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RESEARCH ARTICLE

The characterization of regular ordered $\Gamma-$semigroups in terms of $(\in,\in\vee q_k)-$fuzzy $\Gamma-$ideals

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Abstract

The advancement in the fascinating area of fuzzy set theory has become an area of much interest, generalization of the existing fuzzy subsystems of other algebraic structures is very important to tackle more current real life problems. In this paper, we give more generalized form of regular ordered $\Gamma-$semigroups in terms of $(\in,\in\vee q_k)-$fuzzy $\Gamma-$ideals. Particularly, we characterized left regular, right regular, simple and completely regular ordered $\Gamma-$semigroups in terms of this new notion. Some necessary and sufficient conditions for ordered $\Gamma-$semigroup to be completely regular are provided in this paper.

Keywords: Completely regular; $(\in,\in\vee q_k)-$fuzzy Γ-ideals; simple regular, Regular ordered Γ-semigroup.

INTRODUCTION

The inception of the notion of a fuzzy set, introduced by Zadeh (1965), laid the foundations and frame works as well as gave birth to great researches. This notion has been the subject of great attention to many researchers and consequently a series of interesting results have been published. In fact, the concept of ordered semigroups and Γ-semigroups is a generalization of semigroups. Also the ordered Γ-semigroup is a generalization of Γ-semigroups and this concept was introduced by M.K. Sen (1981). The notion of a bi-ideal was first introduced by Good and Hughes (1952). Kazancı and Yamak (2008) introduced the concept of a generalized fuzzy bi-ideal in semigroups where they studied $(\in,\in\vee q)-$fuzzy bi-ideal in a semigroup. The invention of idea of quasi-coincidence of a fuzzy point with a fuzzy set played a very vital role to generate some different types of fuzzy subgroups. A new generalization of fuzzy subgroup types was introduced in earlier papers (Bhakat & Das, 1992, 1996) using the combined notions of "belonginess" and "quasi-coincidence" of fuzzy point and fuzzy set. Hence $(\in,\in\vee q)-$fuzzy subgroup is an important generalization of Rosenfeld's fuzzy subgroup. Changphas (2012) Studied the characterization of when a Γ-semigroup G is an intra-regular semigroup based on bi-ideals, quasi-ideals and left (right) ideals of G. Abbasi and Basar (2013) studied some classical properties of the ordered Γ-semigroup. Jun, Khan, and Shabir (2009) introduced and studied the new sort of fuzzy bi-ideals called $(\alpha,\beta)-$fuzzy bi-ideals and with some interesting characterisations of ordered semigroups in terms of this $(\alpha,\beta)-$fuzzy bi-ideals and also gave the characterisations of regular and intra-regular ordered semigroups using $(\in,\in\vee q)-$fuzzy bi-ideals. Gambo *et al.* presented the new generalization of bi $\Gamma-$ideals in ordered $\Gamma-$semigroups. Ma, Zhan, Davvaz, and Jun (2008) introduce the notion of $(\in,\in\vee q)-$interval-valued fuzzy ideals of BCI-algebras and investigate some of their related properties with some characterization theorems of generalized interval-valued fuzzy ideals. Algebraic structures play a prominent role in mathematics with wide ranging of applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces and the likes. Khan *et al.* (2014) characterized some fuzzy filters and give more generalizations of classifications for different classes of fuzzy Γ-ideals that include regular, weakly regular and intra-regular of ordered Γ-semigroups of the form $(\in,\in\vee q_k)$. Due to these motivating facts, researchers explore and review various concepts and results from the realm of semigroups in broader framework of fuzzy settings. Using the Khan's idea, we come up with this new generalization of bi ideal in Gambo *et al.* (2017) and explore the new form with characterization of this bi $\Gamma-$ideals in terms of regular, completetly regular and simple $\Gamma-$ideals of ordered $\Gamma-$semigroups and also prove that in ordered $\Gamma-$semigroup G, the intersection of bi Γ-ideals of the form $(\in,\in\vee q_k)$ is also a bi $\Gamma-$ideal of the form $(\in,\in\vee q_k)$.

PRELIMINARIES

We give some basic definitions and results that are used in obtaining the results of this paper.

Let G and Γ be two nonempty sets. Then G is called a $\Gamma-$semigroup if G satisfies $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a,b,c\in G$ and $\alpha,\beta\in\Gamma$. A nonempty subset S of a $\Gamma-$semigroup G is called a sub $\Gamma-$semigroup of G if $a\alpha b\in S$ for all $a,b,c\in S$ and $\alpha\in\Gamma$. For any nonempty subsets A,B of G, $A\Gamma B=\{a\alpha b : a\in A, b\in B \text{ and } \alpha\in\Gamma\}$ (M.K. Sen, 1981; M. K. Sen & Saha, 1986). Since the invention of the defintions of $\Gamma-$semigroups then many researches are carried out in this direction of generalizations.

Definition 2.1 (F. M. Khan, Sarmin, & Khan, 2013)

A nonempty subset B of G is called a generalized bi Γ-ideal of G if the following conditions hold for all $a,b\in G$:

(b3) $a\le b\in B\to a\in B,$

(b4) $B\Gamma G\Gamma B\subseteq B.$

Definition 2.2 (Tang & Xie, 2013)

Let A be a fuzzy subset of an ordered semigroup G is called $(\in,\in\vee q_k)-$fuzzy left (resp. right) ideal of G If for all $t\in(0,1]$ and $a,b\in G,$ the following conditions hold: A satisfies the following two conditions, then A is called $(\in,\in\vee q_k)-$fuzzy generalized bi Γ-ideal of G :

(c1) $a\le b\Rightarrow A(a)\ge A(b),$ and

(c2) $b_t\in A, a\in G\Rightarrow (ab)_t\in\vee q_k A\big(resp. (ba)_t\in\vee q_k A\big).$

A fuzzy subset A on an ordered semigroup G is called fuzzy ideal if it is both $(\in,\in\vee q_k)-$fuzzy left and $(\in,\in\vee q_k)-$fuzzy right ideal of $G.$

MAIN RESULTS

In this section, we give some characterizations of regular ordered $\Gamma-$semigroups G in a more generalized form as proven in the following theorems and propositions. Throughout this section G will represent ordered $\Gamma-$semigroup while $\overset{-k}{\lambda}$ reperesent the lower part of the characteristic function and denoting $a^2=a\Gamma a$ unless or otherwise stated.

Definition 3.1 $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideals

Given a fuzzy subset I of $G,$ then λ is called $(\in,\in\vee q_k)$-fuzzy bi Γ-ideal of G for all $a,b,c\in G,\ \alpha,\beta\in\Gamma$ and $t_1,t_2\in(0,1]$ if the following three conditions are satisfied:

(1) $b_t\in I\to a_t\in\vee q_k I$ such that $a\le b,$

(2) $a_{t_1}\in I,\ b_{t_2}\in I\to(a\alpha b)_{\min\{t_1,t_2\}}\in\vee q_k I,$

(3) $a_{t_1}\in I,\ c_{t_2}\in I\to(a\alpha b\beta c)_{\min\{t_1,t_2\}}\in\vee q_k I.$

Example 3.2

Consider the set $G=\{a,b,c,d\}$ and defined a $\Gamma=\{\alpha\}$ with a mapping defined by $G\times\Gamma\times G\to G$ with a relation $\le\ :=\{(a,a),(b,b),(c,c),(d,d),(a,b),(a,c),(c,d)\}.$ The operation defined is given in the following Cayley table:

α	a	b	c	d
a	a	a	a	a
b	b	b	b	b
c	a	a	a	c
d	a	a	a	d

And defined a fuzzy subset λ of G as:

$$\lambda: G\to[0,1]\,|\,x\to\lambda(x)=\begin{cases}0.7, & if\ x=a,\\ 0.6, & if\ x=b,\\ 0.5, & if\ x=c,\\ 0.3, & if\ x=d.\end{cases}$$

Then,
$$U(\lambda;t)=\begin{cases}\varnothing, & if\ 0.7<t\le 1,\\ \{a,b\}, & if\ 0.5<t\le 0.6,\\ \{a,b,c\}, & if\ 0.3<t\le 0.5,\\ G, & if\ 0<t\le 0.3.\end{cases}$$

Then λ is $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal of G for all $t\in\left(0,\frac{1-k}{2}\right]$ and $k=0.4$

The necessary and sufficient conditions for G to be completely regular are provided in the following theorem.

Definition 3.3

An ordered $\Gamma-$semigroup G is said to be left (resp. right) regular if for every $a\in G$ there exist an $x\in G$ and $\alpha,\beta\in\Gamma$ such that $a\le x\alpha a\beta a$ (resp. $a\le a\beta a\alpha x$) and G is called a regular if and only if the left and right regular are idempotent that is $a\le a\alpha x\beta a$ and is completely regular if with $a^2=a\Gamma a$ then $a\in\left(a^2\Gamma G\Gamma a^2\right].$

Theorem 3.4

A $(\in,\in\vee q_k)-$fuzzy bi Γ-ideal of an ordered $\Gamma-$semigroup G is completely regular if and only if $\overset{-k}{\lambda}(a)=\overset{-k}{\lambda}(a^2)$ for every $a\in G.$

Proof: Suppose $a\in G,$ but from the hypothesis G is completely regular, hence $a\in\left(a^2\Gamma G\Gamma a^2\right].$ Thus, $\exists x\in G$ such that $a\le a^2\alpha x\beta a^2.$ So also λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideals of $G.$

$$\begin{aligned}\lambda(a)&\ge\min\left\{\lambda\left(a^2\alpha x\beta a^2\right),\frac{1-k}{2}\right\}\\ &\ge\min\left\{\min\left\{\lambda(a^2),\lambda(a^2),\frac{1-k}{2}\right\},\frac{1-k}{2}\right\}\\ &=\min\left\{\lambda(a^2),\frac{1-k}{2}\right\}\end{aligned}$$

$$\geq \min\left\{\min\left\{\lambda(a),\lambda(a),\frac{1-k}{2}\right\},\frac{1-k}{2}\right\}$$
$$= \min\left\{\lambda(a),\frac{1-k}{2}\right\}.$$

Hence, $\min\left\{\lambda(a),\frac{1-k}{2}\right\} \geq \min\left\{\lambda(a^2),\frac{1-k}{2}\right\} \geq \min\left\{\lambda(a),\frac{1-k}{2}\right\}$ and then we have $\overset{-k}{\lambda}(a) \geq \overset{-k}{\lambda}(a^2) \geq \overset{-k}{\lambda}(a)$ which shows that $\overset{-k}{\lambda}(a) = \overset{-k}{\lambda}(a^2)$.

$(\Leftarrow)$ Given $a \in G$ and with bi $\Gamma-$ideal B and from the hypothesis G it is completely regular then we have $B(a^2) = \left(a^2 \cup a^4 \cup a^2\Gamma G\Gamma a^2\right]$ of G generated by a^2.

Hence,

$$\overset{-k}{\chi}_{B(a^2)} : G \to [0,1] \mid x \mapsto \overset{-k}{\chi}_{B(a^2)}(a) = \begin{cases} \frac{1-k}{2}, & \text{if} \quad a \in B(a^2), \\ 0, & \text{if} \quad x \notin B(a^2), \end{cases}$$

is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal~~s~~ of G. From the hypothesis, $\overset{-k}{\chi}_{B(a^2)}(a^2) = \frac{1-k}{2}$ and therefore, $\overset{-k}{\chi}_{B(a^2)}(a) = \frac{1-k}{2}$. Hence, $a \in B(a^2)$ thus $a \leq a^2$ or $a \leq a^4$ for some $x \in G$, $a \leq a^2\alpha x\beta a^2$ if $a \leq a^2$, then

$$a \leq a^2 \leq a\alpha a \leq a^2\alpha a^2 = a\alpha a\alpha a^2$$

$$a\alpha a\alpha a^2 \leq a^2\alpha a\alpha a^2 \in a^2\Gamma G\Gamma a^2$$ and $a \in \left(a^2\Gamma G\Gamma a^2\right]$.

In the same way, for $a \leq a^4$ or $a \leq a^2\alpha x\beta a^2$, we have $a \in \left(g^2\alpha G\beta h^2\right]$ for some $g,h \in G$, and a,b Î G which shows that G is completely regular. ■

The conditions for G to be semilattice of left and right simple semigroups are given in the following theorem.

Definition 3.5

An ordered $\Gamma-$semigroup G is said to be left (resp. right) simple if G has no proper left (rep. right) $\Gamma-$ideals. Similarly, An ordered $\Gamma-$semigroup G is said to be fuzzy simple if every fuzzy $\Gamma-$ideal of G is a constant function.

Theorem 3.6

Given any $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal of an ordered $\Gamma-$semigroup G. Then $\overset{-k}{\lambda}(a) = \overset{-k}{\lambda}(a^2)$ and $\overset{-k}{\lambda}(a\alpha b) = \overset{-k}{\lambda}(b\alpha a)$ for some $a,b \in G$ and $\alpha \in \Gamma$ if and only if G is a semilattice of left and right simple $\Gamma-$.semigroups.

Proof: Suppose λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal, hence from the hypothesis there exists a family $\{G_i : i \in M\}$ with M sublattice of left and right simple subsemigroup of G such that for two disjoint in the sublattice then $G = \bigcup_{i\in M} G_i$, $G_iG_j \subseteq G_{ij} \forall i,j \in M$. Next we show that $\overset{-k}{\lambda}(a) \geq \overset{-k}{\lambda}(a^2)\ \forall a \in G$. Since $A \subseteq G$ then it implies $a \in \left(a^2\Gamma G\Gamma a^2\right]$. Let $a \in G$ then, there exists a semilattice M such that $a \in G_i$. But G_i is both left and right simple then it shows that $(G_i\Gamma a] = G_i$ and $(a\Gamma G_i] = G_i$, hence, $G_i = (a\Gamma G_i] = (a\Gamma(G_i\Gamma a]] = (a\Gamma G_i\Gamma a]$ but $a \in (a\Gamma G\Gamma a]$, thus, $\exists x \in G_i$ such that $a \leq a\alpha x\beta a$. Since $x \in G_i$, $x \leq a\alpha y\beta a$ for some $y \in G_i$. Thus,

$$a \leq a\alpha x\beta a \leq a\alpha(a\alpha y\beta a)\beta a$$

$$a \leq a^2\alpha y\beta a^2 \in a\Gamma G_i\Gamma a \subseteq a\Gamma G\Gamma a$$ and $a \in \left(a^2\Gamma G\Gamma a^2\right]$.

Let $a,b \in G$ and $\alpha \in \Gamma$ from the above $\overset{-k}{\lambda}(a\alpha b) \geq \overset{-k}{\lambda}((a\alpha b)^2) = \overset{-k}{\lambda}((a\alpha b)^4)$. Moreover, it can be written as:

$$(a\alpha b)^4 = (a\alpha b)^2(a\alpha b)^2 = (a\alpha b)(a\alpha b)(a\alpha b)(a\alpha b).$$
$$= (a\alpha b)(a\alpha b)(a\alpha b)(a\alpha b).$$

$$= (a\alpha b\alpha a)(b\alpha a\alpha b\alpha a\alpha b) \in B(a\alpha b\alpha a)B(b\alpha a\alpha b\alpha a\alpha b).$$
$$\subseteq \left(B(a\alpha b\alpha a)B(b\alpha a\alpha b\alpha a\alpha b)\right].$$
$$= \left(B(b\alpha a\alpha b\alpha a\alpha b)B(a\alpha b\alpha a)\right].$$
$$= \left(B(b\alpha a\alpha b\alpha a\alpha b)\left(B(a\alpha b\alpha a)^2\right]\right].$$
$$= \left(B(b\alpha a\alpha b\alpha a\alpha b)\left(B(a\alpha b\alpha a)B(a\alpha b\alpha a)\right]\right].$$
$$= \left(B(b\alpha a\alpha b\alpha a\alpha b)B(a\alpha b\alpha a)B(a\alpha b\alpha a)\right]$$

$(by((A]] = (A])$.

$$\subseteq \left(((b\alpha a\alpha b\alpha a\alpha b)G(a\alpha b\alpha a)(a\alpha b\alpha a)(a\alpha b\alpha a)\right]\right].$$
$$\subseteq \left(b\alpha a\alpha b\alpha a\alpha b\alpha G\alpha a\alpha b\alpha a\right].$$

Then, $(a\alpha b)^4 \leq (b\alpha a\alpha b\alpha a\alpha b)z(a\alpha b\alpha a)$ for some $z \in G$.

From the hypothesis G is completely regular, hence $a \in \left(a^2\Gamma G\Gamma a^2\right]$. Thus, $\exists x \in G$ such that $a \leq a^2\alpha x\beta a^2$. So also λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideals of G. Since λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideals of G.

Hence,

$$\lambda((a\alpha b)^4) \geq \min\left\{(b\alpha a\alpha b\alpha a\alpha b)z(a\alpha b\alpha a),\frac{1-k}{2}\right\}$$
$$= \min\left\{\min\left\{\lambda(b\alpha a),\lambda(b\alpha a),\frac{1-k}{2}\right\},\frac{1-k}{2}\right\}$$
$$= \min\left\{\lambda(b\alpha a),\frac{1-k}{2}\right\}.$$

Therefore, $\min\left\{\lambda((a\alpha b)^4),\frac{1-k}{2}\right\} \geq \min\left\{\lambda(a\alpha b),\frac{1-k}{2}\right\}$.

$\min\left\{\lambda((a\alpha b)^4),\frac{1-k}{2}\right\} \geq \min\left\{\lambda(b\alpha a),\frac{1-k}{2}\right\}$ and thus, $\overset{-k}{\lambda}((a\alpha b)^4) \geq \overset{-k}{\lambda}(b\alpha a), \overset{-k}{\lambda}(b\alpha a) = \overset{-k}{\lambda}((b\alpha a)^2)$ $\overset{-k}{\lambda}((a\alpha b)^4) \geq \overset{-k}{\lambda}(b\alpha a)$.

In a similar way, we have $\overset{-k}{\lambda}(b\alpha a) = \overset{-k}{\lambda}(a\alpha b)$.

$(\Leftarrow)$ Conversely, G is a semilattice, then it is enough to show that every left (resp. right) ideal of G is an ideal of G. Suppose L is a left ideal of G and let $a \in L, \alpha \in \Gamma$ and let $t \in G$. Since L is a left ideal of G, then

$$\overset{-k}{\chi} : G \to [0,1] \mid t\alpha a \mapsto \overset{-k}{\chi}_L(t\alpha a) = \begin{cases} \frac{1-k}{2}, & \text{if} \quad t\alpha a \in L, \\ 0, & \text{if} \quad t\alpha a \notin L. \end{cases}$$

Hence, is a $(\in, \in \vee q_k)-$fuzzy left $\Gamma-$ideals of G and from the hypothesis, we have $\overset{-k}{\chi}_L(t\alpha a) = \overset{-k}{\chi}_L(a\alpha t)$. Hence, $t\alpha a \in G\Gamma L \subseteq L$, we have $\overset{-k}{\chi}_L(t\alpha a) = \frac{1-k}{2}$, thus $t\alpha a \in L$. That is $L\Gamma G \subseteq L$, thus L is a right left $\Gamma-$ideals of G. If $a^2 \in L$, then $\overset{-k}{\chi}_L(a^2) = \overset{-k}{\chi}_L(a)$.

This implies $\overset{-k}{\chi}_L(a) = \frac{1-k}{2}$ and so $a \in L$. Hence the proof.

■

The result below shows that the family of intersection of $(\in, \in \vee q_k)-$fuzzy bi Γ-ideal is also a $(\in, \in \vee q_k)-$fuzzy bi Γ-ideal of G.

Proposition 3.7

Let $\{\lambda_i : i \in I\}$ be a family of a $(\in, \in \vee q_k)-$fuzzy bi $\Gamma-$ideal of an ordered $\Gamma-$semigroup G, then the intersection over $i \in I$ of the family is also an $(\in, \in \vee q_k)-$fuzzy bi $\Gamma-$ideal of an ordered $\Gamma-$semigroup G.

Proof: Assume $\{\lambda_i\}_{i \in I}$ is a family of $(\in, \in \vee q_k)-$fuzzy bi $\Gamma-$ideal of G, and suppose $\exists a, b \in G, \alpha, \beta \in \Gamma$ such that $a \le b$ then,

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a) = \min_{i \in I} \lambda_i(a) \ge \min\Big\{\lambda_{\substack{i \\ i \in I}}(b), \frac{1-k}{2}\Big\}.$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a) = \min\Big\{\Big(\bigcap_{i \in I} \lambda_i\Big)(b), \frac{1-k}{2}\Big\}$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b) = \min_{i \in I} \lambda_i(a\alpha b) \ge \min\Big\{\lambda_{\substack{i \\ i \in I}}(a), \lambda_{\substack{i \\ i \in I}}(b), \frac{1-k}{2}\Big\}$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b) = \min\Big\{\min_{i \in I}\Big\{\lambda_i(a), \frac{1-k}{2}\Big\}, \min_{i \in I}\Big\{\lambda_i(b), \frac{1-k}{2}\Big\}\Big\}$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b) = \min\Big\{\Big(\bigcap_{i \in I} \lambda_i\Big)(a), \Big(\bigcap_{i \in I} \lambda_i\Big)(b), \frac{1-k}{2}\Big\}.$$

Given $a, b, c \in G$ and $\alpha, \beta \in \Gamma$ then we have

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b\beta c) = \min_{i \in I} \lambda_i(a\alpha b\beta c)$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b\beta c) \ge \min\Big\{\lambda_{\substack{i \\ i \in I}}(a), \lambda_{\substack{i \\ i \in I}}(c), \frac{1-k}{2}\Big\}$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b\beta c) \ge \min\Big\{\min_{i \in I}\Big\{\lambda_i(a), \frac{1-k}{2}\Big\}, \min_{i \in I}\Big\{\lambda_i(c), \frac{1-k}{2}\Big\}\Big\}$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b\beta c) \ge \min\Big\{\Big(\bigcap_{i \in I} \lambda_i\Big)(a), \Big(\bigcap_{i \in I} \lambda_i\Big)(c), \frac{1-k}{2}\Big\}$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b\beta c) = \min_{i \in I} \lambda_i(a\alpha b\beta c)$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b\beta c) \ge \min\Big\{\lambda_{\substack{i \\ i \in I}}(a), \lambda_{\substack{i \\ i \in I}}(c), \frac{1-k}{2}\Big\}$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b\beta c) \ge \min\Big\{\min_{i \in I}\Big\{\lambda_i(a), \frac{1-k}{2}\Big\}, \min_{i \in I}\Big\{\lambda_i(c), \frac{1-k}{2}\Big\}\Big\}$$

$$\Big(\bigcap_{i \in I} \lambda_i\Big)(a\alpha b\beta c) \ge \min\Big\{\Big(\bigcap_{i \in I} \lambda_i\Big)(a), \Big(\bigcap_{i \in I} \lambda_i\Big)(c), \frac{1-k}{2}\Big\}$$

Thus, $\bigcap_{i \in I} \lambda_i$ is a $(\in, \in \vee q_k)-$fuzzy bi $\Gamma-$ideal of G. ■

Proposition 3.8

Given two fuzzy bi $\Gamma-$ideal of the type $(\in, \in \vee q_k)$ say R and S of G. Then $\min\{R, S\}$ is also a $(\in, \in \vee q_k)-$fuzzy bi $\Gamma-$ideal of G.

Proof: Suppose R and S are fuzzy bi $\Gamma-$ideal of the type $(\in, \in \vee q_k)$. Let $a, b \in G$ and $\alpha \in \Gamma$ with $a \le b$, then

$$(R \wedge^k S)^-(a) = \min\Big\{R(a), S(a), \frac{1-k}{2}\Big\}$$

$$(R \wedge^k S)^-(a) \ge \min\Big\{\min\Big\{R(b), \frac{1-k}{2}\Big\}, \min\Big\{S(b), \frac{1-k}{2}\Big\}, \frac{1-k}{2}\Big\}$$

$$(R \wedge^k S)^-(a) \ge \min\Big\{\min\Big\{R(b), S(b), \frac{1-k}{2}\Big\}, \frac{1-k}{2}\Big\}$$

$$(R \wedge^k S)^-(a) \ge \min\Big\{(R \wedge^k S)^-(b), \frac{1-k}{2}\Big\} \quad \text{for all } a, b \in G$$

with $a \le b$.

To prove the second condition, suppose $a, b \in G$, $\alpha \in \Gamma$ hence,

$$(R \wedge^k S)^-(a\alpha b) = \min\Big\{R(a\alpha b), S(a\alpha b), \frac{1-k}{2}\Big\},$$

$$(R \wedge^k S)^-(a\alpha b) \ge \min\Big\{\min\Big\{R(a), S(a), \frac{1-k}{2}\Big\}, \Big\{R(b), S(b), \frac{1-k}{2}\Big\}, \frac{1-k}{2}\Big\},$$

$$(R \wedge^k S)^-(a\alpha b) \ge \min\Big\{(R \wedge^k S)^-(a), (R \wedge^k S)^-(b), \frac{1-k}{2}\Big\},$$

Suppose $a, b, c \in G$, $\alpha, \beta \in \Gamma$ hence,

$$(R \wedge^k S)^-(a\alpha b\beta c) = \min\Big\{R(a\alpha b\beta c), S(a\alpha b\beta c), \frac{1-k}{2}\Big\},$$

$$(R \wedge^k S)^-(a\alpha b\beta c) \ge \min\Big\{\min\Big\{R(a), S(a), \frac{1-k}{2}\Big\}, \Big\{R(c), S(c), \frac{1-k}{2}\Big\}, \frac{1-k}{2}\Big\},$$

$$(R \wedge^k S)^-(a\alpha b\beta c) \ge \min\Big\{(R \wedge^k S)^-(a), (R \wedge^k S)^-(c), \frac{1-k}{2}\Big\}.$$

Hence, $(R \wedge^k S)^-$ is a $(\in, \in \vee q_k)-$fuzzy bi $\Gamma-$ideal of G. ■

CONCLUSION

Many classical notions and results of the theory of semigroups have been extended and generalized. Having this motivation, it is naturally significant to generalize the results of semigroups to Γ-semigroups as Γ-semigroups is a generalization of semigroups. In this paper we characterized regular ordered semigroup of the type $(\in, \in \vee q_k)$ in ordered $\Gamma-$semigroup and give some necessary conditions to be completely regular and conditions of semilattice with both left and right simple on ordered $\Gamma-$semigroup are proved and finally, the family of intersection of $(\in, \in \vee q_k)-$fuzzy bi Γ-ideal are considered.

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