6th International Graduate Conference on Engineering, Science & Humanities
(IGCESH 2016)

CONFERENCE PROCEEDINGS

15th - 17th August 2016

Organized by
UTM Postgraduate Student Society (PGSS-UTM)

In collaboration with
School of Graduate Studies,
Universiti Teknologi Malaysia

Email: igcesh2016@utm.my
Tel: +607-5537903 (office)
Fax: +607-5537800
Website: sps.utm.my/igcesh2016
THE PROBABILITY THAT A METACYCLIC 5-GROUP ELEMENT FIXES A SET BY CONJUGATION

Siti Norziahidayu Amzee Zamri*  
Nor Haniza Sarmin  
Sanhan Muhammad Salih Khasraw

1, 2 Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia,  
Johor Bahru, Johor, MALAYSIA.  
(E-mail: norzisan@gmail.com, nhs@utm.my)

3 Department of Mathematics, College of Education, Salahaddin University-Erbil, Kurdistan Region, IRAQ.  
(E-mail: sanhan.khasraw@su.edu.krd)

ABSTRACT

The probability that an element of a group fixes a set was introduced in 2013. Let G be a metacyclic 5-group and Ω the set of all subsets of commuting elements of G in the form of \((x, y)\) such that \(\text{lcm}(|x|, |y|) = 5\). In this research, the probability that an element of a metacyclic 5-group fixes a set Ω is determined by using a group action on a set which is conjugation.

Key words: Commutativity degree, Metacyclic 5-group, Conjugation action

INTRODUCTION

In 1944, Miller [1] introduced a concept of commutativity degree which is defined as the probability that a pair of two randomly chosen elements \((x, y)\) from a group G commute. The definition is given in the following.

Definition 1.1: Let G be a finite group. The commutativity degree is the probability that two random elements \((x, y)\) in G commute, defined as follows:

\[
P(G) = \frac{\left| \{(x, y) \in G \times G | xy = yx\} \right|}{|G|^2}.
\]

In 1965, Erdos and Turan [2] investigated several problems based on the concept of commutativity degree on symmetric groups. Later on, Gustafson [3] showed that the probability of a random pair of elements can be computed by dividing the number of conjugacy classes with the size of the group. He also showed that \(P(G) \leq \frac{5}{8}\).
In 1979, Sherman [4] extended the concept of commutativity degree by introducing the probability of an automorphism of a finite group which fixes an arbitrary element with the following definition:

**Definition 1.2:** Let $G$ be a group. Let $X$ be a non-empty set of $G$ where $G$ is a group permutation of $X$. Then the probability of an automorphism of a group fixes a random element $X$ is defined as follows:

$$P_G(X) = \frac{|\{(g,x) | gx = x \forall g \in G, x \in X\}|}{|X||G|}.$$ 

In 2013, Omer et al. [5] extended the probability given by Sherman [4] by introducing the probability that a group element fixes a set with the following definition:

**Definition 1.3:** Let $G$ be a group. Let $S$ be a set of all subsets of commuting elements of size two in $G$ where $G$ acts on $S$ by conjugation. Then the commutativity degree of an element of a group fixes a set is given as follows:

$$P_G(S) = \frac{|\{(g,s) | gS = S \forall g \in G, s \in S\}|}{|S||G|}.$$ 

The probability given by Omer et al. [5] can also be obtained using the following theorem:

**Theorem 3.1:** Let $G$ be a finite group and let $X$ be the set of elements of $G$ of size two in the form of $(a,b)$ where $a$ and $b$ commute. Let $S$ be the set of all subsets of commuting elements of $G$ of size two and $G$ acts on $S$ by conjugation. Then the probability that an element of a group fixes a set is given by $P_G(S) = \frac{K}{|S|}$, where $K$ is the number of conjugacy classes of $S$ in $G$.

Throughout this research, the probability that an element of a metacyclic 5-group fixes a set by conjugation will be computed using Theorem 3.1

**MAIN RESULTS**

Our main result from this research is given in the following:

**Main Theorem:** Let $G$ be a metacyclic 5-group such that $G \cong \langle a,b | a^{\alpha \delta} = b^{\beta \delta} = 1, [b,a] = a^{\alpha \delta - \beta} \rangle$ where $\alpha, \beta, \delta \in \mathbb{N}$, $\delta \leq \alpha < 2\delta$, $\delta \leq \beta$. 

354
\(\delta \leq \min\{\alpha - 1, \beta\}\). Let \(\Omega\) be the set of all subsets of commuting elements of \(G\) in the form of \((x, y)\) and \(lcm(|x|, |y|) = 5\) and \(G\) acts on \(\Omega\) by conjugation.

Then, the probability that an element of \(G\) fixes a set \(\Omega\),

\[
P_G(\Omega) = \begin{cases} 
\frac{2}{15}, & \text{when } \alpha > \beta = \delta, \\
1, & \text{otherwise.}
\end{cases}
\]

**CONCLUSION**

In this research, the probability that a metacyclic 5-group element fixes a set \(\Omega\) by the conjugation action has been computed. The probability is found to be depending on the size of the conjugacy classes and the size of the set \(\Omega\).

**Acknowledgment:** The authors would like to express their appreciation to Universiti Teknologi Malaysia (UTM) for the support of the Research University Grant (GUP) Vote No. 08H07.

**REFERENCES**


