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*Full Length Research Paper*

# The nonabelian tensor square ($G\otimes G$) of symplectic groups and projective symplectic groups

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**The determination of $G\otimes G$ for linear groups was mentioned as an open problem by Brown et al. (1987). Hannebauer focused on the nonabelian tensor square of $SL(2, q)$, $PSL(2, q)$, $GL(2, q)$ and $PGL(2, q)$ for all $q \geq 5$ and $q = 9$ in a contribution of 1990. The aim of this paper is to determine the nonabelian tensor square $G\otimes G$ for these groups up to isomorphism by the use of the commutator subgroup and Schur multiplier.**

**Key words:** 2000 mathematics subject classification, Primary, 20J99, secondary, 20J06, 19C09, and phrases, commutator subgroup, Schur multiplier, nonabelian tensor square.

## INTRODUCTION

For a group $G$, the nonabelian tensor square $G\otimes G$ is the group generated the symbols $g\otimes h$ and defined by the relations

$$gg'\otimes h=({}^{g}g'\otimes{}^{g}h)(g\otimes h),\ g\otimes hh'=(g\otimes h)({}^{h}g\otimes{}^{h}h').$$

for all $g, g', h, h' \in G$, where ${}^{g}g'=gg'g^{-1}$. The nonabelian tensor square is a special case of the nonabelian tensor product which has its origin in homotopy theory and was introduced by Brown and Loday (1984, 1987). The exterior square $G\wedge G$ is obtained by imposing the additional relations $g\otimes g=1_{\otimes}$ for all $g\in G$ on $G\otimes G$. The commutator map induces homomorphisms $\kappa : g\otimes h \in G\otimes G \rightarrow \kappa(g\otimes h)= [g, h] \in G'$ and $\kappa' : g\wedge h \in G\wedge G \rightarrow \kappa'(g\wedge h)= [g, h] \in G'$ and $J_2(G)=\ker(\kappa)$. The results of Brown and Loday (1984, 1987) give the commutative diagram given as in Figure 1 with exact rows and central extensions as columns, where $G'$ is the commutator subgroup of $G$, $M(G)$ is the multiplicator of $G$ and $\Gamma$ is Whitehead's quadratic function (Whitehead, 1950).

The determination of $G\otimes G$ for $G = GL(2, q)$ and other linear groups was mentioned as an open problem by Brown et al. (1987) and was pointed out in a more general form in (Kappe, 1999). In the latter paper, there is a list of open problems on the computation of the nonabelian tensor square of finite groups. Among these, there is the problem to find an explicit value of the nonabelian tensor square of linear groups. Hannebauer (1990) determined the nonabelian tensor square of $SL(2, q)$, $PSL(2, q)$, $GL(2, q)$ and $PGL(2, q)$ for all $q \geq 5$ and $q = 9$. Later, Erfanian et al. (2008) determined the nonabelian tensor square of $SL(n, q)$, $PSL(n, q)$, $GL(n, q)$ and $PGL(n, q)$ for all $n, q \geq 2$. This work continues the investigations in the same area, focusing on symplectic groups and projective symplectic groups. We also determine this structure for special linear groups and projective special linear groups, but the method used for computing this structure is different from the method that has been used by Erfanian et al. (2008). As an application we determine the Schur multiplier of these groups. The epicentre and exterior centre of these groups are also determined in the sense of Beyl et al. (1979).

We will prove the following two main theorems:

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