given by

A) $\{3\,\boldsymbol{i} + 3\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ m

B) $\{-3\,\boldsymbol{i} - 3\,\boldsymbol{j} - 3\,\boldsymbol{k}\}$ m

C) $\{5\,\boldsymbol{i} + 7\,\boldsymbol{j} + 9\,\boldsymbol{k}\}$ m

D) $\{-3\,\boldsymbol{i} + 3\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ m

E) $\{4\,\boldsymbol{i} + 5\,\boldsymbol{j} + 6\,\boldsymbol{k}\}$ m

2. A force vector, $\boldsymbol{F}$, directed along a line defined by PQ is given by

A) $(\boldsymbol{F}/\,\mathrm{F})\,\boldsymbol{r}_{PQ}$ B) $\boldsymbol{r}_{PQ}/\mathrm{r}_{PQ}$

C) $\mathrm{F}(\boldsymbol{r}_{PQ}/\mathrm{r}_{PQ})$ D) $\mathrm{F}(\mathrm{r}_{PQ}/\boldsymbol{r}_{PQ})$

End of the Lecture

Let Learning Continue