MOMENT ABOUT AN AXIS

Today's Objectives:

Students will be able to determine the moment of a force about an axis using

a) scalar analysis, and,

b) vector analysis.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Scalar Analysis
- Vector Analysis
- Example Problem
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. When determining the moment of a force about a specified axis, the axis must be along _______________.

   A) the x axis B) the y axis C) the z axis

   D) any line in 3-D space E) any line in the x-y plane

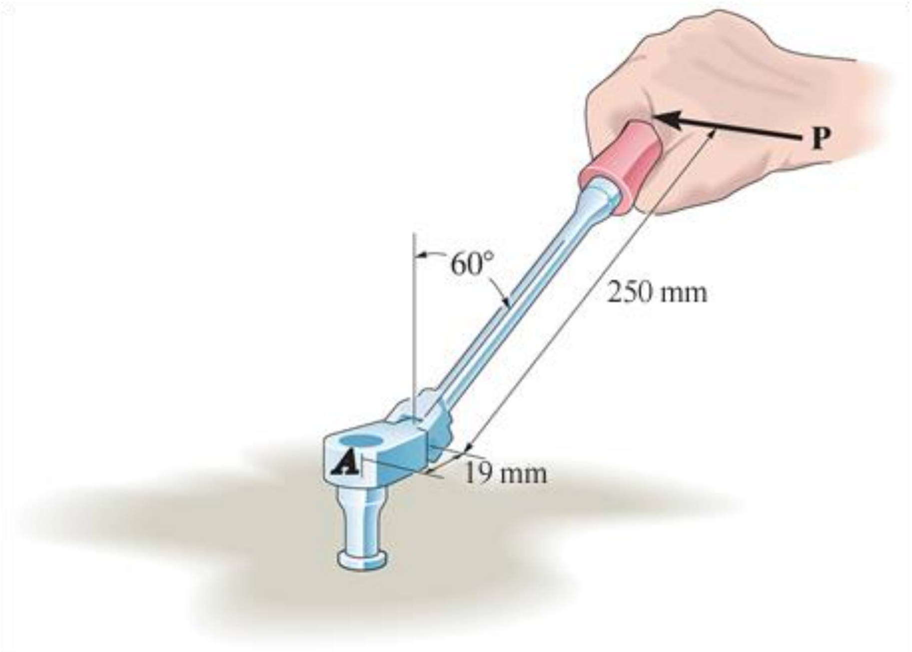
2. The triple scalar product $\mathbf{u} \cdot (\mathbf{r} \times \mathbf{F})$ results in

   A) a scalar quantity (+ or -). B) a vector quantity.

   C) zero. D) a unit vector.

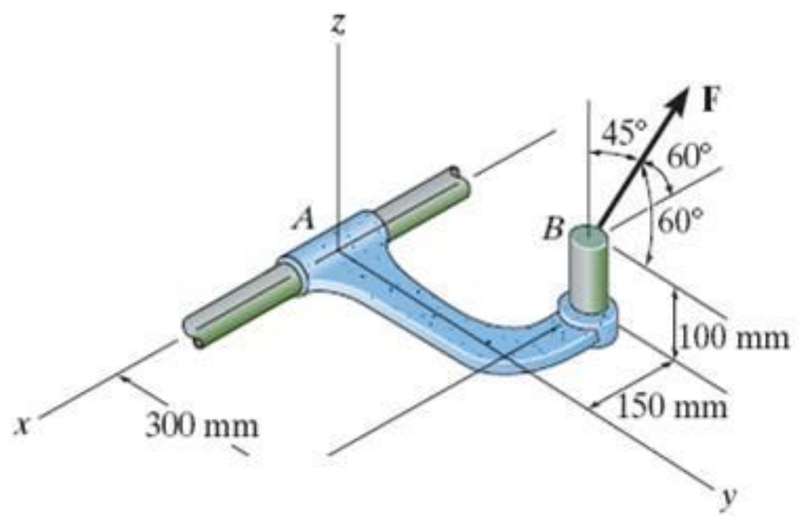
   E) an imaginary number.

APPLICATIONS



With the force ***P***, a person is creating a moment $\boldsymbol{M_A}$ using this flex-handle socket wrench. Does all of $\boldsymbol{M_A}$ act to turn the socket? How would you calculate an answer to this question?

APPLICATIONS (continued)



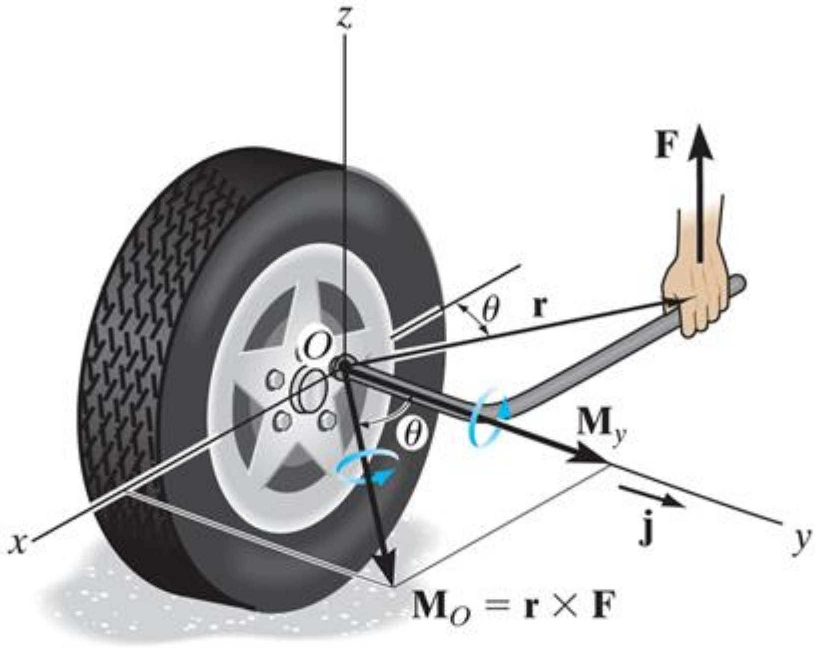
Sleeve A of this bracket can provide a maximum resisting moment of 125 N·m about the x-axis. How would you determine the maximum magnitude of ***F*** before turning about the x-axis occurs?

SCALAR ANALYSIS

Recall that the moment of a scalar force about any point O is $M_O = F\ d_O$ where d_O is the perpendicular (or shortest) distance from the point to the force's line of action. This concept can be extended to find the moment of a force about an axis.

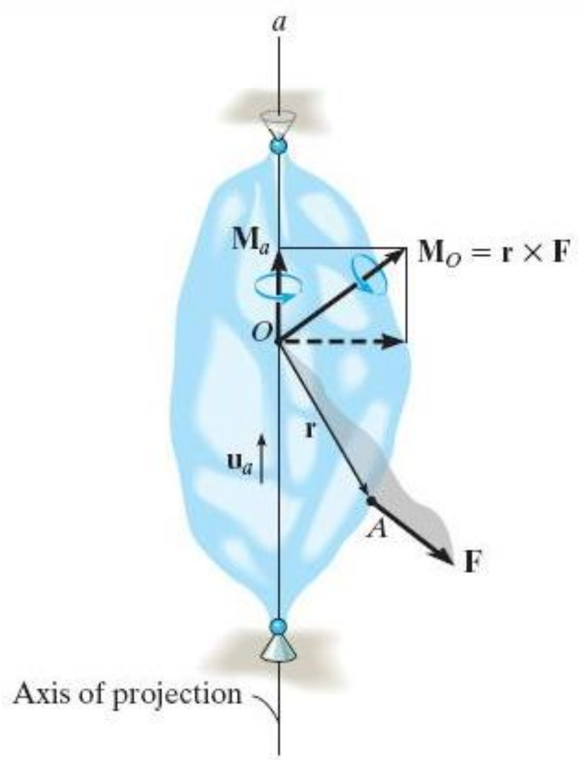
Finding the moment of a force about an axis can help answer the types of questions we just considered.

SCALAR ANALYSIS (continued)



In the figure above, the moment about the y-axis would be $M_y = F_z\,(d_x) = F\,(r \cos \theta)$. However, unless the force can easily be broken into components and the "d_x" found quickly, such calculations are not always trivial and vector analysis may be much easier (and less likely to produce errors).

VECTOR ANALYSIS



Our goal is to find the moment of ***F*** (the tendency to rotate the body) about the *a*-axis.

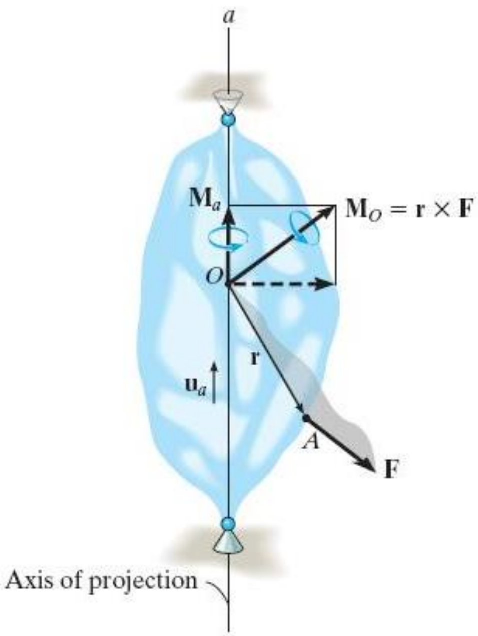
First compute the moment of ***F*** about any arbitrary point O that lies on the *a*-axis using the cross product.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

Now, find the component of $\boldsymbol{M_O}$ along the *a*-axis using the dot product.

$$M_a = \boldsymbol{u_a} \cdot \boldsymbol{M_O}$$

VECTOR ANALYSIS (continued)



M_a can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The above equation is also called the triple scalar product.

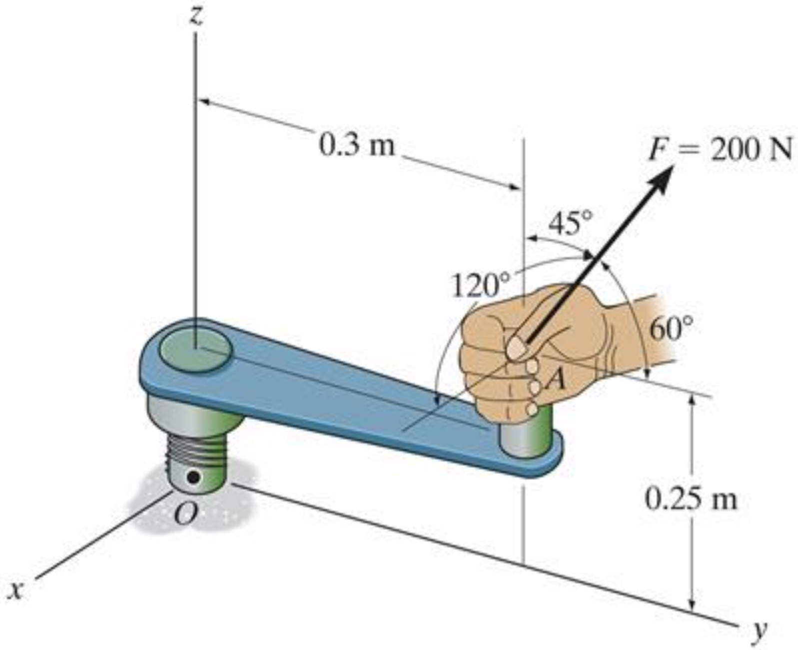
In the this equation,

$\boldsymbol{u_a}$ represents the unit vector along the *a*-axis,

$\boldsymbol{r}$ is the position vector from any point on the *a*-axis to any point A on the line of action of the force, and

$\boldsymbol{F}$ is the force vector.

EXAMPLE



Given: A force is applied to the tool as shown.

Find: The magnitude of the moment of this force about the x axis of the value.

Plan:

1) Use M_z = ***u*** • (***r*** × ***F***).

2) First, find ***F*** in Cartesian vector form.

3) Note that ***u*** = 1 ***i*** in this case.

4) The vector ***r*** is the position vector from O to A.

EXAMPLE (continued)

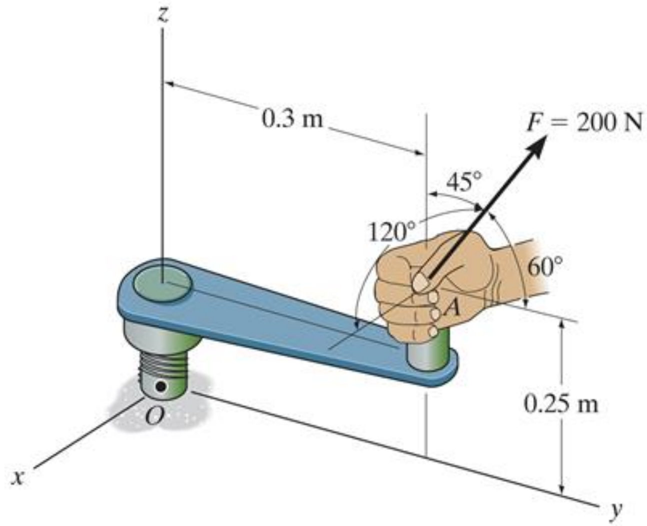
Solution:

$\boldsymbol{u} = 1\ \boldsymbol{i}$

$\boldsymbol{r}_{OA} = \{0\ \boldsymbol{i} + 0.3\ \boldsymbol{j} + 0.25\ \boldsymbol{k}\}$ m

$\boldsymbol{F} = 200\ (\cos 120\ \boldsymbol{i} + \cos 60\ \boldsymbol{j} + \cos 45\ \boldsymbol{k})$ N

$= \{-100\ \boldsymbol{i} + 100\ \boldsymbol{j} + 141.4\ \boldsymbol{k}\}$ N



Now find $M_z = \boldsymbol{u} \bullet (\boldsymbol{r}_{OA} \times \boldsymbol{F})$

$$M_z = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix} = 1\{0.3\ (141.4) - 0.25\ (100)\}\ \text{N·m}$$

$M_z = 17.4$ N·m CCW

CONCEPT QUIZ

1. The vector operation $(\boldsymbol{P} \times \boldsymbol{Q}) \bullet \boldsymbol{R}$ equals

   A) $\boldsymbol{P} \times (\boldsymbol{Q} \bullet \boldsymbol{R})$.

   B) $\boldsymbol{R} \bullet (\boldsymbol{P} \times \boldsymbol{Q})$.

   C) $(\boldsymbol{P} \bullet \boldsymbol{R}) \times (\boldsymbol{Q} \bullet \boldsymbol{R})$.

   D) $(\boldsymbol{P} \times \boldsymbol{R}) \bullet (\boldsymbol{Q} \times \boldsymbol{R})$.

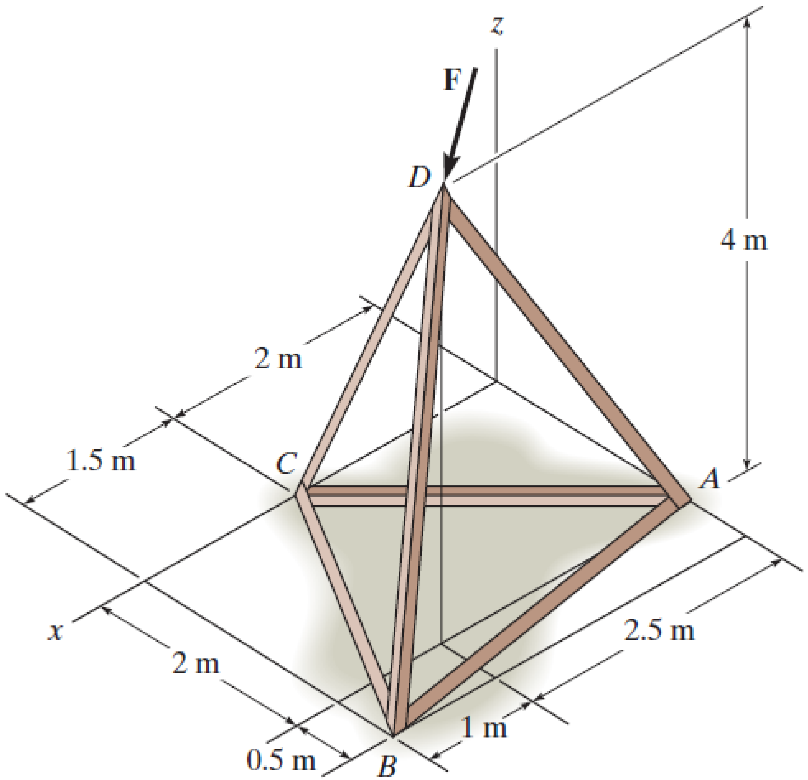
CONCEPT QUIZ (continued)

2. The force ***F*** is acting along DC. Using the triple scalar product to determine the moment of ***F*** about the bar BA, you could use any of the following position vectors except ______.

   A) $\mathbf{r}_{BC}$ B) $\mathbf{r}_{AD}$

   C) $\mathbf{r}_{AC}$ D) $\mathbf{r}_{DB}$

   E) $\mathbf{r}_{BD}$



GROUP PROBLEM SOLVING

Given: The force of F = 30 N acts on the bracket.
$\alpha = 60°$, $\beta = 60°$, $\gamma = 45°$.

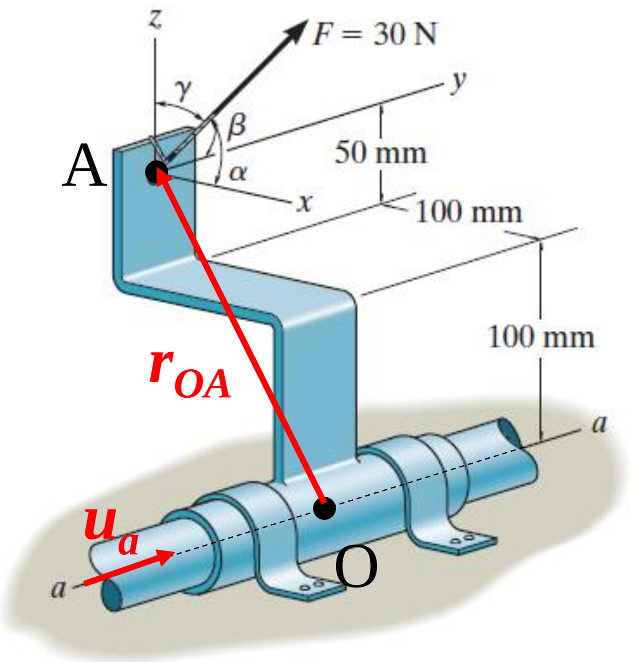
Find: The moment of ***F*** about the a-a axis.

Plan:

1) Find $\boldsymbol{u_a}$ and $\boldsymbol{r_{OA}}$

2) Find ***F*** in Cartesian vector form.

3) Use $M_a = \boldsymbol{u_a} \bullet (\boldsymbol{r_{OA}} \times \boldsymbol{F})$

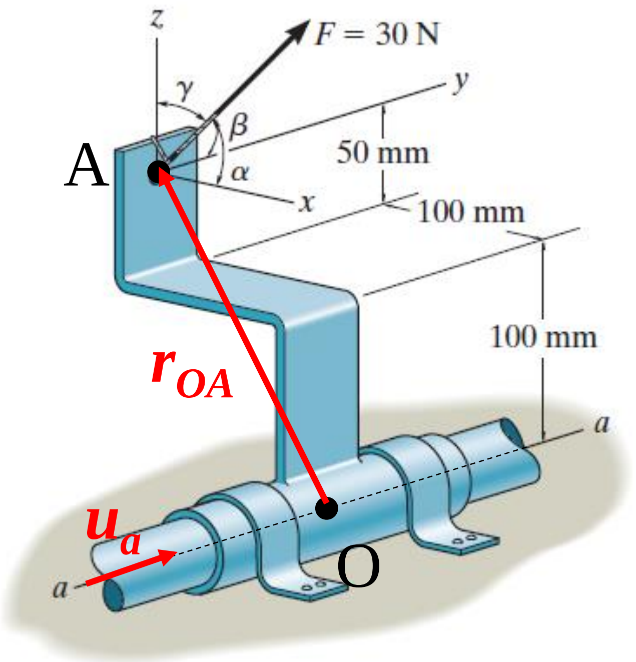


GROUP PROBLEM SOLVING (continued)

Solution:

$\boldsymbol{u}_a = \boldsymbol{j}$

$\boldsymbol{r}_{OA} = \{-0.1\ \boldsymbol{i} + 0.15\ \boldsymbol{k}\}$ m



$\boldsymbol{F} = 30\ \{\cos 60°\ \boldsymbol{i} + \cos 60°\ \boldsymbol{j} + \cos 45°\ \boldsymbol{k}\}$ N

$\boldsymbol{F} = \{\ 15\ \boldsymbol{i} + 15\ \boldsymbol{j} + 21.21\ \boldsymbol{k}\}$ N

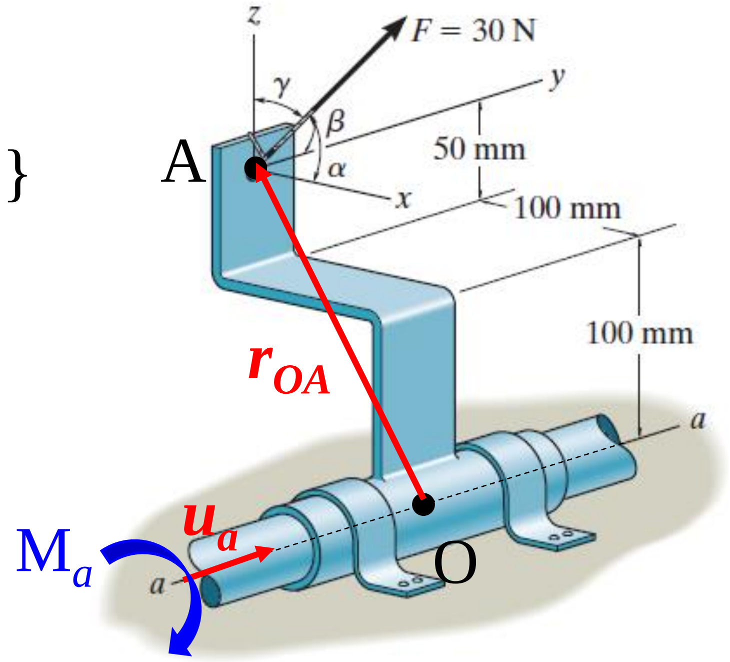
GROUP PROBLEM SOLVING (continued)

Now find the triple product, $M_a = \mathbf{u_a} \bullet (\mathbf{r_{OA}} \times \mathbf{F})$

$$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} \text{ N·m}$$

$$M_a = -1 \{-0.1\,(21.21) - 0.15\,(15)\}$$

$= \underline{4.37 \text{ N·m}}$



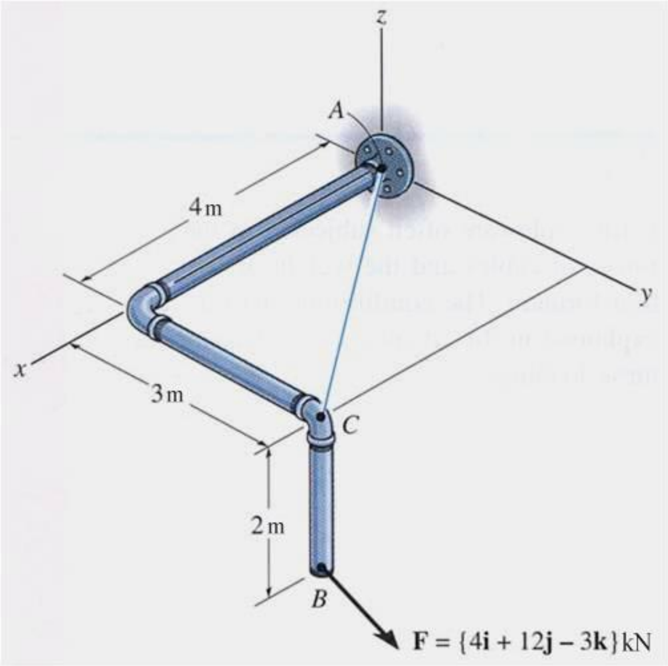
PEARSON

ATTENTION QUIZ

1. For finding the moment of the force ***F*** about the x-axis, the position vector in the triple scalar product should be ___ .

A) $\boldsymbol{r}_{AC}$ B) $\boldsymbol{r}_{BA}$

C) $\boldsymbol{r}_{AB}$ D) $\boldsymbol{r}_{BC}$



2. If ***r*** = {1 ***i*** + 2 ***j***} m and ***F*** = {10 ***i*** + 20 ***j*** + 30 ***k***} N, then the moment of ***F*** about the y-axis is ____ N·m.

A) 10 B) -30

C) -40 D) None of the above.

End of the Lecture

Let Learning Continue