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MATHEMATIQUES

Algèbre

CONJUGACY CLASSES AND COMMUTATIVITY DEGREE OF METACYCLIC 2-GROUPS

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Abstract

In this paper we obtain the exact number of conjugacy classes and commutativity degree of metacyclic 2-groups. In particular, we describe the number of conjugacy classes both in the split and non-split case.

Key words: conjugacy class, commutativity degree, metacyclic group, nilpotency class

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1. Introduction. All groups considered in this research are finite. In the last years many authors achieved significant results but only on the lower and upper bound of the number of conjugacy classes for finite groups including the *p*-groups $[^{1-4}]$. Among these authors, for instance, LOPEZE $[^3]$, in Theorem 1 shows that if A is a maximal abelian subgroup of finite nilpotent group G and $|A| = p^{\alpha}$, then there is an integer $k \geq 0$ such that $k(G) = p^{2\alpha-m} + p^{\beta}(p+1)(p^{m-\alpha}-1)/p^{m-\alpha} + k(p^2-1)(p-1)/p^{m-\alpha}$, where $|G| = p^m$ and $|Z(G)| = p^{\beta}$. For k > 0, this formula is an upper bound for G and does not determine the exact number of conjugacy classes of subgroups of metacyclic *p*-groups $[^{5-7}]$. For example, in $[^7]$ Theorem 1.3, it was shown that if G is any finite split metacyclic *p*-group for an odd prime *p*, that is, $G = H \ltimes K$ for subgroups H and K, and if $|H| = p^{\alpha}$ and $|K| = p^{\alpha+\beta}$, then there exist exactly $(\beta - \alpha + 1)(p^{\alpha+1} - 1)/(p-1) + 4\sum_{i=0}^{\alpha-1} p^i(\alpha+i)$ conjugacy classes of subgroups of G. HETHELYI and KULSHAMMER ($[^6]$ Proposition 3.2) proved if G is a metacyclic *p*-group where *p* is an odd prime number and $|G'| = p^n$,