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CARTESIAN PRODUCT OF INTERVAL-VALUED FUZZY IDEALS IN ORDERED SEMIGROUP

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Abstract. Interval-valued fuzzy set theory is a more generalized theory that can deal with real world problems more precisely than ordinary fuzzy set theory. In this paper, the concepts of interval-valued fuzzy (prime, semiprime) ideal and the Cartesian product of interval-valued fuzzy subsets have been introduced. Some interesting results about Cartesian product of interval-valued fuzzy ideals, interval-valued fuzzy prime ideals, interval-valued fuzzy semiprime ideals, interval-valued fuzzy bi-ideals and interval-valued fuzzy interior ideals in ordered semigroups are obtained. The purport of this paper is to link ordinary ideals with interval-valued fuzzy ideals by means of level subset of Cartesian product of interval-valued fuzzy subsets.

Keywords : Interval-valued fuzzy left (right) ideals, Interval-valued fuzzy bi-ideals, Interval-valued fuzzy interior ideals, Interval-valued fuzzy prime ideals, Interval-valued fuzzy semiprime ideals.
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1. Introduction

The concept of fuzzy relation on a set was defined by Zadeh [19, 20]. Further, in 1985 Bhattacharya and Mukherjee [1] considered fuzzy relations in group

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and investigated conditions under which the fuzzy relation becomes a fuzzy subgroup of $G \times G$ for any group G. In addition, Malik and Mordeson [14] studied rings and groups by the properties fuzzy relations and investigated that $f_1 \times f_2$ is a fuzzy left (right) ideal of $R \times R$ for any two fuzzy left (right) ideals f_1 and f_2 of a ring . Moreover, Rosenfeld [16] has developed the study of fuzzy graphs by using fuzzy relations on fuzzy sets. Furthermore, Yeh and Bang applied the concept of fuzzy relation and its applications to cluster analysis [18].

In mathematics, an ordered semigroup is a semigroup together with a partial order that is compatible with the semigroup operation. For any two ordered semigroups $(S_1, ., \leq_{S_1})$ and $(S_2, ., \leq_{S_2})$ the Cartesian product $S_1 \times S_2$ forms a semigroup under the coordinate wise multiplication [15]. In addition, Changphas [3] defined an ordered relation $\leq$ on $S_1 \times S_2$ by $(a_1, a_2) \leq (b_1, b_2)$ if and only if $a_1 \leq b_1$ and $a_2 \leq b_2$ for all $(a_1, a_2), (b_1, b_2) \in S_1 \times S_2$. In which it follows that $S_1 \times S_2$ is an ordered semigroup under the ordered relation defined by Chanphas [3]. The fundamental concept of a fuzzy set, introduced by Zadeh [19], provides a natural frame-work for generalizing several basic notions of algebra. The study of fuzzy sets in semigroups was introduced by Kuroki [10, 11]. In addition, Majumder and Sardar [13] further studied semigroups and presented some interesting results about Cartesian product of fuzzy ideals, fuzzy prime ideals and fuzzy semiprime ideals. The concept of a fuzzy bi-ideal in ordered semigroups was initiated by Kehayopulu and Tsingelis [6], and discussed some basic properties of fuzzy bi-ideals. Further, Esroy et al., [4] considered a commutative ring with identity and studied the Cartesian product of fuzzy prime ideals and fuzzy semiprime ideals.

In this paper, we considered ordered semigroup and interval-valued fuzzy set [21] and defined Cartesian product of interval-valued fuzzy ideals and interval-valued fuzzy (prime, semi prime, interior and bi-) ideals. Some interesting results about Cartesian product of different types of interval-valued fuzzy ideals in ordered semigroups are obtained by applying interval-valued fuzzy set theory. In addition, ordinary ideals are linked with interval-valued fuzzy ideals by means of level subset of Cartesian product of two interval-valued fuzzy subsets.

2. Preliminaries

Throughout this paper an ordered semigroup will be denoted by S. By an ordered semigroup (or po-semigroup) we mean a $(S,\ .,\ \leq)$ structure in which the following are satisfied for all $x,\ a,\ b \in S$:

(OS1) $(S,\ .)$ is a semigroup,

(OS2) $(S,\ \leq)$ is a poset,

(OS3) $a \leq b \Rightarrow a \cdot x \leq b \cdot x$ and $x \cdot a \leq x \cdot b$.

In what follows, $x \cdot y$ is simply denoted by xy for all $x, y \in S$.

Definition 2.1. *([5]) A non-empty subset A of S is called a subsemigroup of S if $A^2 \subseteq A$.*

Definition 2.2. *([7]) A subsemigroup A of S is called a bi-ideal of S if the following conditions hold:*

(i) For all $a, b \in S$ and $a \leq b \in A \rightarrow a \in A$.
(ii) $ASA \subseteq A$.

Definition 2.3. *([8]) A subsemigroup A of S is called a bi-ideal of S if the following conditions hold:*

(iii) For all $a, b \in S$ and $a \leq b \in A \rightarrow a \in A$.
(iv) $SA \subseteq A$ and $AS \subseteq A$.

Definition 2.4. *([8]) A non-empty subset P of S is called prime if $AB \subseteq P \Rightarrow A \subseteq P$ or $B \subseteq P$ for all $A, B \subseteq S$.*

Definition 2.5. *([8]) A non-empty subset P of S is called prime if $A^2 \subseteq P \Rightarrow A \subseteq P$ for all $A \subseteq S$.*

Definition 2.6. *([10]) A function $f : S \rightarrow [0, 1]$ is called a fuzzy set in S.*

Definition 2.7. *([5]) If $f(xy) \geq \min\{f(x), f(y)\}$ for all $x, y \in S$, then f is called a fuzzy subsemigroup of S.*

Definition 2.8. *([7]) A fuzzy set f in S is called a fuzzy bi-ideal if the following hold for all $x, y, z \in S$:*

(v) $f(xy) \geq \min\{f(x), f(y)\}$,
(vi) $x \leq y \Rightarrow f(x) \geq f(y)$,
(vii) $f(xyz) \geq \min\{f(x), f(z)\}$.

Definition 2.9. *([6]) A fuzzy set f in S is called a fuzzy bi-ideal if the following hold for all $x, y, z \in S$:*

(viii) $x \leq y \Rightarrow f(x) \geq f(y)$,
(ix) $f(xy) \geq f(y)$, $f(xy) \geq f(x)$.

By an interval number $\widetilde{a}$ we mean an interval $[a^-, a^+]$ where $0 \leq a^- \leq a^+ \leq 1$ and the set of all closed sub interval numbers is denoted by $D[0, 1]$. The interval $[a, a]$ can be simply identified by the number $a \in [0, 1]$.

For the interval numbers $\widetilde{a}_i = [a_i^-, a_i^+]$, $\widetilde{b}_i = [b_i^-, b_i^+] \in D[0, 1]$, $i \in I$, we define;

$(\forall i \in I)$ (r $\max\{\widetilde{a}_i, \widetilde{b}_i\} = [\max(a_i^-, b_i^-), \max(a_i^+, b_i^+)]$),
$(\forall i \in I)$ (r $\min\{\widetilde{a}_i, \widetilde{b}_i\} = [\min(a_i^-, b_i^-), \min(a_i^+, b_i^+)]$),
r $\inf \widetilde{a}_i = \left[\bigwedge_{i \in I} a_i^-, \bigwedge_{i \in I} a_i^+\right]$, r $\sup \widetilde{a}_i = \left[\bigvee_{i \in I} a_i^-, \bigvee_{i \in I} a_i^+\right]$ and

- $\widetilde{a}_1 \leq \widetilde{a}_2 \Longleftrightarrow a_1^- \leq a_2^-$ and $a_1^+ \leq a_2^+$,
- $\widetilde{a}_1 = \widetilde{a}_2 \Longleftrightarrow a_1^- = a_2^-$ and $a_1^+ = a_2^+$,
- $\widetilde{a}_1 < \widetilde{a}_2 \Longleftrightarrow \widetilde{a}_1 \leq \widetilde{a}_2$ and $\widetilde{a}_1 \neq \widetilde{a}_2$,
- $k\widetilde{a}_i = [ka_i^-, ka_i^+]$, whenever $0 \leq k \leq 1$.

Then, it is clear that $(D[0,1], \leq, \vee, \wedge)$ forms a complete lattice with $\widetilde{0} = [0,0]$ as its least element and $\widetilde{1} = [1,1]$ as its greatest element.

The interval-valued fuzzy subsets provide a more adequate description of uncertainty than the traditional fuzzy subsets; it is therefore important to use interval-valued fuzzy subsets in applications. One of the main applications of fuzzy subsets is fuzzy control, and one of the most computationally intensive part of fuzzy control is the "defuzzification". Since a transition to interval-valued fuzzy subsets usually increase the amount of computations, it is vitally important to design faster algorithms for the corresponding defuzzification.

Next, some basic concepts of interval-valued fuzzy set theory are given in the following lines.

Definition 2.10. *([2]) An interval-valued fuzzy subset A defined on a set X is represented as $A = \{(x, \widetilde{f}_A(x) = [f_A^-(x), f_A^+(x)]) : x \in X\}$, where $f_A^-(x), f_A^+(x)$ are fuzzy subsets such that $0 \leq f_A^-(x) \leq f_A^+(x) \leq 1$. The interval $\widetilde{f}_A(x) = [f_A^-(x), f_A^+(x)]$ denotes the degree of the membership of an element x in A and $D[0,1]$ is the set of all closed subintervals of the unit closed interval $[0,1]$. If $f_A^-(x) = f_A^+(x) = a$, then $0 \leq a \leq 1$ and therefore $\widetilde{f}_A(x) = [a,a]$ and this shows that fuzzy set is a special case of an interval-valued fuzzy subset. For the sake of convenience it is assumed that $[a,a] \in D[0,1]$ and hence $\widetilde{f}_A(x) \in D[0,1]$ for all $x \in X$.*

Definition 2.11. *([9]) Let A be an interval-valued fuzzy subset of an ordered semigroup S. Then, for every $\widetilde{0} \leq \widetilde{t} \leq \widetilde{1}$ the crisp set $U(A; \widetilde{t}) = \{x \in S : \widetilde{f}_A(x) \geq \widetilde{t}\}$ is called a level subset of A.*

Definition 2.12. *([17]) An interval-valued fuzzy subset A of S is called an interval-valued fuzzy ideal of S if the following conditions hold for all $x, y \in S$:*

(x) $x \leq y \Rightarrow \widetilde{f}_A(x) \geq \widetilde{f}_A(y)$,
(xi) $\widetilde{f}_A(xy) \geq \widetilde{f}_A(y)$ and $\widetilde{f}_A(xy) \geq \widetilde{f}_A(x)$.

Definition 2.13. *([17]) An interval-valued fuzzy subset A of S is called an interval-valued fuzzy interior ideal of S if :*

(xii) $x \leq y \Rightarrow \widetilde{f}_A(x) \geq \widetilde{f}_A(y)$,
(xiii) $\widetilde{f}_A(xyz) \geq \widetilde{f}_A(y)$.
for all $x, y, z \in S$.

Definition 2.14. *([17]) An interval-valued fuzzy subset A of S is called an interval-valued fuzzy bi-ideal of S if the following conditions hold for all $x, y \in S$:*

(xiv) $x \leq y \Rightarrow \widetilde{f}_A(x) \geq \widetilde{f}_A(y)$,
(xv) $\widetilde{f}_A(xy) \geq r\min\left\{\widetilde{f}_A(x),\ \widetilde{f}_A(y)\right\}$,
(xvi) $\widetilde{f}_A(xyz) \geq r\min\left\{\widetilde{f}_A(x),\ \widetilde{f}_A(z)\right\}$.

3. Cartesian Product

In this section, the Cartesian products of interval-valued fuzzy ideal, interval-valued fuzzy prime ideal, interval-valued fuzzy semiprime ideal, interval-valued fuzzy bi-ideal, and interval-valued fuzzy interior ideal of ordered semigroup are defined.

Definition 3.1. *An interval-valued fuzzy ideal A of an ordered semigroup S is called an interval-valued fuzzy prime ideal of if $\widetilde{f}_A(xy) \leq r\max\left\{\widetilde{f}_A(x),\ \widetilde{f}_A(y)\right\}$ for all $x,\ y \in S$.*

Definition 3.2. *An interval-valued fuzzy ideal A of an ordered semigroup S is called an interval-valued fuzzy semiprime ideal of if $\widetilde{f}_A(x) \geq \widetilde{f}_A(x^2)$ for all $x \in S$.*

Definition 3.3. *The Cartesian product of two interval-valued fuzzy subsets $A = \{(x, \widetilde{f}_A(x)) : x \in S\}$ and $B = \{(x, \widetilde{f}_B(x)) : x \in S\}$ of an ordered semigroup S is defined as $A \times B = \{((x,y), (\widetilde{f}_A \times \widetilde{f}_B)(x,y)) : (x,y) \in S \times S\}$, where $(\widetilde{f}_A \times \widetilde{f}_B)(x,y) = r\min\left\{\widetilde{f}_A(x),\ \widetilde{f}_A(y)\right\}$ for all $x,\ y \in S$.*

Definition 3.4. *Consider two interval-valued fuzzy subsets A and B of an ordered semigroup S. Then the crisp set $U(A \times B; \widetilde{t}) = U(A; \widetilde{t}) \times U(B; \widetilde{t})$ is called a level subset of $A \times B$ for $\widetilde{0} \leq \widetilde{t} \leq \widetilde{1}$.*

Theorem 3.5. *The Cartesian product of two interval-valued fuzzy ideals of an ordered semigroup S is an interval-valued fuzzy ideal of $S \times S$.*

Proof. Let A and B be interval-valued fuzzy ideals of an ordered semigroup S and consider $(a,b), (c,d) \in A \times B \subseteq S \times S$ such that $(a,b) \leq (c,d)$. Then

$$\begin{aligned}\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b) &= r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_B(b)\right\} \\ &\geq r\min\left\{\widetilde{f}_A(c),\ \widetilde{f}_B(d)\right\} \\ &\quad \text{(since } A \text{ and } B \text{ are interval-valued fuzzy ideals)} \\ &= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(c,d).\end{aligned}$$

It follows that $\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b) \geq \left(\widetilde{f}_A \times \widetilde{f}_B\right)(c,d)$. Next consider

$$\begin{aligned}\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)(c,d) &= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(ac,bd)\\ &= r\min\left\{\widetilde{f}_A(ac),\ \widetilde{f}_B(bd)\right\}\\ &\geq r\min\left\{\widetilde{f}_A(c),\ \widetilde{f}_B(d)\right\}\\ &\quad \text{(since } A \text{ and } B \text{ are interval-valued fuzzy ideals)}\\ &= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(c,d),\end{aligned}$$

that is $\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)(c,d) \geq \left(\widetilde{f}_A \times \widetilde{f}_B\right)(c,d)$. Similarly, it can be shown that $\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)(c,d) \geq \left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)$ and hence $A \times B$ is an interval-valued fuzzy ideal of $S \times S$. □

Lemma 3.6. *The Cartesian product of any two interval-valued fuzzy prime ideals of an ordered semigroup S is an interval-valued fuzzy prime ideal of $S \times S$.*

Proof. Let A and B be interval-valued fuzzy prime ideals of S and $(a,b),(c,d) \in A \times B \subseteq S \times S$. Consider

$$\begin{aligned}\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)(c,d) &= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(ac,bd)\\ &= r\min\left\{\widetilde{f}_A(ac),\ \widetilde{f}_B(bd)\right\}\\ &\leq r\min\left\{\begin{array}{c} r\max\left\{\widetilde{f}_A(a),\ \widetilde{f}_A(c)\right\}, \\ r\max\left\{\widetilde{f}_B(b),\ \widetilde{f}_B(d)\right\}\end{array}\right\}\\ &\quad (A,\ B \text{ are interval-valued fuzzy prime ideals})\\ &= r\max\left\{\begin{array}{c} r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_B(b)\right\}, \\ r\min\left\{\widetilde{f}_A(c),\ \widetilde{f}_B(d)\right\}\end{array}\right\}\\ &= r\max\left\{\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b),\ \left(\widetilde{f}_A \times \widetilde{f}_B\right)(c,d)\right\},\end{aligned}$$

it follows that

$$\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)(c,d) \geq r\max\left\{\begin{array}{c}\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b), \\ \left(\widetilde{f}_A \times \widetilde{f}_B\right)(c,d)\end{array}\right\},$$

also by *Theorem 1* $A \times B$ is an interval-valued fuzzy ideal. Hence $A \times B$ is an interval-valued fuzzy prime ideal of $S \times S$. □

Lemma 3.7. *The Cartesian product of any two interval-valued fuzzy semiprime ideals of an ordered semigroup S is an interval-valued fuzzy semiprime ideal of $S \times S$.*

Proof. Let $(a,b) \in A \times B \subseteq S \times S$, where A and B are interval-valued fuzzy semiprime ideals of S and consider

$$\begin{aligned}
\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b) &= r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_B(b)\right\} \\
&\leq r\min\left\{\widetilde{f}_A(a^2),\ \widetilde{f}_B(b^2)\right\} \\
&\quad (A,\ B \text{ are interval-valued fuzzy semiprime ideals}) \\
&= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(a^2, b^2) \\
&= \left(\widetilde{f}_A \times \widetilde{f}_B\right)((a,b)\,(a,b)) \\
&= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)^2,
\end{aligned}$$

it follows that $\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b) \leq \left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)^2$. The rest of the proof follows from *Theorem 1* and hence $A \times B$ is an interval-valued fuzzy semiprime ideal of $S \times S$. □

Theorem 3.8. *The Cartesian product of two interval-valued fuzzy interior ideals of an ordered semigroup S is an interval-valued fuzzy interior ideal of $S \times S$.*

Proof. Let $(a,b), (c,d), (l,m) \in A \times B \subseteq S \times S$, where A and B are interval-valued fuzzy interior ideals of S. Consider

$$\begin{aligned}
\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)\,(l,m)\,(c,d) &= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(alc, bmd) \\
&= r\min\left\{\widetilde{f}_A(alc),\ \widetilde{f}_B(bmd)\right\} \\
&\geq r\min\left\{\widetilde{f}_A(l),\ \widetilde{f}_B(m)\right\} \\
&\quad (A,\ B \text{ are interval-valued fuzzy interior ideals}) \\
&= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(l,m),
\end{aligned}$$

in which it follows that $\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)\,(l,m)\,(c,d) \geq \left(\widetilde{f}_A \times \widetilde{f}_B\right)(l,m)$. The remaing part of the proof follows from *Theorem 1*. Thus, $A \times B$ is an interval-valued fuzzy interior ideal of $S \times S$. □

Theorem 3.9. *If A and B are interval-valued fuzzy bi-ideals of S, then their Cartesian product $A \times B$ is an interval-valued fuzzy bi-ideal of $S \times S$.*

Proof. Let $(a,b),(c,d),(g,h) \in A \times B \subseteq S \times S$, where A and B are interval-valued fuzzy interior ideals of S. Consider

$$\begin{aligned}
\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)\,(c,d) &= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(ac,bd) \\
&= r\min\left\{\widetilde{f}_A(ac),\ \widetilde{f}_B(bd)\right\} \\
&\geq r\min\left\{\begin{array}{c} r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_A(c)\right\}, \\ r\min\left\{\widetilde{f}_B(b),\ \widetilde{f}_B(d)\right\} \end{array}\right\} \\
&\quad (A,\ B \text{ are interval-valued fuzzy bi-ideals}) \\
&= r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_A(c),\ \widetilde{f}_B(b),\ \widetilde{f}_B(d)\right\} \\
&= r\min\left\{\begin{array}{c} r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_B(b)\right\}, \\ r\min\left\{\widetilde{f}_A(c),\ \widetilde{f}_B(d)\right\} \end{array}\right\} \\
&= r\min\left\{\begin{array}{c} \left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b), \\ \left(\widetilde{f}_A \times \widetilde{f}_B\right)(c,d) \end{array}\right\}.
\end{aligned}$$

Again, consider

$$\begin{aligned}
\left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b)\,(c,d)\,(g,h) &= \left(\widetilde{f}_A \times \widetilde{f}_B\right)(acg,bdh) \\
&= r\min\left\{\widetilde{f}_A(acg),\ \widetilde{f}_B(bdh)\right\} \\
&\geq r\min\left\{\begin{array}{c} r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_A(g)\right\}, \\ r\min\left\{\widetilde{f}_B(b),\ \widetilde{f}_B(h)\right\} \end{array}\right\} \\
&\quad (A,\ B \text{ are interval-valued fuzzy bi-ideals}) \\
&= r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_A(g),\ \widetilde{f}_B(b),\ \widetilde{f}_B(h)\right\} \\
&= r\min\left\{\begin{array}{c} r\min\left\{\widetilde{f}_A(a),\ \widetilde{f}_B(b)\right\}, \\ r\min\left\{\widetilde{f}_A(g),\ \widetilde{f}_B(h)\right\} \end{array}\right\} \\
&= r\min\left\{\begin{array}{c} \left(\widetilde{f}_A \times \widetilde{f}_B\right)(a,b), \\ \left(\widetilde{f}_A \times \widetilde{f}_B\right)(g,h) \end{array}\right\}.
\end{aligned}$$

The remaing part of the proof follows from *Theorem 1*. Thus, $A \times B$ is an interval-valued fuzzy bi-ideal of $S \times S$. □

Conclusion: In the world of contemporary mathematics, the use of algebraic structures in computer science, control theory and fuzzy automata theory always gain the interest of researchers. Algebraic structures particularly semigroups play a key role in such applied branches. Further, the fuzzification of several subsystems of semigroups are used in various models involving uncertainties. In this paper, we have considered the structure of ordered semigroup and defined Cartesian product of two interval-valued fuzzy ideals and interval-valued fuzzy (prime, semi prime, interior and bi-) ideals. Some interesting results about Cartesian product of different types of interval-valued fuzzy ideals in ordered semigroups are presented by applying interval-valued fuzzy set theory. In addition, ordinary ideals are linked with interval-valued fuzzy ideals by means of level subset of Cartesian product of two interval-valued fuzzy subsets.

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