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Consistent Polycyclic Presentation of a Bieberbach Group with a Nonabelian Point Group

Siti Afiqah Mohammad[1], Nor Haniza Sarmin[2], and Hazzirah Izzati Mat Hassim[3]

[1]*Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia, Email: afiqahmohammad91@gmail.com*
[2]*Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia, Email: nhs@utm.my*
[3]*Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia, Email: hazzirah@utm.my*

Abstract. Research on the nonabelian tensor square of a group is requisite on finding the other homological functors. One of the methods to explicate the nonabelian tensor square is to ensure the presentation of the group is polycyclic and to prove its consistency. In this research, the polycyclic presentation of a Bieberbach group with the quaternion point group of order eight is shown to be consistent.

Keywords: nonabelian tensor square, Bieberbach group
PACS: 02.20.Bb

INTRODUCTION

The nonabelian tensor square of a group is essential in determining the other properties of the group including its homological functors. These properties have been studied over the years using mathematical approach. Masri [1] in 2009 had focused on Bieberbach groups with cyclic point group of order two and elementary abelian 2-group. The method of converting the matrix representation of a group into its polycyclic presentation has been used and the presentation is checked to be consistent before its nonabelian tensor square can be computed. Then, Mohd Idrus [2] in 2011 considered the Bieberbach groups with dihedral point group of order eight. In 2014, Tan *et al.* [3] had found the consistency polycyclic presentation for Bieberbach group with symmetry point group of order six.

In this research, a crystallographic group named Bieberbach with a nonabelian point group, namely the quaternion group of order eight, is considered by referring to the Crystallographic, Algorithms and Table (CARAT) package [4]. This group is of dimension six. By using the technique developed by Blyth and Morse [5], the polycyclic presentation of the Bieberbach group with a quaternion point group is shown to be polycyclic. Then the nonabelian tensor is determined using the method developed for polycyclic groups. It is crucial to show the consistency of the presentation before its nonabelian tensor square can be determined.

PRELIMINARIES

The following are some definitions that are used throughout this research.

Definition 1 [6] A Bieberbach Group
A Bieberbach group G is a torsion free group given by a short exact sequence

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$$L \to G \to P$$

where L is a free abelian normal subgroup of G of finite rank, called lattice of G, P is a finite group called the point group of G and G/L is isomorphic to P. The point group G acts on L by conjugation in G.

Definition 2 [7] Polycyclic Presentation

Let F_n be a free group on generators $g_1,...,g_n$ and R be a set of relations of a group G. The relations of a polycyclic presentation have the form:

$$g_i^{e_i} = g_{i+1}^{x_{i,i+1}} ... g_n^{x_{i,n}} \qquad \text{for } i \in I,$$

$$g_j^{-1} g_i g_j = g_{j+1}^{y_{i,j,j+1}} ... g_n^{y_{i,j,n}} \qquad \text{for } j < i,$$

$$g_j g_i g_j^{-1} = g_{j+1}^{z_{i,j,j+1}} ... g_n^{z_{i,j,n}} \qquad \text{for } j < i \text{ and } j \notin I$$

for some $I \subseteq \{1,...,n\}, e_i \in \mathbb{N}$ for $i \in I$ and $x_{i,j}, y_{i,j,k}, z_{i,j,k} \in \mathbb{Z}$ for all i, j and k.

Definition 3 [7] Consistent Polycyclic Presentation

Let G be a group generated by $g_1,...,g_n$ and $e_i, e_j \in \mathbb{N}$. The consistency of the relations in G can be determined using the following consistency relations.

$$g_k(g_j g_i) = (g_k g_j) g_i \qquad \text{for } k > j > i,$$

$$(g_j^{e_j}) g_i = g_j^{e_j^{-1}} (g_j g_i) \qquad \text{for } j > i, j \in I,$$

$$g_j(g_i^{e_i}) = (g_j g_i) g_i^{e_i^{-1}} \qquad \text{for } j > i, i \in I,$$

$$(g_i^{e_i}) g_i = g_i (g_i^{e_i}) \qquad \text{for } i \in I,$$

$$g_j = (g_j g_i^{-1}) g_i \qquad \text{for } j > i, i \notin I.$$

The following is the matrix representation of a Bieberbach group with a quaternion point group of order eight, denoted as G, given in the CARAT package:

G is generated by $a_0, a_1, l_1, l_2, l_3, l_4, l_5, l_6$ where

$$a_0 = \begin{bmatrix} \frac{0}{4} & \frac{-4}{4} & \frac{4}{4} & \frac{-4}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{4}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{-4}{4} & \frac{-4}{4} & \frac{0}{4} & \frac{4}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{0}{4} & \frac{-4}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{4}{4} & \frac{0}{4} & \frac{-1}{4} \\ \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{4}{4} & \frac{1}{4} \\ \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{4}{4} \end{bmatrix}, \quad a_1 = \begin{bmatrix} \frac{-4}{4} & \frac{-4}{4} & \frac{0}{4} & \frac{4}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{4}{4} & \frac{4}{4} & \frac{-4}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{0}{4} & \frac{4}{4} & \frac{-4}{4} & \frac{4}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{-4}{4} & \frac{0}{4} & \frac{-4}{4} & \frac{4}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{4}{4} & \frac{-2}{4} \\ \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{4}{4} & \frac{0}{4} & \frac{0}{4} \\ \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{0}{4} & \frac{4}{4} \end{bmatrix}, \quad l_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$l_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, l_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, l_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$l_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, l_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

Therefore, based on Definition 2, the polycyclic presentation of G is established as in (2) below:

$$G = \left\langle a, b, c, l_1, l_2, l_3, l_4, l_5, l_6 \;\middle|\; \begin{array}{l} a^2 = cl_6,\ b^2 = cl_5 l_6^{-1},\ b^a = bcl_5^{-2} l_6^2,\ c^2 = l_5 l_6^{-1},\ c^a = cl_5^{-1} l_6,\ c^b = c, \\ l_1^a = l_1^{-1} l_2 l_4^{-1},\ l_1^b = l_3^{-1}, l_1^c = l_1^{-1},\ l_2^a = l_1^{-1} l_2 l_3,\ l_2^b = l_1^{-1} l_3^{-1} l_4^{-1},\ l_2^c = l_2^{-1}, \\ l_3^a = l_2^{-1} l_3^{-1} l_4^{-1},\ l_3^b = l_3,\ l_3^c = l_3^{-1},\ l_4^a = l_1 l_3 l_4,\ l_4^b = l_1^{-1} l_2 l_3, l_4^c = l_4^{-1}, \\ l_5^a = l_6, l_5^b = l_5, l_5^c = l_5, l_6^a = l_5, l_6^b = l_6, l_6^c = l_6,\ l_j^{l_i} = l_j,\ l_j^{l_i^{-1}} = l_j \\ \text{for } j > i,\ 1 \le i, j \le 6 \end{array} \right\rangle. \tag{2}$$

In this research, by using Definition 3, the polycyclic presentation given in (2) is shown to be consistent.

Computing the Consistency of Polycyclic Presentation of a Bieberbach Group with a Quaternion Point Group of Order Eight

In this section, the polycyclic presentation (2) of a Bieberbach group with a quaternion point group of order eight is shown to be consistent. The proof is given in the following theorem.

Main Theorem
The polycyclic presentation (2) of a Bieberbach group with a quaternion point group of order eight is consistent.

Proof The presentation of a Bieberbach group with a quaternion point group of order eight denoted as *G,* as given in (2) is polycyclic. Next, the presentation is shown to be consistent. By Definition 2, *G* is generated by $a, b, c, l_1, l_2, l_3, l_4, l_5, l_6$.Let $g_1 = a,\ g_2 = b,\ g_3 = c,\ g_4 = l_1,\ g_5 = l_2,\ g_6 = l_3,\ g_7 = l_4,\ g_8 = l_5,\ g_9 = l_6$. Based on Definition 3, there are five relations that need to be proven. For the first consistency check, the following relations hold:

i) $c(ba) = (cb)a$,
ii) $l_1(cb) = (l_1c)b$,
iii) $l_1(ca) = (l_1c)a$,
iv) $l_1(ba) = (l_1b)a$,
v) $l_2(l_1c) = (l_2l_1)c$,
vi) $l_2(l_1b) = (l_2l_1)b$,
vii) $l_2(l_1a) = (l_2l_1)a$,
viii) $l_2(cb) = (l_2c)b$,
ix) $l_2(ca) = (l_2c)a$,
x) $l_2(ba) = (l_2b)a$,
xi) $l_3(l_2l_1) = (l_3l_2)l_1$,
xii) $l_3(l_2c) = (l_3l_2)c$,
xiii) $l_3(l_2b) = (l_3l_2)b$,
xiv) $l_3(l_2a) = (l_3l_2)a$,
xv) $l_3(l_1c) = (l_3l_1)c$,
xvi) $l_3(l_1b) = (l_3l_1)b$,
xvii) $l_3(l_1a) = (l_3l_1)a$,
xviii) $l_3(cb) = (l_3c)b$,
xix) $l_3(ca) = (l_3c)a$,
xx) $l_3(ba) = (l_3b)a$,
xxi) $l_4(l_3l_2) = (l_4l_3)l_2$,
xxii) $l_4(l_3l_1) = (l_4l_3)l_1$,
xxiii) $l_4(l_3c) = (l_4l_3)c$,
xxiv) $l_4(l_3b) = (l_4l_3)b$,
xxv) $l_4(l_3a) = (l_4l_3)a$,
xxvi) $l_4(l_2l_1) = (l_4l_2)l_1$,
xxvii) $l_4(l_2c) = (l_4l_2)c$,
xxviii) $l_4(l_2b) = (l_4l_2)b$,
xxix) $l_4(l_2a) = (l_4l_2)a$,
xxx) $l_4(l_1c) = (l_4l_1)c$,
xxxi) $l_4(l_1b) = (l_4l_1)b$,
xxxii) $l_4(l_1a) = (l_4l_1)a$,
xxxiii) $l_4(cb) = (l_4c)b$,
xxxiv) $l_4(ca) = (l_4c)a$,
xxxv) $l_4(ba) = (l_4b)a$,
xxxvi) $l_5(l_4l_3) = (l_5l_4)l_3$,
xxxvii) $l_5(l_4l_2) = (l_5l_4)l_2$,
xxxviii) $l_5(l_4l_1) = (l_5l_4)l_1$,
xxxix) $l_5(l_4c) = (l_5l_4)c$,
xl) $l_5(l_4b) = (l_5l_4)b$,
xli) $l_5(l_4a) = (l_5l_4)a$,
xlii) $l_5(l_3l_2) = (l_5l_3)l_2$,
xliii) $l_5(l_3l_1) = (l_5l_3)l_1$,
xliv) $l_5(l_3c) = (l_5l_3)c$,
xlv) $l_5(l_3b) = (l_5l_3)b$,
xlvi) $l_5(l_3a) = (l_5l_3)a$,
xlvii) $l_5(l_2l_1) = (l_5l_2)l_1$,
xlviii) $l_5(l_2c) = (l_5l_2)c$,
xlix) $l_5(l_2b) = (l_5l_2)b$,
l) $l_5(l_2a) = (l_5l_2)a$,
li) $l_5(l_1c) = (l_5l_1)c$,
lii) $l_5(l_1b) = (l_5l_1)b$,
liii) $l_5(l_1a) = (l_5l_1)a$,
liv) $l_5(cb) = (l_5c)b$,
lv) $l_5(ca) = (l_5c)a$,
lvi) $l_5(ba) = (l_5b)a$,
lvii) $l_6(l_5l_4) = (l_6l_5)l_4$,
lviii) $l_6(l_5l_3) = (l_6l_5)l_3$,
lix) $l_6(l_5l_5) = (l_6l_5)l_2$,
lx) $l_6(l_5l_1) = (l_6l_5)l_1$,
lxi) $l_6(l_5c) = (l_6l_5)c$,
lxii) $l_6(l_5b) = (l_6l_5)b$,
lxiii) $l_6(l_5a) = (l_6l_5)a$,
lxiv) $l_6(l_4l_3) = (l_6l_4)l_3$,
lxv) $l_6(l_4l_2) = (l_6l_4)l_2$,
lxvi) $l_6(l_4l_1) = (l_6l_4)l_1$,
lxvii) $l_6(l_4c) = (l_6l_4)c$,
lxviii) $l_6(l_4b) = (l_6l_4)b$,
lxix) $l_6(l_4a) = (l_6l_4)a$,
lxx) $l_6(l_3l_2) = (l_6l_3)l_2$,
lxxi) $l_6(l_3l_1) = (l_6l_3)l_1$,
lxxii) $l_6(l_3c) = (l_6l_3)c$,
lxxiii) $l_6(l_3b) = (l_6l_3)b$,
lxxiv) $l_6(l_3a) = (l_6l_3)a$,
lxxv) $l_6(l_2l_1) = (l_6l_2)l_1$,
lxxvi) $l_6(l_2c) = (l_6l_2)c$,
lxxvii) $l_6(l_2b) = (l_6l_2)b$,
lxxviii) $l_6(l_2a) = (l_6l_2)a$,
lxxix) $l_6(l_1c) = (l_6l_1)c$,
lxxx) $l_6(l_1b) = (l_6l_1)b$,
lxxxi) $l_6(l_1a) = (l_6l_1)a$,
lxxxii) $l_6(cb) = (l_6c)b$,
lxxxiii) $l_6(ca) = (l_6c)a$,
lxxxiv) $l_6(ba) = (l_6b)a$.

By the polycyclic presentation of *G,* as given in (2),

For i),

$$c\left(ba\right) = cabcl_5^{-2}l_6^2 = acl_5^{-1}l_6bcl_5^{-2}l_6^2 = acbl_5^{-1}l_6cl_5^{-2}l_6^2 = acbl_5^{-1}cl_6l_5^{-2}l_6^2$$
$$= acbcl_5^{-1}l_6l_5^{-2}l_6^2 = abc^2l_5^{-3}l_6^3.$$
$$(cb)a = bacl_5^{-1}l_6 = abcl_5^{-2}l_6^2cl_5^{-1}l_6 = abc^2l_5^{-2}l_6^2l_5^{-1}l_6 = abc^2l_5^{-3}l_6^3.$$

For ii),

$$l_1(cb) = l_1bc = bl_3^{-1}c = bcl_3.$$
$$(l_1c)b = cl_1^{-1}b = cbl_3 = bcl_3.$$

For iii),

$$l_1(ca) = l_1acl_5^{-1}l_6 = al_1^{-1}l_2l_4^{-1}cl_5^{-1}l_6 = al_1^{-1}l_2cl_4l_5^{-1}l_6 = al_1^{-1}cl_2^{-1}l_4l_5^{-1}l_6$$
$$= acl_1l_2^{-1}l_4l_5^{-1}l_6.$$
$$(l_1c)a = cl_1^{-1}a = cal_4l_2^{-1}l_1 = acl_5^{-1}l_6l_4l_2^{-1}l_1 = acl_1l_2^{-1}l_4l_5^{-1}l_6.$$

The rest of the relations xxxvi) until lxxxiv) can be shown in a similar manner.

Next, the relations of G are shown to satisfy the second consistency relation. The following relations hold:

i) $b^2a=b(ba)$, ii) $c^2a=c(ca)$, iii) $c^2b=c(cb)$

By polycyclic presentation (2),

For i),

$$b^2a = cl_5l_6^{-1}a = cl_5al_5^{-1} = cal_6l_5^{-1} = acl_5^{-1}l_6l_6l_5^{-1} = acl_5^{-2}l_6^2.$$
$$b(ba) = babcl_5^{-2}l_6^2 = abcl_5^{-2}l_6^2bcl_5^{-2}l_6^2 = abcl_5^{-2}bl_6^2cl_5^{-2}l_6^2 = abcbl_5^{-2}l_6^2cl_5^{-2}l_6^2$$
$$= ab^2cl_5^{-2}l_6^2cl_5^{-2}l_6^2 = ab^2cl_5^{-2}cl_6^2l_5^{-2}l_6^2 = ab^2c^2l_5^{-2}l_6^2l_5^{-2}l_6^2$$
$$= acl_5l_6^{-1}l_5l_6^{-1}l_5^{-2}l_6^2l_5^{-2}l_6^2 = acl_5^{-2}l_6^2.$$

The rest of the relations can be shown in a similar way.

Next, for the third consistency relations, the following relations hold:

i) $ba^2=(ba)a$,	viii) $l_6a^2=(l_6a)a$,	xv) $l_6b^2=(l_6b)b$,
ii) $ca^2=(ca)a$,	ix) $cb^2=(cb)b$,	xvi) $l_1c^2=(l_1c)c$,
iii) $l_1a^2=(l_1a)a$,	x) $l_1b^2=(l_1b)b$,	xvii) $l_2c^2=(l_2c)c$,
iv) $l_2a^2=(l_2a)a$,	xi) $l_2b^2=(l_2b)b$,	xviii) $l_3c^2=(l_3c)c$,
v) $l_3a^2=(l_3a)a$,	xii) $l_3b^2=(l_3b)b$,	xix) $l_4c^2=(l_4c)c$,
vi) $l_4a^2=(l_4a)a$,	xiii) $l_4b^2=(l_4b)b$,	xx) $l_5c^2=(l_5c)c$,
vii) $l_5a^2=(l_5a)a$,	xiv) $l_5b^2=(l_5b)b$,	xxi) $l_6c^2=(l_6c)c$,

By (2),

For i),

$$ba^2 = bcl_6.$$
$$(ba)a = abcl_5^{-2}l_6^2a = abcl_5^{-1}l_5^{-1}l_6l_6a = abcl_5^{-1}l_5^{-1}l_6al_5 = abcl_5^{-1}l_5^{-1}al_5l_5$$
$$= abcl_5^{-1}al_6^{-1}l_5^2 = abcal_6^{-1}l_6^{-1}l_5^2 = abacl_5^{-1}l_6l_6^{-2}l_5^2 = abacl_5l_6^{-1}$$
$$= aabcl_5^{-2}l_6^2cl_5l_6^{-1} = a^2bc^2l_5^{-2}l_6^2l_5l_6^{-1} = a^2bc^2l_5^{-1}l_6 = cl_6bl_5l_6^{-1}l_5^{-1}l_6$$
$$= cbl_6 = bcl_6.$$

For ii) until xxi) the relations are shown to be true.

Next, the relations of G are shown to satisfy the forth consistency relation. The following relations hold:

i) $a^2a=aa^2$ ii) $b^2b=bb^2$ iii) $c^2c=cc^2$

By relations (2),
For i),

$$a^2a = cl_6a = cal_5 = acl_5^{-1}l_6l_5^{-1} = acl_6.$$

$$aa^2 = acl_6.$$

The relations for ii) and iii) are shown to be true.

Lastly, the relations of G are shown to satisfy the fifth consistency relation. The following relations hold:

i) $l_2=(l_2\,l_1^{-1})\,l_1$,
ii) $l_3=(l_3\,l_1^{-1})\,l_1$,
iii) $l_4=(l_4\,l_1^{-1})\,l_1$,
iv) $l_5=(l_5\,l_1^{-1})\,l_1$,
v) $l_6=(l_6\,l_1^{-1})\,l_1$,
vi) $l_3=(l_3\,l_2^{-1})\,l_2$,
vii) $l_4=(l_4\,l_2^{-1})\,l_2$,
viii) $l_5=(l_5\,l_2^{-1})\,l_2$,
ix) $l_6=(l_6\,l_2^{-1})\,l_2$,
x) $l_4=(l_4\,l_3^{-1})\,l_3$,
xi) $l_5=(l_5\,l_3^{-1})\,l_3$,
xii) $l_6=(l_6\,l_3^{-1})\,l_3$,
xiii) $l_5=(l_5\,l_4^{-1})\,l_4$,
xiv) $l_6=(l_6\,l_4^{-1})\,l_4$,
xv) $l_6=(l_6\,l_5^{-1})\,l_5$.

All 15 relations above are true since l_1, l_2, l_3, l_4, l_5 and l_6 commute with each other. Since the presentation of G satisfies the consistency relations given in Definition 3, then G has a consistent polycyclic presentation.

CONCLUSION

In this research, the polycyclic presentations of a Bieberbach group with a quaternion point group of order eight is shown to be consistent. This polycyclic presentation which is consistent is needed in finding the nonabelian tensor squares of the group.

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