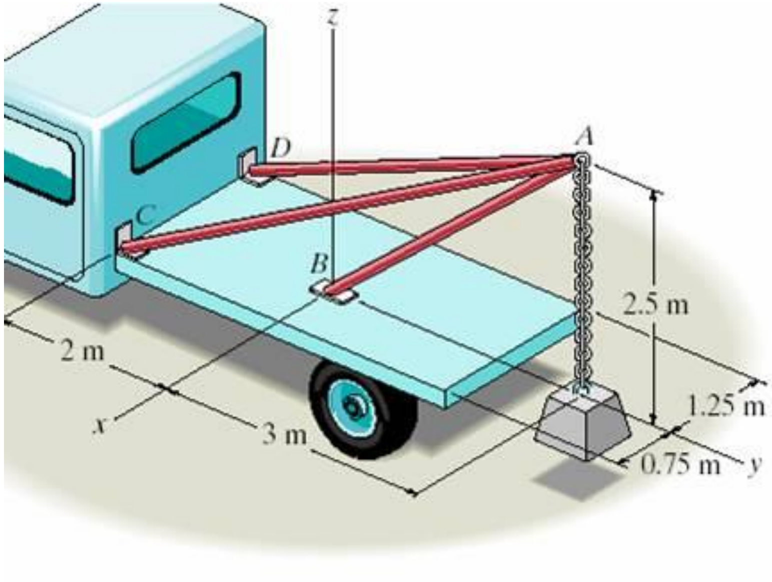
THREE-DIMENSIONAL FORCE SYSTEMS

Today's Objectives:

Students will be able to solve 3-D particle equilibrium problems by

a) Drawing a 3-D free body diagram, and,

b) Applying the three scalar equations (based on one vector equation) of equilibrium.



In-class Activities:

- Check Homework
- Reading Quiz
- Applications
- Equations of Equilibrium
- Example Problems
- Concept Questions
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. Particle P is in equilibrium with five (5) forces acting on it in 3-D space. How many scalar equations of equilibrium can be written for point P?

   A) 2 B) 3 C) 4

   D) 5 E) 6

2. In 3-D, when a particle is in equilibrium, which of the following equations apply?

   A) $(\Sigma F_x)\ \boldsymbol{i} + (\Sigma F_y)\ \boldsymbol{j} + (\Sigma F_z)\ \boldsymbol{k} = 0$

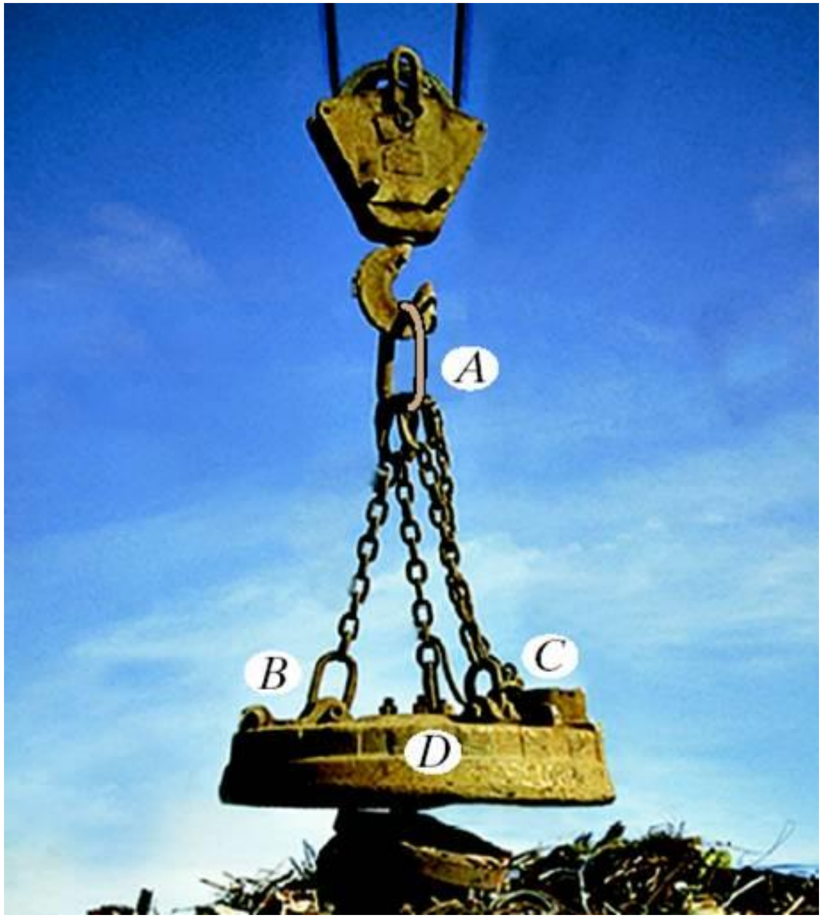
   B) $\Sigma \boldsymbol{F} = 0$

   C) $\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$

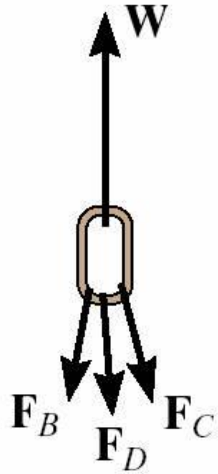
   D) All of the above.

   E) None of the above.

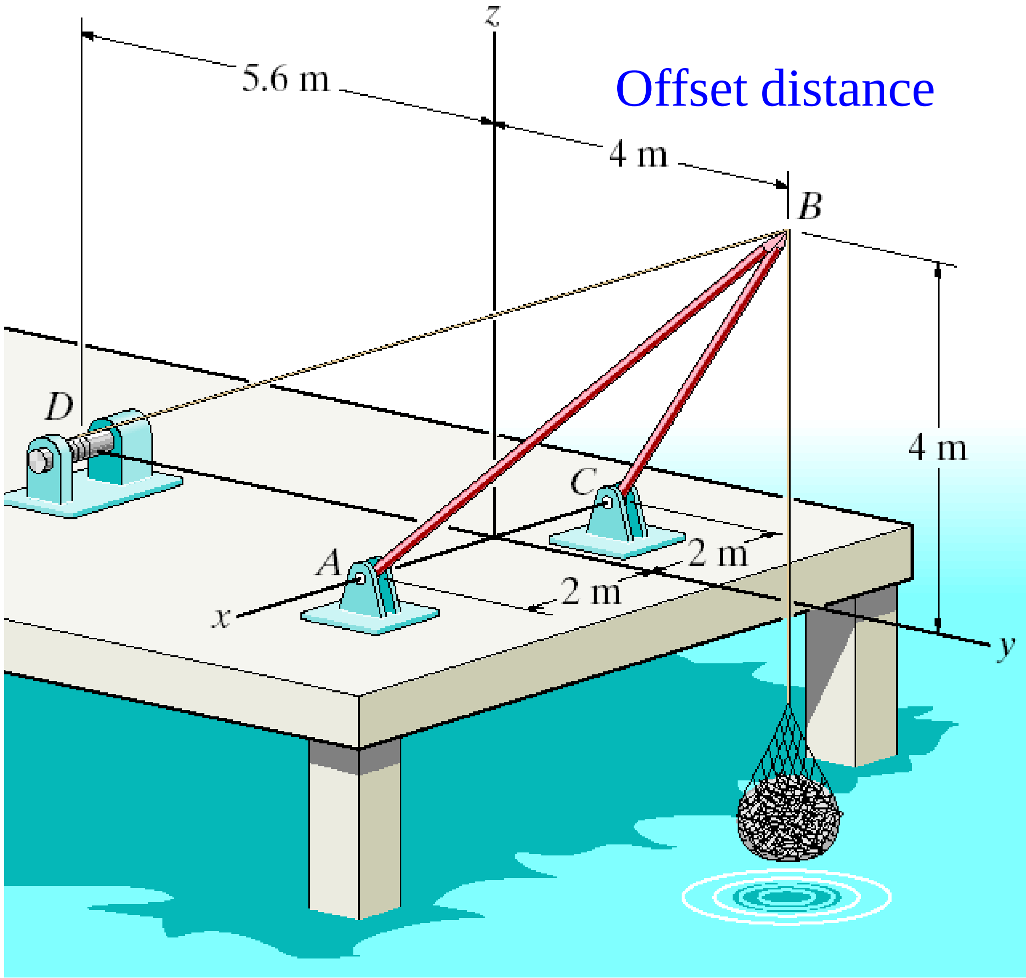
APPLICATIONS



You know the weight of the electromagnet and its load. But, you need to know the forces in the chains to see if it is a safe assembly. How would you do this?



APPLICATIONS (continued)

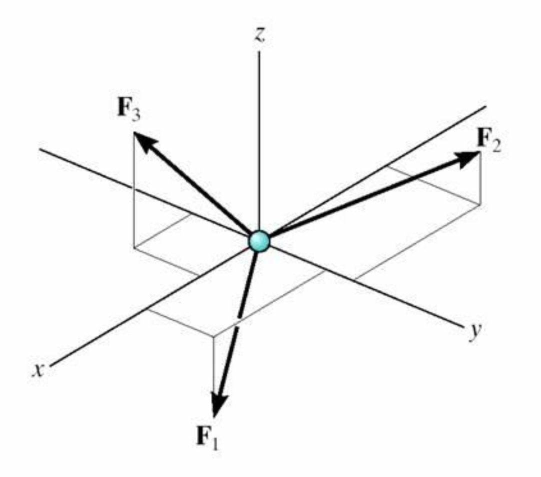


This shear-leg derrick is to be designed to lift a maximum of 200 kg of fish.

How would you find the effect of different offset distances on the forces in the cable and derrick legs?

THE EQUATIONS OF 3-D EQUILIBRIUM

When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero ($\Sigma \boldsymbol{F} = 0$).



This equation can be written in terms of its x, y and z components. This form is written as follows.

$$(\Sigma F_x)\, \boldsymbol{i} + (\Sigma F_y)\, \boldsymbol{j} + (\Sigma F_z)\, \boldsymbol{k} = 0$$

This vector equation will be satisfied only when

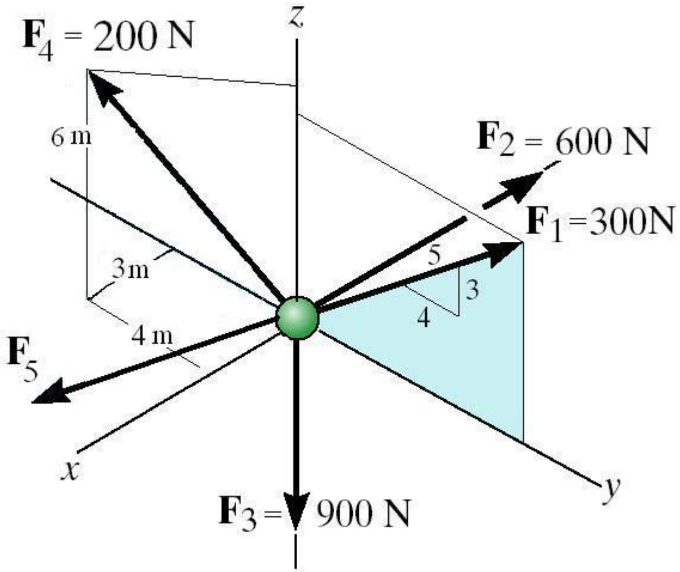
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

These equations are the three scalar equations of equilibrium. They are valid for any point in equilibrium and allow you to solve for up to three unknowns.

EXAMPLE I

Given: The four forces and geometry shown.

Find: The force ***F***$_5$ required to keep particle O in equilibrium.



Plan:

1) Draw a FBD of particle O.

2) Write the unknown force as

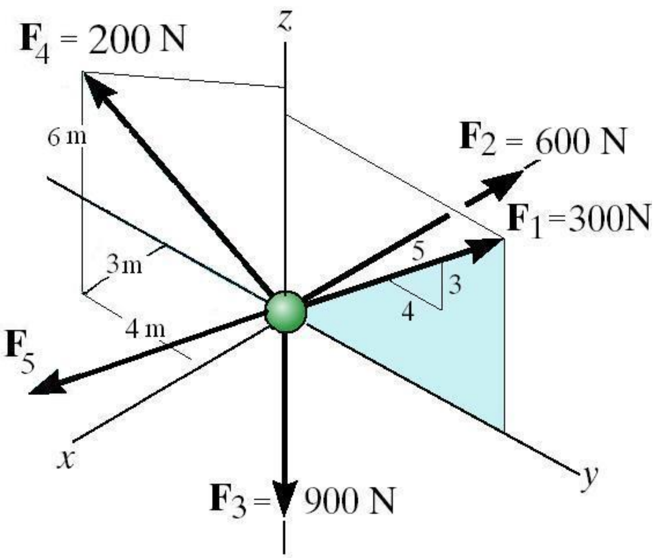
$$\boldsymbol{F_5} = \{F_x \, \boldsymbol{i} + F_y \, \boldsymbol{j} + F_z \, \boldsymbol{k}\} \text{ N}$$

3) Write ***F***$_1$, ***F***$_2$, ***F***$_3$, ***F***$_4$, and ***F***$_5$ in Cartesian vector form.

4) Apply the three equilibrium equations to solve for the three unknowns F_x, F_y, and F_z.

EXAMPLE I (continued)

Solution:



$\boldsymbol{F_1} = \{300(4/5)\ \boldsymbol{j} + 300\ (3/5)\ \boldsymbol{k}\}$ N

$\boldsymbol{F_1} = \{240\ \boldsymbol{j} + 180\ \boldsymbol{k}\}$ N

$\boldsymbol{F_2} = \{-600\ \boldsymbol{i}\}$ N

$\boldsymbol{F_3} = \{-900\ \boldsymbol{k}\}$ N

$\boldsymbol{F_4} = F_4\ (\boldsymbol{r_B}/\ r_B)$

$= 200\ \text{N}\ [(3\boldsymbol{i} - 4\ \boldsymbol{j} + 6\ \boldsymbol{k})/(3^2 + 4^2 + 6^2)^{½}]$

$= \{76.8\ \boldsymbol{i} - 102.4\ \boldsymbol{j} + 153.6\ \boldsymbol{k}\}$ N

$\boldsymbol{F_5} = \{ F_x\ \boldsymbol{i} - F_y\ \boldsymbol{j} + F_z\ \boldsymbol{k}\}$ N

EXAMPLE I (continued)

Equating the respective ***i***, ***j***, ***k*** components to zero, we have

$\Sigma F_x = 76.8 - 600 + F_x = 0$; solving gives $F_x = 523.2$ N

$\Sigma F_y = 240 - 102.4 + F_y = 0$; solving gives $F_y = -137.6$ N

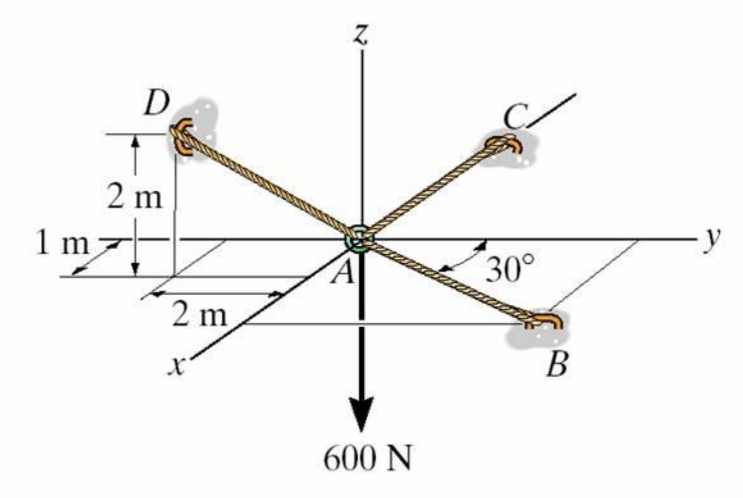
$\Sigma F_z = 180 - 900 + 153.6 + F_z = 0$; solving gives $F_z = 566.4$ N

Thus, $\boldsymbol{F_5} = \{523\, \boldsymbol{i} - 138\, \boldsymbol{j} + 566\, \boldsymbol{k}\}$ N

Using this force vector, you can determine the force's magnitude and coordinate direction angles as needed.

EXAMPLE II

Given: A 600 N load is supported by three cords with the geometry as shown.



Find: The tension in cords AB, AC and AD.

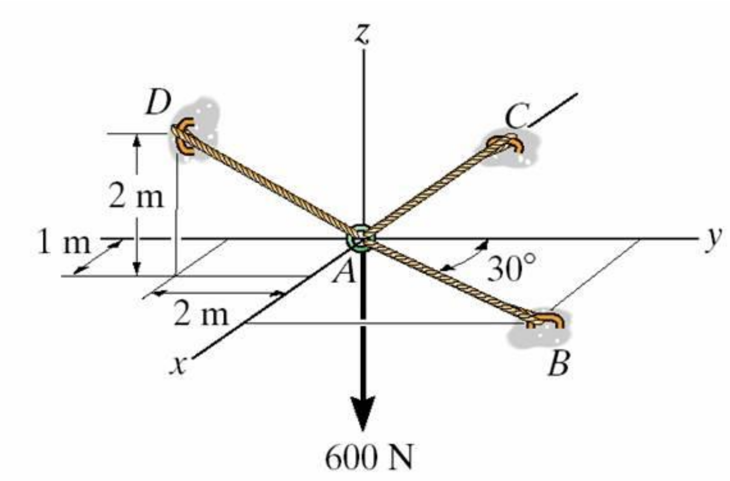
Plan:

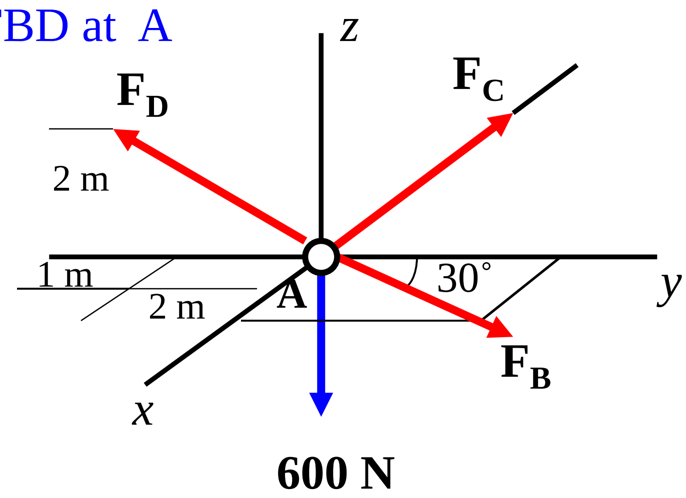
1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

2) Represent each force in its Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

EXAMPLE II (continued)





$\boldsymbol{F_B} = F_B(\sin 30°\ \boldsymbol{i} + \cos 30°\ \boldsymbol{j})$ N

$= \{0.5\ F_B\ \boldsymbol{i} + 0.866\ F_B\ \boldsymbol{j}\}$ N

$\boldsymbol{F_C} = -F_C\ \boldsymbol{i}$ N

$\boldsymbol{F_D} = F_D(\boldsymbol{r_{AD}}/r_{AD})$

$= F_D\{(1\ \boldsymbol{i} - 2\ \boldsymbol{j} + 2\ \boldsymbol{k}) / (1^2 + 2^2 + 2^2)^{½}\}$ N

$= \{0.333\ F_D\ \boldsymbol{i} - 0.667\ F_D\ \boldsymbol{j} + 0.667\ F_D\ \boldsymbol{k}\}$ N

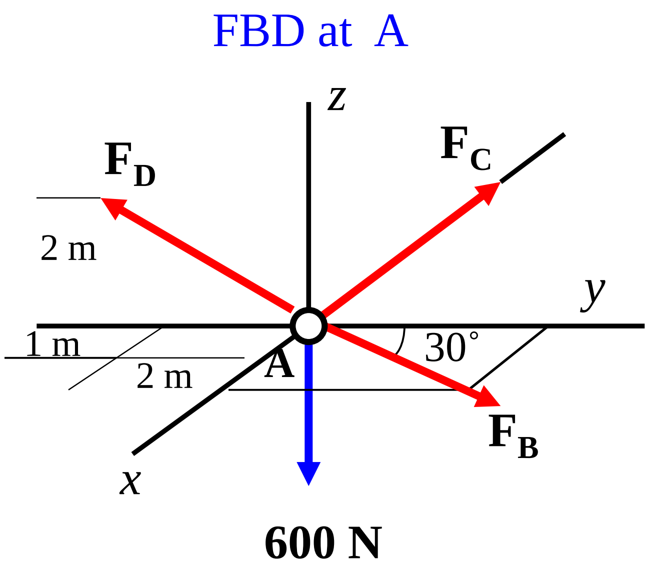
EXAMPLE II (continued)

Now equate the respective ***i***, ***j***, and ***k*** components to zero.

$$\sum F_x = 0.5\ F_B - F_C + 0.333\ F_D = 0$$

$$\sum F_y = 0.866\ F_B - 0.667\ F_D = 0$$

$$\sum F_z = 0.667\ F_D - 600 = 0$$



Solving the three simultaneous equations yields

$F_C = 646$ N (since it is positive, it is as assumed, e.g., in tension)

$F_D = 900$ N

$F_B = 693$ N

CONCEPT QUIZ

1. In 3-D, when you know the direction of a force but not its magnitude, how many unknowns corresponding to that force remain?

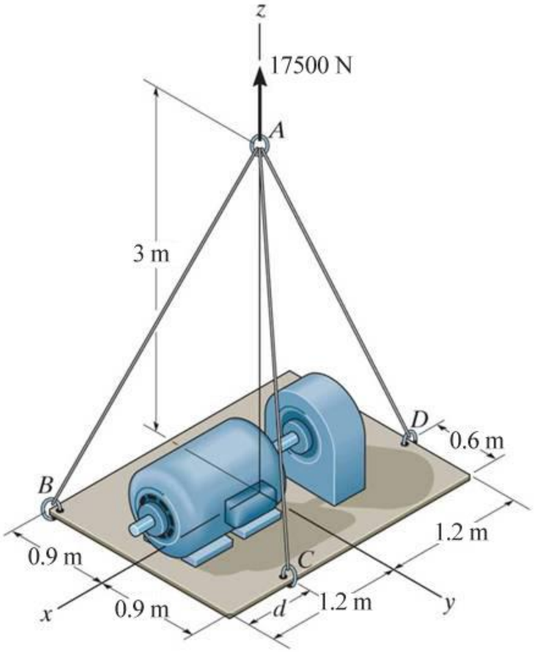
   A) One B) Two C) Three D) Four

2. If a particle has 3-D forces acting on it and is in static equilibrium, the components of the resultant force (ΣF_x, ΣF_y, and ΣF_z) ___ .

   A) have to sum to zero, e.g., -5 ***i*** + 3 ***j*** + 2 ***k***
   B) have to equal zero, e.g., 0 ***i*** + 0 ***j*** + 0 ***k***
   C) have to be positive, e.g., 5 ***i*** + 5 ***j*** + 5 ***k***
   D) have to be negative, e.g., -5 ***i*** - 5 ***j*** - 5 ***k***

GROUP PROBLEM SOLVING

Given: A 17500-N (≈ 1750-kg) motor and plate, as shown, are in equilibrium and supported by three cables and d = 1.2 m



Find: Magnitude of the tension in each of the cables.

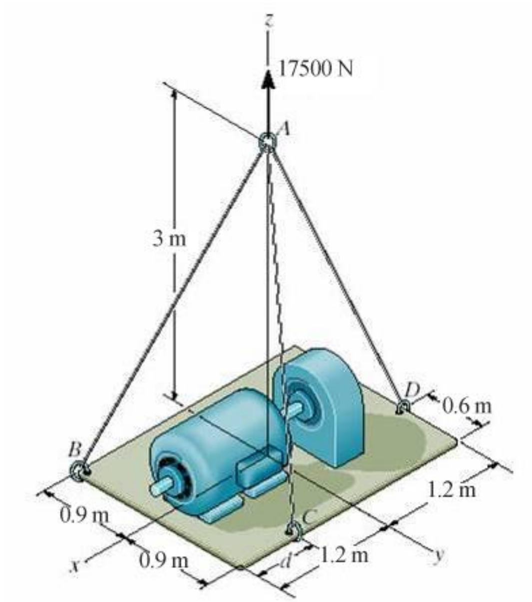
Plan:

1) Draw a free-body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

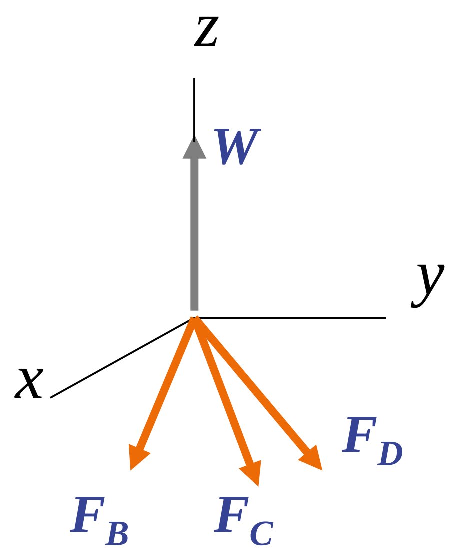
2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

GROUP PROBLEM SOLVING (continued)



FBD of Point A



$\boldsymbol{W}$ = load or weight of unit = 17500 $\boldsymbol{k}$ N

$\boldsymbol{F_B} = F_B(\boldsymbol{r_{AB}}/r_{AB}) = F_B\{(1.2\,\boldsymbol{i} - 0.9\,\boldsymbol{j} - 3\,\boldsymbol{k}) / (3.354)\}$ N

$\boldsymbol{F_C} = F_C(\boldsymbol{r_{AC}}/r_{AC}) = F_C\{(0.9\,\boldsymbol{j} - 3\,\boldsymbol{k}) / (3.132)\}$ N

$\boldsymbol{F_D} = F_D(\boldsymbol{r_{AD}}/r_{AD}) = F_D\{(-1.2\,\boldsymbol{i} + 0.3\,\boldsymbol{j} - 3\,\boldsymbol{k}) / (3.245)\}$ N

GROUP PROBLEM SOLVING (continued)

The particle A is in equilibrium, hence

$\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} + \boldsymbol{W} = 0$

Now equate the respective ***i***, ***j***, ***k*** components to zero (i.e., apply the three scalar equations of equilibrium).

$$\sum F_x = (1.2/\,3.354)F_B - (1.2/\,3.245)F_D = 0 \qquad (1)$$

$$\sum F_y = (-\,0.9/\,3.354)F_B + (0.9/\,3.132)F_C + (0.3/\,3.245)F_D = 0 \qquad (2)$$

$$\sum F_z = (-\,3/\,3.354)F_B - (3/\,3.132)F_C - (3/\,3.245)F_D + 17500 = 0 \qquad (3)$$

Solving the three simultaneous equations gives the forces

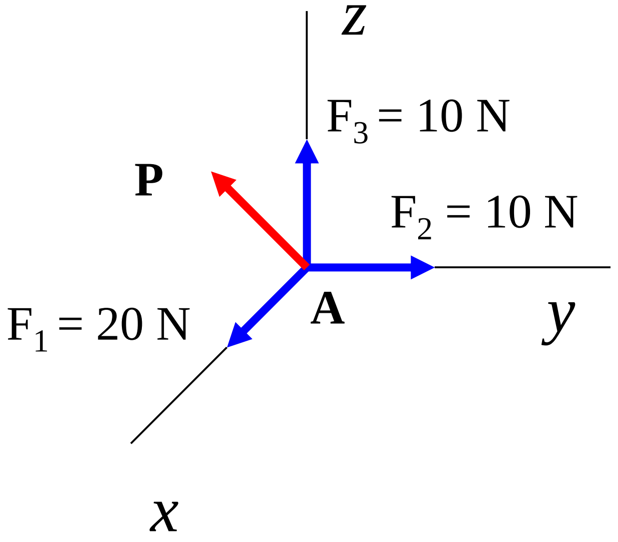
$F_B = 7337$ N

$F_C = 4568$ N

$F_D = 7098$ N

ATTENTION QUIZ

1. Four forces act at point A and point A is in equilibrium. Select the correct force vector ***P***.



A) {-20 ***i*** + 10 ***j*** – 10 ***k***}N

B) {-10 ***i*** – 20 ***j*** – 10 ***k***} N

C) {+ 20 ***i*** – 10 ***j*** – 10 ***k***}N

D) None of the above.

2. In 3-D, when you don't know the direction or the magnitude of a force, how many unknowns do you have corresponding to that force?

A) One B) Two C) Three D) Four

End of the Lecture

Let Learning Continue