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CONSISTENCY OF POLYCYCLIC PRESENTATIONS FOR CRYSTALLOGRAPHIC GROUPS WITH QUATERNION EXTENSION

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ABSTRACT

The nonabelian tensor square of a group is essential in finding the other properties of the group including its homological invariants. The step needed in explicating the nonabelian tensor square of a group is first to ensure that the presentation of the group is polycyclic and consistent. In this research, the polycyclic presentations of some crystallographic groups with quaternion extension are shown to be consistent.

Key words: Crystallographic group, Polycyclic Presentations, Quaternion Extension

INTRODUCTION

In mathematical view, a crystallographic group is the description on the symmetrical pattern of a crystal. It is a symmetry group which has configuration in space. A crystallographic group is the torsion free space group. It is an extension of a free abelian group of finite rank by a finite point group. Research on homological invariants has been increasing in number since it is related to the study of the properties of the crystal using mathematical approach. One of them is research on the nonabelian tensor square of the group. The nonabelian tensor square is requisite in determining the other properties of the group. The groups being considered are taken from Crystallographic, Algorithms and Table (CARAT) package [1]. By using the technique developed by Blyth and Morse [2], these groups are transformed from matrix representation to polycyclic before the nonabelian tensor square being computed. It is crucial to perform the consistency check for those polycyclic presentations so that we can proceed to find the homological invariants of the group. Masri [3] used this technique on exploring crystallographic group with cyclic point group of order two while of order three and five by Mat Hassim [4]. Crystallographic groups with dihedral extension have grabbed attention of Mohd Idrus [5] and Wan Mohd Fauzi [6]. Furthermore, Tan *et al.* [7] has proved the consistency for crystallographic group with symmetric point group of order six. Recently, Mohammad [8] has verified the consistency of the first crystallographic group with quaternion extension. Therefore, in this research, the presentations of second, third and fourth of Crystallographic group with quaternion extension will be proved to be consistent.

MAIN RESULTS

The following theorems are our main results and will be shown to be consistent.

Theorem 1

Let $Q_2(6)$ be the second crystallographic group with quaternion extension and its polycyclic presentation is given as in the following:

$$Q_2(6)=\left\langle a,b,c,l_1,l_2,l_3,l_4,l_5,l_6 \left| \begin{array}{l} a^2=cl_2l_3l_4l_6,\ b^2=cl_5l_6^{-1}, b^a=bcl_1^{-1}l_2l_5^{-2}l_6^2,\ c^2=l_5l_6^{-1}, \\ c^a=cl_2l_4l_5^{-1}l_6,\ c^b=c,\ l_1^a=l_1^{-1}l_2l_4^{-1},\ l_1^b=l_3^{-1}, l_1^c=l_1^{-1}, \\ l_2^a=l_1^{-1}l_2l_3,\ l_2^b=l_1^{-1}l_3^{-1}l_4^{-1}, l_2^c=l_2^{-1},\ l_3^a=l_2^{-1}l_3^{-1}l_4^{-1},\ l_3^b=l_1, \\ l_3^c=l_3^{-1},\ l_4^a=l_1l_3l_4,\ l_4^b=l_1^{-1}l_2l_3, l_4^c=l_4^{-1},\ l_5^a=l_6, l_5^b=l_5, \\ l_5^c=l_5, l_6^a=l_5,\ l_6^b=l_6,\ l_6^c=l_6, l_j^{l_i}=l_j,\ l_j^{l_i^{-1}}=l_j \\ \text{for } j>i,\ 1\le i,j\le 6 \end{array}\right.\right\rangle. \quad (1)$$

Then, the polycyclic presentation is consistent.

Theorem 2

Let $Q_3(6)$ be the third crystallographic group with quaternion extension and its polycyclic presentation is given as in the following:

$$Q_3(6)=\left\langle a,b,c,l_1,l_2,l_3,l_4,l_5,l_6 \left| \begin{array}{l} a^2=cl_3^{-1}l_5,\ b^2=cl_2^{-1}l_4^{-1}l_5^{-1}l_6^{-1},\ b^a=bcl_2l_4^2l_5^2l_6^2, \\ c^2=l_2^{-1}l_4^{-1}l_5^{-1}l_6^{-1},\ c^a=cl_3^{-1}l_4l_5l_6, c^b=c,\ l_1^a=l_3^{-1}, \\ l_1^b=l_4^{-1}, l_1^c=l_1^{-1},\ l_2^a=l_4,\ l_2^b=l_3^{-1},\ l_2^c=l_2^{-1},\ l_3^a=l_1, \\ l_3^b=l_2,\ l_3^c=l_3^{-1},\ l_4^a=l_2^{-1},\ l_4^b=l_1, l_4^c=l_4^{-1}, l_5^a=l_1l_4^{-1}l_6^{-1}, \\ l_5^b=l_2l_4l_5,\ l_5^c=l_1l_2l_3^{-1}l_4l_5,\ l_6^a=l_1^{-1}l_4^{-1}l_5^{-1},\ l_6^b=l_1^{-1}l_3l_6, \\ l_6^c=l_1^{-1}l_2l_3l_4l_6,\ l_j^{l_i}=l_j,\ l_j^{l_i^{-1}}=l_j \text{ for } j>i,\ 1\le i,j\le 6 \end{array}\right.\right\rangle. \quad (2)$$

Then, the polycyclic presentation is consistent.

Theorem 3

Let $Q_4(6)$ be the fourth crystallographic group with quaternion extension and its polycyclic presentation is given as in the following:

$$Q_4(6)=\left\langle a,b,c,l_1,l_2,l_3,l_4,l_5,l_6 \left| \begin{array}{l} a^2=cl_6,\ b^2=cl_5l_6^{-1},\ b^a=bcl_5^{-2}l_6^2,\ c^2=l_5l_6^{-1}, \\ c^a=cl_5^{-1}l_6,\ c^b=c,\ l_1^a=l_4^{-1},\ l_1^b=l_3^{-1}, l_1^c=l_1^{-1}, \\ l_2^a=l_3,\ l_2^b=l_4^{-1},\ l_2^c=l_2^{-1},\ l_3^a=l_2^{-1},\ l_3^b=l_1, \\ l_3^c=l_3^{-1},\ l_4^a=l_1,\ l_4^b=l_2, l_4^c=l_4^{-1},\ l_5^a=l_6,\ l_5^b=l_5, \\ l_5^c=l_5,\ l_6^a=l_5,\ l_6^b=l_6,\ l_6^c=l_6,\ l_j^{l_i}=l_j,\ l_j^{l_i^{-1}}=l_j \\ \text{for } j>i,\ 1\le i,j\le 6 \end{array}\right.\right\rangle. \quad (3)$$

Then, the polycyclic presentation is consistent.

CONCLUSION

In this research, the polycyclic presentations of the second, third and fourth Crystallographic groups with quaternion extension are shown to be consistent. These polycyclic presentations which are consistent are needed in finding the nonabelian tensor squares of the group.

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