NEW TYPES OF GENERALIZED FUZZY BI Γ-IDEALS IN ORDERED Γ-SEMIGROUPS

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ABSTRACT.: *In this paper, we introduce a new concept of fuzzy bi Γ-ideal of an ordered Γ-semigroup G called a (λ,θ)-fuzzy bi Γ-ideal of G. Fuzzy bi Γ-ideals of type (λ,θ) are the generalization of ordinary fuzzy bi Γ-ideals of an ordered Γ-semigroup G. A new characterization of ordered Γ-semigroups in terms of a (λ,θ)-fuzzy bi Γ-ideal is given. We show that $U(\mu,t)$ is a bi Γ-ideal if and only if the fuzzy subset μ is a (λ,θ)-fuzzy bi Γ-ideal of G for all $t\in(\lambda,\theta]$. Similarly, A is a bi Γ-ideal if and only if the characteristic function μ_A of A is a (λ,θ)-fuzzy bi Γ-ideal of G. Finally, the characterization of regular ordered Γ-semigroups in terms of (λ,θ)-fuzzy bi Γ-ideal is given.*

Keywords: Ordered Γ-semigroups; regular ordered Γ-semigroups; fuzzy sets; fuzzy bi Γ-*ideal*; (λ,θ)-fuzzy bi Γ-*ideal.*

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1. INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines like formal languages, fuzzy automata, coding theory, theoretical physics and control engineering. In this regards, ordered semigroups and related structures are extensively investigated in terms of fuzzy notions (see [1]). In 1965, Zadeh [2] introduced a pioneering paper on fuzzy sets. Due to the increasing number of high-quality research articles on fuzzy sets and related concepts that have been published in different indexed journals, workshops and conferences, researchers are motivated to study fuzzy set theory. The notion of fuzzy sets in algebraic structures has been initiated by Rosenfeld [3] in 1971. Kehayopulu and Tsingelis **[4]** investigated fuzzy bi-ideals in ordered semigroups and discussed some important results of ordered semigroups in terms of their fuzzy bi-ideals (Also see [5, 6]). Rosenfeld's idea of fuzzy subgroup [3] has played a sparking role for researchers to investigate similar type of generalizations of the existing fuzzy subsystems of other algebraic structures. With this objective in view, Sen and Saha [7] initiated the concept of a Γ-semigroup, and established a relation between regular Γ-semigroup and Γ-group (see also [8]). Kwon and Lee introduced the concept of Γ-ideals and weakly prime Γ-ideals in ordered Γ-semigroups in [9], and established the basic properties of ordered Γ-semigroups in terms of weakly prime Γ-ideals. In [10], Iampan gave the concept of (0-)minimal and maximal ordered bi-ideals in ordered Γ-semigroups, and give some characterizations of (0-)minimal and maximal ordered bi-ideals in ordered Γ-semigroups (also see [11, 12, 13, 14]). Davvaz et al. [15] studied fuzzy Γ-hypernear-rings and investigated some important results in terms of this notion. Khan *et al*. [16] characterized ordered Γ-semigroups in terms of fuzzy interior Γ-ideals and discussed some interesting properties of fuzzy interior Γ-ideals in ordered Γ semigroups.

The concept of fuzzy subfield with thresholds has been given by Yuan [17]. Yao [18] discussed a (λ,μ)-fuzzy subrings and a (λ,μ)-fuzzy ideals. In [19], (λ,μ)-fuzzy ideals in semigroups are studied (also see **[**20, 21]).

In this paper, we deal with a more generalized form of the fuzzy bi Γ-ideals. We introduce the concept of a (λ,θ)-fuzzy bi-ideal of G. We show that every fuzzy bi Γ-ideal is a (λ,θ)-fuzzy bi Γ-ideals of G. We also discuss that the level subset $U(\mu;t)(\phi)$ is a bi Γ-ideal if and only if μ is a (λ,θ)-fuzzy bi Γ-ideal of G for all $t\in(\lambda,\theta]$. We prove that a non-empty subset A of G is a bi Γ-ideal if and only if a characteristic function μ_A of A is a (λ,θ)-fuzzy bi Γ-ideal of G. Finally, regular ordered semigroups are also characterized in terms of this new notion.

2. Preliminaries

Let $G=\{x,y,z,...\}$ and $\Gamma=\{\alpha,\beta,\gamma,...\}$ be any two non-empty sets. If there is a function $G\times\Gamma\times G\to G$ such that $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x,y,z\in G$ and $\alpha,\beta\in\Gamma$ then G is called *a* Γ*-semigroup* [7]. By an *ordered* Γ*-semigroup* we mean a Γ-semigroup G at the same time a poset $(G,\le)$ satisfying the following condition:

$a\le b\Rightarrow a\alpha x\le b\alpha x$ and $x\beta a\le x\beta b$ for all $a,b,x\in G$ and $\alpha,\beta\in\Gamma$.

For $A\subseteq G$, we denote $(A]:=\{t\in G\,|\,t\le h$ for some $h\in A\}$ If $A=\{a\}$, then we write $(a]$ instead of $(\{a\}]$.

For $A, B \subseteq G$, we denote

$A\Gamma B := \{a\alpha b \mid a \in A, b \in B, \alpha \in \Gamma\}$.

Throughout the paper G will denote the ordered Γ-semigroup unless otherwise stated.

A non-empty subset A of an ordered Γ-semigroup G is called a *left* (resp. *right*) Γ-*ideal* [7] of G if it satisfies

(i) $(\forall a, b \in G)(\forall b \in A)(a \le b \Rightarrow a \in A)$,

(ii) $G\Gamma A \subseteq A$ (resp. $A\Gamma G \subseteq A$) .

If A is both left Γ-ideal and a right Γ-ideal of G then A is called Γ-ideal of G.

A non-empty subset A of an ordered Γ-semigroup G is called a *bi* Γ-*ideal* [14] of G if it satisfies

(i) $(\forall a, b \in G)(\forall b \in A)(a \le b \Rightarrow a \in A)$,

(ii) $A\Gamma G\Gamma A \subseteq A$.

An ordered Γ-semigroup G is *regular* [16] if for every $a \in G$ and $\alpha, \beta \in \Gamma$ there exists $x \in G$ such that $a \le a\alpha x\beta a$ or equivalently, (i) $a \in (a\Gamma G\Gamma a]$ for all $a \in G$ and (ii) $A \subseteq (A\Gamma G\Gamma A]$ for all $A \subseteq G$. An ordered Γ-semigroup G is called a *left* (resp. *right*) *simple* [16] if for every left (resp. right) Γ-ideal A of G we have $A = G$. An ordered Γ-semigroup G is called *simple* if it is both left and right simple.

Now we recall some fuzzy logic concepts.

A *fuzzy subset* μ of a universe X is a function from X into a unit closed interval $[0,1]$ of real numbers, i.e., $\mu : X \to [0,1]$.

For a non-empty subset A of G, the *characteristic function* μ_A of A is a fuzzy subset of G defined by

$$\mu_A = \begin{cases} 1, \text{ if } x \in A, \\ 0, \text{ if } x \notin A. \end{cases}$$

If A is a non-empty subset of G and $a \in G$. Then,

$A_a = \{(y, z) \in G \times G \mid a \le y\alpha z \text{ where } \alpha \in \Gamma\}$.

If μ and F are two fuzzy subsets of G. Then the product $\mu \circ F$ of μ and F is defined by:

$$\mu \circ F : G \to [0,1] \mid a \mapsto (\mu \circ F)(a) = \begin{cases} \bigvee_{(y,z) \in A_a} (\mu(y) \wedge F(z)) \text{ if } A_a \ne \phi \\ 0 \qquad\qquad \text{if } A_a = \phi. \end{cases}$$

2.1 Definition

A fuzzy subset μ of an ordered Γ-semigroup $(G, \Gamma, \le)$ is called a *fuzzy left* (resp. *right*) Γ-*ideal* of G if it satisfies

(i) $(\forall x, y \in G)(x \le y \Rightarrow \mu(x) \ge \mu(y))$.

(ii)

$(\forall x, y \in G)(\alpha \in \Gamma)(\mu(x\alpha y) \ge \mu(y))$(resp. $\mu(x\alpha y) \ge \mu(x)$).

If μ is both a fuzzy left and fuzzy right Γ-ideal of G then it is called a fuzzy Γ-ideal of G.

2.2 Definition

A fuzzy subset μ of an ordered Γ-semigroup $(G, \Gamma, \le)$ is called a *fuzzy bi* Γ-*ideal* of G if it satisfies

(i) $(\forall x, y \in G)(\alpha \in \Gamma)(\mu(x\alpha y) \ge \mu(x) \wedge \mu(y))$,

(ii)

$(\forall x, y, z \in G)(\beta, \gamma \in \Gamma)(\mu(x\beta y\gamma z) \ge \mu(x) \wedge \mu(z))$,

(iii) $(\forall x, y \in G)(x \le y \Rightarrow \mu(x) \ge \mu(y))$.

2.3 Lemma [14]

A non-empty subset A of an ordered Γ-semigroup G is a bi Γ-ideal if and only if a characteristic function μ_A of A is a fuzzy bi Γ-ideal of G.

For a fuzzy subset μ of G and $t \in (0,1]$, the *crisp set* $U(\mu; t) := \{x \in G \mid \mu(x) \ge t\}$ is called the *level subset* of μ.

2.4 Theorem [14].

A fuzzy subset μ of G is a fuzzy bi Γ-ideal of G if and only if each non-empty level subset $U(\mu; t)$ is a bi Γ-ideal of G for all $t \in (0,1]$.

3. (λ, θ)-fuzzy bi Γ-ideals

In this section, we introduce some new types of fuzzy bi Γ-ideals in ordered Γ-semigroups called (λ, θ)-fuzzy bi Γ-ideals and investigate different characterization theorems in term of this notion.

In what follows, let $\lambda, \theta \in [0,1]$ be such that $0 \le \lambda < \theta \le 1$. Both λ and θ are arbitrary but fixed.

Now, we introduce the concept of a (λ, θ)-fuzzy bi Γ-ideal of an ordered Γ-semigroup G in the following definition.

3.1 Definition

A fuzzy subset μ of G is called a (λ, θ)-*fuzzy left* (resp. *right*) Γ-*ideal* of G if it satisfies the following conditions:

(1) $(\forall x, y \in G$ with $x \le y)(\max\{\mu(x), \lambda\} \ge \min\{\mu(y), \theta\})$.

(2) $(\forall x, y \in G)(\forall \alpha \in \Gamma)(\mu(x\alpha y) \vee \lambda\} \ge \mu(y) \wedge \theta)$ (resp. $\mu(x\alpha y) \vee \lambda\} \ge \mu(x) \wedge \theta$)

3.2 Definition

A fuzzy subset μ of G is called a (λ, θ)-*fuzzy bi* Γ-*ideal* of G if it satisfies the following conditions:

(B1) $(\forall x, y \in G$ with $x \le y)(\max\{\mu(x), \lambda\} \ge \min\{\mu(y), \theta\})$.

(B2) $(\forall x, y \in G)(\forall \alpha \in \Gamma)(\mu(x\alpha y) \vee \lambda\} \ge \mu(x) \wedge \mu(y) \wedge \theta)$.

(B3) $(\forall x, y, z \in G)(\forall \beta, \gamma \in \Gamma)$
$(\mu(x\beta y\gamma z) \vee \lambda\} \geq \mu(x) \wedge \mu(z) \wedge \theta)$

3.3 Theorem

If a fuzzy subset μ of an ordered Γ-semigroup G is a (λ,θ)-fuzzy bi Γ-ideal of G, then the set $\mu_{\bar{\lambda}}$ is a bi Γ-ideal of G.

Proof Assume that μ is a (λ,θ)-fuzzy bi Γ-ideal of G. Let $x, y \in G$ such that $x \leq y$, $y \in \mu_{\bar{\lambda}}$. Then $\mu(y) > \lambda$. Since μ is a (λ,θ)-fuzzy bi Γ-ideal therefore $\mu(x) \vee \lambda \geq \mu(y) \wedge \theta > \lambda \wedge \theta = \lambda$. Hence $\mu(x) > \lambda$. It implies that $x \in \mu_{\bar{\lambda}}$. If there exist $x, y \in G$ and $\alpha \in \Gamma$ such that $x, y \in \mu_{\bar{\lambda}}$. Then $\mu(x) > \lambda$ and $\mu(y) > \lambda$. Since μ is a (λ,θ)-fuzzy bi Γ-ideal of G, then $\mu(x\alpha y) \vee \lambda \geq \mu(x) \wedge \mu(y) \wedge \theta > \lambda \wedge \lambda \wedge \theta = \lambda$. Hence $\mu(x\alpha y) > \lambda$. It shows that $x\alpha y \in \mu_{\bar{\lambda}}$ for all $x, y \in G$ and $\alpha \in \Gamma$. let $x, y, z \in G$ and $\alpha, \beta \in \Gamma$ such that $x, z \in \mu_{\bar{\lambda}}$. Then $\mu(x) > \lambda$ and $\mu(z) > \lambda$. Since μ is a (λ,θ)-fuzzy bi Γ-ideal of G, then $\mu(x\alpha y\beta z) \vee \lambda \geq \mu(x) \wedge \mu(z) \wedge \theta > \lambda \wedge \lambda \wedge \theta = \lambda$. Hence $\mu(x\alpha y\beta z) > \lambda$ It shows that $x\alpha y\beta z \in \mu_{\bar{\lambda}}$ for all $x, y, z \in G$ and $\alpha, \beta \in \Gamma$. Hence $\mu_{\bar{\lambda}}$ is a bi Γ-ideal of G.

3.4 Theorem

A non empty subset A of an ordered Γ-semigroup G is a bi Γ-ideal of G if and only if a fuzzy subset μ of G defined as follows:

$$\mu(x) = \begin{cases} \geq \theta \text{ for all } x \in A, \\ \lambda \text{ for all } x \notin A, \end{cases}$$

is a (λ,θ)-fuzzy bi Γ-ideal of G.

Proof Let A be a bi Γ-ideal of G. If there exist $x, y \in G$ such that $x \leq y$ and $y \in A$ then $x \in A$. Hence $\mu(x) \geq \theta$. Therefore $\mu(x) \vee \lambda \geq \theta = \mu(y) \wedge \theta$. If $y \notin A$ then $\mu(y) \wedge \theta = \lambda$. Thus $\mu(x) \vee \lambda \geq \lambda = \mu(y) \wedge \theta$. let $x, y \in G$ and $\alpha \in \Gamma$ such that $x, y \in A$. Hence $\mu(x) \geq \theta$ and $\mu(y) \geq \theta$. Therefore $\mu(x\alpha y) \vee \lambda \geq \theta = \mu(x) \wedge \mu(y) \wedge \theta$. Now if $x \notin A$ or $y \notin A$ then $\mu(x) \wedge \mu(y) \wedge \theta = \lambda$. Thus $\mu(x\alpha y) \vee \lambda \geq \lambda = \mu(x) \wedge \mu(y) \wedge \theta$. Assume $x, y, z \in G$ and $\alpha, \beta \in \Gamma$ such that $x, z \in A$. Then $\mu(x) \geq \theta$ and $\mu(z) \geq \theta$. Hence $\mu(x\alpha y\beta z) \vee \lambda \geq \theta = \mu(x) \wedge \mu(z) \wedge \theta$. If $x \notin A$ or $z \notin A$ then $\mu(x) \wedge \mu(z) \wedge \theta = \lambda$. Thus $\mu(x\alpha y\beta z) \vee \lambda \geq \lambda = \mu(x) \wedge \mu(z) \wedge \theta$. Consequently μ is a (λ,θ)-fuzzy bi Γ-ideal of G.

3.5 Proposition

If $\{F_i : i \in I\}$ is a family of (λ,θ)-fuzzy bi Γ-ideals of an ordered Γ-semigroup G. Then $\bigcap_{i\in I} F_i$ is an (λ,θ)-fuzzy bi Γ-ideal of G.

Proof Let $\{F_i\}_{i\in I}$ be a family of (λ,θ)-fuzzy bi Γ-ideals of G. Let $x, y \in G$, $\alpha \in \Gamma$ and $x \leq y$. Then

$$(\bigcap_{i\in I} F_i)(x) \vee \lambda = \bigwedge_{i\in I} F_i(x) \vee \lambda \geq \bigwedge_{i\in I} (F_i(y) \wedge \theta)$$
$$= \left(\bigcap_{i\in I} F_i\right)(y) \wedge \theta$$

and

$$\left(\bigcap_{i\in I} F_i\right)(x\alpha y) \vee \lambda$$
$$= \bigwedge_{i\in I} F_i(x\alpha y) \vee \lambda \geq \bigwedge_{i\in I} (F_i(x) \wedge F_i(y) \wedge \theta)$$
$$= \left(\bigwedge_{i\in I} (F_i(x) \wedge \theta) \wedge \bigwedge_{i\in I} (F_i(y) \wedge \theta)\right)$$
$$= \left(\bigcap_{i\in I} F_i\right)(x) \wedge \left(\bigcap_{i\in I} F_i\right)(y) \wedge \theta$$

Let $x, y, z \in G$ and $\beta, \gamma \in \Gamma$. Then,

$$\left(\bigcap_{i\in I} F_i\right)((x\beta y\gamma z) \vee \lambda = \bigwedge_{i\in I} F_i((x\beta y\gamma z) \vee \lambda \geq \bigwedge_{i\in I} (F_i(x) \wedge F_i(z) \wedge \theta)$$
$$= \left(\bigwedge_{i\in I} (F_i(x) \wedge \theta) \wedge \bigwedge_{i\in I} (F_i(z) \wedge \theta)\right)$$
$$= \left(\bigcap_{i\in I} F_i\right)(x) \wedge \left(\bigcap_{i\in I} F_i\right)(z) \wedge \theta.$$

Thus $\bigcap_{i\in I} F_i$ is an (λ,θ)-fuzzy bi Γ-ideal of G.

3.6 Theorem

A fuzzy subset μ of an ordered Γ-semigroup G is a fuzzy bi Γ-ideal of G if and only if each non-empty level subset $U(\mu;t)$ is a bi Γ-ideal of G for all $t \in (\lambda,\theta]$.

Proof Let μ be a (λ,θ)-fuzzy bi Γ-ideal of G. If there exist $x, y \in G$, $t \in (\lambda,\theta]$ with $x \leq y$ such that $y \in U(\mu;t)$. Then $\mu(y) \geq t$, since μ is a (λ,θ)-fuzzy bi Γ-ideal of G. Therefore $\mu(x) \vee \lambda \geq \mu(y) \wedge \theta = t \wedge \theta = t > \lambda$. It implies that $\mu(x) \geq t$. Thus $x \in U(\mu;t)$. Let $x, y \in G$, $\alpha \in \Gamma$ and $x, y \in U(\mu;t)$ where $t \in (\lambda,\theta]$. Then $\mu(x) \geq t$ and $\mu(y) \geq t$, since μ is a (λ,θ)-fuzzy bi Γ-ideal of G. Hence

$\mu(x\alpha y)\vee\lambda\geq\mu(x)\wedge\mu(z)\wedge\theta=t\wedge t\wedge\theta=t>\lambda$. It implies that $\mu(x\alpha y)\geq t$. Thus $x\alpha y\in U(\mu;t)$. Now assume $x,y,z\in G$, $\alpha,\beta\in\Gamma$ and $x,z\in U(\mu;t)$ where $t\in(\lambda,\theta]$. Then $\mu(x)\geq t$ and $\mu(z)\geq t$, since μ is a (λ,θ)-fuzzy bi Γ-ideal of G. Therefore $\mu(x\alpha y\beta z)\vee\lambda\geq\mu(x)\wedge\mu(z)\wedge\theta=t\wedge t\wedge\theta=t>\lambda$. It implies that $\mu(x\alpha y\beta z)\geq t$. Thus $x\alpha y\beta z\in U(\mu;t)$. Hence $U(\mu;t)$ is a bi Γ-ideal of G for all $t\in(\lambda,\theta]$.

Conversely, assume that $U(\mu;t)$ is a bi Γ-ideal of G for all $t\in(\lambda,\theta]$. If there exist $x,y\in G$ with $x\leq y$ such that $\mu(x)\vee\lambda<\mu(y)\wedge\theta$, then there exists $t\in(\lambda,\theta]$ such that $\mu(x)\vee\lambda<t\leq\mu(y)\wedge\theta$. This shows that $\mu(y)\geq t$ and $\mu(x)<t$ so $y\in U(\mu;t)$, since $U(\mu;t)$ is a bi Γ-ideal of G. Therefore $x\in U(\mu;t)$, but this is a contradiction to $\mu(x)<t$. Thus $\mu(x)\vee\lambda\geq\mu(y)\wedge\theta$. If $x,y\in G$ and $\alpha\in\Gamma$ such that $\mu(x\alpha y)\vee\lambda<\mu(x)\wedge\mu(y)\wedge\theta$, then there exists $t\in(\lambda,\theta]$ such that $\mu(x\alpha y)\vee\lambda<t\leq\mu(x)\wedge\mu(y)\wedge\theta$. This shows that $\mu(x)\geq t$, $\mu(y)\geq t$ and $\mu(x\alpha y)<t$ so $x,y\in U(\mu;t)$, since $U(\mu;t)$ is a bi Γ-ideal of G. Therefore $x\alpha y\in U(\mu;t)$, but this is a contradiction to $\mu(x\alpha y)<t$. Thus $\mu(x\alpha y)\vee\lambda\geq\mu(x)\wedge\mu(y)\wedge\theta$ for all $x,y\in G$ and $\alpha\in\Gamma$. Now let $x,y,z\in G$ and $\alpha,\beta\in\Gamma$ such that $\mu(x\alpha y\beta z)\vee\lambda<\mu(x)\wedge\mu(z)\wedge\theta$, then there exists $t\in(\lambda,\theta]$ such that $\mu(x\alpha y\beta z)\vee\lambda<t\leq\mu(x)\wedge\mu(z)\wedge\theta$. This shows that $\mu(x)\geq t$, $\mu(z)\geq t$ and $\mu(x\alpha y\beta z)<t$ so $x,z\in U(\mu;t)$, since $U(\mu;t)$ is a bi Γ-ideal of G. Therefore $x\alpha y\beta z\in U(\mu;t)$, but this is a contradiction to $\mu(x\alpha y\beta z)<t$. Hence $\mu(x\alpha y\beta z)\vee\lambda\geq\mu(x)\wedge\mu(z)\wedge\theta$ for all $x,y,z\in G$ and $\alpha,\beta\in\Gamma$. Consequently μ is a (λ,θ)-fuzzy bi Γ-ideal of G.

3.7 Theorem

Let μ be a fuzzy subset of G. If μ is a (λ,θ)-fuzzy bi Γ-ideal, then the following conditions hold:

(1) $(\forall x,y\in G)(\mu(x)\vee\lambda\geq\mu(y)\wedge\theta$ with $x\leq y)$,

(2) $(\forall x,y,z\in G)$ $(\forall\alpha\in\Gamma)$ $(\mu(x\alpha y)\vee\lambda\geq\mu(x)\wedge\mu(y)\wedge\theta)$.

(3) $(\forall x,y,z\in G)$ $(\forall\beta,\gamma\in\Gamma)$ $(\mu(x\beta y\gamma z)\vee\lambda\geq\mu(x)\wedge\mu(z)\wedge\theta)$.

If $\lambda=0$ and $\theta=1$ then we have the following corollary.

3.8 Corollary

Every fuzzy bi Γ-ideal μ of an ordered Γ-semigroup G is a (λ,θ)-fuzzy bi Γ-ideal of G.

3.9 Lemma

A non-empty subset A of an ordered Γ-semigroup G is a bi Γ-ideal if and only if a characteristic function μ_A of A is a (λ,θ)-fuzzy bi Γ-ideal of G.

Proof Suppose that A is a bi Γ-ideal of G and μ_A is a characteristic function of A. Let $x,y\in G$ with $x\leq y$.

If $y\in A$ then $\mu_A(y)=1$. Since A is bi Γ-ideal and $x\leq y$ such that $y\in A$ then $x\in A$. Therefore $\mu_A(x)=1$. As $\lambda<\theta$ so we have $\mu_A(x)\vee\lambda\geq\mu_A(y)\wedge\theta$.

If $y\notin A$ then $\mu_A(y)=0$. Hence $\mu_A(x)\vee\lambda\geq\mu_A(y)\wedge\theta$.

Now let $x,y\in G$ and $\alpha\in\Gamma$ then we discuss the following Cases:

Case 1 if $x,y\in A$, then $\mu_A(x)=1=\mu_A(y)$. Since A is bi Γ-ideal and $x,y\in A$, $\alpha\in\Gamma$. Therefore $x\alpha y\in A$. Hence $\mu_A(x\alpha y)=1$. Also $\lambda<\theta$. Thus $\mu_A(x\alpha y)\vee\lambda\geq\mu_A(x)\wedge\mu_A(y)\wedge\theta$.

Case 2 if $x,y\notin A$, then $\mu_A(x)=0=\mu_A(y)$. Hence $\mu_A(x\alpha y)\vee\lambda\geq\mu_A(x)\wedge\mu_A(y)\wedge\theta$.

Case 3 if $x\in A$ and $y\notin A$ then $\mu_A(x)=1$ and $\mu_A(y)=0$. Thus $\mu_A(x0\wedge\mu_A(y)\wedge\theta=0$. Therefore $\mu_A(x\alpha y)\vee\lambda\geq\mu_A(x)\wedge\mu_A(y)\wedge\theta$.

Similarly for $x\notin A$ and $y\in A$ then $\mu_A(x)=0$ and $\mu_A(y)=1$. It implies that $\mu_A(x)\wedge\mu_A(y)\wedge\theta=0$. Hence in any case $\mu_A(x\alpha y)\vee\lambda\geq\mu_A(x)\wedge\mu_A(y)\wedge\theta$. for all $x,y\in G$ and $\alpha\in\Gamma$. If there exist $x,y,z\in G$ and $\alpha,\beta\in\Gamma$ then we discuss the following Cases:

Case 1 if $x,z\in A$, then $\mu_A(x)=1=\mu_A(z)$. Since A is bi Γ-ideal and $x,z\in A$, $\alpha,\beta\in\Gamma$. Therefore $x\alpha y\beta z\in A$. Hence $\mu_A(x\alpha y\beta z)=1$. Also $\lambda<\theta$. Thus $\mu_A(x\alpha y\beta z)\vee\lambda\geq\mu_A(x)\wedge\mu_A(z)\wedge\theta$.

Case 2 if $x,z\notin A$, then $\mu_A(x)=0=\mu_A(z)$. Hence $\mu_A(x\alpha y\beta z)\vee\lambda\geq\mu_A(x)\wedge\mu_A(z)\wedge\theta$.

Case 3 if $x\in A$ and $z\notin A$ then $\mu_A(x)=1$ and $\mu_A(z)=0$. Thus $\mu_A(x)\wedge\mu_A(z)\wedge\theta=0$. Therefore $\mu_A(x\alpha y\beta z)\vee\lambda\geq\mu_A(x)\wedge\mu_A(z)\wedge\theta$.

Similarly for $x \notin A$ and $z \in A$ then $\mu_A(x) = 0$ and $\mu_A(z) = 1$. It implies that $\mu_A(x) \wedge \mu_A(z) \wedge \theta = 0$. Thus in any case

$\mu_A(x\alpha y\beta z) \vee \lambda \geq \mu_A(x) \wedge \mu_A(z) \wedge \theta$. for all $x, y, z \in G$ and $\alpha, \beta \in \Gamma$. Consequently μ_A is a (λ, θ)-fuzzy bi Γ-ideal of G.

Conversely, assume that μ_A is a (λ, θ)-fuzzy bi Γ-ideal of G. Suppose that $x, y \in G$ such that $x \leq y$ and $y \in A$. Then $\mu_A(y) = 1$, since μ_A is a (λ, θ)-fuzzy bi Γ-ideal so

$$\mu_A(x) \vee \lambda \geq \mu_A(y) \wedge \theta$$
$$= 1 \wedge \theta = \theta.$$

Since $\lambda < \theta$ therefore $\mu_A(x) \geq \theta$. Hence $x \in A$. If there exist $x, y \in G$ and $\alpha \in \Gamma$ such that $x, y \in A$. Then $\mu_A(x) = 1 = \mu_A(y)$, since μ_A is a (λ, θ)-fuzzy bi Γ-ideal so

$$\mu_A(x\alpha y) \vee \lambda \geq \mu_A(x) \wedge \mu_A(y) \wedge \theta$$
$$= 1 \wedge 1 \wedge \theta = \theta.$$

That is, $\mu_A(x\alpha y) \geq \theta$. Hence $x\alpha y \in A$. Now let $x, y, z \in G$ and $\beta, \gamma \in \Gamma$ such that $x, z \in A$. Then $\mu_A(x) = 1 = \mu_A(z)$, since μ_A is a (λ, θ)-fuzzy bi Γ-ideal so

$$\mu_A(x\beta y\gamma z) \vee \lambda \geq \mu_A(x) \wedge \mu_A(z) \wedge \theta$$
$$= 1 \wedge 1 \wedge \theta = \theta.$$

It implies that $\mu_A(x\beta y\gamma z) \geq \theta$. Hence $x\beta y\gamma z \in A$. Therefore A is a bi Γ-ideal of G.

4. Regular ordered Γ-semigroups in terms of (λ, θ)-fuzzy bi Γ-ideals

In this section, we introduce the notion of μ_λ^θ fuzzy subset of G. We prove that if μ is a (λ, θ)-fuzzy bi Γ-ideals of G, then μ_λ^θ is a fuzzy bi Γ-ideal of G. We also characterize regular ordered Γ-semigroups by the properties of their (λ, θ)-fuzzy bi Γ-ideals.

4.1 Definition

Let μ be a fuzzy subset of an ordered Γ-semigroup G, we define the fuzzy subset μ_λ^θ of G as follows:

$$\mu_\lambda^\theta(x) = \{\mu(x) \wedge \theta\} \vee \lambda \text{ for all } x \in G.$$

4.2 Definition

Let μ and F be fuzzy subsets of an ordered Γ-semigroup G. Then we define the fuzzy subsets $\mu \wedge_\lambda^\theta F, \mu \vee_\lambda^\theta F$ and $\mu \circ_\lambda^\theta F$ of G as follows:

$$(\mu \wedge_\lambda^\theta F)(x) = \{(\mu \wedge F)(x) \wedge \theta\} \vee \lambda,$$
$$(\mu \vee_\lambda^\theta F)(x) = \{(\mu \vee F)(x) \wedge \theta\} \vee \lambda,$$
$$(\mu \circ_\lambda^\theta F)(x) = \{(\mu \circ F)(x) \wedge \theta\} \vee \lambda,$$

for all $x \in G$.

4.3 Lemma

Let μ and F be fuzzy subsets of an ordered Γ-semigroup G. Then the following holds.

(1) $(\mu \wedge_\lambda^\theta F) = (\mu_\lambda^\theta \wedge F_\lambda^\theta)$.

(2) $(\mu \vee_\lambda^\theta F) = (\mu_\lambda^\theta \vee F_\lambda^\theta)$.

(3) $(\mu \circ_\lambda^\theta F) = (\mu_\lambda^\theta \circ F_\lambda^\theta)$.

Proof The proofs are straightforward.

4.4 Definition

If χ_A is the characteristic function of A, then $(\chi_A)_\lambda^\theta$ is defined as:

$$(\chi_A)_\lambda^\theta(x) := \begin{cases} \theta \text{ if } x \in A, \\ \lambda \text{ if } x \notin A. \end{cases}$$

4.5 Lemma

Let A and B be non-empty subsets of an ordered Γ-semigroup G. Then the following hold:

(1) $(\chi_A \wedge_\lambda^\theta \chi_B) = (\chi_{A \cap B})_\lambda^\theta$.

(2) $(\chi_A \vee_\lambda^\theta \chi_B) = (\chi_{A \cup B})_\lambda^\theta$.

(3) $(\chi_A \circ_\lambda^\theta \chi_B) = (\chi_{(A\Gamma B]})_\lambda^\theta$.

Proof The proofs of (1) and (2) are obvious. For the proof of (3) let $x \in (A\Gamma B]$. Then $\chi_{(A\Gamma B]}(x) = 1$ and hence $\{\chi_{(A\Gamma B]}(x) \wedge \theta\} \vee \lambda = \{1 \wedge \theta\} \vee \lambda = \theta$. It implies that $(\chi_{(A\Gamma B]})_\lambda^\theta(x) = \theta$. Since $x \in (A\Gamma B]$, we have $x \leq a\alpha b$ for some $a \in A$, $b \in B$ and $\alpha \in \Gamma$. Then $(a, b) \in A_x$ and $A_x \neq \phi$. Thus

$$(\chi_A \circ_\lambda^\theta \chi_B)(x) = \{(\chi_A \circ \chi_B)(x) \wedge \theta\} \vee \lambda$$
$$= \left[\left\{\bigvee_{(y,z) \in A_x} (\chi_A(y) \wedge \chi_B(z))\right\} \wedge \theta\right] \vee \lambda$$
$$\geq [\{(\chi_A(a) \wedge \chi_B(b))\} \wedge \theta] \vee \lambda.$$

Since $a \in A$ and $b \in B$, we have $\chi_A(a) = 1$ and $\chi_B(b) = 1$ and so

$$(\chi_A \circ_\lambda^\theta \chi_B)(x) \geq [\{(\chi_A(a) \wedge \chi_B(b))\} \wedge \theta] \vee \lambda$$
$$= [\{(1 \wedge 1)\} \wedge \theta] \vee \lambda$$
$$= [1 \wedge \theta] \vee \lambda = \theta \vee \lambda = \theta$$

Thus, $(\chi_A \circ_\lambda^\theta \chi_B)(x) = \theta = (\chi_{(A\Gamma B]})_\lambda^\theta(x)$. Let $x \notin (A\Gamma B]$, then $\chi_{(A\Gamma B]}(x) = 0$ and hence,

$\{\chi_{(A\Gamma B]}(x)\wedge\theta\}\vee\lambda=(0\wedge\theta)\vee\lambda=\lambda.$ So $(\chi_{(A\Gamma B]})^{\theta}_{\lambda}(x)=\lambda.$ Let $(y,z)\in A_x.$ Then

$$(\chi_A\circ^{\theta}_{\lambda}\chi_B)(x)=\{(\chi_A\circ\chi_B)(x)\wedge\theta\}\vee\lambda$$
$$=\left[\left\{\bigvee_{(y,z)\in A_x}(\chi_A(y)\wedge\chi_B(z))\right\}\wedge\theta\right]\vee\lambda$$

Since $(y,z)\in A_x$, then $x\le y\beta z$ for $\beta\in\Gamma$. If $y\in A$ and $z\in B$, then $y\beta z\in A\Gamma B$ and so $x\in(A\Gamma B]$. This is a contradiction. If $y\notin A$ and $z\in B$, then

$$\left[\left\{\bigvee_{(y,z)\in A_x}(\chi_A(y)\wedge\chi_B(z))\right\}\wedge\theta\right]\vee\lambda$$
$$=\left[\left\{\bigvee_{(y,z)\in A_x}(0\wedge 1)\right\}\wedge\theta\right]\vee\lambda=\lambda.$$

Hence, $(\chi_A\circ^{\theta}_{\lambda}\chi_B)(x)=\lambda=(\chi_{(A\Gamma B]})^{\theta}_{\lambda}(x).$ Similarly, for $y\in A$ and $z\notin B$, we have

$$(\chi_A\circ^{\theta}_{\lambda}\chi_B)(x)=\lambda=(\chi_{(A\Gamma B]})^{\theta}_{\lambda}(x).$$

4.6 Theorem

The characteristic function $(\chi_A)^{\theta}_{\lambda}$ of A is a (λ,θ)-fuzzy bi Γ-ideal if and only if A is a bi Γ-ideal of $G.$

Proof Suppose that A is a bi Γ-ideal of $G.$ Then by Lemma 3.9, $(\chi_A)^{\theta}_{\lambda}$ is a (λ,θ)-fuzzy bi Γ-ideal of $G.$ Conversely, assume that $(\chi_A)^{\theta}_{\lambda}$ is a (λ,θ)-fuzzy bi Γ-ideal of $G.$ Let $x,y\in G$ and $\alpha\in\Gamma$ be such that $x,y\in A.$ It implies that $(\chi_A)^{\theta}_{\lambda}(x)=\theta$ and $(\chi_A)^{\theta}_{\lambda}(y)=\theta$ Since $(\chi_A)^{\theta}_{\lambda}$ is a (λ,θ)-fuzzy bi Γ-ideal of $G.$ Therefore

$$\max\{(\chi_A)^{\theta}_{\lambda}(x\alpha y),\lambda\}\ge\min\{(\chi_A)^{\theta}_{\lambda}(x),(\chi_A)^{\theta}_{\lambda}(y),\theta\}$$
$$=\{\theta,\theta,\theta\}=\theta.$$

Since $\lambda<\theta.$ Hence $(\chi_A)^{\theta}_{\lambda}(x\alpha y)=\theta$. It shows that $x\alpha y\in A.$ Now if there exist $x,y,z\in G$ and $\beta,\gamma\in\Gamma$ such that $x,z\in A.$ Then $(\chi_A)^{\theta}_{\lambda}(x)=\theta$ and $(\chi_A)^{\theta}_{\lambda}(z)=\theta$ Since $(\chi_A)^{\theta}_{\lambda}$ is a (λ,θ)-fuzzy bi Γ-ideal of $G.$ We have

$$\max\{(\chi_A)^{\theta}_{\lambda}(x\beta y\gamma),\lambda\}\ge\min\{(\chi_A)^{\theta}_{\lambda}(x),(\chi_A)^{\theta}_{\lambda}(z),\theta\}$$
$$=\{\theta,\theta,\theta\}=\theta.$$

Since $\lambda<\theta.$ Hence $(\chi_A)^{\theta}_{\lambda}(x\beta y\gamma z)=\theta$. Therefore $x\beta y\gamma z\in A.$ Consequently, A is a bi Γ-ideal of $G.$

4.7 Lemma

The characteristic function $(\chi_A)^{\theta}_{\lambda}$ of A is a (λ,θ)-fuzzy left (resp. right) Γ-ideal of an ordered Γ-semigroup G if and only if A is a left (resp. right) Γ-ideal of $G.$

Proof The proof follows from Theorem 4.6.

4.8 Proposition

If μ is a (λ,θ)-fuzzy bi Γ-ideal, then μ^{θ}_{λ} is a fuzzy bi Γ-ideal of $G.$

Proof Assume that μ is a (λ,θ)-fuzzy bi Γ-ideal of $G.$ If there exist $x,y,z\in G$ and $\alpha,\beta\in\Gamma,$ then

$$\max\{\mu^{\theta}_{\lambda}(x\alpha y\beta z),\lambda\}$$
$$=\max\{(\{\mu(x\alpha y\beta z)\wedge\theta\}\vee\lambda),\lambda\}=$$
$$\{\mu(x\alpha y\beta z)\wedge\theta\}\vee\lambda$$
$$=\{\mu(x\alpha y\beta z)\vee\lambda\}\wedge\{\theta\vee\lambda\}$$
$$=\{\mu(x\alpha y\beta z)\vee\lambda\}\wedge\theta$$
$$=\{(\mu(x\alpha y\beta z)\vee\lambda)\vee\lambda\}\wedge\theta$$
$$\ge\{(\mu(x)\wedge\mu(z)\wedge\theta)\vee\lambda\}\wedge\theta$$
$$=\{(\mu(x)\wedge\mu(z)\wedge\theta\wedge\theta)\vee\lambda\vee\lambda\}\wedge\theta$$
$$=\{\{(\mu(x)\wedge\theta)\vee\lambda\}\wedge\{(\mu(z)\wedge\theta)\vee\lambda\}\}\wedge\theta$$
$$=\{\mu^{\theta}_{\lambda}(x)\wedge\mu^{\theta}_{\lambda}(z)\}\wedge\theta$$
$$=\min\{\mu^{\theta}_{\lambda}(x),\mu^{\theta}_{\lambda}(z),\theta\}.$$

By similar way we can show the remaining part of the proposition.

4.9 Corollary

If $\{\mu_i\ :\ i\in I\}$ is a family of (λ,θ)-fuzzy bi Γ-ideal of an ordered Γ-semigroups G, then $\bigcap_{i\in I}(\mu_i)^{\theta}_{\lambda}$ is a (λ,θ)-fuzzy bi Γ-ideal of $G.$

4.10 Lemma [16]

An ordered Γ-semigroup G is left (resp. right) simple if and only if $(G\Gamma a]=G$ (resp. $(a\Gamma G]=G$) for every $a\in G.$

4.11 Theorem

An ordered Γ-semigroup G is regular, left and right simple if and only if for every (λ,θ)-fuzzy bi Γ-ideal μ of $G,$ μ^{θ}_{λ} is a constant function.

Proof Assume that G is regular, left and right simple and μ is a (λ,θ)-fuzzy bi Γ-ideal of G. We consider, $E_G=\{e\in G\mid e\le e\alpha e$ for $\alpha\in\Gamma\},$ then $E_G\ne\phi,$ because G is regular, hence for every $a\in G$, there exists $x\in G$ and $\alpha,\beta\in\Gamma$ such that $a\le a\alpha x\beta a$, then $a\alpha x\le(a\alpha x\beta a)\alpha x=(a\alpha x)\beta(a\alpha x)$ and so $a\alpha x\in E_G.$ Let $b,e\in E_G.$ Since G is left and right simple, by Lemma 4.10, it follows that $G=(G\Gamma b]$ and

$G=(b\Gamma G]$. Since $e\in G$, we have $e\in(G\Gamma b]$ and $e\in(b\Gamma G]$, then $e\le x\gamma b$ and $e\le b\delta y$ for some $x,y\in G$ and $\gamma,\delta\in\Gamma$, and we have

$$e\alpha e=(b\delta y)\alpha(x\gamma b)\le b\delta(y\alpha x)\gamma b.$$

Since μ is a (λ,θ)-fuzzy bi Γ-ideal of G, we have

$$\begin{aligned}\mu_\lambda^\theta(e\alpha e)&=\{\mu(e\alpha e)\wedge\theta\}\vee\lambda\\&=\{\mu(e\alpha e)\vee\lambda\}\wedge\{\theta\vee\lambda\}\\&=\{\mu(e\alpha e)\vee\lambda\}\wedge\theta\\&=\{(\mu(e\alpha e)\vee\lambda)\vee\lambda\}\wedge\theta\\&\ge\{(\mu(b\delta(y\alpha x)\gamma b)\wedge\theta)\vee\lambda\}\wedge\theta)\\&=(\mu(b\delta(y\alpha x)\gamma b)\vee\lambda)\wedge(\theta\vee\lambda)\wedge\theta\\&=(\mu(b\delta(y\alpha x)\gamma b)\vee\lambda)\wedge\theta\wedge\theta\\&=(\mu(b\delta(y\alpha x)\gamma b)\vee\lambda)\wedge\theta\\&=(\mu(b\delta(y\alpha x)\gamma b)\vee\lambda)\vee\lambda)\wedge\theta\\&\ge(\{\mu(b)\wedge\mu(b)\wedge\theta\}\vee\lambda)\wedge\theta\\&=(\{\mu(b)\wedge\mu(b)\wedge\theta\}\wedge\theta)\vee(\lambda\wedge\theta)\\&=(\mu(b)\wedge\theta)\vee\lambda\\&=\mu_\lambda^\theta(b).\end{aligned}$$

Thus we have,

$$\mu_\lambda^\theta(e\alpha e)\ge\mu_\lambda^\theta(b)\qquad(*).$$

Since $e\in E_G$, we have $e\le e\alpha e$ and so

$$\begin{aligned}\mu_\lambda^\theta(e)&=\{\mu(e)\wedge\theta\}\vee\lambda\\&=(\mu(e)\vee\lambda)\wedge(\theta\vee\lambda)\\&=(\mu(e)\vee\lambda)\wedge\theta\\&=((\mu(e)\vee\lambda)\vee\lambda)\wedge\theta\\&\ge\{(\mu(e\alpha e)\wedge\theta)\vee\lambda\}\wedge\theta\\&=\{(\mu(e\alpha e)\wedge\theta)\wedge\theta\}\vee\{\lambda\wedge\theta\}\\&=(\mu(e\alpha e)\wedge\theta)\vee\lambda\\&=\mu_\lambda^\theta(e\alpha e),\end{aligned}$$

it follows that $\mu_\lambda^\theta(e)\ge\mu_\lambda^\theta(e\alpha e)$. Thus, by $(*)$, we have $\mu_\lambda^\theta(e)\ge\mu_\lambda^\theta(b)$. On the other hand, since $e\in G$, by Lemma 4.10, we have $(G\Gamma e]=G=(e\Gamma G]$. Since $b\in G$, we have $b\in(G\Gamma e]$ and $b\in(e\Gamma G]$, then $b\le g\beta e$ and $b\le t\gamma e$ for some $g,t\in G$ and $\beta,\gamma\in\Gamma$. Thus, by the same arguments as above, we get $\mu_\lambda^\theta(b)\ge\mu_\lambda^\theta(e)$. It follows that $\mu_\lambda^\theta(e)=\mu_\lambda^\theta(b)$ and hence, μ_λ^θ is constant on E_G.

Now, let $a\in G$, then there exists $x\in G$ and $\alpha,\beta\in\Gamma$ such that $a\le a\alpha x\beta a$. It follows that $a\alpha x\le(a\alpha x\beta a)\alpha x=(a\alpha x)\beta(a\alpha x)$ and $x\beta a\le x\beta(a\alpha x\beta a)=(x\beta a)\alpha(x\beta a)$. Thus, $a\alpha x,x\beta a\in E_G$, it follows by the previous arguments, we get, $\mu_\lambda^\theta(a\alpha x)=\mu_\lambda^\theta(b)=\mu_\lambda^\theta(x\beta a)$. Since $(a\alpha x)\gamma a\delta(x\beta a)=(a\alpha x\gamma a)\delta x\beta a\ge a\alpha x\gamma a\ge a$, we get,

$$\begin{aligned}\mu_\lambda^\theta(a)&=\{\mu(a)\wedge\theta\}\vee\lambda\\&=(\mu(a)\vee\lambda)\wedge(\theta\vee\lambda)\\&=(\mu(a)\vee\lambda)\wedge\theta\\&=(\mu(a)\vee\lambda\vee\lambda)\wedge\theta\\&\ge(\{\mu((a\alpha x)\gamma a\delta(x\beta a))\wedge\theta\}\vee\lambda)\wedge\theta\\&=[\{\mu((a\alpha x)\gamma a\delta(x\beta a))\vee\lambda\}\wedge\{\theta\vee\lambda\}]\wedge\theta\\&=[\{\mu((a\alpha x)\gamma a\delta(x\beta a))\vee\lambda\}\wedge\theta]\wedge\theta\\&=\{\mu((a\alpha x)\gamma a\delta(x\beta a))\vee\lambda\}\wedge\theta\\&=\{\mu((a\alpha x)\gamma a\delta(x\beta a))\vee\lambda\vee\lambda\}\wedge\theta\\&\ge[\{\mu(a\alpha x)\wedge\mu(x\beta a)\wedge\theta\}\vee\lambda]\wedge\theta\\&=[\{\mu(a\alpha x)\wedge\mu(x\beta a)\wedge\theta\}\wedge\theta]\vee[\lambda\wedge\theta]\\&=[\{\mu(a\alpha x)\wedge\mu(x\beta a)\wedge\theta\}\vee\lambda]\\&=[\{\mu(a\alpha x)\wedge\theta\}\vee\lambda]\wedge[\{\mu(x\beta a)\wedge\theta\}\vee\lambda]\\&=\mu_\lambda^\theta(a\alpha x)\wedge\mu_\lambda^\theta(x\beta a)=\mu_\lambda^\theta(b)\end{aligned}$$

Thus $\mu_\lambda^\theta(a)\ge\mu_\lambda^\theta(b)$. Similarly we can show that $\mu_\lambda^\theta(b)\ge\mu_\lambda^\theta(a)$. Therefore $\mu_\lambda^\theta(a)=\mu_\lambda^\theta(b)$ and so, μ_λ^θ is a constant function on G.

Conversely, let $a\in G$. Then obviously, $(G\Gamma a]$ is a bi Γ-ideal of G and Lemma 3.9, the characteristic function

$$\chi_{(G\Gamma a]}:G\to[0,1]\,|\,x\mapsto\chi_{(G\Gamma a]}(x)=\begin{cases}\theta\text{ if }x\in(G\Gamma a],\\\lambda\text{ if }x\notin(G\Gamma a],\end{cases}$$

is an (λ,θ)-fuzzy bi Γ-ideal of G. By hypothesis, since every (λ,θ)-fuzzy bi Γ-ideal is a constant function so for every $x\in G$, we have, $\chi_{(G\Gamma a]}(x)=c$. Let $(G\Gamma a]\subset G$ and take $t\in G$ such that $t\notin(G\Gamma a]$. Then $\chi_{(G\Gamma a]}(t)=\lambda$, on the other hand, since $a\alpha a\in(G\Gamma a]$, we have $\chi_{(G\Gamma a]}(a\alpha a)=\theta$, a contradiction, because every (λ,θ)-fuzzy bi Γ-ideal is a constant function. Thus, $(G\Gamma a]=G$. In a similar way, we have, $(a\Gamma G]=G$, consequently, G is left and right simple. Since $a\in(G\Gamma a]=((a\Gamma G]\Gamma a]=(a\Gamma G\Gamma a]$. Thus, G is regular.

5. CONCLUDING REMARKS.

In this paper, we investigated the more generalized form of fuzzy bi Γ-ideals of an ordered Γ-semigroup G and gave the concept of a (λ,θ)-fuzzy bi Γ-ideal of G. We proved that μ is an ordinary fuzzy bi Γ-ideal for $\lambda=0$ and $\theta=1$. We have also provided the necessary and sufficient conditions for both level subset $U(\mu;t)$ of μ and a characteristic function μ_A of A to be fuzzy bi Γ-ideals of the type (λ,θ). Finally, we characterized regular ordered Γ-semigroups by the properties of their (λ,θ)-fuzzy bi Γ-ideals.

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