Jurnal Teknologi

Full Paper

A GENERALIZATION ON THE NTH COMMUTATIVITY DEGREE OF ALTERNATING GROUPS OF DEGREE 4 AND 5

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Article history
Received
10 February 2015
Received in revised form
21 May 2015
Accepted
12 November 2015

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Graphical abstract

$$P_n(G) = \frac{\left|\left\{(x,y) \in G \times G \middle| x^n y = yx^n\right\}\right|}{|G|^2}.$$

Abstract

The theory of commutativity degree is important in determining the abelianness of a group. The commutativity degree of a finite group G is the probability that a pair of elements chosen randomly from a group G, commute. The concept of commutativity degree can be generalized to the n^{th} commutativity degree of a group which is defined as the probability of commuting the n^{th} power of a randomly chosen element with another random element from the same group. In this research, the n^{th} commutativity degree of alternating groups of degree 4 and 5 are presented.

Keywords: Abelianness; commutativity degree; alternating group

Abstrak

Teori darjah kekalisan tukar tertib adalah sangat penting dalam menentukan keabelanan satu kumpulan. Darjah kekalisan tukar tertib untuk kumpulan terhingga G ialah kebarangkalian dua unsur terpilih secara rawak dalam kumpulan G, kalis tukar tertib. Konsep darjah kekalisan tukar tertib boleh teritlak kepada darjah kekalisan tukar tertib kuasa ke-n suatu kumpulan yang ditakrifkan sebagai kebarangkalian bahawa kuasa ke-n bagi suatu unsur yang dipilih secara rawak berkalis tukar tertib dengan unsur yang lain daripada kumpulan yang sama. Dalam kajian ini, kebarangkalian kekalisan tukar tertib kuasa ke-n bagi kumpulan selang-seli darjah 4 dan 5 dipersembahkan.

Kata kunci: Keabelanan; darjah kekalisan tukar tertib; kumpulan selang-seli

1.0 INTRODUCTION

All groups mentioned in this paper are considered finite. The commutativity degree of a group G is the probability that a selected chosen pair of elements of G commute. It is denoted by $P(G)$. The definition of the commutativity degree is given as follows.

Definition 1.1 [1] The commutativity degree of a group G, denoted as $P(G)$, can be written as

$$P(G) = \frac{\left|\left\{(x,y) \in G \times G \middle| xy = yx\right\}\right|}{|G|^2}.$$

The concept of commutativity degree was first introduced by Miller [2] in 1944. He provided a list of open problems related to the commutativity degree and its generalization. In 1968, Erdos and Turan [3] investigate some problems of statistical group theory and commutativity degree in nonabelian group and introduced the concept of commutativity degree for symmetric groups, S_m. Later, Gustafson [4] and Machale [1] showed that the commutativity degree of all nonabelian groups is less than or equal to $\frac{5}{8}$.

In 2006, Mohd Ali and Sarmin [5] extended the concept of commutativity degree of a group G to the n^{th} commutativity degree of G, denoted as $P_n(G)$, which is the probability that the n^{th} power of a selected element commute with another element of G.

The formal definition of n^{th} commutativity degree is given in the following.

Definition 1.2 [5] The n^{th} commutativity degree of a group G, denoted as $P_n(G)$, is defined as

$$P_n(G) = \frac{\left|\left\{(x,y) \in G \times G \,\middle|\, x^n y = y x^n\right\}\right|}{|G|^2}.$$

Note that for $n = 1$, $P_1(G) = P(G)$. In finding $P_n(G)$, the power of each element in G is gradually raised until the power n is achieved.

There are two approaches on finding the probability that a pair of elements commute. First by using the Cayley Table (or symmetrical 0-1 Table) and second by using the number of conjugacy classes. MacHale [1] used the 0-1 Table to find the probability that two elements commute in a group. In this research, the 0-1 Table is used to determine the n^{th} commutativity degree of a group G.

In this research the n^{th} commutativity degree of alternating groups of degree 4 of order 12 and alternating groups of degree 5 of order 60 are found.

2.0 PRELIMINARIES

In this section, we provide some preliminaries and basic definitions that are needed in this research.

Definition 2.1 [6] Symmetric Group of Degree m

Let A be the finite set $\{1, 2, ..., m\}$. The group of all permutations of A is the symmetric group on m letters, and is denoted by S_m. The order of S_m is $m!$.

Definition 2.2 [7] Alternating Group of Degree m

The set of all even permutation in S_m forms a subgroup of S_m for $m \geq 2$. This subgroup is called the alternating group of degree m, and denoted by A_m. The order of A_m is $\frac{m!}{2}$.

Definition 2.3 [1] The 0-1 Table for a Group G

If $xy = yx$ for all x, y in G, each of the boxes corresponding to xy and yx will be assigned the number 1. In other side, if $xy \neq yx$, the number 0 will be placed in each of these boxes.

3.0 RESULTS AND DISCUSSION

In this section, the results of $P_n(A_m)$, which is the n^{th} commutativity degree of alternating groups of degree m where $m = 4$ and 5 are determined using the 0-1 Table.

Clearly, A_4 is the alternating group of degree 4. The elements of A_4 are (1), (123), (124), (134), (132), (142), (143), (234), (243), (12)(34), (14)(23) and (13)(24). To compute the multiplication table for A_4, we let

$\beta_1 = (1)$	$\beta_7 = (143)$
$\beta_2 = (123)$	$\beta_8 = (234)$
$\beta_3 = (124)$	$\beta_9 = (243)$
$\beta_4 = (134)$	$\beta_{10} = (12)\ (34)$
$\beta_5 = (132)$	$\beta_{11} = (13)\ (24)$
$\beta_6 = (142)$	$\beta_{12} = (14)\ (23)$.

The Cayley table of A_4 is given in the following:

Table 1 The Cayley Table of A_4

·	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}
β_1	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}
β_2	β_2	β_5	β_{11}	β_8	β_1	β_7	β_{12}	β_{10}	β_3	β_4	β_9	β_6
β_3	β_3	β_{12}	β_6	β_{11}	β_4	β_1	β_9	β_2	β_{10}	β_7	β_5	β_8
β_4	β_4	β_3	β_{10}	β_7	β_{12}	β_8	β_1	β_{11}	β_5	β_2	β_6	β_9
β_5	β_5	β_1	β_9	β_{10}	β_2	β_{12}	β_6	β_4	β_{11}	β_8	β_3	β_7
β_6	β_6	β_8	β_1	β_5	β_{11}	β_3	β_{10}	β_{12}	β_7	β_9	β_4	β_2
β_7	β_7	β_{10}	β_2	β_1	β_9	β_{11}	β_4	β_6	β_{12}	β_3	β_8	β_5
β_8	β_8	β_{11}	β_4	β_{12}	β_6	β_{10}	β_2	β_9	β_1	β_5	β_7	β_3
β_9	β_9	β_7	β_{12}	β_3	β_{10}	β_5	β_{11}	β_1	β_8	β_6	β_2	β_4
β_{10}	β_{10}	β_9	β_8	β_6	β_7	β_4	β_5	β_3	β_2	β_1	β_{12}	β_{11}
β_{11}	β_{11}	β_6	β_7	β_9	β_8	β_2	β_3	β_5	β_4	β_{12}	β_1	β_{10}
β_{12}	β_{12}	β_4	β_5	β_2	β_3	β_9	β_8	β_7	β_6	β_{11}	β_{10}	β_1

From Table 3.1, we can produce the 0-1 Table for A_4 as shown in the following.

Table 2 The 0-1 Table for A_4

•	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}	β_{11}	β_{12}
β_1	1	1	1	1	1	1	1	1	1	1	1	1
β_2	1	1	0	0	1	0	0	0	0	0	0	0
β_3	1	0	1	0	0	1	0	0	0	0	0	0
β_4	1	1	0	1	0	0	1	0	0	0	0	0
β_5	1	0	1	0	1	0	0	0	0	0	0	0
β_6	1	0	0	1	0	1	0	0	0	0	0	0
β_7	1	0	0	0	0	0	1	0	0	0	0	0
β_8	1	0	0	0	0	0	0	1	1	0	0	0
β_9	1	0	0	0	0	0	0	1	1	0	0	0
β_{10}	1	0	0	0	0	0	0	0	0	1	1	1
β_{11}	1	0	0	0	0	0	0	0	0	1	1	1
β_{12}	1	0	0	0	0	0	0	0	0	1	1	1

From Table 3.2, 48 pairs of elements commute with each other. Therefore, $P(A_4) = \frac{48}{144} = \frac{1}{3}$.

In Table 3.3 and Table 3.4, the powers of each element in A_4 are computed up to a certain value (until it can be generalized) and the value of $P_n(A_4)$ is computed for $n = 1, 2, 3, \ldots, 12$.

Table 3 $P_n(A_4)$ for $n = 2, 3, 4, 5$ and 6

$x \in A_4$	x^2	x^3	x^4	x^5	x^6
β_1	$(\beta_1)^2 = \beta_1$	$(\beta_1)^3 = \beta_1$	$(\beta_1)^4 = \beta_1$	$(\beta_1)^5 = \beta_1$	$(\beta_1)^6 = \beta_1$
β_2	$(\beta_2)^2 = \beta_5$	$(\beta_2)^3 = \beta_1$	$(\beta_2)^4 = \beta_2$	$(\beta_2)^5 = \beta_5$	$(\beta_2)^6 = \beta_1$
β_3	$(\beta_3)^2 = \beta_6$	$(\beta_3)^3 = \beta_1$	$(\beta_3)^4 = \beta_3$	$(\beta_3)^5 = \beta_6$	$(\beta_3)^6 = \beta_1$
β_4	$(\beta_4)^2 = \beta_7$	$(\beta_4)^3 = \beta_1$	$(\beta_4)^4 = \beta_4$	$(\beta_4)^5 = \beta_7$	$(\beta_{44})^6 = \beta_1$
β_5	$(\beta_5)^2 = \beta_2$	$(\beta_5)^3 = \beta_1$	$(\beta_5)^4 = \beta_5$	$(\beta_5)^5 = \beta_2$	$(\beta_5)^6 = \beta_1$
β_6	$(\beta_6)^2 = \beta_3$	$(\beta_6)^3 = \beta_1$	$(\beta_6)^4 = \beta_6$	$(\beta_6)^5 = \beta_3$	$(\beta_6)^6 = \beta_1$
β_7	$(\beta_7)^2 = \beta_4$	$(\beta_7)^3 = \beta_1$	$(\beta_7)^4 = \beta_7$	$(\beta_7)^5 = \beta_4$	$(\beta_7)^6 = \beta_1$
β_8	$(\beta_8)^2 = \beta_9$	$(\beta_8)^3 = \beta_1$	$(\beta_8)^4 = \beta_8$	$(\beta_8)^5 = \beta_9$	$(\beta_8)^6 = \beta_1$
β_9	$(\beta_9)^2 = \beta_8$	$(\beta_9)^3 = \beta_1$	$(\beta_9)^4 = \beta_9$	$(\beta_9)^5 = \beta_8$	$(\beta_9)^6 = \beta_1$
β_{10}	$(\beta_{10})^2 = \beta_1$	$(\beta_{10})^3 = \beta_{10}$	$(\beta_{10})^4 = \beta_1$	$(\beta_{10})^5 = \beta_{10}$	$(\beta_{10})^6 = \beta_1$
β_{11}	$(\beta_{11})^2 = \beta_1$	$(\beta_{11})^3 = \beta_{11}$	$(\beta_{11})^4 = \beta_1$	$(\beta_{11})^5 = \beta_{11}$	$(\beta_{11})^6 = \beta_1$
β_{12}	$(\beta_{12})^2 = \beta_1$	$(\beta_{12})^3 = \beta_{12}$	$(\beta_{12})^4 = \beta_1$	$(\beta_{12})^5 = \beta_{12}$	$(\beta_{12})^6 = \beta_1$
	$P_2(A_4) = \frac{1}{2}$	$P_3(A_4) = \frac{5}{6}$	$P_4(A_4) = \frac{1}{2}$	$P_5(A_4) = \frac{1}{3}$	$P_6(A_4) = 1$

Table 4 $P_n(A_4)$ for $n = 7, 8, 9, 10, 11$ and 12

x^7	x^8	x^9	x^{10}	x^{11}	x^{12}
$(\beta_1)^7 = \beta_1$	$(\beta_1)^8 = \beta_1$	$(\beta_1)^9 = \beta_1$	$(\beta_1)^{10} = \beta_1$	$(\beta_1)^{11} = \beta_1$	$(\beta_1)^{12} = \beta_1$
$(\beta_2)^7 = \beta_2$	$(\beta_2)^8 = \beta_5$	$(\beta_2)^9 = \beta_1$	$(\beta_2)^{10} = \beta_2$	$(\beta_2)^{11} = \beta_5$	$(\beta_2)^{12} = \beta_1$
$(\beta_3{}^7 = \beta_3$	$(\beta_3)^8 = \beta_6$	$(\beta_3)^9 = \beta_1$	$(\beta_3)^{10} = \beta_3$	$(\beta_3)^{11} = \beta_6$	$(\beta_3)^{12} = \beta_1$
$(\beta_4)^7 = \beta_4$	$(\beta_4)^8 = \beta_7$	$(\beta_4)^9 = \beta_1$	$(\beta_4)^{10} = \beta_4$	$(\beta_4)^{11} = \beta_7$	$(\beta_4)^{12} = \beta_1$
$(\beta_5)^7 = \beta_5$	$(\beta_5)^8 = \beta_2$	$(\beta_5)^9 = \beta_1$	$(\beta_5)^{10} = \beta_5$	$(\beta_5)^{11} = \beta_2$	$(\beta_5)^{12} = \beta_1$
$(\beta_6)^7 = \beta_6$	$(\beta_6)^8 = \beta_3$	$(\beta_6)^9 = \beta_1$	$(\beta_6)^{10} = \beta_6$	$(\beta_6)^{11} = \beta_3$	$(\beta_6)^{12} = \beta_1$
$(\beta_7)^7 = \beta_7$	$(\beta_7)^8 = \beta_4$	$(\beta_7)^9 = \beta_1$	$(\beta_7)^{10} = \beta_7$	$(\beta_7)^{11} = \beta_4$	$(\beta_7)^{12} = \beta_1$
$(\beta_8)^7 = \beta_8$	$(\beta_8)^8 = \beta_9$	$(\beta_8)^9 = \beta_1$	$(\beta_8)^{10} = \beta_8$	$(\beta_8)^{11} = \beta_9$	$(\beta_8)^{12} = \beta_1$
$(\beta_9)^7 = \beta_9$	$(\beta_9)^8 = \beta_8$	$(\beta_9)^9 = \beta_1$	$(\beta_9)^{10} = \beta_9$	$(\beta_9)^{11} = \beta_8$	$(\beta_9)^{12} = \beta_1$
$(\beta_{10})^7 = \beta_{10}$	$(\beta_{10})^8 = \beta_1$	$(\beta_{10})^9 = \beta_{10}$	$(\beta_{10})^{10} = \beta_1$	$(\beta_{10})^{11} = \beta_{10}$	$(\beta_{10})^{12} = \beta_1$
$(\beta_{11})^7 = \beta_{11}$	$(\beta_{11})^8 = \beta_1$	$(\beta_{11})^9 = \beta_{11}$	$(\beta_{11})^{10} = \beta_1$	$(\beta_{11})^{11} = \beta_{11}$	$(\beta_{11})^{12} = \beta_1$
$(\beta_{12})^7 = \beta_{12}$	$(\beta_{12})^8 = \beta_1$	$(\beta_{12})^9 = \beta_{12}$	$(\beta_{12})^{10} = \beta_1$	$(\beta_{12})^{11} = \beta_{12}$	$(\beta_{12})^{12} = \beta_1$
$P_7(A_4) = \frac{1}{3}$	$P_8(A_4) = \frac{1}{2}$	$P_9(A_4) = \frac{5}{6}$	$P_{10}(A_4) = \frac{1}{2}$	$P_{11}(A_4) = \frac{1}{3}$	$P_{12}(A_4) = 1$

From Table 3.3 and Table 3.4, we can generalize the n^{th} commutativity degree of alternating group of degree 4, $P_n(A_4)$ as in the following theorem.

Theorem 3.1 Let A_4 be an alternating group of degree 4. Then for $n, k \in \mathbb{Z}^+$ where $k = 0, 1, 2, \ldots$, $P_n(A_4)$ is given as follows:

$$P_n(A_4) = \begin{cases} \frac{1}{3}, & n = 1 + 6k,\ n = 5 + 6k \\ \frac{1}{2}, & n = 2 + 6k,\ n = 4 + 6k \\ \frac{5}{6}, & n = 3 + 6k \\ 1, & n = 6k \end{cases}$$

Proof For all elements x in A_4, the order of x is 1, 2 or 3. Furthermore, for any $x \in A_4$, $x^6 = e$ and $x^n = e$ for $n = 6k$ where $k \in \mathbb{Z}^+$.

The number of (x, y) where $x \cdot y = y \cdot x$ also equal to the number of (x, y) when $x^5 \cdot y = y \cdot x^5$, $x^7 \cdot y = y \cdot x^7$ and $x^{11} \cdot y = y \cdot x^{11}$.

Now we need to prove that $x^5 \cdot y = y \cdot x^5, x^7 \cdot y = y \cdot x^7$ and $x^{11} \cdot y = y \cdot x^{11}$ can be reduced to $x \cdot y = y \cdot x$.

Suppose $x^6 = e$. This implies $x^{-1} = x^5$. Therefore $x^5 \cdot y = y \cdot x^5$ is the same as $x^{-1} \cdot y = y \cdot x^{-1}$. By cancellation we have $x \cdot y = y \cdot x$.

Next $x^7 \cdot y = y \cdot x^7$ can be written as

$$x \cdot x^6 \cdot y = y \cdot x \cdot x^6$$
$$x \cdot e \cdot y = y \cdot x \cdot e$$
$$x \cdot y = y \cdot x.$$

By the same calculations and argument, it can be shown that $x^{11} \cdot y = y \cdot x^{11}$ can be reduced to $x \cdot y = y \cdot x$.

Next $x^4 \cdot y = y \cdot x^4, x^8 \cdot y = y \cdot x^8$ and $x^{10} \cdot y = y \cdot x^{10}$ are equal to $x^2 \cdot y = y \cdot x^2$ and $x^9 \cdot y = y \cdot x^9$ is equal to $x^3 \cdot y = y \cdot x^3$.

Suppose $x^6 = e$. This implies $(x^2)^{-1} = x^4$. Therefore $x^4 \cdot y = y \cdot x^4$ is the same as $(x^2)^{-1} \cdot y = y \cdot (x^2)^{-1}$. By cancellation we have $x^2 \cdot y = y \cdot x^2$.

Next $x^8 \cdot y = y \cdot x^8$ can be written as

$$x^2 \cdot x^2 \cdot x^4 \cdot y = y \cdot x^2 \cdot x^2 \cdot x^4$$
$$x^2 \cdot e \cdot y = y \cdot x^2 \cdot e$$
$$x^2 \cdot y = y \cdot x^2.$$

By the same calculations and argument, it can be shown that $x^{10} \cdot y = y \cdot x^{10}$ can be reduced to $x^2 \cdot y = y \cdot x^2$.

Next $x^9 \cdot y = y \cdot x^9$ can be written as

$$x^3 \cdot x^3 \cdot x^3 \cdot y = y \cdot x^3 \cdot x^3 \cdot x^3$$
$$x^3 \cdot e \cdot y = y \cdot x^3 \cdot e$$
$$x^3 \cdot y = y \cdot x^3.$$

Clearly x^6 is an identity in A_4 then $x^{12} \cdot y = y \cdot x^{12}$ can also be reduced to $x^6 \cdot y = y \cdot x^6$.

By some calculations,

$x^{1+6k} \cdot y = y \cdot x^{1+6k}$ is equal to $x \cdot y = y \cdot x$.
Suppose $x^{6k} = e$,
then,

$$x^{1+6k} \cdot y = y \cdot x^{1+6k}$$
$$x \cdot x^{6k} \cdot y = y \cdot x \cdot x^{6k}$$
$$x \cdot e \cdot y = y \cdot x \cdot e$$
$$x \cdot y = y \cdot x$$

$x^{5+6k} \cdot y = y \cdot x^{5+6k}$ is equal to $x^5 \cdot y = y \cdot x^5$.
Suppose $x^{6k} = e$,
then,

$$x^{5+6k} \cdot y = y \cdot x^{5+6k}$$
$$x^5 \cdot x^{6k} \cdot y = y \cdot x^5 \cdot x^{6k}$$
$$x^5 \cdot e \cdot y = y \cdot x^5 \cdot e$$
$$x^5 \cdot y = y \cdot x^5$$

$x^{2+6k} \cdot y = y \cdot x^{2+6k}$ is equal to $x^2 \cdot y = y \cdot x^2$.
Suppose $x^{6k} = e$,
then,

$$x^{3+6k} \cdot y = y \cdot x^{3+6k}$$
$$x^3 \cdot x^{6k} \cdot y = y \cdot x^3 \cdot x^{6k}$$
$$x^3 \cdot e \cdot y = y \cdot x^3 \cdot e$$
$$x^3 \cdot y = y \cdot x^3$$

$x^{3+6k} \cdot y = y \cdot x^{3+6k}$ is equal to $x^3 \cdot y = y \cdot x^3$.
Suppose $x^{6k} = e$,
then,

$$x^{2+6k} \cdot y = y \cdot x^{2+6k}$$
$$x^2 \cdot x^{6k} \cdot y = y \cdot x^2 \cdot x^{6k}$$
$$x^2 \cdot e \cdot y = y \cdot x^2 \cdot e$$
$$x^2 \cdot y = y \cdot x^2$$

$x^{4+6k} \cdot y = y \cdot x^{4+6k}$ is equal to $x^4 \cdot y = y \cdot x^4$.
Suppose $x^{6k} = e$,
then,

$$x^{4+6k} \cdot y = y \cdot x^{4+6k}$$
$$x^4 \cdot x^{6k} \cdot y = y \cdot x^4 \cdot x^{6k}$$
$$x^4 \cdot e \cdot y = y \cdot x^4 \cdot e$$
$$x^4 \cdot y = y \cdot x^4$$

Suppose x^{6k} is the identity in A_4 then, clearly $x^{6k} \cdot y = y \cdot x^{6k}$.

Using similar method, we found the generalization of the n^{th} commutativity degree of alternating group of degree 5, $P_n(A_5)$ given as follows.

Theorem 3.2 Let A_5 be an alternating group of degree 5. Then for $n, k \in \mathbb{Z}^+$ where $k = 0, 1, 2, \ldots$, $P_n(A_5)$ is given as follows:

$$P_n(A_5) = \begin{cases} \dfrac{1}{12}, & n = 1+30k,\ n = 7+30k,\ n = 11+30k,\ n = 13+30k, \\ & n = 17+30k,\ n = 19+30k,\ n = 23+30k,\ n = 29+30k \\ \dfrac{1441}{3600}, & n = 3+30k,\ n = 9+30k,\ n = 21+30k,\ n = 27+30k \\ \dfrac{19}{60}, & n = 2+30k,\ n = 4+30k,\ n = 8+30k,\ n = 14+30k, \\ & n = 16+30k,\ n = 22+30k,\ n = 26+30k,\ n = 28+30k \\ \dfrac{1619}{3600}, & n = 5+30k,\ n = 25+30k \\ \dfrac{2281}{3600}, & n = 6+30k,\ n = 12+30k,\ n = 18+30k,\ n = 24+30k \\ \dfrac{2399}{3660}, & n = 10+30k,\ n = 20+30k \\ \dfrac{23}{30}, & n = 15+30k \\ 1, & n = 30+30k \end{cases}$$

4.0 CONCLUSION

As a conclusion, the n^{th} commutativity degree of alternating groups of degree 4 and alternating groups of degree 5 are determined. The 0-1 Table was used in finding $P_n(A_4)$ and $P_n(A_5)$.

Acknowledgement

The authors would like to acknowledge Universiti Teknologi Malaysia (UTM) for the financial funding through the Research University Grant (RUG) Vote No. 10J68 and Ministry of Higher Education (MOHE) Malaysia for their support. The first author would also like to thank Universiti Teknologi MARA (UiTM) for the fellowship scheme.

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