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New Form of Fuzzy Bi Γ-Ideals in Ordered Γ- Semigroup

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Abstract. The structure of ordered Γ-semigroup is a generalization of ordered semigroup. The purpose of this paper is to introduce the notion of $(\in,\in\vee q_k)-$ fuzzy bi Γ-ideals in ordered Γ-semigroup. This new concept is the generalization of fuzzy bi-ideals of ordered semigroup. Further we explore some classifications of different classes such as regular, right regular in terms of $(\in,\in\vee q_k)-$ fuzzy bi Γ-ideals. Particularly, we examine that the concepts of $(\in,\in\vee q_k)-$ fuzzy bi Γ-ideals and $(\in,\in\vee q_k)-$ fuzzy generalized bi Γ-ideals coincide in right regular ordered Γ-semigroup and regular ordered Γ-semigroup.

INTRODUCTION

Since the pioneered of the concept of fuzzy set by Zadeh in [1], many researches were coming up to show the importance of the concept and its application to topology, measure theory, group theory, logic and many other disciplines. Rosenfeld [2] was arguably the first to studied the concept of fuzzy subgroups and come up with many results in groups which were extended to develop the theory of fuzzy subgroups. The concept of a $\Gamma-$semigroup has been introduced by Sen in [3]. Thereafter, Kehayapulu [4] considered the definition of $\Gamma-$semigroup defined by Sen and added the uniqueness condition to show the pass way from ordered semigroups to ordered $\Gamma-$semigroups and clearly describe that the result of ordered semigroup can be transferred to ordered $\Gamma-$semigroup. Good [5] introduced the notion of bi $\Gamma-$ideals of semigroups while in [6] Chinram studied some properties of bi $\Gamma-$ideals in semigroups. The contribution of Bhakat and Das [7], gave the concept of $(\alpha,\beta)-$fuzzy subgroups by using the "belongs to" relation $\in$ and "quasi-coincident with" relation q between a fuzzy point and fuzzy subgroup and introduced the concept of a $(\in,\in\vee q)-$ fuzzy subgroups, where $\alpha,\beta\in\{\in,q,\in,\vee q,\in\wedge q\}$ and $\alpha\neq\in\wedge q$. Hence, $(\in,\in\vee q)-$fuzzy subgroup is a useful generalization of Resenfied's fuzzy subgroups. In addition, a further generalization is given in [7]. The study of fuzzy set theory to ideals in semigroups by Kuroki can be traced in [7, 8]. Kazanci and Yamak [9] studied $(\in,\in\vee q_k)-$ fuzzy bi ideals of a semigroups. Khan *et al.* [10, 11] introduced the concept of generalized bi $\Gamma-$ideals of type (λ,θ) in ordered semigroups. In this paper, we explore some classifications of different classes such as regular, weakly regular and intra-regular ordered $\Gamma-$semigroups in terms of $(\in,\in\vee q_k)-$ fuzzy bi Γ-ideals. Particularly, we examine that the concepts of $(\in,\in\vee q_k)-$ fuzzy bi Γ-ideals and

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$(\in,\in\vee q_k)-$fuzzy generalized bi Γ-ideals coincide in right regular ordered Γ-semigroup and regular ordered Γ-semigroup.

BASIC DEFINITIONS AND PRELIMINARY

In what follows, we state some basic definitions and preliminaries results that will be used in this paper.

Definition 1. [2] *A fuzzy subset* A *defined on a set* X *is represented as* $A=\{(x,\lambda_A(x)),\text{ where } x\in X\}$.

Definition 2. *If* G *and* Γ *are non-empty sets, then a structure* $(G,\Gamma,\leq)$ *is called an ordered* $\Gamma-$*semigroup if:*
(b₁) $(x\alpha y)\beta z=x\alpha(y\beta z)$ *for all* $x,y,z\in G$ *and* $\alpha,\beta\in\Gamma$,
(b₂) $a\leq b\rightarrow a\alpha x\leq b\alpha x$ and $x\beta a\leq x\beta b$ *for all* $a,b,x\in G$ *and* $\alpha,\beta\in\Gamma$.

Definition 3. [12] *A nonempty subset* A *of an ordered semigroup* G *for all* $x,y,z\in G$ *is called a bi-ideal of* G *if*
(b_3) $a\leq b\in A\rightarrow a\in A$,
(b_4) $A\Gamma G\Gamma A\subseteq A$,
(b_5) $A^2\subseteq A$.

MAIN RESULTS

$(\in,\in\vee q_k)-$ Fuzzy Bi $\Gamma-$ Ideals in Ordered $\Gamma-$ Semigroup

In this part we extend the concept of fuzzy bi ideal of ordered semigroup to ordered $\Gamma-$semigroup and introduce the notion of fuzzy bi $\Gamma-$ideals. Throughout this paper ordered $\Gamma-$semigroup will be represented by G and $k\in[0,1)$.

Definition 4. *Given a fuzzy subset* λ *of* G, *then* λ *is called* $(\in,\in\vee q_k)-$*fuzzy bi* $\Gamma-$*ideal of* G *if for all* $a,b,c\in G$, $\alpha,\beta\in\Gamma$ *and* $t_1,t_2\in(0,1]$ *the following three conditions are satisfied:*
(c_1) $b_t\in\lambda\rightarrow a_t\in\vee q_k\lambda$ *such that* $a\leq b$,
(c_2) $a_{t_1}\in\lambda,\ b_{t_2}\in\lambda\rightarrow(a\alpha b)_{\min\{t_1,t_2\}}\in\vee q_k\lambda$,
(c_3) $a_{t_1}\in\lambda,\ c_{t_2}\in\lambda\rightarrow(a\alpha b\beta c)_{\min\{t_1,t_2\}}\in\vee q_k\lambda$.

Theorem 1. *Given* B *as a bi* $\Gamma-$*ideal of* G *and* λ *a fuzzy subset of* G *defined as:*

$$\lambda(a)=:\begin{cases}\geq\dfrac{1-k}{2}, & \text{if } a\in B,\\ 0, & \text{if } a\notin B.\end{cases}$$

Then, λ *is both* $(q,\in\vee q_k)-$*fuzzy bi* $\Gamma-$*ideal and* $(\in,\in\vee q_k)-$*fuzzy bi* $\Gamma-$*ideal of* G.

Proof: Let B be a bi $\Gamma-$ideal of G, λ be a fuzzy subset of G and $a,b\in G$. If $a\leq b$ and $t\in(0,1]$ such that $b_t q\lambda$, then $\lambda(b)+t>1$ and hence $b\in B$. Since $a\leq b\in B$, this implies $a\in B$ and therefore $\lambda(a)\geq\frac{1-k}{2}$. Thus $(a)_t\in\lambda$ whenever $\frac{1-k}{2}\geq t$ and if $\frac{1-k}{2}<t$, then $a_t q_k\lambda$. Hence, $a_t\in\vee q_k\lambda$.

To prove the second condition, we suppose $a,b \in G, t_1,t_2 \in (0,1]$ and $\alpha \in \Gamma$ such that $a_{t_1} q\lambda,\ b_{t_2} q\lambda.$ Then $\lambda(a)+t_1 > 1$ and $\lambda(b)+t_2 > 1$ hence, $a,b \in B.$ By hypothesis, B is a bi $\Gamma-$ideal so we have $a\alpha b \in B$ and therefore, $\lambda(a\alpha b) \geq \frac{1-k}{2}.$ This implies,

$$\lambda(a\alpha b) = \begin{cases} \geq \min\{t_1,t_2\}, & \text{if} \quad \min\{t_1,t_2\} \leq \frac{1-k}{2}, \\ \frac{1-k}{2}, & \text{if} \quad \min\{t_1,t_2\} > \frac{1-k}{2}. \end{cases}$$

It follows that $(a\alpha b)_{\min\{t_1,t_2\}} \in q_k\lambda.$

Next we take $a,b,c \in G$ and $t_1,t_2 \in (0,1]$ such that $a_{t_1} q_k \lambda$ and $c_{t_2} q\lambda.$ Then $\lambda(a)+t_1 > 1$ and $\lambda(c)+t_2 > 1$ hence $a,c \in B.$ But B is given to be a bi $\Gamma-$ideal and therefore it implies that $a\alpha b\beta c \in B$ for $\alpha,\beta \in \Gamma$ and hence $\lambda(a\alpha b\beta c) \geq \frac{1-k}{2}.$ Therefore,

$$\lambda(a\alpha b\beta c) = \begin{cases} \geq \min\{t_1,t_2\}, & \text{if} \quad \min\{t_1,t_2\} \leq \frac{1-k}{2}, \\ > 1-t-k, & \text{if} \quad \min\{t_1,t_2\} > \frac{1-k}{2}. \end{cases}$$

It follows that $(a\alpha b\beta c)_{\min\{t_1,t_2\}} \in \vee q_k\lambda.$ Consequently, λ is also a $(q,\in \vee q_k)-$fuzzy bi $\Gamma-$ideal of $G.$ Similarly, we can show that λ is an $(\in,\in \vee q_k)-$fuzzy bi $\Gamma-$ideal of $G.$ ■

Theorem 2. *Given a fuzzy subset λ of G and $a,b \in G, \alpha,\beta \in \Gamma.$ Then the following two statements are equivalent:*
(1). λ is a $(\in,\in \vee q_k)-$fuzzy bi $\Gamma-$ideal of $G.$
(2). (a) $a \leq b \rightarrow \lambda(a) \geq \min\left\{\lambda(b), \frac{1-k}{2}\right\},$

(b) $\lambda(a\alpha b) \geq \min\left\{\lambda(a), \lambda(b), \frac{1-k}{2}\right\},$

(c) $\lambda(a\alpha b\beta c) \geq \min\left\{\lambda(a), \lambda(c), \frac{1-k}{2}\right\}.$

Proof: $(1) \Rightarrow (2)$: Suppose λ is a $(\in,\in \vee q_k)-$fuzzy bi $\Gamma-$ideal of $G.$ Suppose by contradiction that there exist $a,b \in G,$ with $a \leq b$ such that $\lambda(a) < \min\left\{\lambda(b), \frac{1-k}{2}\right\}.$ Then for some $t \in \left(0, \frac{1-k}{2}\right]$ we have $\lambda(a) < t \leq \min\left\{\lambda(b), \frac{1-k}{2}\right\}.$ It follows that $a_t \bar{\in} \lambda,$ $a_t \bar{q}_k \lambda$ (since $\lambda(a)+t+k < \frac{1-k}{2} + \frac{1-k}{2} + k = 1$) that is $b_t \overline{\in \vee q_k} \lambda,$ therefore a contradiction with Definition 4 (c_1). Hence $\lambda(a) \geq \min\left\{\lambda(b), \frac{1-k}{2}\right\}$ $a,b \in G.$ with $a \leq b.$

Next, we suppose that there exist $a,b \in G$ and $\alpha \in \Gamma$ such that $\lambda(a\alpha b) < \min\left\{\lambda(a), \lambda(b), \frac{1-k}{2}\right\}.$ Then $\lambda(a\alpha b) < t \leq \min\left\{\lambda(a), \lambda(b), \frac{1-k}{2}\right\}$ for some $t \in \left(0, \frac{1-k}{2}\right].$ This inequality implies $a_t \in \lambda,$ $b_t \in \lambda$ but

$(a\alpha b)_t \overline{\in \vee q_k} \lambda$. This contradict with Definition 4 (c_2) and hence $\lambda(a\alpha b) \geq \min\left\{\lambda(a), \lambda(b), \frac{1-k}{2}\right\}$ for all $a, b \in G$ and $\alpha, \beta \in \Gamma$.

Finally, for the third condition we $a, b, c \in G$ and $\alpha, \beta \in \Gamma$. Suppose on the contrary that $\lambda(a\alpha b\beta c) \leq \min\left\{\lambda(a), \lambda(c), \frac{1-k}{2}\right\}$. Then there exist some $t \in \left(0, \frac{1-k}{2}\right]$ such that $\lambda(a\alpha b\beta c) < t \leq \min\left\{\lambda(a), \lambda(c), \frac{1-k}{2}\right\}$. Thus from this inequality we see that $a_t \in \lambda$, $c_t \in \lambda$, but $\lambda(a\alpha b\beta c)_t \overline{\in \vee q_k} \lambda$. Again this is a contradiction to Definition 4 (c_3) and hence $\lambda(a\alpha b\beta c) \geq \min\left\{\lambda(a), \lambda(c), \frac{1-k}{2}\right\}$ for all $a, b, c \in G$ and $\alpha, \beta \in \Gamma$.

(2) $\Rightarrow$ (1): Let λ be a fuzzy subset of G and Conditions (a), (b) and (c) are applied. Consider $a, b \in G$ such that $a \leq b$ and $b_t \in \lambda$ for some $t \in (0,1]$, then $\lambda(b) \geq t$. By Condition (a) above we have;

$$\begin{aligned}\lambda(a) &\geq \min\left\{\lambda(b), \frac{1-k}{2}\right\} \\ &\geq \min\left\{t, \frac{1-k}{2}\right\}.\end{aligned}$$

$$\lambda(a) = \begin{cases} t, & \text{if } \quad t \leq \frac{1-k}{2}, \\ \frac{1-k}{2}, & \text{if } \quad t > \frac{1-k}{2}. \end{cases}$$

From the above inequality, we have $a_t \in \vee q_k \lambda$.

Next, let us consider $\alpha \in \Gamma$ and $a_{t_1}, b_{t_2} \in \lambda$ for some $t_1, t_2 \in (0,1]$. Then $\lambda(a) \geq t_1$ and $\lambda(b) \geq t_2$. Hence, Condition (b) implies that

$$\begin{aligned}\lambda(a\alpha b) &\geq \min\left\{\lambda(a), \lambda(b), \frac{1-k}{2}\right\} \\ &\geq \min\left\{t_1, t_2, \frac{1-k}{2}\right\} \\ &= \begin{cases} \min\{t_1, t_2\}, & \text{if } \quad \min\{t_1, t_2\} \leq \frac{1-k}{2}, \\ \frac{1-k}{2}, & \text{if } \quad \min\{t_1, t_2\} > \frac{1-k}{2}. \end{cases}\end{aligned}$$

It follows that $(a\alpha b)_t \in \vee q_k \lambda$.

Lastly, take $a, b, c \in G$ and $\alpha, \beta \in \Gamma$. If $a_{t_1}, c_{t_2} \in \lambda$ for some $t_1, t_2 \in (0,1]$, then $(a\alpha b\beta c)_t \in \vee q_k \lambda$. By Condition (c) we have;

$$\begin{aligned}\lambda(a\alpha b\beta c) &\geq \min\left\{\lambda(a), \lambda(c), \frac{1-k}{2}\right\} \\ &\geq \min\left\{t_1, t_2, \frac{1-k}{2}\right\}\end{aligned}$$

$$\lambda(a\alpha b\beta c)=\begin{cases}\min\{t_1,t_2\}, & \text{if} \quad \min\{t_1,t_2\}\le \frac{1-k}{2},\\ \frac{1-k}{2}, & \text{if} \quad \min\{t_1,t_2\}> \frac{1-k}{2}.\end{cases}$$

It follows that $(a\alpha b\beta c)_t \in \vee q_k \lambda$.
From the above discussion we conclude that λ satisfies all the three stated conditions of Definition 4 and hence λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal of G. ■

To investigate a relationship between ordinary bi $\Gamma-$ideal and fuzzy bi $\Gamma-$ideal of the form $(\in,\in\vee q_k)$ (Definition 4) we provide the following results.

Theorem 3. *Let λ be a fuzzy subset of G. Then λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal of G if and only if the non-empty level subset $U(\lambda,t)$ is a bi $\Gamma-$ideal of G for all $a,b\in G,\alpha,\beta\in\Gamma$ and $t\in\left(0,\frac{1-k}{2}\right]$.*

Proof: Assume that λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal of G and $a,b,c\in G$ such that $a\le b\in U(\lambda,t)$ is a bi $\Gamma-$ideal of G for all $a,b,c\in G,\alpha,\beta\in\Gamma$ and $t\in\left(0,\frac{1-k}{2}\right]$. Then, $\lambda(b)\ge t$ and from Theorem 2 (2) (a) we have;

$$\begin{aligned}\lambda(a)&\ge \min\left\{\lambda(b),\frac{1-k}{2}\right\}\\ &\ge \min\left\{t,\frac{1-k}{2}\right\}\\ &=t.\end{aligned}$$

It follows that $a_t\in U(\lambda,t)$.

If $\alpha\in\Gamma$ such that $a,b\in U(\lambda,t)$ for some $t\in\left(0,\frac{1-k}{2}\right]$, then $(a\alpha b)_t\in\vee q_k\lambda$. By Theorem 2 (2) (b) we have:

$$\begin{aligned}\lambda(a\alpha b)&\ge \min\left\{\lambda(a),\lambda(b),\frac{1-k}{2}\right\}\\ &\ge \min\left\{t,t,\frac{1-k}{2}\right\}\\ &=\begin{cases}t, & \text{if} \quad t\le\frac{1-k}{2},\\ \frac{1-k}{2}, & \text{if} \quad t>\frac{1-k}{2}.\end{cases}\end{aligned}$$

The above inequality implies that $(a\alpha b)_t\in U(\lambda,t)$.

If $\alpha,\beta\in\Gamma$ and $t\in\left(0,\frac{1-k}{2}\right]$ such that $a,c\in U(\lambda,t)$, then $(a\alpha b\beta c)_t\in U(\lambda,t)$. Indeed: By Theorem 2 (2) (c) we have;

$$\begin{aligned}\lambda(a\alpha b\beta c)&\ge \min\left\{\lambda(a),\lambda(c),\frac{1-k}{2}\right\}\\ &\ge \min\left\{t,t,\frac{1-k}{2}\right\} \text{ but } t\in\left(0,\frac{1-k}{2}\right]\\ &=t,\end{aligned}$$

in which it follows that $(a\alpha b\beta c)_t\in U(\lambda;t)$. Hence, $U(\lambda;t)$ is a bi $\Gamma-$ideal of G.

Conversely, let us suppose that the non-empty level subset $U(\lambda;t)$ for all $t\in\left(0,\frac{1-k}{2}\right]$ is a bi $\Gamma-$ideal of G. Let $a,b\in G$ such that $a\le b$ and $\lambda(a)<\min\left\{\lambda(b),\frac{1-k}{2}\right\}$. Then there exists $t\in\left(0,\frac{1-k}{2}\right]$ such that $\lambda(a)<\min\left\{\lambda(b),\frac{1-k}{2}\right\}$. This inequality implies $b_t\in U(\lambda;t)$, but $a_t\,\overline{\in}\,U(\lambda;t)$ a contradiction (Since $U(\lambda;t)$ is a bi $\Gamma-$ideal). Hence $\lambda(a)\ge\min\left\{\lambda(b),\frac{1-k}{2}\right\}$.

If $a,b\in G$ $\alpha\in\Gamma$ and $\lambda(a\alpha b)<t\le\min\left\{\lambda(a),\lambda(b),\frac{1-k}{2}\right\}$, then for some $t\in\left(0,\frac{1-k}{2}\right]$ we can write $\lambda(a\alpha b)<t\le\min\left\{\lambda(a),\lambda(b),\frac{1-k}{2}\right\}$. From this inequality, we see that $a_t,b_t\in U(\lambda;t)$ but $(a\alpha b)_t\,\overline{\in}\,U(\lambda;t)$, again a contradiction. Hence $\lambda(a\alpha b)\ge\min\left\{\lambda(a),\lambda(b),\frac{1-k}{2}\right\}$.

Again let $a,b,c\in G,\alpha,\beta\in\Gamma$ and $\lambda(a\alpha b\beta c)<\min\left\{\lambda(a),\lambda(c),\frac{1-k}{2}\right\}$. Then $\lambda(a\alpha b\beta c)<t\le\min\left\{\lambda(a),\lambda(c),\frac{1-k}{2}\right\}$ for some $t\in\left(0,\frac{1-k}{2}\right]$ and hence have $a_t,c_t\in U(\lambda;t)$ but $(a\alpha b\beta c)_t\,\overline{\in}\,U(\lambda;t)$. This contradicts the definition of bi $\Gamma-$ideal and therefore $\lambda(a\alpha b\beta c)\ge\min\left\{\lambda(a),\lambda(c),\frac{1-k}{2}\right\}$. In light of the above discussion and Theorem 2 it is concluded that λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal of G.

In this regard, in the following proposition we investigate that in regular ordered $\Gamma-$semigroups, the concepts of $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideals and $(\in,\in\vee q_k)-$fuzzy generalized bi $\Gamma-$ideals coincide.

Proposition 1. *Every fuzzy generalized bi $\Gamma-$ideals of the form $(\in,\in\vee q_k)$ of a regular ordered $\Gamma-$semigroup is also $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideals.*

Proof: Let G be a regular ordered $\Gamma-$semigroup, $a,b\in G$ and $\alpha,\beta\in\Gamma$. Since G is regular so $a\le a\alpha x\beta a$ for some $x\in G$. If λ is $(\in,\in\vee q_k)-$fuzzy generalized bi $\Gamma-$ideals of G. Then, $a\alpha b\le(a\alpha x\beta a)\alpha a$ and hence by hypothesis;

$$\lambda(a\alpha b)\ge\min\left\{\lambda((a\alpha x\beta a)\alpha b),\frac{1-k}{2}\right\}$$

which can also be written in the form:

$$\lambda(a\alpha b)\ge\min\left\{\lambda(a\alpha(x\beta a)\alpha b),\frac{1-k}{2}\right\}$$
$$\ge\min\left\{\lambda(a),\lambda(b),\frac{1-k}{2}\right\}.$$

Therefore, λ is a $(\in,\in\vee q_k)-$fuzzy $\Gamma-$sub semigroup of G and hence λ is a $(\in,\in\vee q_k)-$fuzzy bi $\Gamma-$ideal of G.

CONCLUSION

The structure of ordered $\Gamma-$semigroup is considered important in several areas of mathematics, such as in coding and language theory, automata theory, combinatorics and mathematical analysis. An ordered $\Gamma-$semigroup is a generalization of both ordered semigroup and ordered ternary semigroup. In this paper, we explore some classifications of different classes such as regular and right regular ordered Γ-semigroups in terms of $(\in,\in\vee q_k)-$ fuzzy bi Γ-ideals. Particularly, we examine that the concepts of $(\in,\in\vee q_k)-$ fuzzy bi Γ-ideals and $(\in,\in\vee q_k)-$ fuzzy generalized bi Γ-ideals coincide with right regular ordered $\Gamma-$semigroup.

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