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COMPUTATIONAL POWER OF WEIGHTED SPLICING SYSTEMS

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Abstract

A weighted splicing system is a restriction of splicing systems in which weights are associated with the axioms, and the weight of a string z generated from two strings x and y is computed from the weights of x and y according to some operations defined on the weights. In this paper we study the computational power of weighted splicing systems considering different weighting spaces and cut-points. We also investigate the relationships of different variants of weighted splicing systems.

Key words: DNA computing, DNA computing splicing systems, weighted splicing systems, computational power

2010 Mathematics Subject Classification: 68Q05, 68Q15

1. Introduction. One of the DNA based computing devices namely splicing systems (or H system) came after Head in 1987. Splicing system is a formal model of the cutting and recombination of DNA molecules in the presence of restriction enzymes called splicing operation which is theoretically proposed by Head. Head has initiated the formal analysis of the generative power of recombination behaviours in general and established a new relationship between formal language theory and the study of informational macromolecules [¹]. The process of splicing operation acting on splicing system works as follows: two DNA molecules are cut at specific subsequences depending on specific enzyme that is used and the first part of one molecule which has been cut is connected to the second

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part of the other molecule, and vice versa. Hence, a new well-formed double stranded molecule can be obtained from the process of splicing. However, with the finite sets of axioms and rules, the splicing systems generate only regular languages as shown in [2]. In overcoming the limitation of the usual splicing systems, several restrictions have been considered in [3].

There is another restriction of splicing systems that has been introduced in $[^4]$, called *weighted splicing systems*. Weighted splicing system is actually the usual splicing system with some weights assigned to its axioms. In formal language theory, the idea of using weights has been widely investigated and has been introduced in different forms. For example, the idea of weights in weighted grammar and automata can be found in $[^{5-10}]$.

In this paper, we focus on the computational power of weighted splicing system. Firstly, some necessary definitions and notations from the theories of formal languages and splicing systems are presented. Next, the concepts of weighted splicing systems and threshold languages generated by weighted splicing systems are discussed. For the computational power of weighted splicing systems, we show that some weighted splicing systems with finite components can generate even non-context-free languages. In addition, the relationships of weighted splicing system with different weighting spaces are presented in this section. The last section gives the conclusion, open problems and suggestions for future research in this direction.

In the next section, the preliminaries of the formal languages and splicing system will be discussed.

2. Preliminaries. In this section, some basic terms, notations and formal definition of the theories of formal languages and splicing systems used in this paper are given. For further information, the reader can refer to [11,12].

In this paper, the following notations are used: The symbol \in denotes the membership of an element to a set while the negation of set membership is denoted by $\not\in$. The inclusion is denoted by \subseteq and the strict (proper) inclusion is denoted by \subset . The symbol \varnothing denotes the empty set. The symbols +, \times denote usual addition and multiplication operations, respectively. The symbols \oplus and \otimes denote componentwise addition and componentwise multiplication operations, respectively. The sets of integers and positive rational numbers are denoted by \mathbb{Z} and \mathbb{Q}^+ , respectively. \mathbb{Z}^n denotes the n-dimensional vector space over integers. The set of matrices with integer entities is denoted by \mathbb{M} . The null matrix, i.e., the matrix of which all components are zero, is denoted by \mathbb{O} . The identity $n \times n$ matrix is denoted by \mathbb{I}_n .

The families of recursively enumerable, context-sensitive, context-free, linear, regular and finite languages are denoted by **RE**, **CS**, **CF**, **LIN**, **REG** and **FIN**, respectively. For these language families, the next strict inclusions, named *Chomsky hierarchy*, hold (see $[^{11}]$):

Theorem 2.1 ($[^{11}]$).

$$\mathbf{FIN} \subset \mathbf{REG} \subset \mathbf{LIN} \subset \mathbf{CF} \subset \mathbf{CS} \subset \mathbf{RE}.$$

Further, we briefly cite some basic definitions and results of iterative splicing systems which are needed in the next section.

Let V be an alphabet, and $\#, \$ \notin V$ be two special symbols. A *splicing rule* over V is a string of the form

$$r = u_1 \# u_2 \$ u_3 \# u_4$$
, where $u_1, u_2, u_3, u_4 \in V^*$.

For such a rule $r \in R$ and strings $x, y, z \in V^*$, we write

$$(x,y)\vdash_r z$$

if and only if

$$x = x_1 u_1 u_2 x_2, \ y = y_1 u_3 u_4 y_2, z = x_1 u_1 u_4 y_2,$$

for some $x_1, x_2, y_1, y_2 \in V^*$.

The string z is said to be obtained by splicing x, y, as indicated by the rule r; u_1u_2 and u_3u_4 are called the *sites* of the splicing. We call x the *first term* and y the *second term* of the splicing operation.

An H scheme is a pair $\sigma = (V, R)$, where V is an alphabet and $R \subseteq V^* \# V^* \$ V^* \# V^*$ is a set of splicing rules. For a given H scheme $\sigma = (V, R)$ and a language $L \subseteq V^*$, we write

$$\sigma(L) = \{ z \in V^* \mid (x, y) \vdash_r z,$$

for some $x, y \in L, r \in R \},$

and we define

$$\sigma^*(L) = \bigcup_{i \ge 0} \sigma^i(L)$$

by

$$\sigma^0(L) = L,$$

$$\sigma^{i+1}(L) = \sigma^i(L) \cup \sigma(\sigma^i(L)), i \ge 0.$$

An extended H system is a construct $\gamma = (V, T, A, R)$, where V is an alphabet, $T \subseteq V$ is the *terminal* alphabet, $A \subseteq V^*$ is the set of *axioms*, and $R \subseteq V^* \# V^* \# V^*$ is the set of *splicing rules*. When T = V, the system is said to be non-extended. The language generated by γ is defined by $L(\gamma) = \sigma^*(A) \cap T^*$.

 $\mathbf{EH}(F_1, F_2)$ denotes the family of languages generated by extended H systems $\gamma = (V, T, A, R)$ with $A \in F_1$ and $R \in F_2$, where

$$F_1, F_2 \in \{ FIN, REG, CF, LIN, CS, RE \}.$$

Theorem 2.2 ([³]). The relations in the following table hold, where at the intersection of the row marked with F_1 with the column marked with F_2 there appear either the family $\mathbf{EH}(F_1, F_2)$ or two families F_3 , F_4 such that $F_3 \subset \mathbf{EH}(F_1, F_2) \subseteq F_4$.

$egin{array}{c c} F_2 & \\ F_1 & \\ \end{array}$	FIN	REG	LIN	CF	CS	RE
FIN	REG	\mathbf{RE}	RE	\mathbf{RE}	\mathbf{RE}	\mathbf{RE}
REG	REG	\mathbf{RE}	\mathbf{RE}	\mathbf{RE}	\mathbf{RE}	\mathbf{RE}
LIN	LIN, CF	RE	RE	RE	RE	RE
CF	CF	RE	RE	\mathbf{RE}	RE	RE
CS	RE	RE	RE	\mathbf{RE}	\mathbf{RE}	\mathbf{RE}
RE	RE	RE	RE	\mathbf{RE}	\mathbf{RE}	\mathbf{RE}

In the next section, a brief discussion on weighted splicing system and its operations will be presented.

3. Weighted splicing system. In this section we state the formal definitions for a new variant of splicing system introduced in [4] called weighted splicing system. Weighted splicing systems are splicing systems which are specified with a weighting space and operations over weights are closed in the weighting space. By some cut-points, we can obtain some languages generated by weighted splicing system called threshold languages. Hence, the formal definition for the splicing operation of weighted splicing system and threshold languages given in [4] are stated in the following:

Definition 3.1. A weighted splicing system is a 7-tuple $\gamma = (V, T, A, R, \omega, M, \odot)$, where V, T, R are defined as usual extended H system, M is a weighting space, $\omega : V^* \to M$ is a weight function, \odot is the operation over the weights $\omega(x)$, $x \in V^*$, and A is a subset of $V^* \times M$.

Furthermore, the weighted splicing operation is defined as in the following: **Definition 3.2.** For $(x, \omega(x))$, $(y, \omega(y))$, $(z, \omega(z)) \in V^* \times M$ and $r \in R$,

$$[(x,\omega(x)),(y,\omega(y))]\vdash_r (z,\omega(z))$$

iff $(x, y) \vdash_r z$ and $\omega(z) = \omega(x) \odot \omega(y)$.

For a weighted splicing system $\gamma = (V, T, A, R, \omega, M, \odot)$, the set of weighted strings obtained by splicing strings in A according to splicing rules in R and the weight operation \odot is defined as in the following definition:

Definition 3.3. Let $\gamma = (V, T, A, R, \omega, M, \odot)$ be a weighted splicing system. Then

$$\sigma_{\omega}(A) = \{(z, \omega(z)) : (x, y) \vdash_{r} z \land \omega(z) = \omega(x) \odot \omega(y)$$
$$for \ some \ (x, \omega(x)), (y, \omega(y)) \in A \ and \ r \in R\}.$$

Moreover, for a weighted splicing system $\gamma = (V, T, A, R, \omega, M, \odot)$, the closure of A under splicing with respect to rules in R and the weight operation \odot is defined as in the following:

Definition 3.4. Let $\gamma = (V, T, A, R, \omega, M, \odot)$ be a weighted splicing system. Then

$$\sigma_\omega^*(A) = \bigcup_{i \geq 0} \sigma_\omega^i(A),$$

where

$$\sigma_{\omega}^{i}(A) = \sigma_{\omega}^{i-1}(A) \cup \sigma_{\omega}(\sigma_{\omega}^{i-1}(A)) \text{ for } i = 1, 2, \dots,$$

$$\sigma_{\omega}^{0}(A) = A.$$

Definition 3.5. The weighted language generated by a weighted splicing system $\gamma = (V, T, A, R, \omega, M, \odot)$ is defined as $L_{\omega}(\gamma) = \sigma_{\omega}^*(A)$.

Remark 1. We can consider the different sets and (algebraic) structures as the weighting spaces, for instance, the sets of integers, rational numbers, real numbers, the sets of Cartesian products of the sets of numbers, the set of matrices with integer entries, groups, etc. Then, the operations over weights of strings are defined with respect to the chosen weighting space. In this paper, the sets of integers, positive rational numbers, the set of Cartesian products of integers and the set of matrices with integer entries are considered as the weighting spaces.

Remark 2. A weighted splicing system may generate the same strings with different weights. This "ambiguity" can be eliminated by introducing a second operation over the weights of strings or by defining threshold languages, i.e., the selection of the "successful" subset of the crispy language generated by a weighted splicing system with respect to some cut-points.

Definition 3.6. Let $L_{\omega}(\gamma)$ be the language generated by a weighted splicing system $\gamma = (V, T, A, R, \omega, M, \odot)$. A threshold language $L_{\omega}(\gamma, \star \tau)$ with respect to a threshold (cut-point) $\tau \in M$ is a subset of $L_{\omega}(\gamma)$ defined by

$$L_{\omega}(\gamma, \star \tau) = \{ z \mid (z, \omega(z)) \in \sigma_{\omega}^*(A) \text{ and } \omega(z) \star \tau \},$$

where $\star \in \{=,>,<\}$ is called the mode of $L_{\omega}(\gamma,\star\tau)$.

Remark 3. A threshold can also be considered as a subset of M. Then, the mode for such a threshold is defined as a membership to the threshold set, i.e., for a threshold set $A \subseteq M$, the modes are \in and \notin .

The family of threshold languages generated by weighted splicing systems of type (F_1, F_2) (with a weighting space M and an operation \odot) is denoted by $\omega EH(F_1, F_2)$ ($\omega EH(F_1, F_2, M, \odot)$), where

$$F_1, F_2 \in \{ \mathbf{FIN}, \mathbf{REG}, \mathbf{LIN}, \mathbf{CF}, \mathbf{CS}, \mathbf{RE} \}$$

and

$$(M, \odot) \in \{(\mathbb{Z}, +), (\mathbb{Z}^k, \{\oplus, \otimes\}), (\mathbb{Q}^+, \times), (\mathbb{M}, \oplus)\}.$$

In the next section, the generative power for weighted splicing systems will be discussed.

4. The relationships of weighted splicing system and families of languages in the Chomsky hierarchy. Here, we investigate the generative power of weighted splicing systems with respect to different weighting spaces and cut-points. First, we state the proposition which is immediately obtained from the definitions in Section 3.

Proposition 4.1. For all language families F_1 , $F_2 \in \{ FIN, REG, CF, LIN, CS, RE \}$,

$$\mathbf{EH}(F_1, F_2) \subseteq \omega \mathbf{EH}(F_1, F_2, M, \odot),$$

where

$$(M, \odot) \in \{(\mathbb{Z}, +), (\mathbb{Z}^k, \oplus, \otimes), (\mathbb{Q}_+, \times), (\mathbb{M}, +)\}.$$

Proof 4.1. For any splicing system γ , we define the weighted splicing system γ' , associating weights

- (a) 0 if $(M, \odot) = (\mathbb{Z}, +)$,
- (b) $(0,0,\ldots,0)$ if $(M,\odot) = (\mathbb{Z}^k,\oplus)$,
- (c) (1, 1, ..., 1) if $(M, \odot) = (\mathbb{Z}^k, \otimes)$,
- (d) 1 if $(M, \odot) = (\mathbb{Q}_+, \times)$,
- (e) null matrix $\mathbf{0}$ if $(M, \odot) = (\mathbb{M}, +)$

with each axiom of γ . Then, it is not difficult to see that

- (a) $L_{\omega}(\gamma',=0)=L(\gamma),$
- (b) $L_{\omega}(\gamma', = (0, 0, \dots, 0) = L(\gamma),$
- (c) $L_{\omega}(\gamma',=1)=L(\gamma),$
- (d) $L_{\omega}(\gamma', = \mathbf{0}) = L(\gamma)$.

Next we present the results obtained from Examples 1 and 2 in [4], Theorem 2.2 and Proposition 4.1.

Theorem 4.2. For $F_1 \in \{LIN, CF\}$,

$$\omega EH(\mathbf{FIN}, \mathbf{FIN}, \mathbb{M}, \odot) - EH(F_1, \mathbf{FIN}) \neq \varnothing,$$

where

$$(\mathbf{M}, \odot) \in \{(\mathbb{Z}, +), (\mathbb{Z}^k, \oplus, \otimes), (\mathbb{Q}_+, \times), (\mathbb{M}, +)\}.$$

Theorem 4.3. For $F_1 \in \{LIN, CF\}$,

$$\omega EH(F_1, \mathbf{FIN}, \mathbb{M}, \odot) - EH(F_1, \mathbf{FIN}) \neq \varnothing,$$

where

$$(\mathbf{M}, \odot) \in \{(\mathbb{Z}, +), (\mathbb{Z}^k, \oplus, \otimes), (\mathbb{Q}_+, \times), (\mathbb{M}, +)\}.$$

Theorem 4.4.

$$\mathbf{REG} = \mathbf{EH}(\mathbf{FIN}, \mathbf{FIN}) \subset \omega \mathbf{EH}(\mathbf{FIN}, \mathbf{FIN}, \mathbb{M}, \odot) \subseteq \mathbf{RE},$$

where

$$(M, \odot) \in \{(\mathbb{Z}, +), (\mathbb{Z}^k, \oplus, \otimes), (\mathbb{Q}_+, \times), (\mathbb{M}, +)\}.$$

Similar results can be obtained for the other families of languages in Chomsky hierarchy if we again use Theorem 2.1, Proposition 4.1 and the examples given in [4].

Theorem 4.5. For $F_1 \in \{REG, LIN, CF\}$ and $F_2 \in \{CS, RE\}$,

- (1) $EH(F_1, \mathbf{FIN}) \subset \omega EH(F_1, \mathbf{FIN}, \mathbf{M}, \odot)$
- (2) $EH(F_2, \mathbf{FIN}) = \omega EH(F_2, \mathbf{FIN}, \mathbf{M}, \odot),$

where

$$(M, \odot) \in \{(\mathbb{Z}, +), (\mathbb{Z}^k, \oplus, \otimes), (\mathbb{Q}_+, \times), (\mathbb{M}, +)\}.$$

Further, we study the computational power of weighted splicing systems with specific weighting spaces and cut-points. First, we choose the n-dimensional vector space \mathbb{Z}^n , $n \geq 0$, over integers as the weighting space and the vector addition operation (denoted by \oplus) as the operation over weights. Then, it is clear that

$$\omega EH(\mathbf{FIN}, \mathbf{FIN}, \mathbb{Z}^0, \oplus) = \omega EH(\mathbf{FIN}, \mathbf{FIN}),$$

and for $n \geq 1$, we prove the next theorem.

Theorem 4.6. For $n \geq 1$,

$$\omega EH(\mathbf{FIN},\mathbf{FIN},\mathbb{Z}^n,\oplus)\subseteq \omega EH(\mathbf{FIN},\mathbf{FIN},\mathbb{Z}^{n+1},\oplus).$$

Proof 4.2. Let $L \in \omega EH(\mathbf{FIN}, \mathbf{FIN}, \mathbb{Z}^n, \oplus)$. Then there is a weighted splicing system $\gamma = (V, T, R, A, \omega, M, \oplus), M \subseteq \mathbb{Z}^n$, such that $L = L_{\omega}(\gamma, *\alpha)$.

We construct the weighted splicing system $\gamma' = (V, T, R, A', \omega', M', \oplus), M' \subseteq \mathbb{Z}^{n+1}$, where

$$A' = \{(x, (a_1, a_2, \dots, a_n, a_n)) : (a_1, a_2, \dots, a_n, a_n) \in M' \text{ and } (x, (a_1, a_2, \dots, a_n)) \in A\}.$$

Then, it is not difficult to see that for every splicing operation in γ

$$(x, (a_1, a_2, ..., a_n)), (y, (b_1, b_2, ..., b_n)) \vdash_r (z, (a_1 + b_1, a_2 + b_2, ..., a_n + b_n)),$$

one can construct the similar splicing operation in γ' :

$$(x, (a_1, a_2, ..., a_n, a_n)), (y, (b_1, b_2, ..., b_n, b_n)) \vdash_r (z, (a_1 + b_1, a_2 + b_2, ..., a_n + b_n, a_n + b_n).$$

Since the *n*th and (n+1)th components are same, they fulfill the same cut-point requirement. Thus, $L_{\omega}(\gamma, *\alpha) = L_{\omega}(\gamma', *\alpha)$.

Theorem 4.7. For all families $F_1, F_2 \in \{FIN, REG, CF, LIN, CS\}$,

$$\omega EH(F_1, F_2, \mathbb{Z}, +) \subseteq \omega EH(F_1, F_2, \mathbb{Q}, \times).$$

Proof 4.3. Let γ be a weighted splicing system with the weighting space $M \subseteq \mathbb{Z}$ and the weight operation +, i.e., $\gamma = (V, T, A, R, \omega, M, +)$. We construct the weighted splicing system γ' with weighting space $M' \subseteq \mathbb{Q}$ and the weight operation \times , i.e., $\gamma' = (V, T, A', R, \omega', M', \times)$, where

$$A' = \{ (a, 2^{\omega(a)}) : (a, \omega(a)) \in A \}.$$

It is not difficult to see that if, for any string x, its weight in $L_{\omega}(\gamma)$ is $\omega(x)$, then its weight in $L_{\omega}(\gamma')$ is $\omega'(x) = 2^{\omega(x)}$.

Then, for any cut-point α for the splicing system γ , we choose the cut-point 2^{α} for the splicing system γ' . It follows that $L_{\omega}(\gamma, *\alpha) = L_{\omega}(\gamma', *2^{\alpha})$.

Next, we study the generative capacity of weighted splicing systems over \mathbb{M} . Let ω EH (**FIN**, **FIN**, \mathbb{M}_n , +) denote the family of languages generated by weighted splicing systems with the set of $n \times n$ matrices \mathbb{M}_n and the matrix addition operation +. It is clear that

$$\omega EH(\mathbf{FIN}, \mathbf{FIN}, \mathbb{M}_0, +) = \omega EH(\mathbf{FIN}, \mathbf{FIN}).$$

For $n \ge 1$, we can show that the increase of the size of matrices lead to the infinite hierarchy.

Theorem 4.8. For $n \geq 1$,

$$\omega EH(\mathbf{FIN}, \mathbf{FIN}, \mathbb{M}_n, +) \subseteq \omega EH(\mathbf{FIN}, \mathbf{FIN}, \mathbb{M}_{n+1}, +).$$

Proof 4.4. Let $L \in \omega EH(\mathbf{FIN}, \mathbf{FIN}, \mathbb{M}_n, +)$. Then there is a weighted splicing system $\gamma = (V, T, R, A, M, +), M \subseteq \mathbb{M}_n$, such that $L = L_{\omega}(\gamma, *\alpha)$. We construct $\gamma' = (V, T, R, A', M', +), M' \subseteq \mathbb{M}_{n+1}$, where A' contains the axiom

$$\left(x, \left[\begin{array}{cc} a_{ij} & a_{in} \\ a_{nj} & a_{nn} \end{array}\right]\right), \left[\begin{array}{cc} a_{ij} & a_{in} \\ a_{nj} & a_{nn} \end{array}\right] \in M',$$

for each $(x, [a_{ij}]) \in A$, $[a_{ij}] \in M$. For each splicing operation

$$((x, [a_{ij}]), (y, [b_{ij}])) \vdash_r (z, [a_{ij} + b_{ij}])$$

in γ , we define the following splicing operation in γ' :

$$\left(\left(x, \left[\begin{array}{cc} a_{ij} & a_{in} \\ a_{nj} & a_{nn} \end{array}\right]\right), \left(y, \left[\begin{array}{cc} b_{ij} & b_{in} \\ b_{nj} & b_{nn} \end{array}\right]\right)\right) \vdash_r \left(z, \left[\begin{array}{cc} a_{ij} + b_{ij}a_{in} + b_{in} \\ a_{nj} + b_{nj}a_{nn} + b_{nn} \end{array}\right]\right).$$

Since the *n*th and (n+1)th rows and columns of the matrices in M' are the same, they fulfill the same cut-point requirement. Thus, $L_{\omega}(\gamma, *\alpha) = L_{\omega}(\gamma', *\alpha)$.

Using the same arguments, one can prove the following theorem:

Theorem 4.9. For $n \geq 1$,

$$\omega EH(\mathbf{FIN},\mathbf{FIN},\mathbb{Z}^n,+) \subseteq \omega EH(\mathbf{FIN},\mathbf{FIN},\mathbb{M}_n,+).$$

5. Conclusion. A brief discussion on weighted splicing system which has been introduced in [4] is mentioned in this paper. We establish some new facts on the weighted splicing system and show that an extension of splicing systems with finite components of weights has a higher generative power compared to the usual splicing systems. In some cases of weighted splicing systems, the weights associated to the axioms can generate non-context-free languages. Furthermore, we found that the computational power of the weighted splicing systems had different generative power when different weighting spaces were considered as shown in Theorems 4.6 to 4.8. However, the incomparability problems for the families of threshold languages generated by weighted splicing systems with the families of linear and context-free languages still remain open. Here, the inverse inequality in Theorem 4.3 and the strictness of the second inclusion in Theorem 4.4 is yet to be solved. It may not be possible for the recursively enumerable languages, to be generated by weighted splicing system, since the simulation of the context-sensitivity property of phrase-structure grammar is not possible with weights. On the other hand, linear languages and simple matrix languages (as mentioned in [12]) can most probably be generated by weighted splicing systems, if matrices are used as the weighting space.

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