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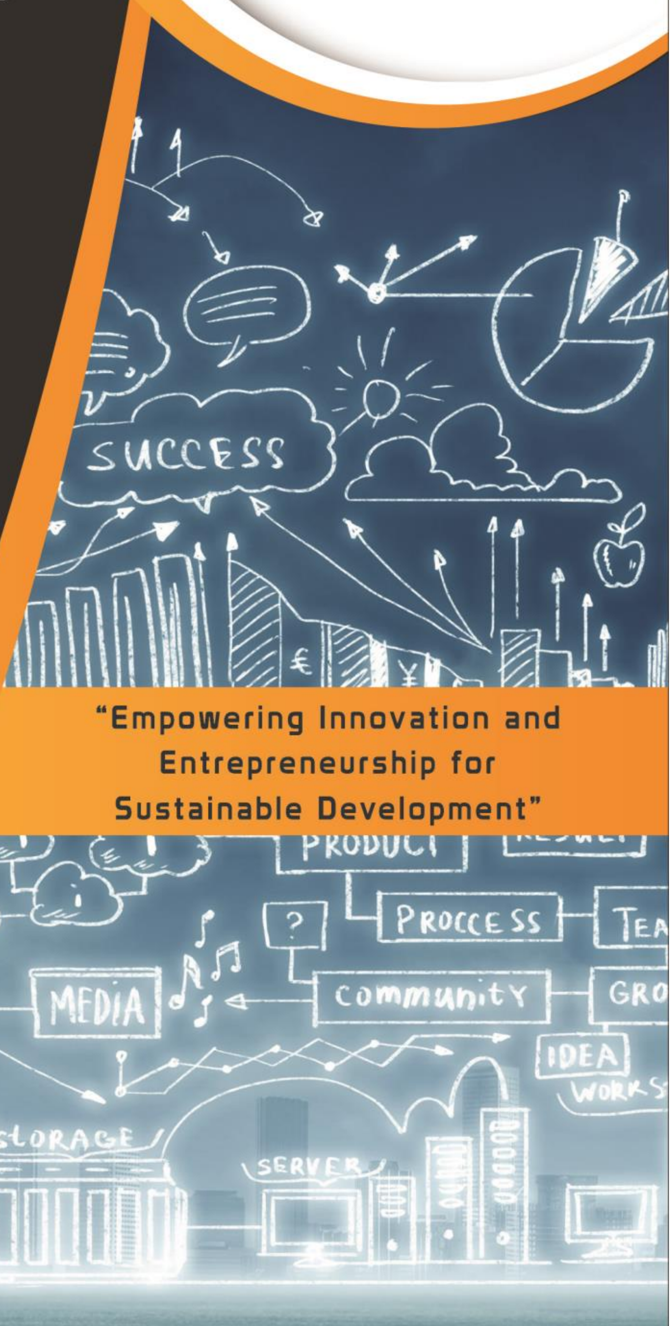

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THE PROBABILITY THAT A METACYCLIC 5-GROUP ELEMENT FIXES A SET BY CONJUGATION

Siti Norziahidayu Amzee Zamri*[1], Nor Haniza Sarmin[2] and Sanhan Muhammad Salih Khasraw[3]

[1, 2] Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, Johor Bahru, Johor, MALAYSIA.
(E-mail: norzisan@gmail.com, nhs@utm.my)
[3] Department of Mathematics, College of Education, Salahaddin University-Erbil, Kurdistan Region, IRAQ.
(E-mail: sanhan.khasraw@su.edu.krd)

ABSTRACT

The probability that an element of a group fixes a set was introduced in 2013. Let G be a metacyclic 5-group and Ω the set of all subsets of commuting elements of G in the form of (x,y) such that $lcm(|x|,|y|)=5$. In this research, the probability that an element of a metacyclic 5-group fixes a set Ω is determined by using a group action on a set which is conjugation.
Key words: Commutativity degree, Metacyclic 5-group, Conjugation action

INTRODUCTION

In 1944, Miller [1] introduced a concept of commutativity degree which is defined as the probability that a pair of two randomly chosen elements (x,y) from a group G commute. The definition is given in the following.

Definition 1.1: Let G be a finite group. The commutativity degree is the probability that two random elements (x,y) in G commute, defined as follows:

$$P(G)=\frac{\left|\{(x,y)\in G\times G \mid xy=yx\}\right|}{|G|^2}.$$

In 1965, Erdos and Turan [2] investigated several problems based on the concept of commutativity degree on symmetric groups. Later on, Gustafson [3] showed that the probability of a random pair of elements can be computed by dividing the number of conjugacy classes with the size of the group. He also showed that $P(G)\le\frac{5}{8}$.

In 1979, Sherman [4] extended the concept of commutativity degree by introducing the probability of an automorphism of a finite group which fixes an arbitrary element with the following definition:

Definition 1.2: Let G be a group. Let X be a non-empty set of G where G is a group permutation of X. Then the probability of an automorphism of a group fixes a random element X is defined as follows:

$$P_G(X) = \frac{\left|\left\{(g,x) \middle| gx = x \ \forall g \in G, x \in X\right\}\right|}{|X||G|}.$$

In 2013, Omer *et al.* [5] extended the probability given by Sherman [4] by introducing the probability that a group element fixes a set with the following definition:

Definition 1.3: Let G be a group. Let S be a set of all subsets of commuting elements of size two in G where G acts on S by conjugation. Then the commutativity degree of an element of a group fixes a set is given as follows:

$$P_G(S) = \frac{\left|\left\{(g,s) \middle| gS = S \ \forall g \in G, s \in S\right\}\right|}{|S||G|}.$$

The probability given by Omer *et al.* [5] can also be obtained using the following theorem:

Theorem 3.1: Let G be a finite group and let X be the set of elements of G of size two in the form of (a,b) where a and b commute. Let S be the set of all subsets of commuting elements of G of size two and G acts on S by conjugation. Then the probability that an element of a group fixes a set is given by $P_G(S) = \frac{K}{|S|}$, where K is the number of conjugacy classes of S in G.

Throughout this research, the probability that an element of a metacyclic 5-group fixes a set by conjugation will be computed using Theorem 3.1

MAIN RESULTS

Our main result from this research is given in the following:

Main Theorem: Let G be a metacyclic 5-group such that $G \cong \left\langle a,b \middle| a^{5^\alpha} = b^{5^\beta} = 1, [b,a] = a^{5^{\alpha-\delta}} \right\rangle$ where $\alpha, \beta, \delta \in \mathbb{N}$, $\delta \le \alpha < 2\delta$, $\delta \le \beta$,

$\delta \le \min\{\alpha-1,\beta\}$. Let Ω be the set of all subsets of commuting elements of G in the form of (x,y) and $lcm(|x|,|y|)=5$ and G acts on Ω by conjugation.

Then, the probability that an element of G fixes a set Ω,

$$P_G(\Omega)=\begin{cases}\frac{2}{15}, & \text{when } \alpha>\beta=\delta, \\ 1, & \text{otherwise.}\end{cases}$$

CONCLUSION

In this research, the probability that a metacyclic 5-group element fixes a set Ω by the conjugation action has been computed. The probability is found to be depending on the size of the conjugacy classes and the size of the set Ω.

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