

**The nonabelian tensor square of Bieberbach group of dimension five with dihedral point group of order eight**

Wan Nor Farhana Wan Mohd Fauzi, Nor'ashiqin Mohd Idrus, Rohaidah Masri, and Nor Haniza Sarmin

Citation: [AIP Conference Proceedings](#) **1605**, 611 (2014); doi: 10.1063/1.4887659

View online: <http://dx.doi.org/10.1063/1.4887659>

View Table of Contents: <http://scitation.aip.org/content/aip/proceeding/aipcp/1605?ver=pdfcov>

Published by the [AIP Publishing](#)

---

**Articles you may be interested in**

[On the abelianization of all Bieberbach groups of dimension four with symmetric point group of order six](#)  
AIP Conf. Proc. **1635**, 479 (2014); 10.1063/1.4903625

[The presentation of the nonabelian tensor square of a Bieberbach group of dimension five with dihedral point group](#)  
AIP Conf. Proc. **1635**, 417 (2014); 10.1063/1.4903616

[On some homological functors of Bieberbach group of dimension four with dihedral point group of order eight](#)  
AIP Conf. Proc. **1602**, 754 (2014); 10.1063/1.4882570

[Homological functor of a torsion free crystallographic group of dimension five with a nonabelian point group](#)  
AIP Conf. Proc. **1602**, 691 (2014); 10.1063/1.4882560

[The Schur multiplier and nonabelian tensor square of some groups of p-power order](#)  
AIP Conf. Proc. **1522**, 1039 (2013); 10.1063/1.4801244

---

# The Nonabelian Tensor Square of Bieberbach Group of Dimension Five with Dihedral Point Group of Order Eight

Wan Nor Farhana Wan Mohd Fauzi<sup>a</sup>, Nor'ashiqin Mohd Idrus<sup>a</sup>,  
Rohaidah Masri<sup>a</sup> and Nor Haniza Sarmin<sup>b</sup>

<sup>a</sup>*Department of Mathematics, Faculty of Science and Mathematics,  
Universiti Pendidikan Sultan Idris, 35900 Tanjong Malim, Perak Darul Ridzuan.*

<sup>b</sup>*Department of Mathematical Sciences, Faculty of Science,  
Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor Darul Takzim.*

**Abstract.** The nonabelian tensor product was originated in homotopy theory as well as in algebraic K-theory. The nonabelian tensor square is a special case of the nonabelian tensor product where the product is defined if the two groups act on each other in a compatible way and their action are taken to be conjugation. In this paper, the computation of nonabelian tensor square of a Bieberbach group, which is a torsion free crystallographic group, of dimension five with dihedral point group of order eight is determined. Groups, Algorithms and Programming (GAP) software has been used to assist and verify the results.

**Keywords:** Nonabelian tensor square; Bieberbach group.

**PACS:** 02.20.-a.

## INTRODUCTION

The nonabelian tensor square  $G \otimes G$  of the group  $G$  is a special case of the nonabelian tensor product  $G \otimes H$  for arbitrary groups  $G$  and  $H$ . The nonabelian tensor square is generated by the symbols  $g \otimes h$  for all  $g, h \in G$  subject to relations

$$gh \otimes k = (g^h \otimes k^h)(h \otimes k) \text{ and } g \otimes hk = (g \otimes k)(g^k \otimes h^k) \quad (1)$$

for all  $g, h, k \in G$  where  $g^h = h^{-1}gh$  denotes the conjugate of  $h$  by  $g$ .

Bieberbach groups are torsion free crystallographic groups. Mohd Idrus *et al.* [1] found that there are 73 centerless Bieberbach groups with dihedral point group of order eight using the Crystallographic Algorithms and Tables (CARAT) package. One of them has dimension 4, eleven of them have dimension 5 and the rest of them have dimension 6.

The main focus of this paper is to compute the nonabelian tensor square of the first Bieberbach group of dimension five with dihedral point group of order eight, denoted by  $B_1(5)$ . The result has been verified using Group, Algorithms and Programming (GAP) software.

## Preliminary Results

This section provides some basic definitions and structural results used in this paper. We start with the definition of polycyclic presentation and consistent polycyclic presentation.

**Definition 1** [2]. Polycyclic Presentation

Let  $F_n$  be a free group on generators  $g_1, \dots, g_n$  and  $R$  be a set of relations of group  $G$ . The relations of a polycyclic presentation  $F_n / R$  have the form:

$$\begin{aligned} g_i^{e_i} &= g_{i+1}^{x_{i,i+1}} \dots g_n^{x_{i,n}} && \text{for } i \leq I, \\ g_j^{-1} g_i g_j &= g_{j+1}^{y_{i,j,j+1}} \dots g_n^{y_{i,j,n}} && \text{for } j \leq i, \\ g_j g_i g_j^{-1} &= g_{j+1}^{z_{i,j,j+1}} \dots g_n^{z_{i,j,n}} && \text{for } j \leq i \text{ and } j \notin I \end{aligned}$$

for some  $I \subseteq \{1, \dots, n\}, e_i \in \mathbb{N}$  for  $i \in I$  and  $x_{i,j}, y_{i,j,k}, z_{i,j,k} \in \mathbb{Z}$  for all  $i, j$  and  $k$ .

**Definition 2** [2]. Consistent Polycyclic Presentation

Let  $G$  be a group generated by  $g_1, \dots, g_n$ . The consistency of the relation in  $G$  can be determined using the following consistency relations:

$$\begin{aligned} g_k(g_j g_i) &= (g_k g_j) g_i && \text{for } k > j > i, \\ (g_i^{e_i}) g_i &= g_j^{e_j^{-1}} (g_j g_i) && \text{for } j > i, j \in I, \\ g_j(g_i^{e_i}) &= (g_j g_i) g_i^{e_i^{-1}} && \text{for } j > i, i \in I, \\ (g_i^{e_i}) g_i &= g_i(g_i^{e_i}) && \text{for } i \notin I \\ g_j &= (g_j g_i^{-1}) g_i && \text{for } j > i, i \notin I. \end{aligned}$$

There are some methods that have been used by earlier researchers in computing the nonabelian tensor square  $G \otimes G$  of a group  $G$ . In [3-5] they used crossed pairing method to compute the nonabelian tensor square by determining a unique homomorphism  $\Phi^*: G \otimes G \rightarrow L$ . Brown *et al.* in [6] used the definition given in (1) to compute  $G \otimes G$  for all group  $G$  of order at most 30 by forming the finite presentation. In [7], Ellis and Leonard give a computer algorithm, Cayley program, which is capable of determining the nonabelian tensor squares of bigger groups.

In this paper, we use the structural results of the group  $\nu(G)$  which is a subgroup of  $[G, G^\varphi]$ , the commutator calculus, and a computer algebraic system GAP [8] to compute the nonabelian tensor square of the group  $B_1(5)$ . The structural results of  $\nu(G)$ , which is polycyclic, was studied by Ellis and Leonard [7], Rocco [9] and has been extended by Blyth and Morse [10], and is given as follows.

**Definition 3** [7, 9]. Let  $G$  be a group with presentation  $\langle G | R \rangle$  and let  $G^\varphi$  be an isomorphic copy of  $G$  via the mapping  $\varphi: g \rightarrow g^\varphi$  for all  $g \in G$ . The group  $\nu(G)$  is defined to be

$$\nu(G) = \langle G, G^\varphi | R, R^\varphi, {}^x[g, h^\varphi] = [{}^xg, ({}^xh)^\varphi] = {}^{x\varphi}[g, h^\varphi], \forall x, g, h \in G \rangle.$$

Ellis and Leonard [7] and Rocco [9] have shown that  $\nu(G)$  is isomorphic to the nonabelian tensor square of the group  $G$  as given in the following theorem.

**Theorem 1** [9].

Let  $G$  be a group. The map  $\sigma: G \otimes G \rightarrow [G, G^\varphi] \triangleleft \nu(G)$  defined by  $\sigma(g \otimes h) = [g, h^\varphi]$  for all  $g, h$  in  $G$  is an isomorphism.

Beside the commutator calculus, the following lemmas about the structural results of the group  $\nu(G)$  are used in the computation of the nonabelian tensor square of  $B_1(5)$ .

**Lemma 1** [9, 10]. Let  $G$  be a group. The following relations hold in  $\nu(G)$  :

- (i)  $[g_1, g_2^\rho]^{[g_3, g_4^\rho]} = [g_1, g_2^\rho]^{[g_3, g_4]}$  for all  $g_1, g_2, g_3, g_4$  in  $G$ ;
- (ii)  $[g_1, g_2^\rho, g_3] = [g_1, g_2, g_3^\rho] = [g_1, g_2^\rho, g_3^\rho]$  and  
 $[g_1^\rho, g_2, g_3] = [g_1^\rho, g_2, g_3^\rho] = [g_1^\rho, g_2^\rho, g_3]$  for all  $g_1, g_2, g_3$  in  $G$ ;
- (iii)  $[g_1, [g_2, g_3]^\rho] = [g_2, g_3, g_1^\rho]^{-1}$  for all  $g_1, g_2, g_3$  in  $G$ ;
- (iv)  $[g, g^\rho]$  is a central in  $\nu(G)$  for all  $g$  in  $G$ ;
- (v)  $[g_1, g_2^\rho][g_2, g_1^\rho]$  is a central in  $\nu(G)$  for all  $g_1, g_2$  in  $G$ ;
- (vi)  $[g, g^\rho] = 1$  for all  $g$  in  $G'$ .

**Lemma 2 [9, 10].** Let  $x$  and  $y$  be element of  $G$  such that  $[x, y] = 1$ . Then in  $\nu(G)$ ,

- (i)  $[x^n, y^\rho] = [x, y^\rho]^n = [x, (y^\rho)^n]$  for all integer  $n$ ;
- (ii)  $[x^n, (y^m)^\rho][y^m, (x^n)^\rho] = ([x, y^\rho][y, (x^\rho)])^{nm}$
- (iii)  $[x, y^\rho]$  is a central in  $\nu(G)$ .
- (iv) If  $x$  and  $y$  are torsion free elements of order  $o(x)$  and  $o(y)$ , then  $o([x, y^\rho])$  divides the gcd  $(o(x), o(y))$

**Lemma 3 [9, 10].** Let  $g_1, g_2, g_3$  and  $g_4$  be elements of a group  $G$ . Then in  $\nu(G)$ ,

- (i)  $[[g_1, g_2^\rho], [g_2, g_1^\rho]] = 1$ ;
- (ii)  $[[g_1, g_2][g_3, g_4]^\rho] = [[g_1, g_2^\rho], [g_3, g_4^\rho]]$ ;
- (iii)  $[g_1^n, g_2^\rho][g_2, (g_1^n)^\rho] = [g_1, (g_2^\rho)^n][g_2^\rho, g_1^\rho] = ([g_1, g_2^\rho][g_2, g_1^\rho])^n$ ;
- (iv)  $[g_1, (g_2^n, g_3^m)^\rho][g_2^n, g_3^m, g_1^\rho] = ([g_1, g_2^\rho][g_2, g_1^\rho])^n \cdot ([g_1, g_3^\rho], [g_3, g_1^\rho])^m$ ;
- (v)  $[g_1^n g_2^m, g_3^\rho][g_3, (g_1^n g_2^m)^\rho] = ([g_1, g_3^\rho][g_3, g_1^\rho])^n \cdot ([g_2, g_3^\rho], [g_3, g_2^\rho])^m$ ;
- (vi)  $[g_2, g_3^\rho]^{g_1} = [g_2, g_3^\rho]^{g_1^\rho}$ .

In order to determine the structure of  $G \otimes G$ , we compute  $\nu(G)$  using the following proposition that had been proved in [10].

**Proposition 1** (Proposition 20 [10])

Let  $G$  be a polycyclic group with a polycyclic generating sequence  $g_1, \dots, g_k$ . Then  $[G, G^\rho]$ , a subgroup of  $\nu(G)$ , is generated by

$$[G, G^\rho] = \langle [g_i, g_i^\rho], [g_i^\varepsilon, (g_j^\rho)^\delta], [g_i^\varepsilon, (g_j^\rho)^\delta][g_j^\delta, (g_i^\rho)^\varepsilon] \rangle$$

for  $1 \leq i < j \leq k$ , where

$$\varepsilon = \begin{cases} 1 & \text{if } |g_i| < \infty \\ \pm 1 & \text{if } |g_i| = \infty \end{cases} \quad \text{and} \quad \delta = \begin{cases} 1 & \text{if } |g_j| < \infty \\ \pm 1 & \text{if } |g_j| = \infty \end{cases}$$

## Computing The Nonabelian Tensor Square Of $B_1(5)$

A consistent polycyclic presentation of the group  $B_1(5)$ , which is determined based on Definition 1 and 2, is given as follows:

$$B_1(5) = \left\langle a, b, c, l_1, l_2, l_3, l_4, l_5 \left| \begin{array}{l} a^2 = l_2^{-1}, b^2 = l_1 l_2^{-1} l_4, c^2 = l_3^{-1}, \\ b^a = c l_2^{-1}, c^a = b l_1^{-1}, c^b = c l_1 l_2^{-1} l_3 l_4 l_5^{-1}, \\ l_1^a = l_1^{-1}, l_2^a = l_2, l_3^a = l_4^{-1}, l_4^a = l_3^{-1}, l_5^a = l_5^{-1}, \\ l_1^b = l_2^{-1}, l_2^b = l_1^{-1}, l_3^b = l_3^{-1}, l_4^b = l_4, l_5^b = l_5^{-1}, \\ l_1^c = l_2, l_2^c = l_1, l_3^c = l_3, l_4^c = l_4^{-1}, l_5^c = l_5^{-1}, \\ l_j^i = l_j, l_j^{i^{-1}} = l_j, \text{ for } j > i, 1 \leq i, j \leq 5. \end{array} \right. \right. \quad (2)$$

To compute the nonabelian tensor square  $B_1(5) \otimes B_1(5)$  of  $B_1(5)$  means that we need to get its independent generators and its presentation. In this paper, we focus on the calculation of getting the independent generators only.

By Proposition 1, the subgroup  $[B_1(5), B_1(5)^\circ]$  which is isomorphic to  $B_1(5) \otimes B_1(5)$  by Theorem 1 is generated by the set of 148 generators as in (3) where some of them are identities and some of them are products of other generators. The set given in (3) below needs to be reduced to only contain the independent generators where they can be obtained by using the relation of  $B_1(5)$ , commutator calculus, and Lemma 1-3.

$$\begin{aligned} & \{[a, a^\circ], [b, b^\circ], [c, c^\circ], [l_1, l_1^\circ], [l_2, l_2^\circ], [l_3, l_3^\circ], [l_4, l_4^\circ], [l_5, l_5^\circ], [a^{\pm 1}, (b^\circ)^{\pm 1}], [a^{\pm 1}, (c^\circ)^{\pm 1}], [a^{\pm 1}, (l_1^\circ)^{\pm 1}], \\ & [a^{\pm 1}, (l_2^\circ)^{\pm 1}], [a^{\pm 1}, (l_3^\circ)^{\pm 1}], [a^{\pm 1}, (l_4^\circ)^{\pm 1}], [a^{\pm 1}, (l_5^\circ)^{\pm 1}], [b^{\pm 1}, (c^\circ)^{\pm 1}], [b^{\pm 1}, (l_1^\circ)^{\pm 1}], [b^{\pm 1}, (l_2^\circ)^{\pm 1}], \\ & [b^{\pm 1}, (l_3^\circ)^{\pm 1}], [b^{\pm 1}, (l_4^\circ)^{\pm 1}], [b^{\pm 1}, (l_5^\circ)^{\pm 1}], [c^{\pm 1}, (l_1^\circ)^{\pm 1}], [c^{\pm 1}, (l_2^\circ)^{\pm 1}], [c^{\pm 1}, (l_3^\circ)^{\pm 1}], [c^{\pm 1}, (l_4^\circ)^{\pm 1}], \\ & [c^{\pm 1}, (l_5^\circ)^{\pm 1}], [l_1^{\pm 1}, (l_2^\circ)^{\pm 1}], [l_1^{\pm 1}, (l_3^\circ)^{\pm 1}], [l_1^{\pm 1}, (l_4^\circ)^{\pm 1}], [l_1^{\pm 1}, (l_5^\circ)^{\pm 1}], [l_2^{\pm 1}, (l_3^\circ)^{\pm 1}], [l_2^{\pm 1}, (l_4^\circ)^{\pm 1}], \\ & [l_2^{\pm 1}, (l_5^\circ)^{\pm 1}], [l_3^{\pm 1}, (l_4^\circ)^{\pm 1}], [l_3^{\pm 1}, (l_5^\circ)^{\pm 1}], [l_4^{\pm 1}, (l_5^\circ)^{\pm 1}], [a, b^\circ][b, a^\circ], [a, c^\circ][c, a^\circ], [a, l_1^\circ][l_1, a^\circ], \\ & [a, l_2^\circ][l_2, a^\circ], [a, l_3^\circ][l_3, a^\circ], [a, l_4^\circ][l_4, a^\circ], [a, l_5^\circ][l_5, a^\circ], [b, c^\circ][c, b^\circ], [b, l_1^\circ][l_1, b^\circ], [b, l_2^\circ][l_2, b^\circ], \\ & [b, l_3^\circ][l_3, b^\circ], [b, l_4^\circ][l_4, b^\circ], [b, l_5^\circ][l_5, b^\circ], [c, l_1^\circ][l_1, c^\circ], [c, l_2^\circ][l_2, c^\circ], [c, l_3^\circ][l_3, c^\circ], [c, l_4^\circ][l_4, c^\circ], \\ & [c, l_5^\circ][l_5, c^\circ], [l_1, l_2^\circ][l_2, l_1^\circ], [l_1, l_3^\circ][l_3, l_1^\circ], [l_1, l_4^\circ][l_4, l_1^\circ], [l_1, l_5^\circ][l_5, l_1^\circ], [l_2, l_3^\circ][l_3, l_2^\circ], [l_2, l_4^\circ][l_4, l_2^\circ], \\ & [l_2, l_5^\circ][l_5, l_2^\circ], [l_3, l_4^\circ][l_4, l_3^\circ], [l_3, l_5^\circ][l_5, l_3^\circ], [l_4, l_5^\circ][l_5, l_4^\circ]\}. \end{aligned} \quad (3)$$

Lemma 4 to 6, lead us to the independent generators of  $B_1(5) \otimes B_1(5)$ . The proofs of the lemmas are quite length, hence in this paper we just give examples of some of the calculations.

**Lemma 4.** Let  $G$  be the Bieberbach group of dimension five with the dihedral point group of order eight, which has a polycyclic presentation as (2) and generators as in the set (3). All generators in the form of commutators that have negatives power can be written as integer powers of its' positive commutators or some other positive commutators.

*Proof:* By the relation of  $B_1(5)$ , comutator calculus, and Lemma 1-3, we have:

$$\begin{aligned} [a, l_1^{-\circ}] &= [l_1^{-1}, [a, l_1^\circ]][a, l_1^\circ]^{-1} = [l_1^{-1}, l_1^{2\circ}][a, l_1^\circ]^{-1} = [l_1^{-1}, l_1^\circ]^2 [a, l_1^\circ]^{-1} = [a, a^\circ]^{-8} [a, l_1^\circ]^{-1} = [a, l_1^\circ]^{-1}. \\ [c, l_1^{-\circ}] &= [l_1^{-1}, [c, l_1^\circ]][c, l_1^\circ]^{-1} = [l_1^{-1}, (l_2^{-1} l_1)^\circ][c, l_1^\circ]^{-1} = [l_1^{-1}, l_1^\circ][l_1^{-1}, l_2^{-\circ}][[l_1^{-1}, l_2^{-1}], l_1^\circ][c, l_1^\circ]^{-1} \\ &= [l_1, l_1^\circ]^{-1} [l_1, l_2^\circ][c, l_1^\circ]^{-1} = [a, a^\circ]^{-4} [c, l_1^\circ]^{-1}. \end{aligned}$$

All others negative powers commutators can be shown in similar manner.

**Lemma 5.** Let  $G$  be the Bieberbach group of dimension five with the dihedral point group of order eight, which has a polycyclic presentation as given in (2) and generators as in (3). Then  $[l_1, l_4^\rho], [l_1, l_5^\rho], [l_2, l_3^\rho], [l_2, l_4^\rho], [l_2, l_5^\rho], [l_3, l_4^\rho], [l_3, l_5^\rho], [l_4, l_5^\rho]$  and all commutator in the form of  $[l_i, l_j^\rho][l_j, l_i^\rho], i < j$  are equal to the identity.

*Proof:* Again, by the relation of  $B_1(5)$ , comutator calculus, Lemma 1-3, we have the following example of calculation:

$$[l_1, l_2^\rho] = [l_1, l_2^{-\rho}]^{-1} = [l_1, a^{2\rho}]^{-1} = ([l_1, a^\rho][l_1, a^\rho]^\rho)^{-1} = 1$$

Some other commutators in the form  $[l_i, l_j], i < j$  can be proved equals to identity in similar manner.

Hence, with these we also have all  $[l_i, l_j^\rho][l_j, l_i^\rho], i < j$ , equals to the identity.

**Lemma 6.** Let  $G$  be the Bieberbach group of dimension five with the dihedral point group of order eight, which has a polycyclic presentation as in (2) and generators as in the set (3). The commutators

$$\begin{aligned} & [l_1, l_1^\rho], [l_4, l_4^\rho], [b, b^\rho], [a, l_1^\rho], [a, l_2^\rho], [a, l_4^\rho], [a, l_5^\rho], [b, c^\rho], [b, l_2^\rho], [b, l_4^\rho], [b, l_5^\rho], [c, l_2^\rho], \\ & [c, l_3^\rho], [c, l_5^\rho], [a, b^\rho][b, a^\rho], [a, l_1^\rho][l_1, a^\rho], [a, l_2^\rho][l_2, a^\rho], [a, l_3^\rho][l_3, a^\rho], [a, l_4^\rho][l_4, a^\rho], \\ & [b, c^\rho][c, b^\rho], [b, l_1^\rho][l_1, b^\rho], [b, l_2^\rho][l_2, b^\rho], [b, l_3^\rho][l_3, b^\rho], [b, l_4^\rho][l_4, b^\rho], [c, l_1^\rho][l_1, c^\rho], \\ & [c, l_2^\rho][l_2, c^\rho], [c, l_3^\rho][l_3, c^\rho] \text{ and } [c, l_4^\rho][l_4, c^\rho] \end{aligned}$$

can be written as products of other generators.

*Proof.* Since we have  $a$  commutes with  $l_2$ ,  $b$  commutes with  $l_4$  and  $c$  commutes with  $l_3$ , then we have the following examples of calculations:

$$[a, l_2^\rho] = [a, l_2^{-\rho}]^{-1} = [a, a^{2\rho}]^{-1} = [a, a^\rho]^{-2}.$$

Using similar argument, we have  $[l_1, l_1^\rho] = [a, a^\rho]^4$  and  $[l_4, l_4^\rho] = [c, c^\rho]^4$ . Furthermore,

$$[a, l_3^\rho][l_3, a^\rho] = [a, c^{-2\rho}][c^{-2}, a^\rho] = ([a, c^\rho][c, a^\rho])^{-2} = ([a, c^\rho][c, a^\rho])^2.$$

Next, Theorem 2 gives the result of the independent generators of the nonabelian tensor square  $B_1(5) \otimes B_1(5)$ .

**Theorem 2.** Let  $G$  be the Bieberbach group of dimension five with the dihedral point group of order eight which has a polycyclic presentation as (2) and generators as in (3). Then the nonabelian tensor square  $B_1(5) \otimes B_1(5)$  of the group  $B_1(5)$  is generated by the following set:

$$\{a \otimes a, c \otimes c, a \otimes c, a \otimes l_3, b \otimes l_1, b \otimes l_3, c \otimes l_1, c \otimes l_4, l_1 \otimes l_3, (a \otimes c)(c \otimes a)\}.$$

*Proof.* From Lemma 4-6 elements of set (3) is reduced to

$$\{a \otimes a, c \otimes c, a \otimes c, a \otimes l_3, b \otimes l_1, b \otimes l_3, c \otimes l_1, c \otimes l_4, l_1 \otimes l_3, (a \otimes c)(c \otimes a)\}.$$

## VERIFICATION USING GAP

In this research, we use GAP to assist our computation.

The following commands give the output of the nonabelian tensor square of  $B_1(5)$ .

```
gap> ts:=NonAbelianTensorSquare(G);  
Pcp-group with orders [ 2, 0, 0, 0, 0, 0, 0, 4, 8, 8 ]
```

To verify this set of generators is true or not, we check using the following GAP commands:

```
.  
. .  
gap> list:= [t1, t3, t7, t8, t10, t14, t16, t18, t20, t23];;  
gap> CommutatorSubgroup(L, R)=Subgroup(Nu, list);;  
true
```

## Conclusion

In this paper, the nonabelian tensor square is computed manually using the structural results of the group  $\nu(G)$ , commutator calculus and then verified by using GAP. We can see that the nonabelian tensor square of the first Bieberbach group of dimension five with dihedral point group of order eight,  $B_1(5)$ , is generated by ten elements. This results can further be used to find other useful properties of  $B_1(5)$  such as the homological functors of the group.

## ACKNOWLEDGEMENTS

The authors would like to thank Universiti Pendidikan Sultan Idris (UPSI) for the funding of this research under Vote # RACE 2012-0144-101-62 and UTM Vote No. 00M04.

## REFERENCES

1. Nor'ashiqin Mohd Idrus, "Bieberbach Groups with Finite Point Groups", Ph.D. Thesis, Universiti Teknologi Malaysia, 2011.
2. B. Eick, and W. Nickel, *Journal of Algebra* **320(2)**, 927-944 (2008).
3. L. C. Kappe, N. H. Sarmin, and M.P. Visscher, *Glasgow Math. J.* **41**, 417-430 (1999).
4. M. R. Bacon, *Glasgow Math. J.* **36(3)**, 291-296 (1994).
5. R. D. Blyth, R.F. Morse and J.A. Redden, "On Computation The Nonabelian Tensor Squares of The Free 2-Engel Groups", Proceeding of the Edinburgh Mathematical Society 47, 2004, pp. 305-323.
6. R. Brown, D. L. John and E. F. Robertson, *Journal of Algebra* **111**, 177-202 (1987).
7. G. Ellis and F. Leonard, Proc. Roy. Irish Acad. Sect. A **95(2)**, 137-147 (1995).
8. The GAP Group. GAP- *Group, Algorithms, and Programming. Version 4.4.11*, 2008. (<http://www.gap-system.org/>).
9. N. R. Rocco, *Bol. Soc. Brasil. Mat. (N.S.)* **22(1)**, 63-79 (1991).
10. R. D. Blyth and R.F. Morse, *Journal of Algebra* **321**, 2139-2148 (2009).