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THE ENERGY OF CONJUGACY CLASS GRAPH OF DIHEDRAL GROUPS

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ABSTRACT

The energy of a graph Γ, which is denoted by $\varepsilon(\Gamma)$, is defined to be the sum of the absolute values of the eigenvalues of its adjacency matrix. In this paper we present the concepts of conjugacy class graph of dihedral groups and found the general formula for the energy of the graph. the general formulas for the energy of conjugacy class graph of dihedral groups are found. For *n* an odd integer, $\varepsilon\left(\Gamma_{D_{2n}}\right) = n - 3$, while for *n* and $\frac{n}{2}$ even integers $\varepsilon\left(\Gamma^{cl}_{D_{2n}}\right) = n$ and if *n* is even integer and $\frac{n}{2}$ is odd integer then $\varepsilon\left(\Gamma^{cl}_{D_{2n}}\right) = n$-2.

Key words: Energy of graph, Conjugacy Class Graph, Eigenvalues and Dihedral Groups.

INTRODUCTION

Let Γ be a graph with vertex-set $V(\Gamma) = \{1, \ldots, n\}$ and edge set $E(\Gamma) = \{e_1, \ldots, e_n\}$. The adjacency matrix of Γ, denoted by $A(\Gamma)$, is an $n \times n$ matrix defined as follows: the rows and the columns of $A(\Gamma)$ are indexed by $V(\Gamma)$. If $i \neq j$, then the (i, j)-entry of $A(\Gamma)$ is 0 for nonadjacent vertices i and j, and the (i, j)-entry is 1 for adjacent i and j. The (i,i)-entry of $A(\Gamma)$ is 0 for $i = 1, \ldots, n$. [1]. Let Γ be a simple graph, *A* be its adjacency matrix and $\lambda_1, \lambda_2, \ldots, \lambda_r$ be the eigenvalues of the graph Γ. By eigenvalues of the graph Γ we mean the eigenvalues of its adjacency matrix. The energy of Γ is defined as the sum of absolute values of its eigenvalues [2]. The energy of graph was first defined by Ivan Gutman in 1978. It is used in chemistry to approximate the total π- electron energy of molecules. The vertices of the graph represented the carbon atoms while the single edge between each pair of distinct vertices represented the hydrogen bonds between the carbon atoms [3]. Recently there are many researches in constructing a graph by a group, for instance one can refer to the work by Bertram *et al*. in [4]. This paper consists of three parts. The first section is the introduction of the energy of the graph which is constructed by a group, followed by some fundamental concepts and definitions related to conjugacy classes and conjugacy class graph. The second section consists of some previous results which are used in this paper. Our main results are presented in the third section, in which we compute the eigenvalues and the energy of the conjugacy class graphs of dihedral groups, and lastly the general formulas for the energy of conjugacy class graphs of dihedral groups of order $2n$, are found. Suppose G is a finite group. Two elements a and b of G are called conjugate if there exists an element $g \in G$ with $gag^{-1} = b$. The conjugacy class is an equivalence relation and therefore partition G into some equivalence classes. This means that every element of the group G belongs to precisely one conjugacy class. The equivalence class that contains the element $a \in G$ is $cl(a) = \{gag^{-1} : g \in G\}$ and is called the conjugacy class of a. The class number of G is the number of distinct (non equivalent) conjugacy classes and we denote it by $K(G)$. Some definitions which are used in this paper are given in the following.

Definition 1.1 [4]
Let G be a finite group and let $Z(G)$ be the center of G. The vertices of conjugacy class graph of G are non-central conjugacy classes of G i.e. $|V(G)| = K(G) - |Z(G)|$, where $K(G)$ is the class number of G. Two vertices are adjacent if their cardinalities are not coprime (i.e. have common factor).

Definition 1.2 [5]
Let Γ be a simple graph. Then Γ is called a complete graph with n vertices and is denoted by K_n if there is an edge between any two arbitrary vertices.

PRELIMINARIES

In this section, some previous results on the energy of graph are presented which are used in this paper.

Proposition 2.1[6]
If the graph Γ consists of (disconnected) components Γ_1 and Γ_2, then the energy of Γ is $\varepsilon(\Gamma) = \varepsilon(\Gamma_1) + \varepsilon(\Gamma_2)$ and if one component of the graph Γ is Γ_1 and other components are isolated vertices, then $\varepsilon(\Gamma) = \varepsilon(\Gamma_1)$.

Proposition 2.2 [7]
A totally disconnected graph has zero energy, while the complete graph K_n with the maximum possible number of edges (among graph on n vertices) has an energy $2(n-1)$.

Proposition 2.3 [1]
Let Γ be a complete graph namely K_n, then the eigenvalues of K_n are $\lambda = n - 1$ (with multiplicity 1) and $\lambda = -1$ (with multiplicity $n-1$).

MAIN RESULTS

In this section we present our main results, namely the energy of the conjugacy class graph of all dihedral groups.

Proposition 3.1
Let G be a dihedral group of order $2n$. Then the conjugacy class graphs of D_{2n} are as

follows:

1. For n odd integer: the conjugacy class graph $\Gamma^{cl}_{D_{2n}}$ is a complete graph joined with isolated vertices, namely $\Gamma^{cl}_{D_{2n}} = K_{\frac{n-1}{2}}$.

2. For n even integer and $\frac{n}{2}$ also even integer: the conjugacy class graph $\Gamma^{cl}_{D_{2n}}$ is a complete graph of the form $\Gamma^{cl}_{D_{2n}} = K_{\frac{n+2}{2}}$.

3. For n even integer and $\frac{n}{2}$ odd integer: the conjugacy class graph $\Gamma^{cl}_{D_{2n}}$ is union of the complete graphs of the form $\Gamma^{cl}_{D_{2n}} = K_{\frac{n-2}{2}} \cup K_2$.

Theorem 3.1

Let G be a dihedral group of order $2n$, where n is an odd integer, i.e. $G = D_{2n} \cong \langle a, b : a^n = b^2 = 1, bab = a^{-1} \rangle$ and let $\Gamma^{cl}_{D_{2n}}$ be its conjugacy class graph. Then the energy of the graph $\Gamma^{cl}_{D_{2n}}$ is $\varepsilon\left(\Gamma^{cl}_{D_{2n}}\right) = n - 3$.

Theorem 3.2

Let G be a dihedral group of order $2n$, where n and $\frac{n}{2}$ are even integers, ie $G = D_{2n} \cong \langle a, b : a^n = b^2 = 1, bab = a^{-1} \rangle$ and let $\Gamma^{cl}_{D_{2n}}$ be its conjugacy class graph. Then the energy of the graph $\Gamma^{cl}_{D_{2n}}$, $\varepsilon\left(\Gamma^{cl}_{D_{2n}}\right) = n$.

Theorem 3.3

Let G be a dihedral group of order $2n$, where n is even integer and $\frac{n}{2}$ is odd integer, $G \cong D_{2n} = \langle a, b : a^n = b^2 = 1, bab = a^{-1} \rangle$ and let $\Gamma^{cl}_{D_{2n}}$ be its conjugacy class graph, then the energy of the graph $\Gamma_{D_{2n}}$ is $\varepsilon\left(\Gamma^{cl}_{D_{2n}}\right) = n$-2.

CONCLUSION

In this paper, the general formulas for the energy of conjugacy class graph of dihedral groups are found. For n an odd integer, $\varepsilon\left(\Gamma_{D_{2n}}\right) = n - 3$, while for n and $\frac{n}{2}$ even integers $\varepsilon\left(\Gamma^{cl}_{D_{2n}}\right) = n$ and if n is even integer and $\frac{n}{2}$ is odd integer then $\varepsilon\left(\Gamma^{cl}_{D_{2n}}\right) = n$-2.

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