Jurnal
Teknologi

Full Paper

ON GRAPHS ASSOCIATED TO CONJUGACY CLASSES OF SOME THREE-GENERATOR GROUPS

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Article history
Received
4 May 2015
Received in revised form
10 October 2016
Accepted
10 December 2016

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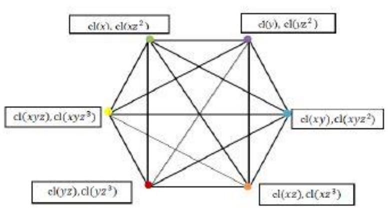
Graphical abstract



Abstract

A graph consists of points which are called vertices, and connections which are called edges, which are indicated by line segments or curves joining certain pairs of vertices. In this paper, four types of graphs which are the commuting graph, non-commuting graph conjugate graph and the conjugacy class graph for some three-generator groups are discussed. Some of the graph properties are also found which include the independent number, chromatic number, clique number and dominating number.

Keywords: Graph, conjugacy class, independent number, chromatic number, clique number, dominating number

Abstrak

Sebuah graf terdiri daripada beberapa titik yang dipanggil bucu, dan sambungannya dipanggil sisi, yang ditunjukkan oleh segmen garisan atau lengkung yang menyambungkan sesetengah bucu tertentu. Dalam kajian ini, empat jenis graf iaitu graf berulang-alik, graf bukan berulang-alik, graf konjugat dan graf kelas konjugat bagi beberapa kumpulan berpenjana-3 dibincangkan. Beberapa sifat graf juga dijumpai termasuklah nombor tak bebasa, nombor berkroma, nombor kelompok dan nombor dominasi.

Kata kunci: Graf, kelas konjugat, nombor tak bebasa, nombor berkroma, nombor kelompok, nombor dominasi

1.0 INTRODUCTION

The study of graph theory has actually begun many years ago. There are many real-life problems that can be related to graph theory, one can refer to [1-5]. One of the problems is the electric network which was discovered by Kirchoff in 1847 [6]. From the electric network's problem, Kirchoff developed the basic concepts and theorem related to trees in graphs. To solve the system of simultaneous linear equation which gives the current in each branch and around each circuit of an electric network, Kirchoff replaced each electrical network by its underlying graph.

Marcus [7] stated that a graph consists of points which are called vertices, and connections which are called edges, which are indicated by line segments or curves joining certain pairs of vertices. In 2013, Ilangovan [8] computed the number of conjugacy classes of two groups of nilpotency class 2 and applied the results to find some graphs. In addition, there are many researches which relate both the group and graph theory as in [9-11].

This paper is structured as follows: the first part is the introduction, the second part describes the methodology used in this research while the third part includes the results and discussion.

2.0 METHODOLOGY

In this section, some basic definitions in graph theory are included. The methodology used in this research will also be included.

Definition 2.1 [7] Graph
A graph consists of points, which are called vertices and connections, which are called edges and which are indicated by line segments of curves joining certain pairs of vertices.

Definition 2.2 [7] Complete Graphs
If all of the vertices in a graph are adjacent to each other, then the graph is called a complete graph. The symbol K_n is used to denote a complete graph with *n* vertices.

Definition 2.3 [7] Subgraphs
A subgraph of a graph *Γ* is a graph that is contained within *G*. All vertices and edges of the subgraphs must be included in *G*.

Four types of graph are in the scope of this research. The definitions for each graph are stated in the following. In all cases, we assume that G is a finite group and *Z*(*G*) is the center of *G*.

Definition 2.4 [12] Commuting Graph
The commuting graph of a group, denoted by Γ_G^{comm}, is the simple undirected graph whose vertices are the non-central elements of G and two distinct vertices x and y are adjacent if and only if $xy = yx$.

Definition 2.5 [13] Non-commuting Graph
Let G be a non-abelian group and let *Z*(*G*) be the center of G. Associate a graph Γ_G^{NC} (called non-commuting graph of G) with G as follows: Take G\Z(G) as the vertices of Γ_G^{NC} and join two distinct vertices vertices x and y, whenever $xy \neq yx$.

Definition 2.6 [14] Conjugate Graph
Assign a graph to a finite nonabelian group *G* with vertex set G\Z(G) such that two distinct vertices join by an edge if they are conjugate. The conjugate graph is denoted by Γ_G^{conj}.

Definition 2.7 [15] Conjugacy Class Graph
A conjugacy class graph, Γ_G^{Cl}, is a graph whose vertice are the non-central conjugacy classes of *G*, i.e $|V(\Gamma_G^{Cl})| = K_G - |Z(G)|$, in which K_G is the number of conjugacy classes in *G*. Two vertices are adjacent if their cardinalities are not coprime, which means that the greatest common divisor (gcd) of the number of vertices in the group is not equal to one.

Some graph properties studied in this research are limited to the following:

Definition 2.8 [13] Independent Number
A subset *X* of the vertices of *Γ* is called an independent set if the induced subgraph on *X* has no edges. The maximum size of an independent set in a graph *Γ* is called the independence number of *Γ* and denoted by α(*Γ*).

Definition 2.9 [15] Chromatic Number
A *k*-vertex coloring of a graph *Γ* is an assignment of *k* colors to the vertices of *Γ* such that no two adjacent vertices have the same color. The vertex chromatic number χ(*Γ*) of a graph *Γ*, is the minimum *k* for which *Γ* has a *k*-vertex coloring.

Definition 2.10 [13] Clique Number
A subset *X* of the vertices of *Γ* is called a clique if the induced subgraph on *X* is a complete graph. The maximum size of a clique in a graph *Γ* is called the clique number of *Γ* and denoted by ω(*Γ*).

Definition 2.11 [14] Dominating Number
For a graph *Γ* and a subset *S* of vertices, denote by $N_\Gamma[S]$ the set of vertices in *Γ* which are in *S* or adjacent to a vertex in S. If $N_\Gamma[S] = V(\Gamma)$, then *S* is called a dominating set for *Γ*. The dominating number γ(*Γ*) of *Γ* is the minimum size of a dominating set of the vertices of *Γ*.

The classification of groups of order p^4 is given by Kim in [16]. Since this paper focuses on some graphs associated to conjugacy classes of 3-generator groups, thus, our research scope is on 3-generator groups of order 16. All 3-generator groups of order 16 are presented as below:

$H_1 = \langle x, y, z \mid x^2 = y^2 = z^4 = 1, [x,z] = [y,z] = 1, [x,y] = z^2 \rangle$,
$H_2 = \langle x, y, z \mid x^4 = y^2 = z^2 = 1, [x,z] = [y,z] = 1, x^y = x^3 \rangle$,
$H_3 = \langle x, y, z \mid x^4 = y^2 = z^2 = 1, [x, y] = z, [x,z] = [y, z] = 1 \rangle$.

The objectives of this research are to find four types of graphs, namely the commuting graph, non-commuting graph, conjugate graph and conjugacy class graph. In order to find these graphs, we first find their conjugacy classes and then we used their definitions. We also applied the definitions to determine the graphs properties.

3.0 RESULTS AND DISCUSSION

This section provides the results of the commuting graph, non-commuting graph, conjugate graph and conjugacy class graph together with their properties for three groups as given in the end of Section 1.

3.1 The Commuting Graph of 3-generator Groups and Their Properties

In this section, the conjugacy classes of 3-generator groups of order 16 are computed, starting with the dihedral group of order 16, namely D_8.

Theorem 3.1 Let $H_1 = \langle x, y, z \mid x^2 = y^2 = z^4 = 1, [x,z] = [y,z] = 1, [x,y] = z^2 \rangle$ be a 3-generator group of order 16. Then, the commuting graph of H_1, $\Gamma_{H_1}^{comm} = \cup_{i=1}^{3} K_4$.

Proof: The conjugacy classes of H_1 are listed as follows:

(i) $cl(1) = \{1\}$,
(ii) $cl(x) = \{x, xz^2\} = cl(xz^2)$,
(iii) $cl(y) = \{y, yz^2\} = cl(yz^2)$,
(iv) $cl(z) = \{z\}$,
(v) $cl(z^2) = \{z^2\}$,
(vi) $cl(z^3) = \{z^3\}$,
(vii) $cl(xy) = \{xy, xyz^2\} = cl(xyz^2)$,
(viii) $cl(xz) = \{xz, xz^3\} = cl(xz^3)$,
(ix) $cl(yz) = \{yz, yz^3\} = cl(yz^3)$,
(x) $cl(xyz) = \{xyz, xyz^3\} = cl(xyz^3)$.

Since an element is in the center if and only if its conjugacy class has one element, therefore the center of H_1, $Z(H_1) = \{1, z, z^2, z^3\}$. Based on Definition 2.4 and since the order of H_1 is 16 and the order of $Z(H_1)$ is 4, hence, the number of vertices of the commuting graph H_1 is 12.

Next, the commuting elements are traced. Thus, the commuting graph of H_1 can be presented in Figure 1, which is a union of three complete graphs of K_4.

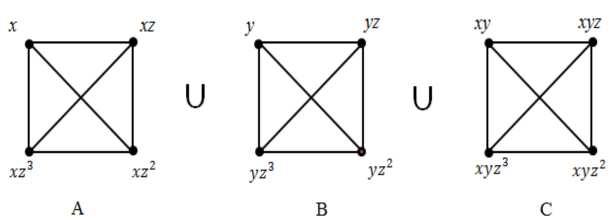


Figure 1 The commuting graph of H_1, $\Gamma_{H_1}^{comm} = \cup_{i=1}^{3} K_4$

Next, some properties of the commuting graph of H_1 are discussed.

Proposition 3.1 Let $H_1 = \langle x, y, z \mid x^2 = y^2 = z^4 = 1, [x,z] = [y,z] = 1, [x,y] = z^2 \rangle$ be a 3-generator group of order 16. Then, the independent number of the commuting graph of H_1, $\alpha(\Gamma_{H_1}^{comm}) = 3$.

Proof: From Definition 2.8 and Figure 1, the maximum independent set = $\{a, b, c\}$ where $a \in A$, $b \in B$ and $c \in C$. Since the independent number is the number of vertices in maximum independent set, thus, $\alpha(\Gamma_{H_1}^{comm}) = 3$. ■

Proposition 3.2 Let $H_1 = \langle x, y, z \mid x^2 = y^2 = z^4 = 1, [x,z] = [y,z] = 1, [x,y] = z^2 \rangle$ be a 3-generator group of order 16. Then, the chromatic number of the commuting graph of H_1, $\chi(\Gamma_{H_1}^{comm}) = 4$.

Proof: Recall from Definition 2.9, the chromatic number of the commuting graph of H_1, $\chi(\Gamma_{H_1}^{comm})$ is 4 since all the four vertices are adjacent in each component in K_4. Thus, they need to have different color of vertices as shown in Figure 2.

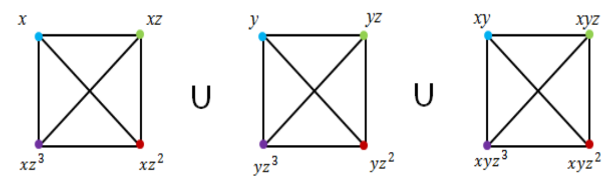


Figure 2 The commuting graph of H_1, $\Gamma_{H_1}^{comm} = \cup_{i=1}^{3} K_4$. with different colors of vertices ■

Proposition 3.3 Let $H_1 = \langle x, y, z \mid x^2 = y^2 = z^4 = 1, [x,z] = [y,z] = 1, [x,y] = z^2 \rangle$ be a 3-generator group of order 16. Then, the clique number of the commuting graph of H_1 is $\omega(\Gamma_{H_1}^{comm}) = 4$.

Proof: In order to prove this proposition, Definition 2.10 is used. The clique number is the size of the largest complete subgraph. From Figure 1, the largest subgraph of H_1 is K_4. Therefore, the clique number, $\omega(\Gamma_{H_1}^{comm}) = 4$. ■

Proposition 3.4 Let $H_1 = \langle x, y, z \mid x^2 = y^2 = z^4 = 1, [x,z] = [y,z] = 1, [x,y] = z^2 \rangle$ be a 3-generator group of order 16. Then, the dominating number of the commuting graph of H_1, $\gamma(\Gamma_{H_1}^{comm}) = 3$.

Proof: Based on Figure 1, one vertex is needed to connect the other vertices in a complete graph of K_4. Since there are three complete graph of K_4, three vertices are needed to connect all vertices in the commuting graph. Therefore, the minimum size of vertex which can connect itself and other vertices in K_4 is 3. Thus, the dominating number, $\gamma(\Gamma_{H_1}^{comm}) = 3$. ■

Theorem 3.2 Let $H_2 = \langle x, y, z \mid x^4 = y^2 = z^2 = 1, [x,z] = [y,z] = 1, x^y = x^3 \rangle$ be a 3-generator group of order 16. Then, the commuting graph of H_2, $\Gamma_{H_2}^{comm} = \cup_{i=1}^{3} K_4$.

Proof: The same steps are used to prove this theorem. The conjugacy classes of H_2 are listed below:

(i) $cl(1) = \{1\}$,
(ii) $cl(x) = \{x, x^3\} = cl(x^3)$,
(iii) $cl(x^2) = \{x^2\}$,
(iv) $cl(xy) = \{xy, x^3y\} = cl(x^3y)$,
(v) $cl(xz) = \{xz, x^3z\} = cl(x^3z)$,
(vi) $cl(y) = \{y, x^2y\} = cl(x^2y)$,
(vii) $cl(z) = \{z\}$,
(viii) $cl(x^2z) = \{x^2z\}$,
(ix) $cl(yz) = \{yz, x^2yz\} = cl(x^2yz)$,
(x) $cl(xyz) = \{xyz, x^3yz\} = cl(x^3yz)$. ■

Since the result is the same as H_1, the properties of their graph are also the same.

Theorem 3.3 Let H_3= <x, y, z | x^4= y^2= z^2= 1, [x, y] = z, [x,z] = [y, z] = 1 > be a 3-generator group of order 16. Then, the commuting graph of H_3, $\Gamma_{H_3}^{comm} = \cup_{i=1}^{3} K_4$.

Proof: The same steps are used to prove this theorem. The conjugacy classes of H_3 are listed below:

(i) cl(1) = {1},
(ii) cl(x) = {x, xz} = cl(xz),
(iii) cl(x^2) = {x^2},
(iv) cl(xy) = {xy, x^3z} = cl(x^3z),
(v) cl(y) = {y, yz} = cl(yz),
(vi) cl(z) ={z},
(vii) cl(xy) = {xy, xyz} = cl(xyz),
(viii) cl(x^2y) = {x^2y, x^2yz} = cl(x^2yz),
(ix) cl(x^3y) = {x^3y, x^3yz} = cl(x^3yz),
(x) cl(x^2z) = {x^2z}. ■

Since the result is the same as H_1, the properties of the commuting graph of H_3 are also the same as H_1.

3.2 The Non-Commuting Graph of 3-generator Groups and Their Properties

Theorem 3.4 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the non-commuting graph of H_1 is given as in Figure 3 below.

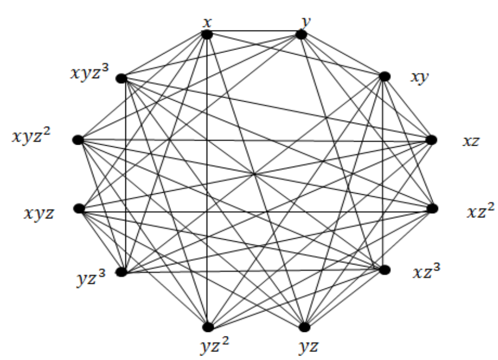


Figure 3 The non-commuting graph of H_1

Proof: Based on Definition 2.5, the vertices of non-commuting graph, Γ_G^{NC}, is calculated. Since the order of H_1 is 16 and the order of the center is 4, hence, $|V(\Gamma_{H_1})| = 12$. Then, the non-commuting elements are determined. From Figure 3, the elements which are adjacent means that the elements are do not commute. ■

Proposition 3.7 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the independent number of the non-commuting graph of H_1, $\alpha(\Gamma_{H_1}^{NC}) = 4$.

Proof: By Definition 2.8 and Theorem 3.4, one of the maximum independent sets is {x, xz, xz^2, xz^3} where x, xz, xz^2, xz^3 ∈ H_1. Since the independent number is the number of vertices in a maximum independent set, thus, $\alpha(\Gamma_{H_1}^{NC}) = 4$.

Proposition 3.8 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the chromatic number of the non-commuting graph of H_1 is $\chi(\Gamma_{H_1}^{NC}) = 12$.

Proof: As stated in Definition 2.9, the chromatic number $\chi(\Gamma_{H_1}^{NC})$ can be determined through the smallest number of colors needed to color the vertices so that no two adjacent vertices own the same color. Thus, the chromatic number of non-commuting graph of H_1 is 12 since all twelve vertices must have different colors as shown in Figure 4.

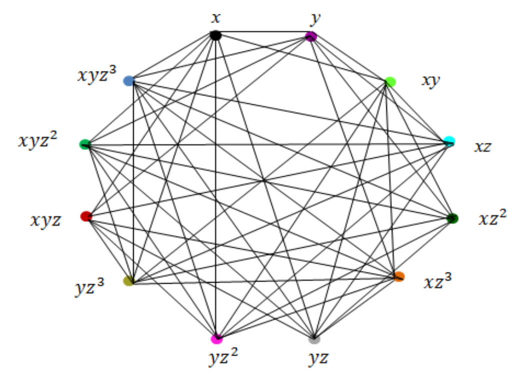


Figure 4 The non-commuting graph of H_1 with 12 different colors of vertices

Proposition 3.9 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the clique number of the non-commuting graph of H_1 is $\omega(\Gamma_{H_1}^{NC}) = 3$. ■

Proof: Based on Definition 2.10 and Figure 4 in Theorem 3.4, the non-commuting graph is not complete. However, by removing the vertices which are not adjacent in the graph, a complete graph known as K_3 = {x, y, xyz} is found. Thus, the largest complete subgraph in the conjugate graph of H_1 is K_3. Therefore, the clique number, $\omega(\Gamma_{H_1}^{NC}) = 3$.

Proposition 3.10 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the dominating number of the non-commuting graph of H_1 is $\gamma(\Gamma_{H_1}^{NC}) = 2$. ■

Proof: By using Definition 2.11, the minimum size of vertex which can connect itself and other vertices in H_1 is 2. Thus, the dominating number, $\gamma(\Gamma_{H_1}^{NC}) = 2$. ■

Theorem 3.5 Let H_2= <x, y, z | x^4= y^2= z^2= 1, [x,z] = [y,z] = 1, x^y= x^3> be a 3-generator group of order 16. Then, the non-commuting graph of H_2 is given as in Figure 5.

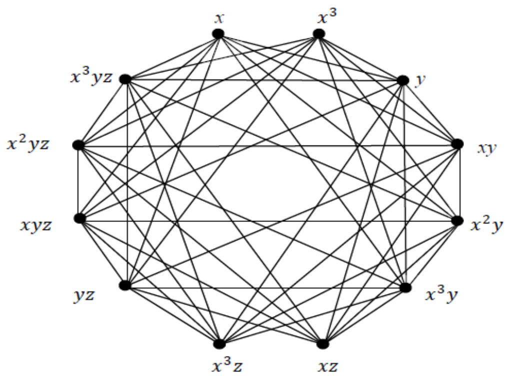


Figure 5 The non-commuting graph of H_2

Proof: The proof is similar as Theorem 3.4. ■

Since the non-commuting graph of H_2 is the same as H_1, therefore, the properties of the non-commuting graph are also the same.

Theorem 3.6 Let H_3= <x, y, z | x^4= y^2= z^2= 1, [x, y] = z, [x,z] = [y, z] = 1 > be a 3-generator group of order 16. Then, the non-commuting graph of H_3 is illustrated in Figure 6.

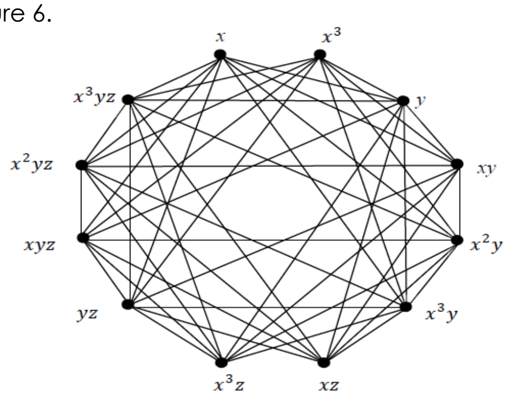


Figure 6 The non-commuting graph of H_3

Proof: The proof is similar as Theorem 3.4. ■

Since the non-commuting graph of H_3 is the same as H_1, therefore, the properties of the non-commuting graph are also the same.

3.3 The Conjugate Graph of 3-generator Groups of Order 16 and Their Properties

Theorem 3.7 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the conjugate graph of H_1, $\Gamma_{H_1}^{conj} = \cup_{i=1}^{6} K_2$, that is, a union of six complete graphs with two vertices.

Proof: Using Definition 2.6, the order of the group H_1 is 16 while the order of the center of the group is 4. Therefore, the order of the vertices of the conjugate graph is equal to 16 – 4 = 12. Based on the same definition, two vertices are adjacent if they are conjugate. Any two elements in a group G which belong to the same conjugacy class are conjugate. Recall from the conjugacy classes of H_1 listed in the proof of Theorem 3.1, the number of edges in the conjugate graph with 12 vertices is equal to 6. The conjugate graph of H_1 is presented in Figure 7:

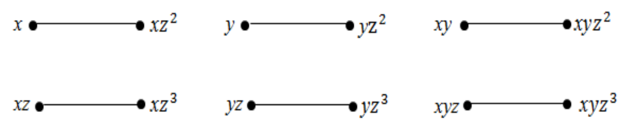


Figure 7 The conjugate graph of the group H_1 ■

There are four properties of the conjugate graph of H_1 given in the following four propositions.

Proposition 3.13 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the independent number of the conjugate graph of H_1, $\alpha(\Gamma_{H_1}^{conj}) = 6$.

Proof: By Definition 2.8 and Figure 7, one of the maximum independent sets is {x, y, xy, xz, yz, xyz}. Since the number of vertices of the maximum independent set is 6, thus, the independent number of H_1 , $\alpha(\Gamma_{H_1}^{conj}) = 6$.■

Proposition 3.14 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the chromatic number of the conjugate graph of H_1 is $\chi(\Gamma_{H_1}^{conj}) = 2$.

Proof: Based on Definition 2.9, the chromatic number $\chi(\Gamma_{H_1}^{conj})$ of the conjugate graph H_1 is 2 since the two adjacent vertices in K_2 have different colors of vertices as shown in Figure 8.

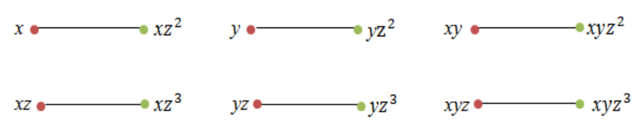


Figure 8 The conjugate graph of the group H_1 with different colors of vertices ■

Proposition 3.15 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the clique number of the conjugate graph of H_1 is $\omega(\Gamma_{H_1}^{conj}) = 2$.

Proof: By Definition 2.10 and Figure 7, the largest complete subgraph in the conjugate graph of H_1 is K_2. Therefore, the clique number, $\omega(\Gamma_{H_1}^{conj}) = 2$.■

Proposition 3.16 Let H_1= <x, y, z | x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2> be a 3-generator group of order 16. Then, the dominating number of the conjugate graph of H_1, denoted by $\gamma(\Gamma_{H_1}^{conj})$ is equal to 6.

Proof: The proof is based on Definition 2.11. The minimum size of vertex which can connect itself and other vertices in $\Gamma_{H_1}^{conj}$ is 6. Thus, the dominating number, $\gamma(\Gamma_{H_1}^{conj})= 6$.■

Theorem 3.8 Let $H_2= <x, y, z \mid x^4= y^2= z^2= 1, [x,z] = [y,z] = 1, x^y= x^3>$ be a 3-generator group of order 16. Then, the conjugate graph of H_2 is $\Gamma_{H_2}^{conj} = \cup_{i=1}^{6} K_2$, which is a union of six complete graphs with two vertices.
Proof: The proof is the same as in Theorem 3.7 by referring to the conjugacy classes of H_2 listed in the proof of Theorem 2.2. ■

Theorem 3.9 Let $H_3= <x, y, z \mid x^4= y^2= z^2= 1, [x, y] = z, [x,z] = [y, z] = 1 >$ be a 3-generator group of order 16. Then, the conjugate graph of H_3 is $\Gamma_{H_3}^{conj} = \cup_{i=1}^{6} K_2$, which is a union of six complete graphs with two vertices.
Proof: The proof is similar to the proof of Theorem 3.7. Refer to the conjugacy classes of H_3 in the proof of Theorem 3.3. ■

Since the conjugate graph of H_2 and H_3 are the same as the conjugate graph of H_1, thus, their properties of graph are also the same as H_1.

3.4 The Conjugacy Class Graph of 3-generator Groups of Order 16 and Their Properties

Theorem 3.10 Let $H_1= <x, y, z \mid x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2>$ be a 3-generator group of order 16. Then, the conjugacy class graph of H_1, $\Gamma_{H_1}^{Cl} = K_6$.
Proof: Since the number of conjugacy classes of H_1 is 10 while the order of the center of H_1 is 4. Thus, the number of the vertices with non-central elements of H_1 is 6. Noticed that all conjugacy classes have size 2, their greatest common divisor (gcd) is also 2. Thus, the graph is complete. These six conjugacy classes formed conjugacy class graph of H_1 as drawn in Figure 9.

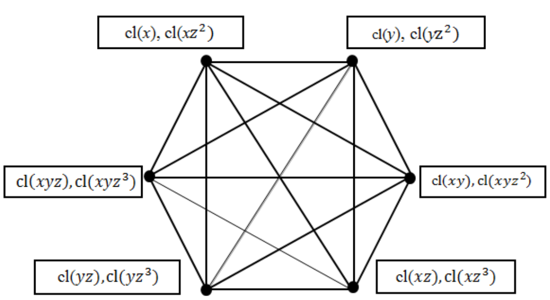


Figure 9 The conjugacy class graph of H_1 is K_6 ■

In the next four propositions, the four properties of the conjugacy class graph are determined.

Proposition 3.19 Let $H_1= <x, y, z \mid x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2>$ be a 3-generator group of order 16. Then, the independent number of the conjugacy class graph of H_1, $\alpha(\Gamma_{H_1}^{Cl}) = 1$.
Proof: Applying Definition 2.8 into Figure 9, the maximum independent set of the conjugacy class graph of H_1 is 1. Since all vertices are connected, the independent number is the number of vertices in a maximum independent set, thus, $\alpha(\Gamma_{H_1}^{Cl}) = 1$. ■

Proposition 3.20 Let $H_1= <x, y, z \mid x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2>$ be a 3-generator group of order 16. Then, the chromatic number of the conjugacy class graph of H_1, $\chi(\Gamma_{H_1}^{Cl}) = 6$.
Proof: Figure 10 shows the graph of K_6 with different colors of vertices. Based on Definition 2.9, the chromatic number $\chi(\Gamma_{H_1}^{Cl})$ of the conjugacy class graph of H_1 is 6. Since all the vertices are connected, six colors are needed. The colors of the six vertices in K_6 are different as shown in Figure 10.

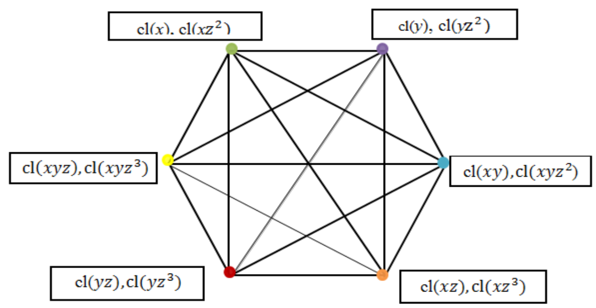


Figure 10 The conjugacy class graph of H_1 with different colors of vertices

Proposition 3.21 Let $H_1= <x, y, z \mid x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2>$ be a 3-generator group of order 16. Then, the clique number of the conjugacy class graph of H_1 is $\omega(\Gamma_{H_1}^{Cl}) = 6$.
Proof: By using Definition 2.10 and since the largest complete subgraph in the conjugate graph of H_1 is K_6. Therefore, the clique number, $\omega(\Gamma_{H_1}^{Cl}) = 6$. ■

Proposition 3.22 Let $H_1= <x, y, z \mid x^2= y^2= z^4= 1, [x,z] = [y,z] = 1, [x,y] = z^2>$ be a 3-generator group of order 16. Then, the dominating number of the conjugacy class graph of H_1, $\gamma(\Gamma_{H_1}^{Cl}) = 1$.
Proof: Using Definition 2.11, the minimum size of vertex which can connect itself with other vertices in K_6 is 1. Thus, the dominating number, $\gamma(\Gamma_{H_1}^{Cl}) = 1$. ■

Theorem 3.11 Let $H_2= <x, y, z \mid x^4= y^2= z^2= 1, [x,z] = [y,z] = 1, x^y= x^3>$ be a 3-generator group of order 16. Then, the conjugacy class graph of H_2 is K_6.
Proof: The proof is the same as the proof in Theorem 3.10. ■

Theorem 3.12 Let $H_3 = \langle x, y, z \mid x^4 = y^2 = z^2 = 1, [x, y] = z, [x,z] = [y, z] = 1 \rangle$ be a 3-generator group of order 16. Then, the conjugacy class graph of H_3 is K_6.

Proof: The proof is the same as the proof in Theorem 2.10. ■

The properties of the conjugacy class graph of H_2 and H_3 are the same as the properties of the conjugacy class graph of H_1.

4.0 CONCLUSION

As a conclusion, the properties of graph for each type of group are simplified in Table 1.

Table 1 The properties of each graph for all 3-generator groups of order 16

H_i (i = 1, 2, 3)				
Type of Graph / **Properties of Graph**	**Commuting graph**	**Non-commuting graph**	**Conjugate graph**	**Conjugacy class graph**
H_i (i = 1, 2, 3)	$\bigcup_{i=1}^{3} K_4$	Not complete (Refer to Figure 3, 5, 6)	$\bigcup_{i=1}^{6} K_2$	K_6
α	3	4	6	1
χ	4	12	2	6
ω	4	3	2	6
γ	3	2	6	1

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Acknowledgement

The first author would like to acknowledge Universiti Teknologi Malaysia, Johor Bahru for her Master Degree and for the Research University Fund (GUP) for vote no 08H07.

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