The homological functor of a Bieberbach group with a cyclic point group of order two

Hazzirah Izzati Mat Hassim, Nor Haniza Sarmin, Nor Muhainiah Mohd Ali, Rohaidah Masri, and Nor'ashiqin Mohd Idrus

Citation: AIP Conference Proceedings **1605**, 672 (2014); doi: 10.1063/1.4887670
View online: http://dx.doi.org/10.1063/1.4887670
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1605?ver=pdfcov
Published by the AIP Publishing

The Homological Functor of a Bieberbach Group with a Cyclic Point Group of Order Two

Hazzirah Izzati Mat Hassim[a], Nor Haniza Sarmin[a], Nor Muhainiah Mohd Ali[a], Rohaidah Masri[b] and Nor'ashiqin Mohd Idrus[b]

[a]*Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia,81310 UTM Johor Bahru, Johor*
[b]*Department of Mathematics, Faculty of Science and Mathematics, Universiti Pendidikan Sultan Idris, 35900 Tg. Malim, Perak*

Abstract. The generalized presentation of a Bieberbach group with cyclic point group of order two can be obtained from the fact that any Bieberbach group of dimension n is a direct product of the group of the smallest dimension with a free abelian group. In this paper, by using the group presentation, the homological functor of a Bieberbach group a with cyclic point group of order two of dimension n is found.

Keywords: Homological functor, Bieberbach group, point group.
PACS: 02.20.-a; 02.20.Hj;

INTRODUCTION

A Bieberbach group is defined as a torsion free crystallographic group whereas a crystallographic group is a discrete subgroup G of the set of isometries of Euclidean space $\mathbb{E}^n$, where the quotient space $\mathbb{E}^n/G$ is compact. Previous researches on crystallographic groups as well as Bieberbach groups can be found in [1- 4]. Rohaidah in [5] computed the nonabelian tensor squares for some Bieberbach groups with cyclic point group of order two, C_2, found in Crystallographic, Algorithms and Tables (CARAT) package [6]. This computer package handles enumeration, construction, recognition, and comparison problems for crystallographic groups up to dimension 6. In [5], the first Bieberbach group with point group C_2 of dimension n is defined as the following:

Definition 1 [5] The groups

$$B_1(n) = B_1(2) \times F_{n-2}^{ab} \text{ for } n \geq 2$$

are Bieberbach groups with point group C_2 of dimension n, where $B_1(2) = \left\langle a, l_1, l_2 \,\middle|\, a^2 = l_2,\ {}^{a}l_1 = l_1^{-1},\ {}^{a}l_2 = l_2,\ {}^{l_1}l_2 = l_2 \right\rangle$ and F_m^{ab} is the free abelian group of rank m.

The notation $B_i(j)$ denotes the i th Bieberbach group with point group C_2 of dimension j. The group $B_1(2)$ has been shown to be polycyclic in [5].

By taking Definition 1 as the basis, the exterior square of $B_1(n)$ is computed in this paper. The exterior square of a group is one of the homological functors, which were originated from homotopy theory. The exterior square of a group G is the factor group $(G \otimes G)/\nabla(G)$ where $G \otimes G$ is the nonabelian tensor square of G while $\nabla(G)$ is the central subgroup of $G \otimes G$ generated by $g \otimes g$, for all $g \in G$.

Some important results from previous researches that are used in the computations of the exterior squares of $B_1(n)$ are presented next.

Proceedings of the 21st National Symposium on Mathematical Sciences (SKSM21)
AIP Conf. Proc. 1605, 672-677 (2014); doi: 10.1063/1.4887670

Definition 2 [7] Let G be a group with presentation $\langle G|R\rangle$ and let G^{φ} be an isomorphic copy of G via the mapping $\varphi: g \to g^{\varphi}$ for all $g \in G$. The group $\nu(G)$ is defined to be

$$\nu(G) = \left\langle G, G^{\varphi} \middle| R, R^{\varphi}, {}^{x}\left[g, h^{\varphi}\right] = \left[{}^{x}g, \left({}^{x}h\right)^{\varphi}\right] = {}^{x^{\varphi}}\left[g, h^{\varphi}\right], \forall x, g, h \in G \right\rangle.$$

Proposition 1 [8] If G is polycyclic, then $\nu(G)$ is polycyclic.

Theorem 1 [9] Let G be a group. The map $\sigma: G \otimes G \to [G, G^{\varphi}] \lhd \nu(G)$ defined by $\sigma(g \otimes h) = \left[g, h^{\varphi}\right]$ for all g and h in G is an isomorphism.

Definition 3 [8] Let G be any group. Then $\tau(G)$ is defined to be the quotient group $\nu(G) / \sigma(\nabla(G))$, where $\sigma: G \otimes G \to \left[G, G^{\varphi}\right]$ is as defined in Theorem 1.

Proposition 2 [8] Let G be any group. The map

$$\hat{\sigma}: G \wedge G \to \left[G, G^{\varphi}\right]_{\tau(G)} \lhd \tau(G)$$

defined by $\sigma(g \wedge h) = \left[g, h^{\varphi}\right]_{\tau(G)}$ is an isomorphism.

For simplicity, since $\tau(G)$ is a subgroup of $\nu(G)$, after this we only denote $\left[g, h^{\varphi}\right]_{\tau(G)}$ as $\left[g, h^{\varphi}\right]$.

Proposition 3 [8] Let G be a polycyclic group with a polycyclic generating sequence $g_1, \ldots, g_k$. Then, $\left[G, G^{\varphi}\right]_{\tau(G)}$, a subgroup of $\tau(G)$, is generated by

$$\left[G, G^{\varphi}\right]_{\tau(G)} = \left\langle \left[g_i^{\delta}, \left(g_j^{\varepsilon}\right)\right], \left[g_j^{\delta}, \left(g_i^{\varepsilon}\right)\right] \right\rangle$$

for $1 \le i < j \le k$, where

$$\varepsilon = \begin{cases} 1 & \text{if } \left|g_j\right| < \infty; \\ \pm 1 & \text{if } \left|g_j\right| = \infty \end{cases} \text{ and } \delta = \begin{cases} 1 & \text{if } \left|g_i\right| < \infty; \\ \pm 1 & \text{if } \left|g_i\right| = \infty. \end{cases}$$

Lemma 1 [10] Let G be a group such that G^{ab} is finitely generated. Assume that G^{ab} is the direct product of the cyclic groups $\langle x_i G' \rangle$, for $i = 1, \ldots, s$. Then, $\nabla(G)$ is generated by the elements of the set $\left\{\left[x_i, x_i^{\varphi}\right], \left[x_i, x_j^{\varphi}\right]\left[x_j, x_i^{\varphi}\right] \middle| 1 \le i < j \le s\right\}$.

A list of commutator calculus that is used in the computations of the exterior square of $B_1(n)$ is given as follows:

$$[x, yz] = [x, y]\,{}^{y}[x, z]; \tag{1}$$

$$\left[x, y^{-1}\right] = {}^{y^{-1}}[x, y]^{-1} = \left[y^{-1}, [x, y]^{-1}\right][x, y]^{-1}; \tag{2}$$

$${}^{z}[x, y] = \left[{}^{z}x, {}^{z}y\right]. \tag{3}$$

The following lemmas record some basic identities related to the group

Lemma 2 [8, 11] Let G be a group. Then the following hold in $\nu(G)$:

(i) $\left[g,g^{\varphi}\right]$ is central in $\nu(G)$ for all g in G;

(ii) $\left[g_1,g_2^{\varphi}\right]\left[g_2,g_1^{\varphi}\right]$ is central in $\nu(G)$ for all g_1,g_2 in G;

(iii) $\left[g,g^{\varphi}\right]=1$ for all g in G';

(iv) $\left[g_1^n,g_2^{\varphi}\right]\left[g_2,\left(g_1^n\right)^{\varphi}\right]=\left[g_1,\left(g_2^n\right)^{\varphi}\right]\left[g_2^n,g_1^{\varphi}\right]=\left(\left[g_1,g_2^{\varphi}\right]\left[g_2,g_1^{\varphi}\right]\right)^n$ for all g_1,g_2 in G and integer n.

Corollary 1 [10] Let G be any group. Then $\left[Z(G),(G')^{\varphi}\right]=1$.

Lemma 3 [5, 8] Let g and h be elements of G such that $[g,h]=1$. Then in $\nu(G)$,

(i) $\left[g^n,h^{\varphi}\right]=\left[g,h^{\varphi}\right]^n=\left[g,\left(h^{\varphi}\right)^n\right]$ for all integers n;

(ii) $\left[g^n,\left(h^m\right)^{\varphi}\right]\left[h^m,\left(g^n\right)^{\varphi}\right]=\left(\left[g,h^{\varphi}\right]\left[h,g^{\varphi}\right]\right)^{nm}$;

(iii) $\left[g,h^{\varphi}\right]$ is in the center of $\nu(G)$.

Lemma 4 [5] Let G and H be groups and let $g\in G$. Suppose ϕ is a homomorphism from G onto H. If $\phi(g)$ has a finite order then $|\phi(g)|$ divides $|g|$. Otherwise the order of $\phi(g)$ equals the order of g.

Lemma 5 [12] Let A, B and C be abelian groups. Consider the ordinary tensor product of two abelian groups. Then,

(i) $C_0\otimes A\cong A$,

(ii) $C_0\otimes C_0\cong C_0$,

(iii) $C_n\otimes C_m\cong C_{\gcd(n,m)}$, for $n,m\in\mathbb{Z}$, and

(iv) $A\otimes(B\times C)=(A\otimes B)\times(A\otimes C)$,

where C_0 is the infinite cyclic group.

Theorem 2 [13] Let G and H be groups such that there is an epimorphism $\varepsilon:G\to H$. Then there exists an epimorphism $\alpha:G\otimes G\to H\otimes H$ defined by $\alpha(g\otimes h)=\varepsilon(g)\otimes\varepsilon(h)$.

THE COMPUTATIONS OF THE EXTERIOR SQUARE OF $B_1(n)$

Based on Definition 1, the consistent polycyclic presentation of $B_1(n)$ is obtained and is given in the following lemma.

Lemma 6 Let $B_1(n)$ be a Bieberbach group with point group C_2 of dimension n. Then,

$$B_1(n)=\left\langle a,l_1,l_2,\ldots,l_n \,\middle|\, a^2=l_2,\ {}^{a}l_1=l_1^{-1},\ {}^{a}l_j=l_j,\ {}^{l_i}l_j=l_j\right\rangle$$

for all $1\le i<j\le n$.

Proof. By Definition 1, $B_1(n)=B_1(2)\times F_{n-2}^{ab}$. Therefore, all elements in F_{n-2}^{ab} commute with elements in $B_1(2)$. Since F_{n-2}^{ab} is the free abelian group of rank $n-2$, then it is generated by $l_3,l_4,\ldots,l_n$. Therefore, ${}^{a}l_j=l_j$, ${}^{l_1}l_j=l_j$ and ${}^{l_2}l_j=l_j$ for a, l_1, $l_2\in B_1(2)$ and $j=3,4,\ldots,n$. Therefore, we have

$$B_1(n)=\left\langle a,l_1,l_2,\ldots,l_n \middle| a^2=l_2,\ {}^{a}l_1=l_1^{-1},\ {}^{a}l_j=l_j,\ {}^{l_i}l_j=l_j \right\rangle$$

where $1\le i<j\le n$. Based on the properties of groups and polycyclic presentations, this presentation is consistent.

Lemma 7 The group $B_1(n)$ has a cyclic derived subgroup and its abelianisation is

$$B_1(n)^{ab}=\left\langle aB_1(n)',l_1B_1(n)',l_jB_1(n)'\right\rangle\cong C_0^{n-1}\times C_2$$

for $3\le j\le n$.

Proof. Based on the relations of $B_1(n)$, $[a,l_1]=l_1^{-2}\ne 1$, $[a,l_j]=1$ and $[l_i,l_j]=1$ for all $1\le i<j\le n$. Therefore, $B_1(n)'=\langle l_1^{-2}\rangle$. Since $B_1(n)$ is torsion free, then $B_1(n)'\cong C_0$.

The abelianisation of $B_1(n)$, denoted as $B_1(n)^{ab}$, is defined to be the quotient group $B_1(n)/B_1(n)'$. Thus, it is generated by $aB_1(n)'$, $l_1B_1(n)'$ and $l_jB_1(n)'$ for all $j=3,4,\ldots,n$. However, $aB_1(n)'\cap l_2B_1(n)'$ is not trivial since $a^2=l_2$ by the relations of $B_1(n)$. Hence $aB_1(n)'=l_2B_1(n)'$. Since $l_1^2\in B_1(n)'$, then the order of $l_1B_1(n)'$ is two. Meanwhile, there is no power of a or l_j is in $B_1(n)'$ and $B_1(n)'$ is generated by elements of infinite order. Thus, , $aB_1(n)'$ and $l_kB_1(n)'$ have infinite order. Therefore,

$$B_1(n)^{ab}=\left\langle aB_1(n)',l_1B_1(n)',l_jB_1(n)'\right\rangle\cong C_0\times C_2\times C_0^{n-2}.$$

Theorem 3 The exterior square of $B_1(n)$ is

$$\begin{aligned}B_1(n)\wedge B_1(n)&=\langle a\wedge l_1,\ a\wedge l_i,\ a\wedge l_n,\ l_1\wedge l_i,\ l_1\wedge l_n,\ l_i\wedge l_j\rangle\\&\cong C_0^{1+\frac{(n-2)(n-1)}{2}}\times C_2^{n-2},\end{aligned}$$

where $1\le i<j\le n$.

Proof. By Proposition 2, $B_1(n)\wedge B_1(n)$ is isomorphic to $\left[B_1(n),B_1(n)^{\varphi}\right]_{\tau(B_1(n))}$. Then, by referring to Proposition 3,

$$\left[B_1(n),B_1(n)^{\varphi}\right]_{\tau(B_1(n))}=\left\langle\left[a^{\pm1},l_i^{\pm\varphi}\right],\left[l_i^{\pm1},a^{\pm\varphi}\right],\left[a^{\pm1},l_n^{\pm\varphi}\right],\left[l_n^{\pm1},a^{\pm\varphi}\right],\left[l_i^{\pm1},l_j^{\pm\varphi}\right],\left[l_j^{\pm1},l_i^{\pm\varphi}\right]\right\rangle$$

where $1\le i<j\le n$. By the definition of exterior square, all elements in $\nabla(B_1(n))$ are trivial in $B_1(n)\wedge B_1(n)$. Since l_i commutes with l_j, then by Lemma 3(i), $[l_i,l_j^{-\varphi}]$, $[l_i^{-1},l_j^{\varphi}]$, $[l_i^{-1},l_j^{-\varphi}]$, $[l_j,l_i^{-\varphi}]$, $[l_j^{-1},l_i^{\varphi}]$ and $[l_i^{-1},l_j^{-\varphi}]$ can be eliminated. The following three cases are now considered.

Case 1 : $i=1$.

By invoking the relations of $B_1(n)$ and the commutator calculus, we obtain the following:

$$\begin{aligned}\left[a,l_1^{-\varphi}\right]&=\left[l_1^{-1},[a,l_1]^{-\varphi}\right]\left[a,l_1^{\varphi}\right]^{-1} && \text{by (2)}\\&=\left[l_1^{-1},\left(l_1^{-2}\right)^{-\varphi}\right]\left[a,l_1^{\varphi}\right]^{-1} && \text{by relations of } B_1(n)\\&=\left[l_1,l_1^{\varphi}\right]^{-2}\left[a,l_1^{\varphi}\right]^{-1} && \text{by Lemma 3 (i)}\end{aligned}$$

$$=\left[a,l_1^{\varphi}\right]^{-1} \qquad \text{since } \left[l_1,l_1^{\varphi}\right]\in\nabla\left(B_1(n)\right) \text{ by Lemma 1.}$$

Similarly, we obtain $\left[a^{-1},l_1^{\varphi}\right]=\left[a,l_1^{\varphi}\right]$ and $\left[a^{-1},l_1^{-\varphi}\right]=\left[a,l_1^{\varphi}\right]^{-1}$. Clearly, $\left[l_1,a^{\varphi}\right]=\left[a,l_1^{\varphi}\right]^{-1}\left[a,l_1^{\varphi}\right]\left[l_1,a^{\varphi}\right]$. By Lemma 1 and Lemma 7, $\left[a,l_1^{\varphi}\right]\left[l_1,a^{\varphi}\right]$ is also in $\nabla\left(B_1(n)\right)$, which implies that it is trivial in $\left[B_1(n),B_1(n)^{\varphi}\right]_{\tau(B_1(n))}$ Therefore, $\left[l_1,a^{\varphi}\right]=\left[a,l_1^{\varphi}\right]^{-1}$. Using similar arguments, it can be shown that $\left[l_1,a^{-\varphi}\right]$, $\left[l_1^{-1},a^{\varphi}\right]$ and $\left[l_1^{-1},a^{-\varphi}\right]$ can be written in terms of $\left[a,l_1^{\varphi}\right]$. Besides, since $a^2=l_2$, by applying the commutator calculus, we obtain

$$\begin{aligned}\left[l_2,l_1^{\varphi}\right]&=\left[a^2,l_1^{\varphi}\right]\\&={}^{a}\left[a,l_1^{\varphi}\right]\left[a,l_1^{\varphi}\right]\\&=\left[a,l_1^{-\varphi}\right]\left[a,l_1^{\varphi}\right]\\&=\left[a,l_1^{\varphi}\right]^{-1}\left[a,l_1^{\varphi}\right]\\&=1.\end{aligned}$$

For all $j=3,4,\ldots,n,$ since $\left[l_1,l_j^{\varphi}\right]\left[l_j,l_1^{\varphi}\right]\in\nabla\left(B_1(n)\right)$ is trivial in $\left[B_1(n),B_1(n)^{\varphi}\right]_{\tau(B_1(n))}$, then $\left[l_j,l_1^{\varphi}\right]=\left[l_1,l_j^{\varphi}\right]^{-1}$.

Case 2 : $i=2$.

Again, since $a^2=l_2$ and $\left[a,a^{\varphi}\right]\in\nabla\left(B_1(n)\right)$ is trivial in $\left[B_1(n),B_1(n)^{\varphi}\right]_{\tau(B_1(n))}$, then $\left[a^{\pm1},l_2^{\pm\varphi}\right]$ and $\left[l_2^{\pm1},a^{\pm\varphi}\right]$ are also trivial in $\left[B_1(n),B_1(n)^{\varphi}\right]_{\tau(B_1(n))}$. Then, since a commutes with l_j in $B_1(n)$, $\left[l_2,l_j^{\varphi}\right]=\left[a,l_j^{\varphi}\right]^{2}$ and $\left[l_j,l_2^{\varphi}\right]=\left[a,l_j^{\varphi}\right]^{-2}$.

Case 3 : $3\le i<j\le n.$

By the relations of $B_1(n)$, l_i and l_j are in the center of $B_1(n)$ for all $3\le i<j\le n.$ Hence, by Lemma 3 (i), $\left[a,l_i^{-\varphi}\right]=\left[a^{-1},l_i^{\varphi}\right]=\left[a,l_i^{\varphi}\right]^{-1}$, $\left[a^{-1},l_i^{-\varphi}\right]=\left[a,l_i^{\varphi}\right]$, $\left[l_i,a^{-\varphi}\right]=\left[l_i^{-1},a^{\varphi}\right]=\left[l_i,a^{\varphi}\right]^{-1}$ and $\left[l_i^{-1},a^{-\varphi}\right]=\left[l_i,a^{\varphi}\right]$. Next, since $\left[a,l_i^{\varphi}\right]\left[l_i,a^{\varphi}\right]\in\nabla\left(B_1(n)\right)$ is trivial in $\left[B_1(n),B_1(n)^{\varphi}\right]_{\tau(B_1(n))}$, then $\left[l_i,a^{\varphi}\right]=\left[a,l_i^{\varphi}\right]^{-1}$. Similarly, all generators $\left[a^{\pm1},l_n^{\pm\varphi}\right]$ and $\left[l_n^{\pm1},a^{\pm\varphi}\right]$ can be written in terms of $\left[a,l_n^{\varphi}\right]$. Furthermore, $\left[l_i^{\pm1},l_j^{\pm\varphi}\right]$ and $\left[l_j^{\pm1},l_i^{\pm\varphi}\right]$ can be simplified to be $\left[l_i,l_j^{\phi}\right]$.

Therefore, the remaining generators of $\left[B_1(n),B_1(n)^{\varphi}\right]_{\tau(B_1(n))}$ are $\left[a,l_1^{\varphi}\right]$, $\left[a,l_i^{\varphi}\right]$, $\left[a,l_n^{\varphi}\right]$, $\left[l_1,l_i^{\varphi}\right]$, $\left[l_1,l_n^{\varphi}\right]$ and $\left[l_i,l_j^{\varphi}\right]$, for all $1\le i<j\le n.$ Then, by Proposition 2, $B_1(n)\wedge B_1(n)$ is generated by $a\wedge l_1$, $a\wedge l_i$, $a\wedge l_n$, $l_1\wedge l_i$, $l_1\wedge l_n$ and $l_i\wedge l_j$ for all $1\le i<j\le n.$ Next, the order of each generators are computed. The mapping $\kappa':B_1(n)\wedge B_1(n)\to B_1(n)'$ gives $\kappa'(a\wedge l_1)=\left[a,l_1\right]$. Since $\left[a,l_1\right]=l_1^{-2}$ in $B_1(n)'$ has infinite order, then by Lemma 4, the order of $a\wedge l_1$ is infinity. Next, we show that the order of $a\wedge l_i$ is also infinite. The following conditions would lead the order of $\left[a,l_i^{\varphi}\right]$ to be finite. Since a and l_i commute with each other in $B_1(n)$, then by Lemma 3(i), $\left[a^r,\left(l_i^s\right)^{\varphi}\right]=\left[a,l_i^{\varphi}\right]^{rs}$ for any integer $r,s.$ However, since $B_1(n)$ is torsion free, then both a and l_i

has infinite order. Next, one power of $\left[a, l_i^{\varphi}\right]$ will give an element in $Z\left(B_1\left(n\right)\right)$ while the other one in the derived subgroup of $B_1\left(n\right)$ by Corollary 1. However, this is not true since there is no power of either a or l_i is in $B_1\left(n\right)'$. Therefore, $a \wedge l_i$ has infinite order. Using similar arguments, the order of $a \wedge l_n$ and $l_i \wedge l_j$ are also infinite.

Since $l_1^2 \in B_1\left(n\right)'$ and $l_i, l_j \in Z\left(B_1\left(n\right)\right)$, then by Corollary 1 and Lemma 3(i), $\left[l_1, l_i^{\varphi}\right]^2 = \left[l_1^2, l_i^{\varphi}\right] = 1$. Thus, without loss of generality, $\left[l_1, l_i^{\varphi}\right]$ has order two, which implies that the order of $l_1 \wedge l_2$ is two by Proposition 2. Similarly, the order of $l_1 \wedge l_n$ is also two. Therefore,

$$B_1\left(n\right) \wedge B_1\left(n\right) = \left\langle a \wedge l_1,\ a \wedge l_i,\ a \wedge l_n,\ l_1 \wedge l_i,\ l_1 \wedge l_n,\ l_i \wedge l_j \right\rangle$$

$$\cong C_0 \times C_0^{n-3} \times C_0 \times C_2^{n-3} \times C_2 \times C_0^{\frac{(n-3)(n-2)}{2}}$$

$$\cong C_0^{1+\frac{(n-2)(n-1)}{2}} \times C_2^{n-2}.$$

CONCLUSION

In this paper, the exterior square of a Bieberbach group of dimension n, namely $B_1\left(n\right)$ is computed. Firstly, the polycyclic presentation of this group is computed and then using the method for computing the exterior square of polycyclic groups, the results of $B_1\left(n\right) \wedge B_1\left(n\right)$ is obtained.

ACKNOWLEDGMENTS

The researchers would like to thank the Ministry of Education Malaysia (MOE) for the financial funding through Research University Grant (RUG) Vote No. 04H13 by Research Management Center UTM and Fundamental Research Grant Scheme (FRGS), Vote No: 2011-0068-101-02 from Research Management Centre UPSI. The first author is also indebted to MOE for her MyPHD Scholarship.

REFERENCES

1. D. R. Farkas, *Rocky Mountain Journal of Mathematics* **11(4)**, 511-551 (1981).
2. H. Hiller, *The American Mathematical Monthly* **93 (10)**, 765-779 (1986).
3. W. Plesken and T. Schulz, *Experimental Mathematics* **9 (3)**, 407- 411 (2000).
4. C. Cid, T. Schulz, *Experimental Mathematics* **10 (1)**, 109-115 (2001).
5. Rohaidah Masri, "The Nonabelian Tensor Squares of Certain Bieberbach Groups with Cyclic Point Group of Order Two", Ph.D. Thesis, Universiti Teknologi Malaysia, 2009.
6. http://wwwb.math.rwth-aachen.de/carat/
7. N. R. Rocco, *Bol. Soc. Brasil. Mat. (N. S.)* **22(1)**, 63-79 (1991).
8. R. D. Blyth and R. F. Morse, *Journal of Algebra* **321**, 2139-2148 (2009).
9. G. Ellis and F. Leonard, *Proc. Roy. Irish Acad. Sect. A.,* **95(2)**, 137-147 (1995).
10. R. D. Blyth, F. Fumagalli and M. Morigi, *Journal of Group Theory,* **13**, 83-94 (2010).
11. R. D. Blyth, P. Moravec and R. F. Morse, *Contemporary Mathematics,* **470**, 27-44 (2008).
12. A. J. Zomorodian, *Topology for Computing,* New York: Cambridge University Press, 2005.
13. R. Brown, D. L. Johnson, E. F. Robertson, *Journal of Algebra* **111**, 177-202 (1987)