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Watson-Crick Petri Net Languages: The Effect of Labeling Strategies

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Abstract. A Watson-Crick automaton is an automaton that works on tapes which are double stranded sequences of symbols related by Watson-Crick complementarity that are similar to the DNA molecules. However, this automaton cannot exploit the other fundamental features of DNA molecules such as the massive parallelism. Watson-Crick automata can be related to a model known as the Petri net. Petri net is a model based on the concepts of asynchronous and concurrent operation by the parts of a system and the realization by the parts can be represented by a graph or a net. From the relation between Watson-Crick automata and Petri net, a new model namely Watson-Crick Petri net has been developed. The language generated by Watson-Crick Petri net is a set of labeled sequences corresponding to the occurrence sequences of the model. In this research, some properties of languages generated by Watson-Crick Petri net are investigated.

Keywords: Watson-Crick, automata, Petri net, language, DNA.
PACS: 87.14.gk

INTRODUCTION

A Watson-Crick Petri net relates Watson-Crick automata and Petri net where the control unit of a Watson-Crick automaton is replaced by a Petri net. The Watson-Crick automaton is an automaton with two reading heads and works on tapes which are double stranded sequences of symbols related by Watson-Crick complementarity similar to the DNA molecules [1-3]; whereas Petri net is a model that is a useful mathematical formalism for modeling concurrent systems and their behaviors [4-6]. The language generated by Watson-Crick Petri net can be determined using the class of labeling functions or the definition of the set of final states. In this paper, we investigate Watson-Crick Petri net language using the class of labeling functions.

PRELIMINARIES

Some definitions regarding Watson-Crick Petri net which will be used throughout this paper are listed in the following.

Definition 1 [7]: Watson-Crick Petri net
A Watson-Crick Petri net is defined as $W=(N,\Sigma,\rho,\ell)$ where $N=(P,T,F,\phi,i,M)$ is a Petri net with final markings where P is the finite set of places, T is the finite set of transitions, $F\subseteq(P\times T)\cup(T\times P)$ is the set of directed arcs, $\phi:F\to\mathbb{N}$ is a weight function on the arcs, i is the initial marking, $M\subseteq\Re(N,i)$ is set of markings which are called final markings, Σ is an alphabet, $\rho\subseteq\Sigma\times\Sigma$ is a symmetric relation, and $\ell:T\to\begin{pmatrix}\Sigma\cup\{\lambda\}\\ \Sigma\cup\{\lambda\}\end{pmatrix}$ is a labeling function.

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Definition 2 [7]: Watson-Crick Petri net language
A Watson-Crick Petri net language is a set of labelled sequences corresponding to occurrence sequences of the Watson-Crick Petri net.

In the next section, various types of languages generated by Watson-Crick Petri net are introduced. The Watson-Crick Petri net languages are then investigated using the class of labeling functions.

MAIN RESULTS

Watson-Crick Petri net languages are determined by two methods, namely labeling functions class and the definition of final markings. In this paper, different types of Watson-Crick Petri net languages are introduced.

Definition 3: Strong free Watson-Crick Petri net language
A strong free Watson-Crick Petri net language (denoted by *s*) generated by a Watson-Crick Petri net *W* determined using the class of labelling functions is a set of languages where any transtitions $t_1, t_2 \in T$ are labeled with $\ell(t_1) = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$, $\ell(t_2) = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ where $a_1 \neq a_2$ and $b_1 \neq b_2$. Also, no transition is labeled with the empty strings, i.e. for any $t \in T$, $\ell(t) = \begin{pmatrix} a \\ b \end{pmatrix}$ where $a \neq \lambda$ and $b \neq \lambda$.

Definition 4: Weak free Watson-Crick Petri net language
A weak free Watson-Crick Petri net language (denoted by *w*) generated by a Watson-Crick Petri net *W* determined using the class of labelling functions is a set of languages where any transtitions $t_1, t_2 \in T$ are labeled with $\ell(t_1) = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$, $\ell(t_2) = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ where $a_1 \neq a_2$ or $b_1 \neq b_2$. Also, for any transition $t \in T$, $\ell(t) = \begin{pmatrix} a \\ b \end{pmatrix}$, either $a \neq \lambda$ or $b \neq \lambda$.

Definition 5: Strong λ - free Watson-Crick Petri net language
A strong λ – free Watson-Crick Petri net language (denoted by $-s\lambda$) generated by a Watson-Crick Petri net *W* determined using the class of labelling functions is a set of languages with no transition labeled with the empty string, i.e. for any $t \in T$, $\ell(t) = \begin{pmatrix} a \\ b \end{pmatrix}$ where $a \neq \lambda$ and $b \neq \lambda$.

Definition 6: Weak λ - free Watson-Crick Petri net language
A weak λ – free Watson-Crick Petri net language (denoted by $-w\lambda$) generated by a Watson-Crick Petri net *W* determined using the class of labelling functions is a set of languages where for any transition $t \in T$, $\ell(t) = \begin{pmatrix} a \\ b \end{pmatrix}$, either $a \neq \lambda$ or $b \neq \lambda$.

Definition 7: Arbitrary Watson-Crick Petri net language
A arbitrary Watson-Crick Petri net language (denoted by λ) generated by a Watson-Crick Petri net *W* determined using the class of labelling functions is a set of languages with no restriction posed on the labeling ℓ function for any transitions.

Definition 8: G-type Watson-Crick Petri net language
A G-type Watson-Crick Petri net language generated by a Watson-Crick Petri net *W* determined using the definition of the set of final states is a set of languages where for a given set $M_0 \subseteq \Re(N, t)$, each marking $\mu \in M$ is greater or equal to any marking M_0.

Definition 9: T-type Watson-Crick Petri net language

A T-type Watson-Crick Petri net language generated by a Watson-Crick Petri net W determined using the definition of the set of final states is a set of languages where M is the set of all terminal markings of N.

Next, the languages generated by Watson-Crick Petri nets determined using the class of labeling functions are discussed. Below we give several examples of different Watson-Crick Petri net languages with respect to different labelling policies.

Case 1: Strong free labelling

The transitions of Watson-Crick Petri net W_1 are labeled with strong free policy such that $\sigma_1(t_1)=\begin{pmatrix} a \\ a \end{pmatrix}$, $\sigma_1(t_2)=\begin{pmatrix} b \\ b \end{pmatrix}$, $\sigma_1(t_3)=\begin{pmatrix} c \\ c \end{pmatrix}$, $\sigma_1(t_4)=\begin{pmatrix} d \\ d \end{pmatrix}$, $\sigma_1(t_5)=\begin{pmatrix} e \\ e \end{pmatrix}$. Figure 1 represents the Watson-Crick Petri net $W_1=(N,\Sigma,\rho,\ell)$ where $P=\{p_1,p_2,p_3,p_4,p_5\}$, $T=\{t_1,t_2,t_3,t_4,t_5\}$ where $t_1=\begin{pmatrix} a \\ a \end{pmatrix}$, $t_2=\begin{pmatrix} b \\ b \end{pmatrix}$, $t_3=\begin{pmatrix} c \\ c \end{pmatrix}$, $t_4=\begin{pmatrix} d \\ d \end{pmatrix}$, $t_5=\begin{pmatrix} e \\ e \end{pmatrix}$, $F=\{(p_1,t_1),(t_1,p_2),(p_2,t_2),(t_2,p_2),(p_2,t_4),(t_2,p_3),(p_3,t_3),(t_3,p_4),(p_4,t_3),(t_4,p_4),(p_4,t_5),(t_5,p_5)\}$, $\phi(x,y)=1$ for all $(x,y)\in P\times T\cup T\times P$, $i=[1,0,...,0]$, $M=[0,...,0,1]$, $\rho=\left\{\begin{pmatrix} a \\ a \end{pmatrix},\begin{pmatrix} b \\ b \end{pmatrix},\begin{pmatrix} c \\ c \end{pmatrix},\begin{pmatrix} d \\ d \end{pmatrix},\begin{pmatrix} e \\ e \end{pmatrix}\right\}$ and $\Sigma=\{a,b,c,d,e\}$. This case is referred to Definition 3.

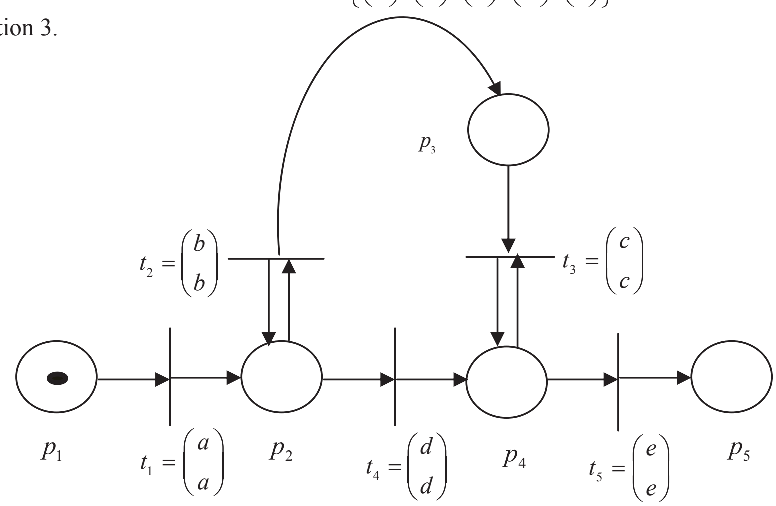

FIGURE 1. Watson-Crick Petri net W_1 with strong free labelling policy.

Therefore, the language generated by Watson-Crick Petri net W_1 is $L(W_1)=\{ab^n dc^n e \mid n\geq 0\}$.

Case 2: Weak free labelling

The transitions of Watson-Crick Petri net W_2 are labeled with weak free labelling policy such that $\sigma_1(t_1)=\begin{pmatrix} a \\ a \end{pmatrix}$, $\sigma_1(t_2)=\begin{pmatrix} b \\ \lambda \end{pmatrix}$, $\sigma_1(t_3)=\begin{pmatrix} \lambda \\ b \end{pmatrix}$, $\sigma_1(t_4)=\begin{pmatrix} b \\ b \end{pmatrix}$, $\sigma_1(t_5)=\begin{pmatrix} c \\ c \end{pmatrix}$. Figure 2 represents the Watson-Crick Petri net $W_2=(N,\Sigma,\rho,\ell)$ where $P=\{p_1,p_2,p_3,p_4,p_5\}$, $T=\{t_1,t_2,t_3,t_4,t_5\}$ where $t_1=\begin{pmatrix} a \\ a \end{pmatrix}$, $t_2=\begin{pmatrix} b \\ \lambda \end{pmatrix}$, $t_3=\begin{pmatrix} \lambda \\ b \end{pmatrix}$, $t_4=\begin{pmatrix} b \\ b \end{pmatrix}$, $t_5=\begin{pmatrix} c \\ c \end{pmatrix}$, $F=\{(p_1,t_1),(t_1,p_2),(p_2,t_2),(t_2,p_2),(p_2,t_4),(t_2,p_3),(p_3,t_3),(t_3,p_4),(p_4,t_3),(t_4,p_4),(p_4,t_5),(t_5,p_5)\}$, $\phi(x,y)=1$ for all $(x,y)\in P\times T\cup T\times P$, $i=[1,0,...,0]$, $M=[0,...,0,1]$, $\rho=\left\{\begin{pmatrix} a \\ a \end{pmatrix},\begin{pmatrix} b \\ b \end{pmatrix},\begin{pmatrix} c \\ c \end{pmatrix}\right\}$ and $\Sigma=\{a,b,c\}$. This case is referred to Definition 4.

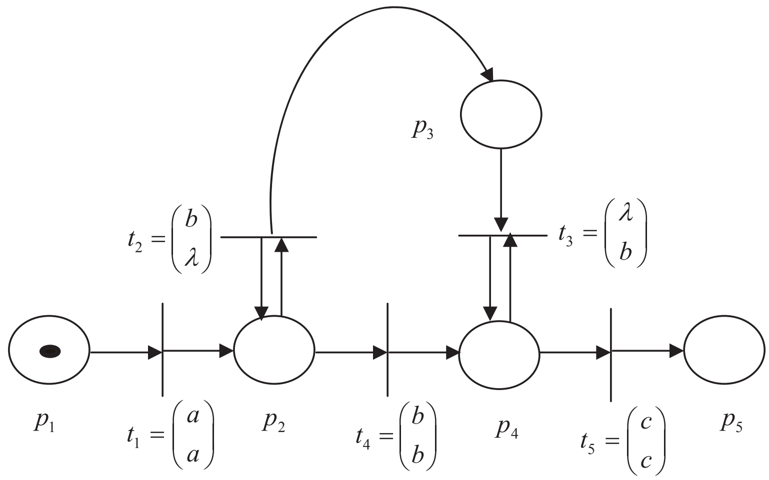

FIGURE 2. Watson-Crick Petri net W_2 with weak free labelling policy

Therefore, the language generated by Watson-Crick Petri net W_2 is $L(W_2) = \{ab^n c \mid n \geq 1\}$.

Case 3: Strong λ−free labelling

The transitions of Watson-Crick Petri net W_3 are labeled with strong λ−free labelling policy such that $\sigma_1(t_1) = \begin{pmatrix} a \\ a \end{pmatrix}$, $\sigma_1(t_2) = \begin{pmatrix} a \\ a \end{pmatrix}$, $\sigma_1(t_3) = \begin{pmatrix} c \\ c \end{pmatrix}$, $\sigma_1(t_4) = \begin{pmatrix} b \\ b \end{pmatrix}$, $\sigma_1(t_5) = \begin{pmatrix} c \\ c \end{pmatrix}$. Figure 3 represents the Watson-Crick Petri net $W_3 = (N, \Sigma, \rho, \ell)$ where $P = \{p_1, p_2, p_3, p_4, p_5\}$, $T = \{t_1, t_2, t_3, t_4, t_5\}$ where $t_1 = \begin{pmatrix} a \\ a \end{pmatrix}$, $t_2 = \begin{pmatrix} a \\ a \end{pmatrix}$, $t_3 = \begin{pmatrix} c \\ c \end{pmatrix}$, $t_4 = \begin{pmatrix} b \\ b \end{pmatrix}$, $t_5 = \begin{pmatrix} c \\ c \end{pmatrix}$, $F =$ $\{(p_1,t_1),(t_1,p_2),(p_2,t_2),(t_2,p_2),(p_2,t_4),(t_2,p_3),(p_3,t_3),(t_3,p_4),(p_4,t_3),(t_4,p_4),(p_4,t_5),(t_5,p_5)\}$, $\phi(x,y) = 1$ for all $(x,y) \in P \times T \cup T \times P$, $i = [1,0,...,0]$, $M = [0,...,0,1]$, $\rho = \left\{ \begin{pmatrix} a \\ a \end{pmatrix}, \begin{pmatrix} b \\ b \end{pmatrix}, \begin{pmatrix} c \\ c \end{pmatrix} \right\}$ and $\Sigma = \{a,b,c\}$. This case is referred to Definition 5.

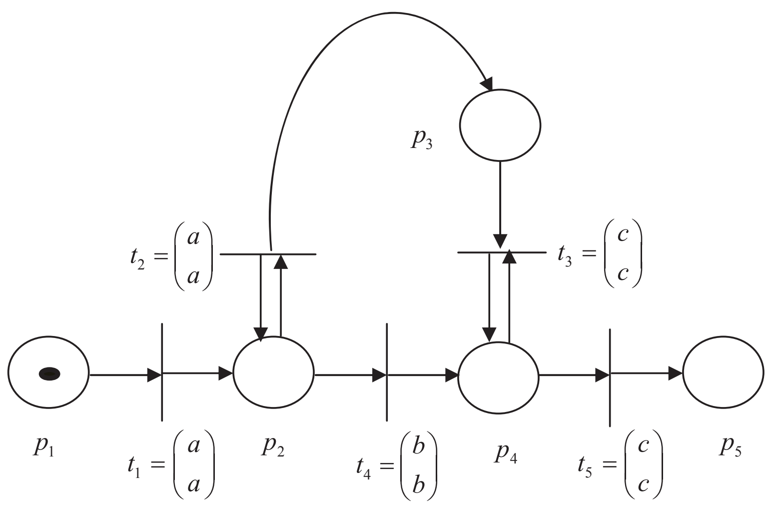

FIGURE 3. Watson-Crick Petri net W_3 with strong λ−free labelling policy.

Therefore, the language generated by Watson-Crick Petri net W_3 is $L(W_3) = \{a^n b c^n \mid n \geq 1\}$.

Case 4: Weak λ−free labelling

The transitions of Watson-Crick Petri net W_4 are labeled with weak λ−free labelling policy such that $\sigma_1(t_1)=\begin{pmatrix}a\\a\end{pmatrix}$, $\sigma_1(t_2)=\begin{pmatrix}b\\\lambda\end{pmatrix}$, $\sigma_1(t_3)=\begin{pmatrix}\lambda\\b\end{pmatrix}$, $\sigma_1(t_4)=\begin{pmatrix}b\\b\end{pmatrix}$, $\sigma_1(t_5)=\begin{pmatrix}a\\a\end{pmatrix}$. Figure 4 represents the Watson-Crick Petri net $W_4=(N,\sum,\rho,\ell)$ where $P=\{p_1,p_2,p_3,p_4,p_5\}$, $T=\{t_1,t_2,t_3,t_4,t_5\}$ where $t_1=\begin{pmatrix}a\\a\end{pmatrix}$, $t_2=\begin{pmatrix}b\\\lambda\end{pmatrix}$, $t_3=\begin{pmatrix}\lambda\\b\end{pmatrix}$, $t_4=\begin{pmatrix}b\\b\end{pmatrix}$, $t_5=\begin{pmatrix}a\\a\end{pmatrix}$, $F=\{(p_1,t_1),(t_1,p_2),(p_2,t_2),(t_2,p_2),(p_2,t_4),(t_2,p_3),(p_3,t_3),(t_3,p_4),(p_4,t_3),(t_4,p_4),(p_4,t_5),(t_5,p_5)\}$, $\phi(x,y)=1$ for all $(x,y)\in P\times T\cup T\times P$, $i=[1,0,...,0]$, $M=[0,...,0,1]$, $\rho=\left\{\begin{pmatrix}a\\a\end{pmatrix},\begin{pmatrix}b\\b\end{pmatrix}\right\}$ and $\Sigma=\{a,b\}$. This case is referred to Definition 6.

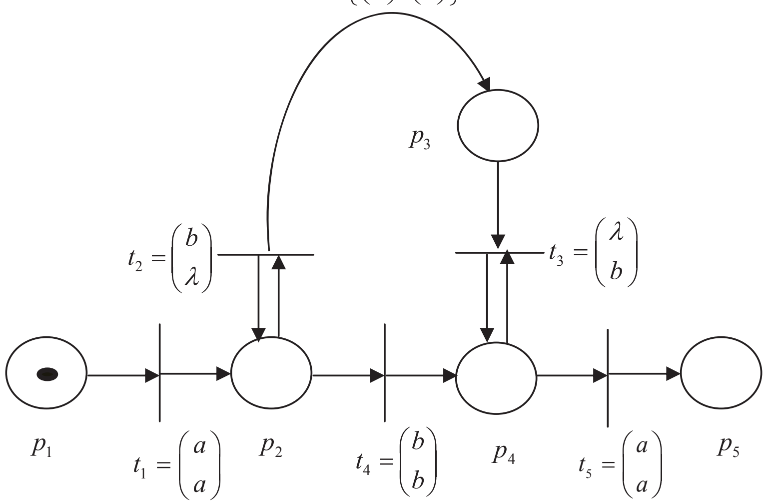

FIGURE 4. Watson-Crick Petri net W_4 with weak λ−free labelling policy.

Therefore, the language generated by Watson-Crick Petri net W_4 is $L(W_4)=\{ab^na\mid n\geq 1\}$.

Case 5: Arbitrary labelling

The transitions of Watson-Crick Petri net W_5 are labeled with arbitrary labeling policy such that $\sigma_1(t_1)=\begin{pmatrix}a\\a\end{pmatrix}$, $\sigma_1(t_2)=\begin{pmatrix}b\\\lambda\end{pmatrix}$, $\sigma_1(t_3)=\begin{pmatrix}\lambda\\b\end{pmatrix}$, $\sigma_1(t_4)=\begin{pmatrix}\lambda\\\lambda\end{pmatrix}$, $\sigma_1(t_5)=\begin{pmatrix}a\\a\end{pmatrix}$. Figure 5 represents the Watson-Crick Petri net $W_5=(N,\sum,\rho,\ell)$ where $P=\{p_1,p_2,p_3,p_4,p_5\}$, $T=\{t_1,t_2,t_3,t_4,t_5\}$ where $t_1=\begin{pmatrix}a\\a\end{pmatrix}$, $t_2=\begin{pmatrix}b\\\lambda\end{pmatrix}$, $t_3=\begin{pmatrix}\lambda\\b\end{pmatrix}$, $t_4=\begin{pmatrix}\lambda\\\lambda\end{pmatrix}$, $t_5=\begin{pmatrix}a\\a\end{pmatrix}$, $F=\{(p_1,t_1),(t_1,p_2),(p_2,t_2),(t_2,p_2),(p_2,t_4),(t_2,p_3),(p_3,t_3),(t_3,p_4),(p_4,t_3),(t_4,p_4),(p_4,t_5),(t_5,p_5)\}$, $\phi(x,y)=1$ for all $(x,y)\in P\times T\cup T\times P$, $i=[1,0,...,0]$, $M=[0,...,0,1]$, $\rho=\left\{\begin{pmatrix}a\\a\end{pmatrix},\begin{pmatrix}b\\b\end{pmatrix}\right\}$ and $\Sigma=\{a,b\}$. This case is referred to Definition 7.

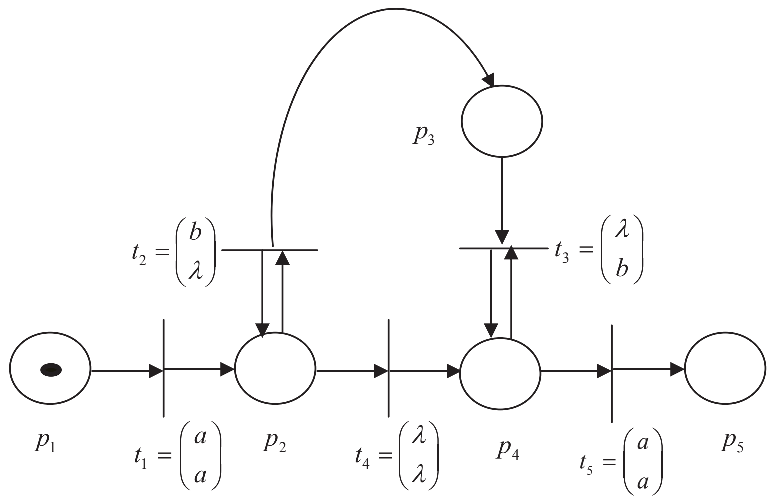

FIGURE 5. Watson-Crick Petri net W_5 with arbitrary labelling policy.

Therefore, the language generated by Watson-Crick Petri net W_5 is $L(W_5) = \{ab^n a | n \geq 0\}$.

CONCLUSION

In this paper, we considered Watson-Crick Petri net languages using the classes of labeling functions. Examples of Watson-Crick Petri net languages with transitions labelled with various labeling policies such as strong free, weak free, strong λ−free, weak λ−free and arbitrary are also presented.

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