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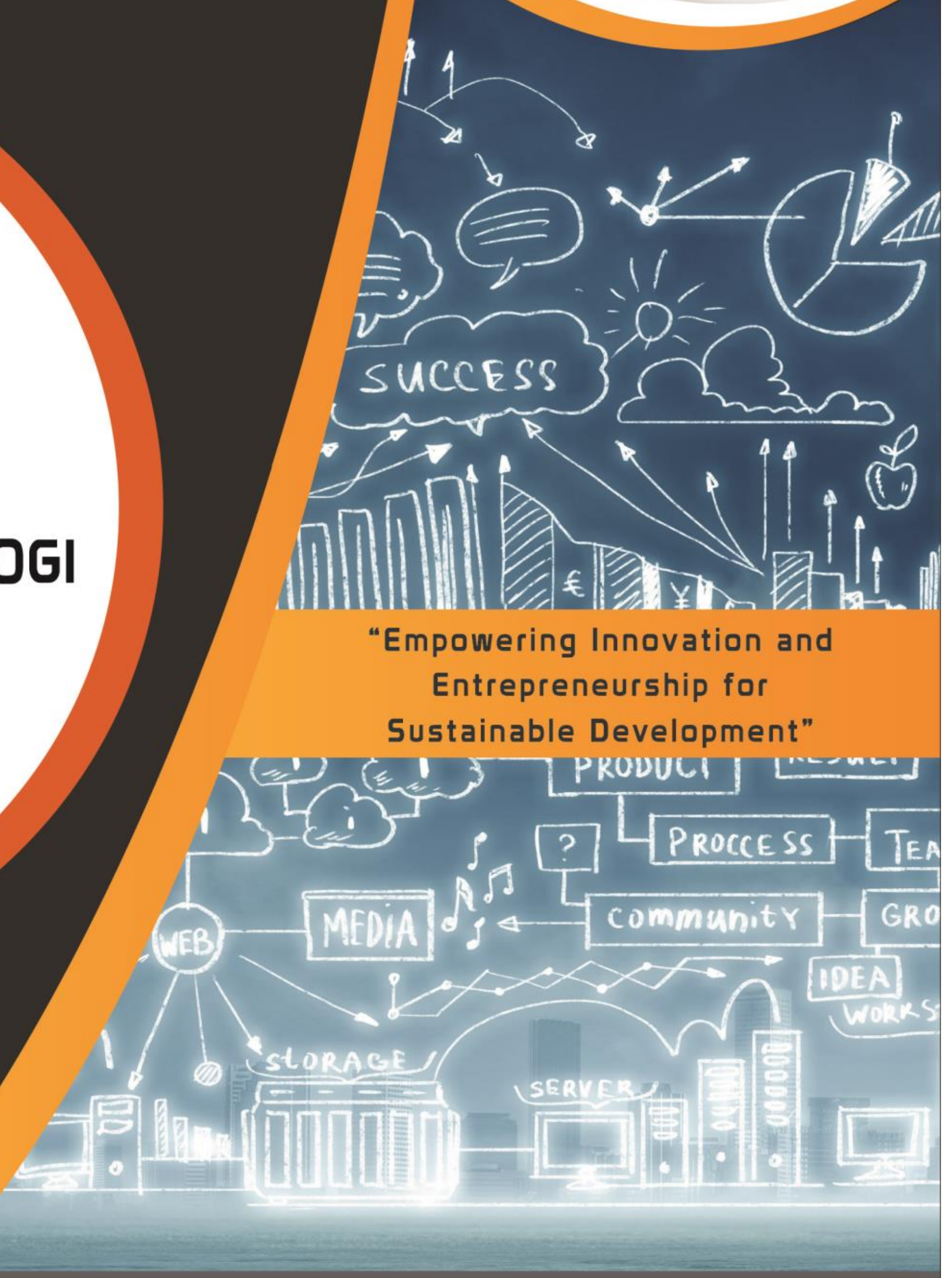

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$(\in,\in\vee q_k)$–FUZZY GENERALIZED BI Γ–IDEALS IN ORDERED Γ–SEMIGROUP

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ABSTRACT

A fuzzy subset A defined on a set X is represented as $A=\{(x,\lambda_A(x)),\text{ where } x\in X\}$. It is not always possible for membership functions of type $\lambda_A : X \to [0,1]$ to associate with each point x in a set X a real number in the closed unit interval ($[0,1]$) without the loss of some useful information. The importance of the ideas of "belongs to" ($\in$) and "quasi coincident with" (q) relations between a fuzzy point and fuzzy set is evident from the research conducted during the past two decades. Ordered Γ-semigroup (generalization of ordered semigroups) play an important role in the broad study of ordered semigroups. In this paper we provide an extension of fuzzy generalized bi Γ-ideals and introduce $(\in,\in\vee q_k)$-fuzzy generalized bi Γ-ideals of ordered Γ-semigroup. The purpose of this paper is to link this new concept with ordinary generalized bi Γ-ideals by using level subset and characteristic function.

Key words: Generalized bi Γ–Ideal, Ordered Γ-semigroup, Fuzzy Point, $(\in,\in\vee q_k)$-Fuzzy generalized bi Γ-Ideal.

INTRODUCTION

Fuzzy algebraic structures of groups have begun in the spearheading paper of Rosenfeld [1] in 1971. He studied the notion of fuzzy subgroups and showed that numerous outcomes in groups can be extended in an elementary manner to develop the theory of fuzzy subgroups after a pioneered paper on fuzzy set theory by Zadeh [2] in 1965. Thereafter, many researchers worked on the fuzzification of various algebraic structures. Sen and Saha [3] were the first to introduce the concept of a Γ-semigroup (a generalization of both semigroup and ternary semigroup). Furthermore, Kwon and Lee [4], further studied po-Γ-semigroup and introduced the concept of weakly prime ideals and provided useful characterizations of weakly prime ideals. The concepts of fuzzy ideals, fuzzy bi-ideals and fuzzy quasi ideals in Γ-semigroups are discussed in [5, 6]. Furthermore, Khan *et al.* [7] introduced the concept of generalized bi Γ-ideals of type (λ, θ) in ordered semigroups. The importance of the ideas of "belongs to" ($\in$) and "quasi coincident with" (q) relations between a fuzzy point and fuzzy set [8] is evident from the research conducted during the past two decades. Jun [9] further generalized the concept of quasi coincident with relations between a fuzzy point and fuzzy set ($x_t qA$) and defined $x_t q_k A$, if $\lambda_A(x) + t + k > 1$, where $k \in [0,1)$. In this paper, we studied the generalized form of fuzzy bi Γ-ideals in ordered Γ-semigroup and introduced the concept of $(\in, \in \vee q_k)$-fuzzy generalized bi Γ-ideals in ordered Γ-semigroup.

BASIC DEFINITIONS AND PRELIMINARIES

Some basic definitions are provided in this section that will be used in this paper.

Definition 2.1 [7]
If G and Γ are non-empty sets, then a structure $(G, \Gamma, \leq)$ is called an ordered Γ-semigroup if:
(b1) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in G$ and $\alpha, \beta \in \Gamma$,
(b2) $a \leq b \rightarrow a\alpha x \leq b\alpha x$ and $x\beta a \leq x\beta b$ for all $a, b, x \in G$ and $\alpha, \beta \in \Gamma$.

Definition 2.2 [7]
A non-empty subset B of G is called a generalized bi Γ-ideal of G if the following conditions hold for all $a, b \in G$:
(b3) $a \leq b \in B \rightarrow a \in B$,
(b4) $B\Gamma G\Gamma B \subseteq B$.

Definition 2.3 [2]
A fuzzy subset A defined on a set X is represented as $A = \{(x, \lambda_A(x)), \text{where } x \in X\}$.

Definition 2.4 [7]
A fuzzy subset A of G is called a fuzzy generalized bi Γ-ideal of G if the following conditions hold for all $x, y, z \in G$ and $\alpha, \beta \in \Gamma$:
(b5) $x \leq y \rightarrow \lambda_A(x) \geq \lambda_A(y)$,
(b6) $\lambda_A(x\alpha y\beta z) \geq \min\{\lambda_A(x), \lambda_A(z)\}$.

Definition 2.5 [10]
Let A be a fuzzy subset and $t \in (0,1]$. Then the crisp set $U(A;t) := \{x \in G : \lambda_A(x) \geq t\}$ is called a level subset of A.
Let t be a fixed point of the interval $(0,1]$ and x be a fixed element of G. Then a fuzzy subset A of G is called a fuzzy point with support x and value t and is denoted by x_t if:

$$\lambda_A(y) = \begin{cases} t, & \text{if} \quad y = x \\ 0, & \text{if} \quad \text{otherwise.} \end{cases}$$

We say that a fuzzy point x_t belongs to a fuzzy subset A if $\lambda_A(x) \geq t$ and is denoted by $x_t \in A$.

If I is a non-empty subset of G, then the characteristic function χ_I of I is a fuzzy subset of G and is defined by: $\chi_I(x) = \begin{cases} 1, & \text{if} \quad x \in I \\ 0, & \text{if} \quad x \notin I. \end{cases}$

MAIN RESULTS

In this section we introduce an extension of fuzzy generalized bi Γ-ideals in ordered Γ-semigroup. Throughout this section G will represent an ordered Γ-semigroup and $k \in (0,1]$.

Definition 3.1
Let A be a fuzzy subset of G. If A satisfies the following two conditions, then A is called $(\in, \in \vee q_k)$-fuzzy generalized bi Γ-ideal of G:

(c1) $y_t \in A \rightarrow x_t \in A$ for all $x, y \in G$ such that $x \le y$ and $t \in (0,1]$,

(c2) $x_{t_1} \in A, z_{t_2} \in A \rightarrow (x\alpha y\beta z)_{\min\{t_1,t_2\}} \in \vee q_k A$ for all $x, y \in G$, $\alpha, \beta \in \Gamma$ and $t_1, t_2 \in (0,1]$.

Theorem 3.1
A fuzzy subset A of G is called $(\in, \in \vee q_k)$-generalized bi Γ-ideal of G if and only if the following conditions hold for all $x, y, z \in G$:

(1) $x \le y \rightarrow \lambda_A(x) \ge \min\left\{\lambda_A(y), \frac{1-k}{2}\right\}$,

(2) $\lambda_A(x\alpha y\beta z) \ge \min\left\{\lambda_A(x), \lambda_A(z), \frac{1-k}{2}\right\}$.

Theorem 3.2
Let $\phi \ne I \subseteq G$ and χ_I be a characteristic function of I. Then the following two statements are equivalent:

(1). I is an ordinary generalized bi Γ-ideal of G.

(2). χ_I is a $(\in, \in \vee q_k)$-fuzzy generalized bi Γ-ideal of G.

Theorem 3.3
The following two statements are equivalent for any fuzzy subset A of G and for all $t \in (0, \frac{1-k}{2}]$:

(1). The non-empty level subset $U(A;t)$ is an ordinary generalized bi Γ-ideal of G.

(2). A is a $(\in, \in \vee q_k)$-fuzzy generalized bi Γ-ideal of G.

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