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Graph Polynomials of the Conjugate Graph of Dihedral Groups

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Abstract. There are various aspects of combinatorial information that are stored in the coefficients of graph polynomials such as the independence polynomial, the matching polynomial and the clique polynomial. The independence polynomial of a graph is defined as a polynomial in which its coefficients are the number of independent sets in the graph. The independent set of a graph is a set of pairwise non-adjacent vertices. The matching polynomial of a graph is the polynomial in which the coefficients are the number of matching sets in the graph. The matching set of a graph is a set of pairwise edges which do not have common vertices. The clique polynomial of a graph is the polynomial in which the coefficients are the number of cliques in the graph. The clique of a graph is the set of pairwise adjacent vertices. Meanwhile, a graph of group G is called conjugate graph if the vertices are non-central elements of G and two distinct vertices are connected if they are conjugate. In this research, the independence polynomial, the matching polynomial and the clique polynomial of the conjugate graph of dihedral groups of order at most twelve are computed.

Keywords: clique polynomial, conjugate graph, dihedral group, independence polynomial, matching polynomial

INTRODUCTION

A graph polynomial is the type of polynomial in which its coefficients store various aspects of combinatorial information concerning a graph. There are some types of graph polynomials that had been introduced and studied by other researchers such as the independence polynomial [1], the matching polynomial [2] and the clique polynomial [1]. The concept of independence polynomial was introduced by Hoede and Li [1] in 1994, together with the concepts of the clique polynomial. Earlier, Farrell [2] in 1979 introduced the concept of the matching polynomial which was actually had been found before that in 1977 by Gutman [3]. However, only simple matching polynomial will be considered in this research and will be referred just as matching polynomial throughout this paper.

Levit and Mandrescu [4] stated that the matching polynomial of a graph is actually identical with the independence polynomial of its line graph. Furthermore, Hoede and Li [1] had also presented that the clique polynomial of a graph is equal to the independence polynomial of its complement graph.

Note that throughout this paper, the graphs mentioned are all simple graphs, and will be referred only as graphs. From [5], we define that a simple graph $\Gamma=(V,E)$ consists of V, a nonempty vertex set and E, a set of edges which are the unordered pairs of vertices from V. Let $u,v\in V$, then the vertices u and v are adjacent to each other in Γ if and only if there is an edge between u and v, i.e. $e=(u,v)\in E$. The edge e is said to be incident with each one of its end vertices, u and v.

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Using the properties of graphs, the algebraic properties of groups can be studied. One type of graph that is associated to groups is the conjugate graph. Erfanian and Tolue [6] in 2012 had introduced the concepts of the conjugate graphs of finite nonabelian groups.

In this research, the independence polynomial, the matching polynomial and the clique polynomial of the conjugate graph are computed for some dihedral groups, given in the following:

i) $G_1 = \langle a,b : a^3 = b^2 = 1, bab = a^{-1} \rangle$, the dihedral group of order 6

ii) $G_2 = \langle a,b : a^4 = b^2 = 1, bab = a^{-1} \rangle$, the dihedral group of order 8

iii) $G_3 = \langle a,b : a^5 = b^2 = 1, bab = a^{-1} \rangle$, the dihedral group of order 10

iv) $G_4 = \langle a,b : a^6 = b^2 = 1, bab = a^{-1} \rangle$, the dihedral group of order 12.

PRELIMINARIES

In this section, some important definitions and theorems on the independence polynomial, the matching polynomial and the clique polynomial of graphs are stated. The fundamental concepts of the conjugate graph of finite groups are also presented.

We start with the basic concepts of graphs and how from the graphs, we can actually compute their polynomials.

Definition 1 [7] **Independent Set, Independence Number**
An independent set is a set of vertices in which no two distinct vertices are adjacent. The maximum number of vertices in an independent set of a graph is called the independence number, denoted by $\alpha(\Gamma)$.

Definition 2 [9] **Matching Set, Matching Number**
Let E be the edge set of the graph Γ. A matching set is the subset $M \subseteq E$, in which there is no common vertex between any two edges in M. The matching number of the graph Γ is the maximum cardinality of the matching set in Γ, denoted by $\beta(\Gamma)$.

Definition 3 [7] **Clique, Clique Number**
Let V be the vertex set of the graph Γ. A clique is the subset $C \subseteq V$, in which every vertex is adjacent to every other vertex. The clique number of the graph Γ is the size of the biggest clique, denoted by $\omega(\Gamma)$.

Definition 4 [8] **Neighborhood, Closed Neighborhood, Empty Graph, Null Graph**
Let (u,v) be an edge of graph Γ. Then the vertex u is called the neighbor of the vertex v. Open neighborhood (or just neighborhood), of v is the set of all vertices adjacent to v, denoted as follows :

$$N(v)=\{u \in V \mid (u,v) \in E, u \neq v\}.$$

Closed neighborhood of v in Γ is the set $N[v] = N(v) \cup \{v\}$.
If the neighborhood of every vertex is empty, then there is no edge in the graph. Such graph is called an empty graph, denoted by E_n. If $n = 0$, then the graph is called null graph, denoted by $E_0 := \varnothing$.

Definition 5 [7] **Complete Graph**
A complete graph, K_n is a graph with n vertices where each pair of distinct vertices is connected by an edge.

Definition 6 [5] **Line Graph**
The line graph of a graph Γ, denoted as $L(\Gamma)$, is defined as the graph containing the edges of Γ as its vertices and two vertices in $L(\Gamma)$ is adjacent if the corresponding edges in Γ have a common vertex.

The following are the fundamental concepts related to the graph polynomials that are used throughout this paper.

Definition 7 [1] **Independence Polynomial**

The independence polynomial of a graph Γ is the polynomial whose coefficients on x^k is given by the number of independent sets of size k in Γ. This is denoted by $I(\Gamma;x)$ as follows:

$$I(\Gamma;x)=\sum_{k=0}^{\alpha(\Gamma)} a_k x^k,$$

where a_k is the number of independent sets of size k in Γ and $\alpha(\Gamma)$ is the independence number of graph Γ.

Theorem 1 [1]

Let Γ_1 and Γ_2 be two disjoint graphs. Then we have the independence polynomial of the union of two graphs as follows:

$$I(\Gamma_1 \cup \Gamma_2;x)=I(\Gamma_1;x)\cdot I(\Gamma_2;x).$$

Proposition 1 [10]

The independence polynomial of a complete graph, K_n is $I(K_n;x)=1+nx$.

Definition 8 [2] **Matching Polynomial**

The matching polynomial of a graph Γ is the polynomial whose coefficients on x^k is given by the number of matching sets of order k in Γ. We denote the polynomial as follows:

$$M(\Gamma;x)=\sum_{k=0}^{\beta(\Gamma)} b_k x^k,$$

where b_k is the number of matching sets of size k in Γ and $\beta(\Gamma)$ is the matching number of graph Γ.

Definition 9 [1] **Clique Polynomial**

The clique polynomial of a graph Γ is the polynomial whose coefficients on x^k is given by the number of cliques of order k in Γ. We denote the polynomial as follows:

$$C(\Gamma;x)=\sum_{k=0}^{\omega(\Gamma)} c_k x^k,$$

where c_k is the number of cliques of size k in Γ and $\omega(\Gamma)$ is the clique number of graph Γ.

Proposition 2 [1, 2, 10]

The independence polynomial, the matching polynomial and the clique polynomial of a null graph are $I(\varnothing;x)=1$, $M(\varnothing;x)=1$ and $C(\varnothing;x)=1$ respectively.

Next, we will state some basic concepts related to the conjugate graph of finite nonabelian groups.

Definition 10 [6] **Conjugate, Conjugacy Class**

Let G be a group and $a,b \in G$. a and b are called conjugate if there exists an element $g \in G$ with $gag^{-1}=b$. For a fixed element $a \in G$, the conjugacy class of a in G is $a^G = cl(a)=\{g \in G : \text{there exists } x \in G, g = xax^{-1}\}$. If the conjugacy class of a contains only one element, then a lies in the center $Z(G)=\{y \in G : xy = yx, x \in G\}$.

Definition 11 [6] **Conjugate Graph**

A conjugate graph Γ_G^c of a group G, is defined as the graph whose vertex set, $V(\Gamma_G^c)$ is non-central elements of G, that is $|V(\Gamma_G^c)|=|G|-|Z(G)|$ in which two distinct vertices are adjacent if they are conjugate.

The aim of this paper is to present the independence polynomial, the matching polynomial and the clique polynomial of the conjugate graph of some dihedral groups G_1, G_2, G_3 and G_4.

RESULTS AND DISCUSSION

This section consists of three parts. The first part presents the independence polynomial of the conjugate graph of dihedral groups G_1, G_2, G_3 and G_4. The second part presents the matching polynomial of the dihedral groups G_1, G_2, G_3 and G_4 and the last part presents the results on the clique polynomial of the dihedral groups G_1, G_2, G_3 and G_4.

The Independence Polynomial of the Conjugate Graph of Dihedral Groups of Order at Most 12

Proposition 1 Let G_1 be the dihedral group of order 6, $G_1 = \langle a,b : a^3 = b^2 = 1, bab = a^{-1} \rangle$. Then the independence polynomial of the conjugate graph of G_1 is $I\left(\Gamma_{G_1}^c; x\right) = 1 + 5x + 6x^2$.

Proof The conjugate graph of G_1 is $\Gamma_{G_1}^c = K_2 \cup K_3$. It consists of the vertex set $V\left(\Gamma_{G_1}^c\right) = \left\{a, a^2, b, ab, a^2b\right\}$ and the edge set $E\left(\Gamma_{G_1}^c\right) = \left\{e_1 = \left(a, a^2\right), e_2 = \left(b, a^2b\right), e_3 = \left(b, ab\right), e_4 = \left(ab, a^2b\right)\right\}$. The graph $\Gamma_{G_1}^c$ has an independence number $\alpha\left(\Gamma_{G_1}^c\right) = 2$. There are six independent sets of size 2 which are $\{a,b\}$, $\{a,ab\}$, $\{a,a^2b\}$, $\{a^2,b\}$, $\{a^2,ab\}$ and $\{a^2,a^2b\}$, and five independent sets of size one in which the sets containing each vertex of G_1, denoted as $\{a\}$, $\{a^2\}$, $\{b\}$, $\{ab\}$ and $\{a^2b\}$. Hence by the definition of the independence polynomial of a graph, we obtain

$$I\left(\Gamma_{G_1}^c; x\right) = \sum_{k=0}^{2} a_k x^k = a_0x^0 + a_1x^1 + a_2x^2 = 1 + 5x + 6x^2. \text{ W}$$

Proposition 2 Let G_2 be the dihedral group of order 8, $G_2 = \langle a,b : a^4 = b^2 = 1, bab = a^{-1} \rangle$. Then the independence polynomial of the conjugate graph of G_2 is $I\left(\Gamma_{G_2}^c; x\right) = 1 + 6x + 12x^2 + 8x^3$.

Proof The conjugate graph of G_2 is $\Gamma_{G_2}^c = K_2 \cup K_2 \cup K_2$. It consists of the vertex set $V\left(\Gamma_{G_2}^c\right) = \left\{a, a^3, b, a^2b, ab, a^3b\right\}$ and the edge set $E\left(\Gamma_{G_2}^c\right) = \left\{e_1 = \left(a, a^3\right), e_2 = \left(b, a^2b\right), e_3 = \left(ab, a^3b\right)\right\}$. The graph $\Gamma_{G_2}^c$ has an independence number $\alpha\left(\Gamma_{G_2}^c\right) = 3$. From the graph, there are eight independent sets of size 3 which are $\{a,b,ab\}$, $\{a,b,a^3b\}$, $\{a,a^2b,ab\}$, $\{a,a^2b,a^3b\}$, $\{a^3,b,ab\}$, $\{a^3,b,a^3b\}$, $\{a^3,a^2b,ab\}$ and $\{a^3,a^2b,a^3b\}$, and the independent sets of size 2 are $\{a,b\}$, $\{a,ab\}$, $\{a,a^2b\}$, $\{a,a^3b\}$, $\{a^3,b\}$, $\{a^3,ab\}$, $\{a^3,a^2b\}$, $\{a^3,a^3b\}$, $\{b,ab\}$, $\{b,a^3b\}$, $\{a^2b,ab\}$ and $\{a^2b,a^3b\}$. Hence by the definition of the independence polynomial of a graph, we obtain

$$I\left(\Gamma_{G_2}^c; x\right) = \sum_{k=0}^{3} a_k x^k = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 = 1 + 6x + 12x^2 + 8x^3. \text{ W}$$

Proposition 3 Let G_3 be the dihedral group of order 10, $G_3 = \langle a,b : a^5 = b^2 = 1, bab = a^{-1} \rangle$. Then the independence polynomial of the conjugate graph of G_3 is $I\left(\Gamma_{G_3}^c; x\right) = 1 + 9x + 24x^2 + 20x^3$.

Proof The conjugate graph of G_3 is $\Gamma_{G_3}^c = K_2 \cup K_2 \cup K_5$ that consists of the vertex set $V\left(\Gamma_{G_3}^c\right) = \left\{a, a^4, a^2, a^3, b, ab, a^2b, a^3b, a^4b\right\}$ and the edge set $E\left(\Gamma_{G_3}^c\right)$. contains the following 12 edges:

$$e_1 = \left(a, a^4\right), \quad e_2 = \left(a^2, a^3\right), \quad e_3 = \left(b, a^4b\right), \quad e_4 = \left(b, ab\right),$$
$$e_5 = \left(ab, a^2b\right), \quad e_6 = \left(a^2b, a^3b\right), \quad e_7 = \left(a^3b, a^4b\right), \quad e_8 = \left(b, a^3b\right),$$
$$e_9 = \left(b, a^2b\right), \quad e_{10} = \left(ab, a^4b\right), \quad e_{11} = \left(a^2b, a^4b\right) \text{ and } \quad e_{12} = \left(ab, a^3b\right).$$

The graph $\Gamma_{G_3}^c$ has an independence number $\alpha\left(\Gamma_{G_3}^c\right) = 3$. There are 20 independent sets of size 3 and 24 independent sets of size 2. Hence, by the definition of the independence polynomial of a graph, we obtain

$$I\left(\Gamma_{G_3}^c; x\right) = \sum_{k=0}^{3} a_k x^k = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 = 1 + 9x + 24x^2 + 20x^3. \text{ W}$$

Proposition 4 Let G_4 be the dihedral group of order 12, $G_4 = \left\langle a, b : a^6 = b^2 = 1, bab = a^{-1}\right\rangle$. Then the independence polynomial of the conjugate graph of G_4 is $I\left(\Gamma_{G_4}^c; x\right) = 1 + 10x + 37x^2 + 60x^3 + 36x^4$.

Proof The conjugate graph of G_4 is $\Gamma_{G_4}^c = K_2 \cup K_2 \cup K_3 \cup K_3$ that consists of the vertex set $V\left(\Gamma_{G_4}^c\right) = \left\{b, a^3ba, a^2b, aba, a, ab, ba, a^2ba, a^3, a^3b\right\}$ and the edge set $E\left(\Gamma_{G_4}^c\right)$ contains the following 8 edges:

$$e_1 = \left(b, a^3ba\right), \quad e_2 = \left(a^2b, aba\right), \quad e_3 = \left(a, ba\right), \quad e_4 = \left(a, ab\right),$$
$$e_5 = \left(ab, ba\right), \quad e_6 = \left(a^2ba, a^3b\right), \quad e_7 = \left(a^2ba, a^3\right) \text{ and } \quad e_8 = \left(a^3, a^3b\right).$$

The graph $\Gamma_{G_4}^c$ has an independence number $\alpha\left(\Gamma_{G_4}^c\right) = 4$ in which there are 36 independent sets of size 4. Next, we obtain 60 independent sets of size 3 and 37 independent sets of size 2. Therefore, by the definition of the independence polynomial of a graph we have

$$I\left(\Gamma_{G_4}^c; x\right) = \sum_{k=0}^{4} a_k x^k = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 = 1 + 10x + 37x^2 + 60x^3 + 36x^4. \text{ W}$$

Remark As we can see, the conjugate graph of the dihedral groups mentioned above is actually the union of some complete graphs. Since the independence polynomial of a complete graph is $I\left(K_n; x\right) = 1 + nx$, we can also use Theorem 1 [1] of the independence polynomial of the union of two graphs in which $I\left(\Gamma_1 \cup \Gamma_2; x\right) = I\left(\Gamma_1; x\right) \cdot I\left(\Gamma_2; x\right)$ to compute the independence polynomial of the conjugate graph of dihedral groups G_1, G_2, G_3 and G_4.

The Matching Polynomial of the Conjugate Graph of Dihedral Groups of Order at Most 12

Proposition 5 Let G_1 be the dihedral group of order 6, $G_1 = \left\langle a, b : a^3 = b^2 = 1, bab = a^{-1}\right\rangle$. Then the matching polynomial of the conjugate graph of G_1 is $M\left(\Gamma_{G_1}^c; x\right) = 1 + 4x + 3x^2$.

Proof The conjugate graph of G_1 is $\Gamma_{G_1}^c = K_2 \cup K_3$. It consists of the vertex set and the edge set as stated in the proof of Proposition 1. The graph $\Gamma_{G_1}^c$ has a matching number $\beta\left(\Gamma_{G_1}^c\right) = 2$. From $E\left(\Gamma_{G_1}^c\right)$, the matching sets of size 2 are $\left\{e_1, e_2\right\}, \left\{e_1, e_3\right\}$ and $\left\{e_1, e_4\right\}$, and the matching sets of size one are the sets containing each edge of G_1, denoted as $\left\{e_1\right\}$, $\left\{e_2\right\}$, $\left\{e_3\right\}$ and $\left\{e_4\right\}$. Hence by the definition of the matching polynomial of a graph, we obtain

$$M\left(\Gamma_{G_1}^c;x\right)=\sum_{k=0}^{2}b_kx^k=b_0x^0+b_1x^1+b_2x^2=1+4x+3x^2. \text{ W}$$

Proposition 6 Let G_2 be the dihedral group of order 8, $G_2=\left\langle a,b:a^4=b^2=1,bab=a^{-1}\right\rangle$. Then the matching polynomial of the conjugate graph of G_2 is $M\left(\Gamma_{G_2}^c;x\right)=1+3x+3x^2+x^3$.

Proof The conjugate graph of G_2 is $\Gamma_{G_2}^c=K_2\cup K_2\cup K_2$. It consists of the vertex set and the edge set as stated in the proof of Proposition 2. The graph $\Gamma_{G_2}^c$ has a matching number $\beta\left(\Gamma_{G_2}^c\right)=3$. There is only one matching set of $\Gamma_{G_2}^c$ of size 3, which is $\{e_1,e_2,e_3\}$. Next, we obtain $\{e_1,e_2\}$, $\{e_1,e_3\}$ and $\{e_2,e_3\}$ as the matching sets of size 2. Hence by the definition of the matching polynomial of a graph, we have

$$M\left(\Gamma_{G_2}^c;x\right)=\sum_{k=0}^{3}b_kx^k=b_0x^0+b_1x^1+b_2x^2+b_3x^3=1+3x+3x^2+x^3. \text{ W}$$

Proposition 7 Let G_3 be the dihedral group of order 10, $G_3=\left\langle a,b:a^5=b^2=1,bab=a^{-1}\right\rangle$. Then the matching polynomial of the conjugate graph of G_3 is $M\left(\Gamma_{G_3}^c;x\right)=1+12x+21x^2+10x^3$.

Proof The conjugate graph of G_3 is $\Gamma_{G_3}^c=K_2\cup K_2\cup K_5$ that consists of the vertex set and the edge set as stated in the proof of Proposition 3. The graph $\Gamma_{G_3}^c$ has a matching number $\beta\left(\Gamma_{G_3}^c\right)=3$. There are 10 matching sets of size 3 and 21 matching sets of size 2. Hence, by the definition of the matching polynomial of a graph, we obtain

$$M\left(\Gamma_{G_3}^c;x\right)=\sum_{k=0}^{3}b_kx^k=b_0x^0+b_1x^1+b_2x^2+b_3x^3=1+12x+21x^2+10x^3. \text{ W}$$

Proposition 8 Let G_4 be the dihedral group of order 12, $G_4=\left\langle a,b:a^6=b^2=1,bab=a^{-1}\right\rangle$. Then the matching polynomial of the conjugate graph of G_4 is $M\left(\Gamma_{G_4}^c;x\right)=1+8x+22x^2+24x^3+9x^4$.

Proof The conjugate graph of G_4 is $\Gamma_{G_4}^c=K_2\cup K_2\cup K_3\cup K_3$ that consists of the vertex set and the edge set as stated in the proof of Proposition 4. The graph $\Gamma_{G_4}^c$ has a matching number $\beta\left(\Gamma_{G_4}^c\right)=4$ in which there are 9 matching sets of size 4. Next, we obtain 24 matching sets of size 3 and 22 matching sets of size 2. Therefore, by the definition of the matching polynomial of a graph we have

$$M\left(\Gamma_{G_4}^c;x\right)=\sum_{k=0}^{4}b_kx^k=b_0x^0+b_1x^1+b_2x^2+b_3x^3+b_4x^4=1+8x+22x^2+24x^3+9x^4. \text{ W}$$

The Clique Polynomial of the Conjugate Graph of Dihedral Groups of Order at Most 12

Proposition 9 Let G_1 be the dihedral group of order 6, $G_1=\left\langle a,b:a^3=b^2=1,bab=a^{-1}\right\rangle$. Then the clique polynomial of the conjugate graph of G_1 is $C\left(\Gamma_{G_1}^c;x\right)=1+5x+4x^2+x^3$.

Proof The conjugate graph of G_1 is $\Gamma_{G_1}^c=K_2\cup K_3$. It consists of the vertex set and the edge set as stated in the proof of Proposition 1. The graph $\Gamma_{G_1}^c$ has a clique number $\omega\left(\Gamma_{G_1}^c\right)=3$. From $E\left(\Gamma_{G_1}^c\right)$, the clique of size 3 is

$\{b, ab, a^2b\}$, and the cliques of size two are $\{a, a^2\}, \{b, ab\}, \{b, a^2b\}$ and $\{ab, a^2b\}$. Hence by the definition of the clique polynomial of a graph, we obtain

$$C\left(\Gamma_{G_1}^c; x\right) = \sum_{k=0}^{3} c_k x^k = c_0x^0 + c_1x^1 + c_2x^2 + c_3x^3 = 1 + 5x + 4x^2 + x^3. \text{ W}$$

Proposition 10 Let G_2 be the dihedral group of order 8, $G_2 = \langle a, b : a^4 = b^2 = 1, bab = a^{-1} \rangle$. Then the clique polynomial of the conjugate graph of G_2 is $C\left(\Gamma_{G_2}^c; x\right) = 1 + 6x + 3x^2$.

Proof The conjugate graph of G_2 is $\Gamma_{G_2}^c = K_2 \cup K_2 \cup K_2$. It consists of the vertex set and the edge set as stated in the proof of Proposition 2. The graph $\Gamma_{G_2}^c$ has a clique number $\omega\left(\Gamma_{G_2}^c\right) = 2$. The cliques of size 2 are the sets $\{b, a^2b\}$ and $\{ab, a^3b\}$, and the cliques of size one are the sets containing each vertex of G_2, denoted as $\{a\}$, $\{a^3\}$, $\{b\}$, $\{a^2b\}$, $\{ab\}$ and $\{a^3b\}$. Hence, by the definition of the clique polynomial of graph, we obtain

$$C\left(\Gamma_{G_2}^c; x\right) = \sum_{k=0}^{2} c_k x^k = c_0x^0 + c_1x^1 + c_2x^2 = 1 + 6x + 3x^2. \text{ W}$$

Proposition 11 Let G_3 be the dihedral group of order 10, $G_3 = \langle a, b : a^5 = b^2 = 1, bab = a^{-1} \rangle$. Then the clique polynomial of the conjugate graph of G_3 is $C\left(\Gamma_{G_3}^c; x\right) = 1 + 9x + 12x^2 + 10x^3 + 4x^4 + x^5$.

Proof The conjugate graph of G_3 is $\Gamma_{G_3}^c = K_2 \cup K_2 \cup K_5$ that consists of the vertex set and the edge set as stated in the proof of Proposition 3. The graph $\Gamma_{G_3}^c$ has a clique number $\omega\left(\Gamma_{G_3}^c\right) = 5$ in which the clique of size 5 is the graph K_5 containing set of vertices $\{b, ab, a^2b, a^3b, a^4b\}$. We obtain the cliques of size 4 which are $\{b, ab, a^2b, a^3b\}$, $\{b, ab, a^2b, a^4b\}$, $\{b, a^2b, a^3b, a^4b\}$ and $\{ab, a^2b, a^3b, a^4b\}$. Next, there are 10 cliques of size 3 and 12 cliques of size 2. Therefore, by the definition of the clique polynomial, we have

$$C\left(\Gamma_{G_3}^c; x\right) = \sum_{k=0}^{5} c_k x^k = c_0x^0 + c_1x^1 + c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 = 1 + 9x + 12x^2 + 10x^3 + 4x^4 + x^5. \text{ W}$$

Proposition 12 Let G_4 be the dihedral group of order 12, $G_4 = \langle a, b : a^6 = b^2 = 1, bab = a^{-1} \rangle$. Then the clique polynomial of the conjugate graph of G_4 is $C\left(\Gamma_{G_4}^c; x\right) = 1 + 10x + 8x^2 + 2x^3$.

Proof The conjugate graph of G_4 is $\Gamma_{G_4}^c = K_2 \cup K_2 \cup K_3 \cup K_3$ that consists of the vertex set and the edge set that are stated in the proof of Proposition 4. The graph $\Gamma_{G_4}^c$ has a clique number $\omega\left(\Gamma_{G_4}^c\right) = 3$. The cliques of size 3 are the sets $\{a, ab, ba\}$ and $\{a^2ba, a^3, a^3b\}$, and there are eight cliques of size 2. Hence, by the definition of the clique polynomial of graph, we obtain

$$C\left(\Gamma_{G_4}^c; x\right) = \sum_{k=0}^{3} c_k x^k = c_0x^0 + c_1x^1 + c_2x^2 + c_3x^3 = 1 + 10x + 8x^2 + 2x^3. \text{ W}$$

CONCLUSION

In conclusion, for the group G_1, the independence polynomial of the conjugate graph is $I\left(\Gamma_{G_1}^c; x\right) = 1 + 5x + 6x^2$, the matching polynomial is $M\left(\Gamma_{G_1}^c;x\right)=1+4x+3x^2$ and the clique polynomial is $C\left(\Gamma_{G_1}^c;x\right)=1+5x+4x^2+x^3$. For the group G_2, the independence polynomial of the conjugate graph is $I\left(\Gamma_{G_2}^c;x\right)=1+6x+12x^2+8x^3$, the matching polynomial is $M\left(\Gamma_{G_2}^c;x\right)=1+3x+3x^2+x^3$ and the clique polynomial is $C\left(\Gamma_{G_2}^c;x\right)=1+6x+3x^2$. For the group G_3, the independence polynomial of the conjugate graph is $I\left(\Gamma_{G_3}^c;x\right)=1+9x+24x^2+20x^3$, the matching polynomial is $M\left(\Gamma_{G_3}^c;x\right)=1+12x+21x^2+10x^3$ and the clique polynomial is $C\left(\Gamma_{G_3}^c;x\right)=1+9x+12x^2+10x^3+4x^4+x^5$. Lastly, for the group G_4, the independence polynomial of the conjugate graph is $I\left(\Gamma_{G_4}^c;x\right)=1+10x+37x^2+60x^3+36x^4$, the matching polynomial is $M\left(\Gamma_{G_4}^c;x\right)=1+8x+22x^2+24x^3+9x^4$ and the clique polynomial is $C\left(\Gamma_{G_4}^c;x\right)=1+10x+8x^2+2x^3$. Note that the matching polynomial of each graph is actually identical with the independence polynomial of its line graph. We can also check that the clique polynomial of each graph is equal to the independence polynomial of its complement graph.

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