The generative capacity of weighted simple and semi-simple splicing systems

Wan Heng Fong, Yee Siang Gan, Nor Haniza Sarmin, and Sherzod Turaev

Citation: AIP Conference Proceedings 1750, 050013 (2016); doi: 10.1063/1.4954601
View online: http://dx.doi.org/10.1063/1.4954601
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1750?ver=pdfcov
Published by the AIP Publishing

Articles you may be interested in
Some characteristics on the generative power of weighted one-sided splicing systems

The concepts of persistent and permanent in non semi-simple DNA splicing system
AIP Conf. Proc. 1605, 586 (2014); 10.1063/1.4887654

Probabilistic simple splicing systems

PSEUDO SEMI‐SIMPLE RINGS
AIP Conf. Proc. 1309, 693 (2010); 10.1063/1.3525194

Highest weights of semisimple Lie algebras
The Generative Capacity of Weighted Simple and Semi-Simple Splicing Systems

Wan Heng Fong\textsuperscript{1,a)}, Yee Siang Gan\textsuperscript{2,b}, Nor Haniza Sarmin\textsuperscript{1,c)} and Sherzod Turaev\textsuperscript{3,d)}

\textsuperscript{1}Department of Mathematical Sciences, Faculty of Science
UniversitiTeknologi Malaysia, 81310 UTM Johor Bahru, Malaysia
\textsuperscript{2}Cambridge A Level, Taylor’s College Sdn Bhd.
No1, Jalan SS15/8 Subang Jaya, 47500 Selangor D.E., Malaysia.
\textsuperscript{3}Department of Computer Science, Kulliyyah of Information and Communication Technology
International Islamic University Malaysia, 53000 Jalan Gombak, Selangor D.E., Malaysia
\textsuperscript{a)Correspondingauthor: fwh@utm.my
\textsuperscript{b)ysgn88@gmail.com
\textsuperscript{c)nhs@utm.my
\textsuperscript{d)sherozd@iium.edu.my

Abstract. The mathematical modelling of splicing systems (H systems) was initiated by Head in 1987. By restricting the splicing rules of splicing systems, some variants of splicing systems such as simple and semi-simple splicing systems have been developed. Due to the limitation on the generative power of the variants of splicing systems, weights have been used as the restrictions in the variants of splicing systems recently, namely weighted one-sided splicing systems, weighted simple splicing systems and weighted semi-simple splicing systems. In this paper, we investigate the generative power of weighted simple and semi-simple splicing systems by considering different and specified weighting spaces and weighting operations. In addition, the generative power of weighted simple and semi-simple splicing systems are generalised by relating their generated threshold languages to the Chomsky hierarchy.

Keywords: weights, simple, semi-simple, splicing systems, generative power
PACS: 07.05.Bx, 89.20.Ff, 02.70.-c

INTRODUCTION

DNA splicing system is a formal model of the recombinant behavior of DNA sequences with the presence of enzymes and ligase. From the recombinant behavior that naturally occur in the DNA molecules, the enzymes act as the splicing rules to cut at specified subsequences of two DNA molecules. Then, the prefix of a DNA molecule is connected to the suffix of another DNA molecule to form a new DNA molecule. This phenomenon which involves the cut and paste activities of the recombinant behavior is known as the splicing operation.

In 1987, the mathematical modelling of splicing system is initiated by Head to generate languages under the framework of formal language theory [1]. According to the splicing operation, computation of a splicing system is complete when a new well-formed double-stranded molecule is obtained from the axioms of a splicing system [2]. From the splicing systems, DNA molecules are modelled as strings. Hence, the formal analysis of the generative power for the recombination behaviors of DNA molecules in splicing systems are studied within the framework of Formal Language Theory [1].

In analysing the generative power of splicing system, Pixton showed that splicing systems with finite sets of axioms and rules generate only regular languages [3]. Hence, extended splicing system, extended splicing system with multisetstyle, etc. have been developed [2, 4, 5]. Also, the results have shown that these splicing systems with finite components are computationally complete.

There is another type of splicing system introduced in [6], called the weighted splicing system. The weights from selected weighting spaces are associated to each axiom of the splicing system and are calculated as follows:
The strand $z$ is generated from two strands $x$ and $y$ with weights $\omega(x)$ and $\omega(y)$ respectively. The results show that the generative power of splicing systems have been increased with weights, since some non-regular languages can be generated by the weighted systems. It is also important to be able to attach certain physical quantities from the weighting spaces to the axioms of the splicing systems. Hence, this provides information on the strings of splicing systems for the development on more efficient tagging algorithm in weighted splicing systems. Further in [7, 8], some variants of weighted splicing systems such as weighted one-sided, simple and semi-simple splicing systems are introduced by associating weights to the axioms of one-sided, simple and semi-simple splicing systems respectively.

In this paper, we investigate the generative power of weighted simple and semi-simple splicing systems by relating the generated threshold languages to the Chomsky hierarchy. From the investigation on the generative power of weighted simple and semi-simple splicing systems, we proved that the generative power of weighted systems is higher than their usual systems. We also show that the selection of weighting spaces and cut-points also affect the generative power of simple and semi-simple splicing systems.

This paper is organised as follows: Section 1 introduces the background of the research. In Section 2, some necessary definitions and notations from formal language theory and splicing system are presented. Next, the formal definitions for weighted simple splicing system, weighted semi-simple splicing system and their threshold languages are given in Section 3. In Section 4, the relation of the threshold languages generated by weighted simple and semi-simple splicing systems with different weighting spaces in the Chomsky hierarchy is investigated.

In the next section, some preliminaries of formal languages and splicing systems are discussed.

Preliminaries

For the basic concepts of formal language theory, the reader may refer to [9, 10] for more information. Throughout the paper, the symbol $\in$ denotes the membership of an element to a set while the negation of set membership is denoted by $\notin$. The inclusion is denoted by $\subseteq$ and the strict (proper) inclusion is denoted by $\subset$. The symbol $\emptyset$ denotes the empty set. The symbols $+, \times, \oplus$ and $\otimes$ denote the usual addition, usual multiplication, componentwise addition and componentwise multiplication operations respectively. The set of integers and positive rational numbers are denoted by $\mathbb{Z}$ and $\mathbb{Q}^+$, respectively. The symbol $\mathbb{Z}^n$ denotes the $n$-dimensional vector space over integers. The set of matrices with integer entries is denoted by $\mathbb{M}$.

Besides, the families of recursively enumerable, context-sensitive, context-free, linear, regular and finite languages are denoted by $\text{RE}$, $\text{CS}$, $\text{CF}$, $\text{LIN}$, $\text{REG}$ and $\text{FIN}$, respectively [9]. For these language families, the next strict inclusions, named Chomsky hierarchy, hold:

**Theorem 1.** [9] $\text{FIN} \subset \text{REG} \subset \text{LIN} \subset \text{CF} \subset \text{CS} \subset \text{RE}$.

Next, the concept of the iteration of splicing systems is explained [2]:

Let $V$ be an alphabet, $V^*$ is the set of all words over an alphabet $V$ and let $\#_1, \$ \notin V$ be two special symbols. A splicing rule over $V$ is a string of the form

\[ r = u_1 \#_1 u_2 S u_3 \#_1 u_4, \text{ where } u_1, u_2, u_3, u_4 \in V^*. \]

For such a rule $r \in R$ and strings $x, y, z \in V^*$, we write

\[ (x, y) \vdash_r z \]

if and only if

\[ x = x_1 u_1 u_2 x_2, y = y_1 u_3 u_4 y_2, z = x_3 u_5 u_4 y_2, \]

for some $x_1, x_2, y_1, y_2 \in V^*$.

The string $z$ is said to be obtained by splicing $x, y$, as indicated by the rule $r$; $u_1, u_2$ and $u_3, u_4$ are called the sites of splicing. We call $x$ the first term and $y$ the second term of the splicing operation.

An $H$ scheme is a pair $\sigma = (V, R)$, where $V$ is an alphabet and $R \subseteq V^* \#_1 V^* \#_1 V^*$ is a set of splicing rules. For a given $H$ scheme $\sigma = (V, R)$ and a language $L \subseteq V^*$, we write

\[ \sigma(L) = \{z \in V^* \mid (x, y) \vdash_r z \text{ for some } x, y \in L, r \in R\}. \]

Then, for $L \subseteq V^*$ we define
\[ \sigma^* (L) = \bigcup_{i \geq 0} \sigma^i (L) \]

by
\[ \sigma^0 (L) = L, \]
\[ \sigma^{i+1} (L) = \sigma^i (L) \cup \sigma (\sigma^i (L)), \quad i \geq 0. \]

**Theorem 2.** [2] The relations in the following table hold, where at the intersection of the row marked with \( F_i \) with the column marked with \( F_j \) there appear either the family \( H(F_i, F_j) \) or two families \( F_3, F_4 \) such that \( F_3 \subseteq H(F_i, F_j) \subseteq F_4 \).

<table>
<thead>
<tr>
<th>( F_1 )</th>
<th>( F_2 )</th>
<th>( \text{FIN} )</th>
<th>( \text{REG} )</th>
<th>( \text{LIN} )</th>
<th>( \text{CF} )</th>
<th>( \text{CS} )</th>
<th>( \text{RE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{FIN} )</td>
<td>( \text{FIN} ), ( \text{REG} )</td>
<td>( \text{FIN} ), ( \text{RE} )</td>
<td>( \text{FIN} ), ( \text{RE} )</td>
<td>( \text{FIN} ), ( \text{RE} )</td>
<td>( \text{FIN} ), ( \text{RE} )</td>
<td>( \text{FIN} ), ( \text{RE} )</td>
<td></td>
</tr>
<tr>
<td>( \text{REG} )</td>
<td>( \text{REG} )</td>
<td>( \text{REG} ), ( \text{RE} )</td>
<td>( \text{REG} ), ( \text{RE} )</td>
<td>( \text{REG} ), ( \text{RE} )</td>
<td>( \text{REG} ), ( \text{RE} )</td>
<td>( \text{REG} ), ( \text{RE} )</td>
<td></td>
</tr>
<tr>
<td>( \text{LIN} )</td>
<td>( \text{LIN} ), ( \text{CF} )</td>
<td>( \text{LIN} ), ( \text{RE} )</td>
<td>( \text{LIN} ), ( \text{RE} )</td>
<td>( \text{LIN} ), ( \text{RE} )</td>
<td>( \text{LIN} ), ( \text{RE} )</td>
<td>( \text{LIN} ), ( \text{RE} )</td>
<td></td>
</tr>
<tr>
<td>( \text{CF} )</td>
<td>( \text{CF} )</td>
<td>( \text{CF} ), ( \text{RE} )</td>
<td>( \text{CF} ), ( \text{RE} )</td>
<td>( \text{CF} ), ( \text{RE} )</td>
<td>( \text{CF} ), ( \text{RE} )</td>
<td>( \text{CF} ), ( \text{RE} )</td>
<td></td>
</tr>
<tr>
<td>( \text{CS} )</td>
<td>( \text{CS} ), ( \text{RE} )</td>
<td>( \text{CS} ), ( \text{RE} )</td>
<td>( \text{CS} ), ( \text{RE} )</td>
<td>( \text{CS} ), ( \text{RE} )</td>
<td>( \text{CS} ), ( \text{RE} )</td>
<td>( \text{CS} ), ( \text{RE} )</td>
<td></td>
</tr>
<tr>
<td>( \text{RE} )</td>
<td>( \text{RE} )</td>
<td>( \text{RE} )</td>
<td>( \text{RE} )</td>
<td>( \text{RE} )</td>
<td>( \text{RE} )</td>
<td>( \text{RE} )</td>
<td></td>
</tr>
</tbody>
</table>

Next, we cite the formal definition for simple splicing system and semi-simple splicing system as follows [11,12]:

A simple splicing system (\( H \) system) is a triple \((V, A, R)\) where \( V \) is an alphabet, \( A \) is a finite language over \( V \), and \( R \) is a set of splicing rules. The symbol \( A \) is called the **axiom** of simple splicing system and the elements of \( R \) are called **markers** in which all rules have the form of \((a, 1; a, 1)\) for some \( a \in A \).

For \( x, y, z \in V^* \) and \( a \in R \), we write

\[ (x, y) \vdash^a z \]

if and only if

\[ x = x_1 a x_2, \quad y = y_1 a y_2, \quad z = x_1 a y_2 \quad \text{for} \quad x_1, x_2, y_1, y_2 \in V^*. \]

A semi-simple splicing system (\( H \) system) is a triple \((V, A, R)\) where \( V \) is an alphabet, \( A \) is a finite language over \( V \), and \( R \) is a set of splicing rules. The symbol \( A \) is also called the **axiom** and the elements of \( R \) are rules of the form \((a, 1; b, 1)\) for \( a, b \in A \).

For \( x, y, z \in V^* \) and \((a, 1; b, 1) \in R\), we write

\[ (x, y) \vdash^{(a, 1; b, 1)} z \]

if and only if

\[ x = x_1 a x_2, \quad y = y_1 b y_2, \quad z = x_1 a y_2 \quad \text{for} \quad x_1, x_2, y_1, y_2 \in V^*. \]

Furthermore, for simple and semi-simple splicing language, \( L \subseteq V^* \), we define

\[ \sigma^* (L) = \bigcup_{i \geq 0} \sigma^i (L) \]

by

\[ \sigma^0 (L) = L, \]
\[ \sigma^{i+1} (L) = \sigma^i (L) \cup \sigma (\sigma^i (L)), \quad i \geq 0, \]
\[ \sigma^* (L) = \lim_{i \to \infty} \sigma^i (L) = \bigcup_{i \geq 0} \sigma^i (L). \]

Then, the language generated by \( \gamma = (V, A, R) \) for simple and semi-simple system is defined similarly by

\[ L(\gamma) = \sigma^*_H (A). \]

Also, \( SH \) and \( SSH \) denote the family of languages generated by simple and semi-simple splicing systems respectively where \( \gamma = (V, A, R) \).

Next, the following theorems illustrate the generative power of simple and semi-simple splicing system.

**Theorem 3.** ([11]) \( \text{FIN} \subseteq SH \subseteq \text{REG} \)

050013-3
Theorem 4. ([12]) If $L$ is a semi-simple splicing language then $L$ is strictly locally testable.

In the next section, the weighted simple and semi-simple splicing systems are discussed.

PROPERTIES OF WEIGHTED SIMPLE AND SEMI-SIMPLE SPLICING SYSTEMS

The weighted simple and semi-simple splicing systems are the usual simple and semi-simple splicing systems respectively which are specified with a weighting space and the operations over weights are closed in its selected weighting space. The formal definitions for weighted simple and semi-simple splicing systems are given in the following.

Definition 1. [7] A weighted simple (or semi-simple) splicing system is a 6-tuple $(V, A, \rho, M, \varnothing)$ where

$V, R$ are defined as for a usual simple (or semi-simple) splicing system,
$A$ is a subset of $V^* \times M$,
$\rho : V^* \to M$ is a weight function,
$M$ is a weighting space, and
$\varnothing$ is the operation over the weights $\rho(x), x \in V^*$.

Next, the weighted simple and semi-simple splicing operation and the languages generated by them are defined.

Definition 2. [7] For a weighted simple splicing system $(V, A, \rho, M, \varnothing)$, the set of weighted strings obtained by splicing strings in $A$ according to splicing rules in $R$ and the weight operation $\rho$ are defined next.

Definition 3. [7] For a weighted simple splicing system $(V, A, \rho, M, \varnothing)$, the set of weighted strings obtained by splicing strings in $A$ according to splicing rules in $R$ and the weight operation $\rho$ are defined next.

Definition 4. [7] Let $(V, A, \rho, M, \varnothing)$ be a weighted simple splicing system. Then

$$\sigma^i_a(A) = \{(z, \rho(z)) : (x, y) \vdash^i z \land \rho(z) = \rho(x) \circ \rho(y) \text{ for some } (x, \rho(x)), (y, \rho(y)) \in A \text{ and } a \in R\}.$$

Definition 5. [7] Let $(V, A, \rho, M, \varnothing)$ be a weighted semi-simple splicing system. Then

$$\sigma^i_a(A) = \{(z, \rho(z)) : (x, y) \vdash^{(a,b)} z \land \rho(z) = \rho(x) \circ \rho(y) \text{ for some } (x, \rho(x)), (y, \rho(y)) \in A \text{ and } a, b \in V, (a, b, l), (a, b, r) \in R\}.$$
Definition 7. [7] The weighted language generated by a weighted simple (or semi-simple) splicing system \( \gamma = (V, A, R, \alpha, M, \varnothing) \) is defined as
\[
L_{\alpha}(\gamma) = \sigma_{\alpha}(A).
\]

Remark 1: For the weighting space, one can consider different sets (and algebraic) structures. In this paper, the sets of integers, positive rational numbers, Cartesian products of integers and matrices with integer entries are considered. Furthermore, the operations over weights in strings are defined with respect to the chosen weighting space.

In order to increase the generative power of the respective weighted splicing systems, the languages generated by weighted simple and semi-simple splicing systems have been considered with some subsets of the weighting space called the thresholds (cut-points). In the following, the formal definitions for threshold languages for weighted simple and semi-simple splicing systems are given.

Definition 8. Let \( L_{\alpha}(\gamma) \) be the language generated by a weighted simple (or semi-simple) splicing system \( \gamma = (V, A, R, \alpha, M, \varnothing) \). A threshold language \( L_{\alpha}(\gamma, \star \tau) \) with respect to a threshold (cut-point) \( \tau \in M \) is a subset of \( L_{\alpha}(\gamma) \) defined by
\[
L_{\alpha}(\gamma, \star \tau) = \{ z | (z, \alpha(z)) \in \sigma_{\alpha}(A) \text{ and } \alpha(z) \star \tau \}
\]
where \( \star \in \{=, >, <\} \) is called the mode of \( L_{\alpha}(\gamma, \star \tau) \).

The family of threshold languages generated by weighted simple and semi-simple splicing systems with a weighting space \( M \) and an operation \( \odot \) is denoted by \( \omega \text{SH}(M, \varnothing) \) and \( \omega \text{SSH}(M, \varnothing) \) respectively, where
\[
(M, \varnothing) \in \{(\mathbb{Z}, +), (\mathbb{Z}, \odot), (\mathbb{Q}, \times), (\mathbb{M}, \odot)\}.
\]

Remark 2: We can also consider a threshold as a subset of \( M \). Then, the mode for such a threshold is defined as a membership to the threshold set, i.e., for a threshold set \( A \subseteq M \), the modes are \( \in \) and \( \notin \).

In the next section, the generative power of weighted simple and semi-simple splicing systems are discussed.

THE GENERATIVE POWER OF WEIGHTED SIMPLE AND SEMI-SIMPLE SPLICING SYSTEMS

First, we cite the following theorems on the generative power of weighted simple and semi-simple splicing systems in the following.

Theorem 5. ([7]) For \( X \in \{S, SS\} \) and weighting spaces \( (M, \varnothing) \in \{(\mathbb{Z}, +), (\mathbb{Z}, \oplus), (\mathbb{Q}, \times), (\mathbb{M}, +)\} \),
\[
XH \subseteq \omega XH(M, \varnothing).
\]

Theorem 6. ([7]) For \( X \in \{S, SS\} \) and \( F_1 \in \{\text{FIN, REG, CF, LIN}\} \),
\[
\omega X(M, \varnothing) - F_1 \neq \varnothing
\]
where \( (M, \varnothing) \in \{(\mathbb{Z}, +), (\mathbb{Z}, \oplus), (\mathbb{Q}, \times), (\mathbb{M}, +)\} \).

Next, we obtained Theorem 7 and Theorem 8 which show that the families of languages generated by weighted simple and semi-simple splicing systems are larger than the \( H \) systems.

Theorem 7. For \( X \in \{S, SS\} \), \( F_1 \in \{\text{FIN, REG, LIN, CF}\} \) and weighting spaces \( (M, \varnothing) \in \{(\mathbb{Z}, +), (\mathbb{Z}, \oplus), (\mathbb{Q}, \times), (\mathbb{M}, +)\} \),
\[
\omega XH(M, \varnothing) - H(F_1, \text{FIN}) \neq \varnothing.
\]

Proof. Let \( X \in \{S, SS\} \), \( F_1 \in \{\text{FIN, REG, LIN, CF}\} \) and weighting spaces \( (M, \varnothing) \in \{(\mathbb{Z}, +), (\mathbb{Z}, \oplus), (\mathbb{Q}, \times), (\mathbb{M}, +)\} \). From Theorem 1, \( \text{FIN} \subseteq \text{REG} \subseteq \text{LIN} \subseteq \text{CF} \subseteq \text{CS} \subseteq \text{RE} \). From Theorem 2, \( H(F_1, \text{FIN}) \in \{\text{FIN, REG, LIN, CF}\} \). From Theorem 6, \( \omega X(M, \varnothing) - F_1 \neq \varnothing \). Hence, this concludes that \( \omega XH(M, \varnothing) - H(F_1, \text{FIN}) \neq \varnothing \), which completes the proof.
Theorem 8. For $X \in \{S, SS\}$, $F_1 \in \{\text{FIN, REG}\}$ and $(M, \circ) \in \{(\mathbb{Z}_+, \oplus, \ominus), (Q^+, \times), (M, +)\}$, 
\[ XH \subset H(F_1, \text{FIN}) \subset \omega XH(M, \circ) \subset \text{RE}. \]

Proof. Let $X \in \{S, SS\}$, $F_1 \in \{\text{FIN, REG}\}$ and weighting spaces $(M, \circ) \in \{(\mathbb{Z}_+, \oplus, \ominus), (Q^+, \times), (M, +)\}$. From Theorem 1, $\text{FIN} \subset \text{LIN} \subset \text{CF} \subset \text{CS} \subset \text{RE}$. From Theorem 3 and Theorem 4, $XH \subset \text{REG} \subset H(F_1, \text{FIN})$. Then by Theorem 5, $H(F_1, \text{FIN}) \subset \omega XH(M, \circ)$. Since $\text{RE}$ is the highest level of family in the Chomsky hierarchy as stated in Theorem 1, this completes the proof.

From Theorem 1, Theorem 2, Theorem 5 and Theorem 7, the following theorem is obtained by using the similar arguments as in the proofs of Theorem 7 and Theorem 8.

Theorem 9. For $X \in \{S, SS\}$, $F_1 \in \{\text{FIN, REG, LIN, CF}\}$ and $F_2 \in \{\text{CS, RE}\}$,
\begin{enumerate}
  \item $\text{REG} = H(F_1, \text{FIN}) \subset \omega XH(M, \circ)$,
  \item $\omega XH(M, \circ) \subset H(F_2, \text{FIN})$,
\end{enumerate}
where $(M, \circ) \in \{(\mathbb{Z}_+, \oplus, \ominus), (Q^+, \times), (M, +)\}$.

Then by Theorem 3, Theorem 4 and Theorem 9, the following theorem holds by the similar arguments as in the proofs of Theorem 7 and Theorem 8.

Theorem 10. For $X \in \{S, SS\}$ and weighting spaces $(M, \circ) \in \{(\mathbb{Z}_+, \oplus, \ominus), (Q^+, \times), (M, +)\}$,
\[ XH \subset \text{REG} \subset \omega XH(M, \circ). \]

CONCLUSION

In this paper, some characteristics and new facts on the weighted simple and semi-simple splicing systems are given. Furthermore, the generative power of weighted simple and semi-simple splicing systems with different weighting spaces and weighting operations are generalised by relating the generated threshold languages to the families of languages in the Chomsky hierarchy. However, weighted simple and semi-simple splicing systems may not generate recursively enumerable languages since the simulation of the context-sensitivity property of phrase-structure grammar is not possible with weights.

ACKNOWLEDGMENTS

The first and third authors would like to thank the Ministry of Education (MOE) and Research Management Centre UTM for the Fundamental Research Grant Scheme Vote No. 4F590 and the fourth author would like to thank MOE for the Fundamental Research Grant Scheme FRGS 13-066-0307.

REFERENCES

10. P. Linz, An Introduction to Formal Languages and Automata, (Jones and Bartlett Publishers,USA, 2006).