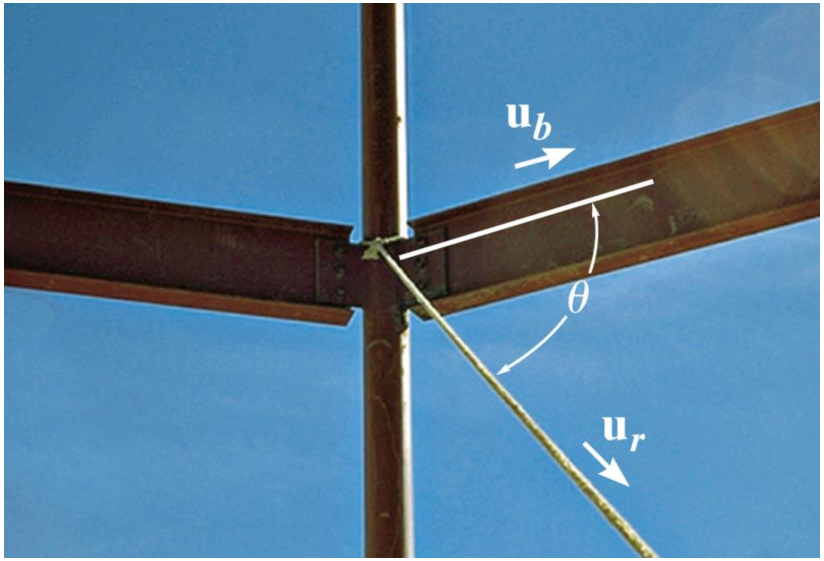
DOT PRODUCT

Today's Objective:

Students will be able to use the vector dot product to:

a) determine an angle between two vectors and,

b) determine the projection of a vector along a specified line.

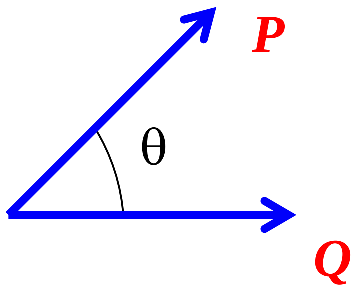


In-Class Activities:

- Check Homework
- Reading Quiz
- Applications / Relevance
- Dot product - Definition
- Angle Determination
- Determining the Projection
- Example Problem
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The dot product of two vectors ***P*** and ***Q*** is defined as



A) P Q sin θ   B) P Q cos θ

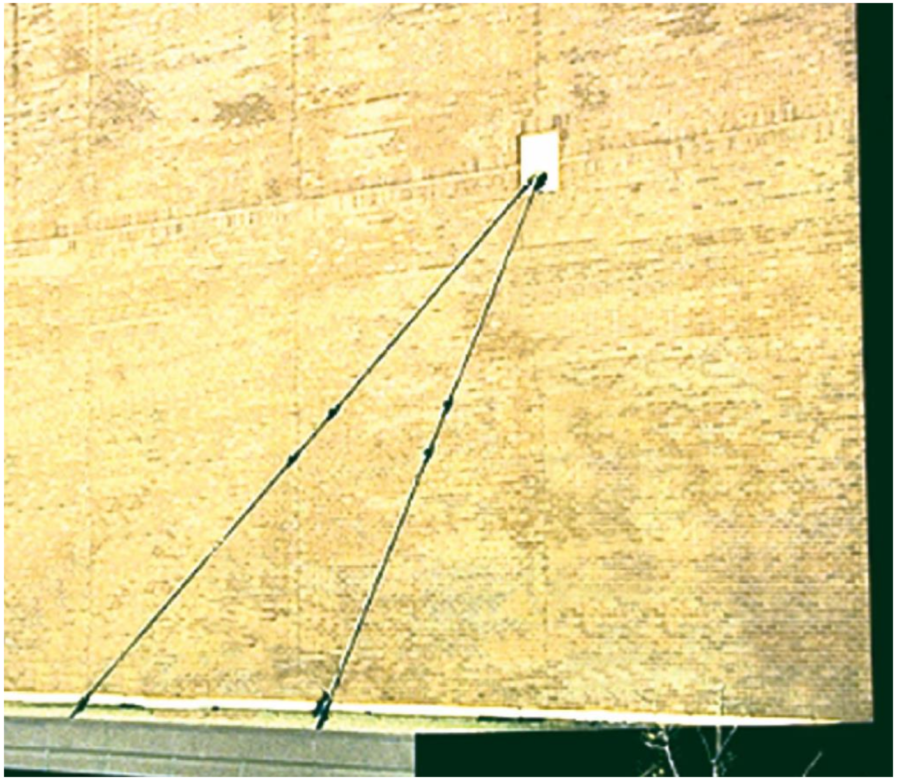
C) P Q tan θ   D) P Q sec θ

2. The dot product of two vectors results in a __________ quantity.

A) Scalar   B) Vector

C) Complex   D) Zero

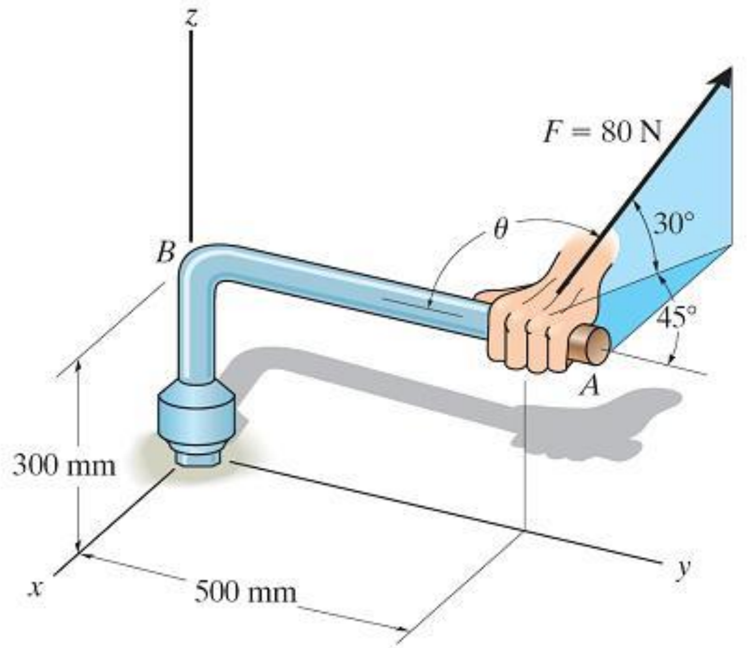
APPLICATIONS



If you know the physical locations of the four cable ends, how could you calculate the angle between the cables at the common anchor?

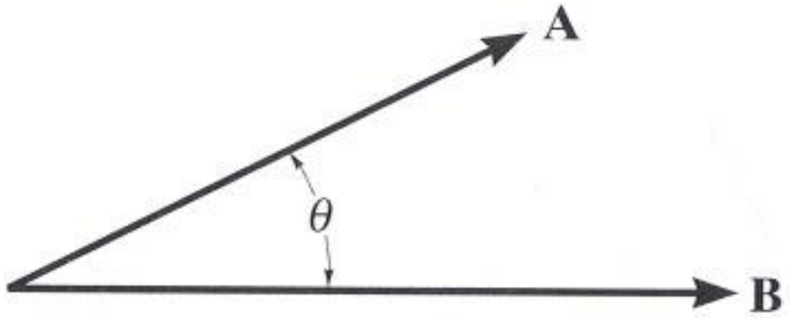
PEARSON

APPLICATIONS (continued)



For the force ***F*** applied to the wrench at Point A, what component of it actually helps turn the bolt (i.e., the force component acting perpendicular to arm AB of the pipe)?

DEFINITION



The dot product of vectors ***A*** and ***B*** is defined as ***A***•***B*** = A B cos θ.

The angle θ is the smallest angle between the two vectors and is always in a range of 0° to 180°.

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors.

DOT PRODUCT DEFINITON (continued)

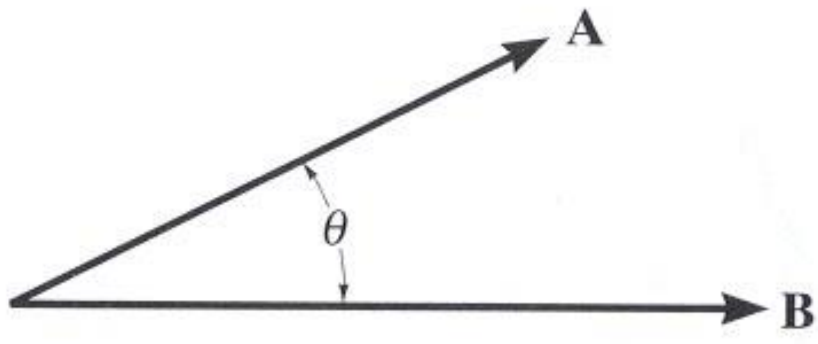
Examples: By definition, $\boldsymbol{i} \bullet \boldsymbol{j} = 0$

$$\boldsymbol{i} \bullet \boldsymbol{i} = 1$$

$$\boldsymbol{A} \bullet \boldsymbol{B} = (A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}) \bullet (B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



For these two vectors in Cartesian form, one can find the angle by

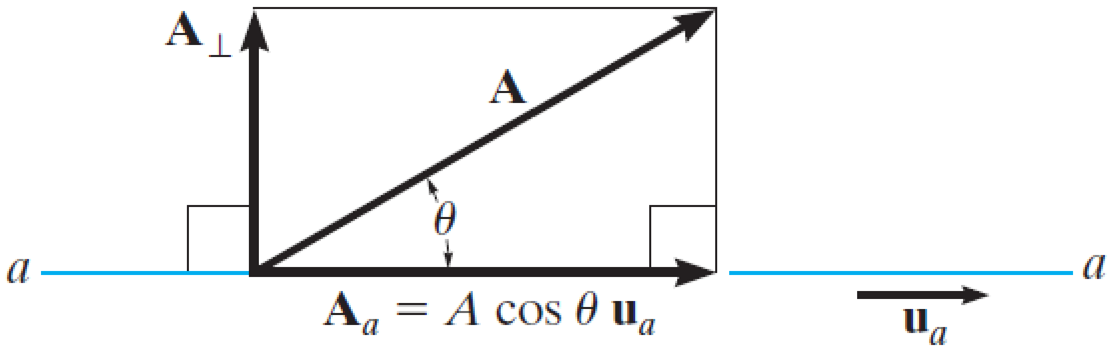
a) Find the dot product, $\boldsymbol{A} \bullet \boldsymbol{B} = (A_x B_x + A_y B_y + A_z B_z)$,

b) Find the magnitudes (A & B) of the vectors ***A*** & ***B***, and

c) Use the definition of dot product and solve for θ, i.e.,

$\theta = \cos^{-1} [(\boldsymbol{A} \bullet \boldsymbol{B})/(A\,B)]$, where $0^\circ \leq \theta \leq 180^\circ$.

DETERMINING THE PROJECTION OF A VECTOR



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, $\boldsymbol{u_a}$ along line aa

2. Find the scalar projection of $\boldsymbol{A}$ along line aa by

$$A_{\|} = \boldsymbol{A} \bullet \boldsymbol{u_a} = A_x u_x + A_y u_y + A_z u_z$$

DETERMINING THE PROJECTION OF A VECTOR (continued)

3. If needed, the projection can be written as a vector, $\mathbf{A}_{||}$, by using the unit vector $\mathbf{u}_a$ and the magnitude found in step 2.

$$\mathbf{A}_{||} = A_{||}\,\mathbf{u}_a$$

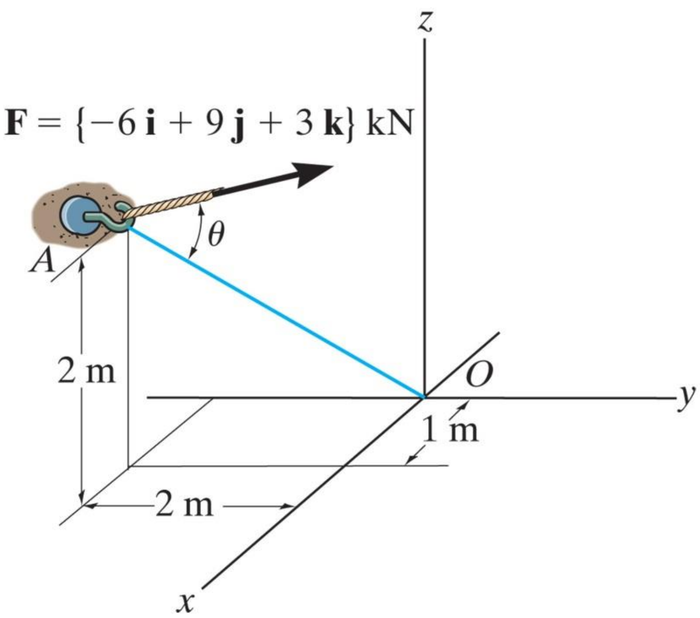
4. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_{\perp} = (A^2 - A_{||}^2)^{1/2} \text{ and}$$

$$\mathbf{A}_{\perp} = \mathbf{A} - \mathbf{A}_{||}$$

(rearranging the vector sum of $\mathbf{A} = \mathbf{A}_{\perp} + \mathbf{A}_{||}$)

EXAMPLE I



Given: The force acting on the hook at point A.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.

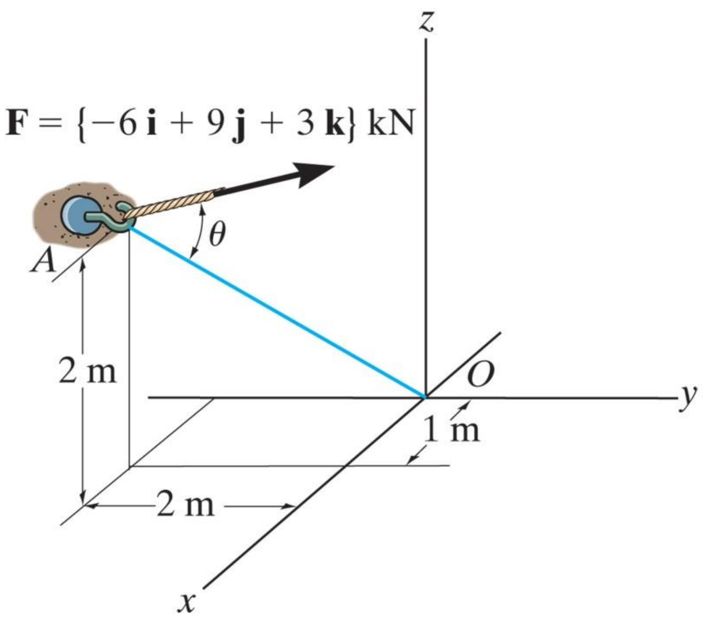
Plan:

1. Find $\boldsymbol{r}_{AO}$

2. Find the angle $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{AO})/(F \; r_{AO})\}$

3. Find the projection via $F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u}_{AO}$ (or $F \cos \theta$)

EXAMPLE I (continued)



$\boldsymbol{r}_{AO} = \{-1\,\boldsymbol{i} + 2\,\boldsymbol{j} - 2\,\boldsymbol{k}\}$ m

$r_{AO} = \{(-1)^2 + 2^2 + (-2)^2\}^{1/2} = 3$ m

$\boldsymbol{F} = \{-6\,\boldsymbol{i} + 9\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ kN

$F = \{(-6)^2 + 9^2 + 3^2\}^{1/2} = 11.22$ kN

$\boldsymbol{F} \bullet \boldsymbol{r}_{AO} = (-6)(-1) + (9)(2) + (3)(-2) = 18$ kN·m

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{AO})/(F\, r_{AO})\}$

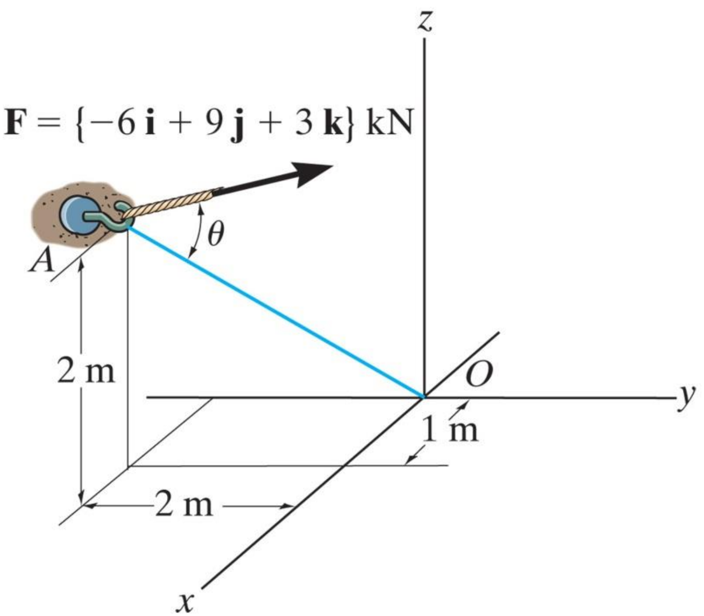
$\theta = \cos^{-1}\{18 / (11.22 \times 3)\} = \underline{57.67°}$

EXAMPLE I (continued)

$\boldsymbol{u}_{AO} = \boldsymbol{r}_{AO} / r_{AO} = (-1/3)\,\boldsymbol{i} + (2/3)\,\boldsymbol{j} + (-2/3)\,\boldsymbol{k}$

$F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u}_{AO} = (-6)(-1/3) + (9)(2/3) + (3)(-2/3) = \underline{6.00\text{ kN}}$

Or: $F_{AO} = F\cos\theta = 11.22\cos(57.67°) = \underline{6.00\text{ kN}}$



CONCEPT QUIZ

1. If a dot product of two non-zero vectors is 0, then the two vectors must be _______________ to each other.

   A) Parallel (pointing in the same direction)

   B) Parallel (pointing in the opposite direction)

   C) Perpendicular

   D) Cannot be determined.

2. If a dot product of two non-zero vectors equals -1, then the vectors must be _________ to each other.

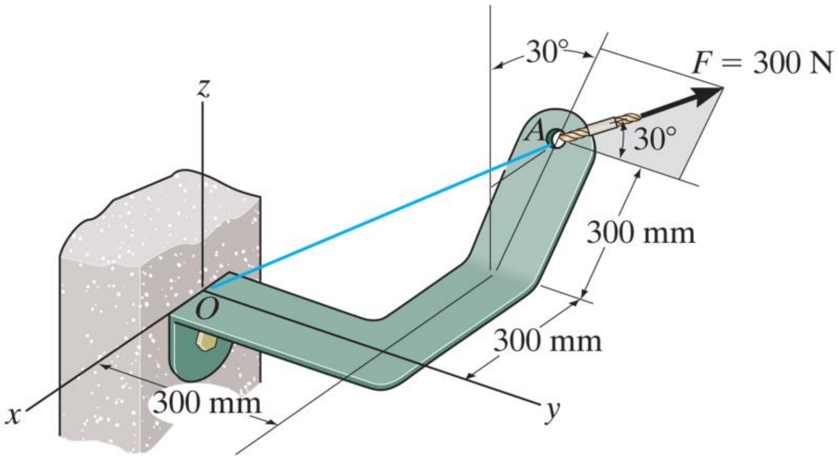
   A) Collinear but pointing in the opposite direction

   B) Parallel (pointing in the opposite direction)

   C) Perpendicular

   D) Cannot be determined.

GROUP PROBLEM SOLVING



Given: The 300 N force acting on the bracket.

Find: The magnitude of the projected component of this force acting along line OA

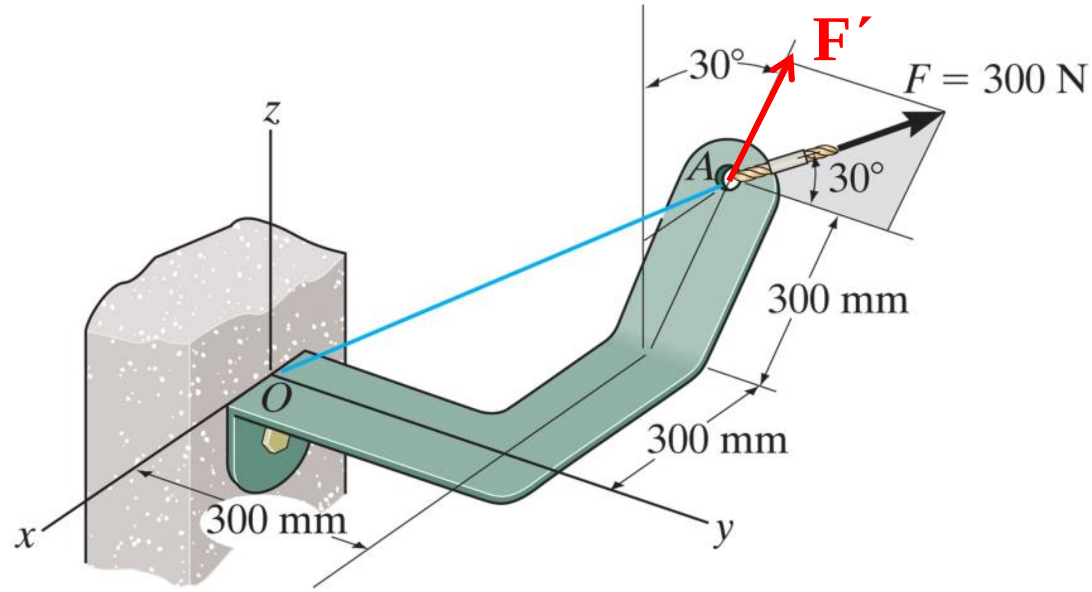
Plan:

1. Find $\boldsymbol{r}_{OA}$ and $\boldsymbol{u}_{OA}$

2. Find the angle $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{OA})/(\mathrm{F} \times \mathrm{r}_{\mathrm{OA}})\}$

3. Then find the projection via $\mathrm{F}_{\mathrm{OA}} = \boldsymbol{F} \bullet \boldsymbol{u}_{OA}$ or $\mathrm{F}\ (1) \cos \theta$

GROUP PROBLEM SOLVING (continued)

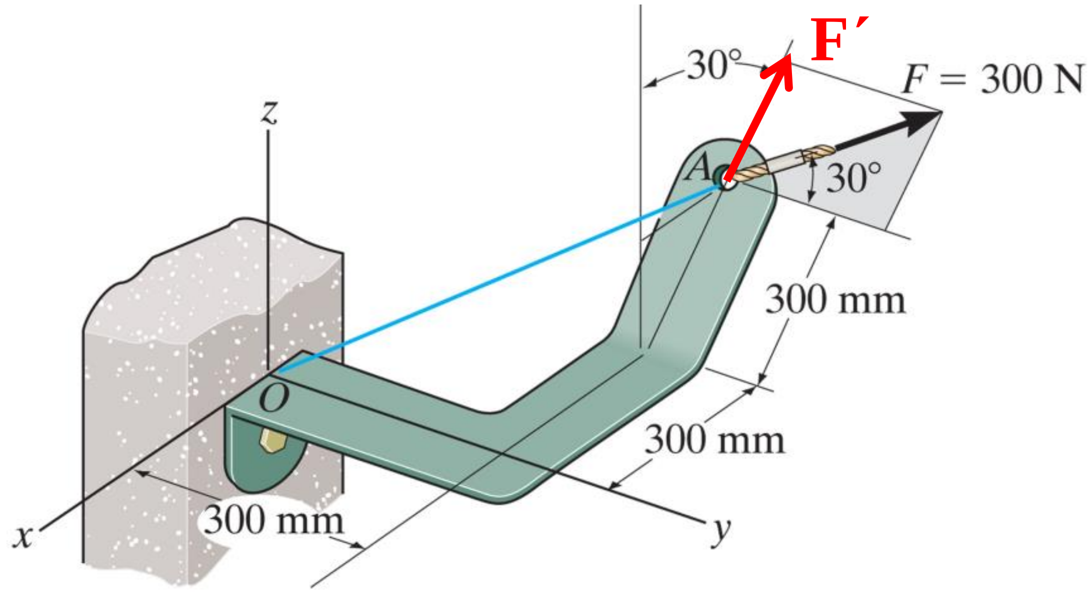


$\boldsymbol{r}_{OA} = \{-0.450\,\boldsymbol{i} + 0.300\,\boldsymbol{j} + 0.260\,\boldsymbol{k}\}$ m

$r_{OA} = \{(-0.450)^2 + 0.300^2 + 0.260^2\}^{1/2} = 0.60$ m

$\boldsymbol{u}_{OA} = \boldsymbol{r}_{OA} / r_{OA} = \{-0.75\,\boldsymbol{i} + 0.50\,\boldsymbol{j} + 0.433\,\boldsymbol{k}\}$

GROUP PROBLEM SOLVING (continued)



$F' = 300 \sin 30° = 150$ N

$\boldsymbol{F} = \{-150 \sin 30° \boldsymbol{i} + 300 \cos 30° \boldsymbol{j} + 150 \cos 30° \boldsymbol{k}\}$ N

$\boldsymbol{F} = \{-75 \boldsymbol{i} + 259.8 \boldsymbol{j} + 129.9 \boldsymbol{k}\}$ N

$F = \{(-75)^2 + 259.8^2 + 129.9^2\}^{1/2} = 300$ N

GROUP PROBLEM SOLVING (continued)

$\boldsymbol{F} \bullet \boldsymbol{r}_{OA} = (-75)(-0.45) + (259.8)(0.30) + (129.9)(0.26)$

$= 145.5 \text{ N·m}$

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{OA})/(F \times r_{OA})\}$

$\theta = \cos^{-1}\{145.5/(300 \times 0.60)\} = \underline{36.1°}$

The magnitude of the projected component of ***F*** along line OA will be

$F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u}_{OA}$

$= (-75)(-0.75) + (259.8)(0.50) + (129.9)(0.433)$

$= \underline{242 \text{ N}}$

Or

$F_{OA} = F \cos\theta = 300 \cos 36.1° = \underline{242 \text{ N}}$

ATTENTION QUIZ

1. The dot product can be used to find all of the following except _____ .

   A) sum of two vectors

   B) angle between two vectors

   C) component of a vector parallel to another line

   D) component of a vector perpendicular to another line

2. Find the dot product of the two vectors ***P*** and ***Q***.

   P = {5 ***i*** + 2 ***j*** + 3 ***k***} m

   Q = {-2 ***i*** + 5 ***j*** + 4 ***k***} m

   A) -12 m B) 12 m C) 12 m ^{2}

   D) -12 m ^{2} E) 10 m ^{2}

End of the Lecture

Let Learning Continue