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> **Editors** Mustafa Bayram **Aydin Secer**

Istanbul Gelisim University, Istanbul, Turkey

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TABLE OF CONTENTS

MESSAGE FROM CHAIRMAN	iv
Chairs	v
Co-Chairs	v
Coordinator-Secrataries	v
Members of Organizing Committee	v
Scientific Committee	vi
Local Committee	vii
Pleneary Speakers	vii
Management and valorization of urban solid waste in landfills: case of the TEC of the cir of Batna Haddad Louiza and Aouachria Zeroual	1
Galerkin vector solution of Kelvin problem for a mixture of two linear elastic solids Emre Kurt and M. Salih Dokuz	
A new machine learning approach to house price estimation Changchun Wang and Hui Wu	19 . 19
On twisted sums of Schreier spaces and James- Schreier spaces Haifa Bin Jebreen	27
Covariance matrix estimation of an elliptically symmetric distribution in high dimension setting Anis M. Haddouche, Fatiha Mezoued and Dominique Fourdrinier	35
Global existence results for semilinear perturbed fractional differential equations with infinite delay Sara Litimein and Atika Matallah	41
Three-point value problem for a class of Hadamart fractional differential equations F. Berhoun and Z. Malki	59
Spectrum of cayley graphs of dihedral groups and their energy Amira Fadina Ahmad Fadzil, Nor Haniza Sarmin and Ahmad Erfanian	65

Closure properties of static Watson-Crick regular grammars Aqilahfarhana Abdul Rahman, Wan Heng Fong, Nor Haniza Sarmin, Sherzod Turaev and Nurul	71
Liyana Mohamad Zulkufli	71
The topological indices of non-commuting graph for quasidihedral group Nur Idayu Alimon, Nor Haniza Sarmin, and Ahmad Erfanian	77 77
Common fixed point theorems for generalized fuzzy homotopic mappings in Q-fuzzy metric space Seema Mehra	.c 85 85
Solar cell parameter estimation using Hybrid Nelder-Mead and Big Bang Big Cruncl optimization algorithms Omer Gonul and Osman Kaan Erol	h 95 95
Study of turbulent flow through a thrust reverser Rayane Dellali and Mahfoud Kadja	101 101
A combined semi-supervised classification approach for text categorization: A case study for movie reviews Nur Uylas Sati	y 113 113
Generalisations of DNA splicing languages with one restriction enzyme using automata Wan Heng Fong, Nurul Izzaty Ismail and Nor Haniza Sarmin	
The mathematical modelling of DNA splicing system with Palindromic and Non-Palindron restriction enzymes Nurul Izzaty Ismail, Wan Heng Fong, Nor Haniza Sarmin	nic 1 27 127
Existence of solutions for elliptic Kirchhoff equations in \mathbb{R}^N Atika Matallah and Sara Litimein	139 139
Existence of solutions for a nonhomogeneous p-Laplacain elliptic equation with critical Hardy-Sobolev exponent Atika Matallah and Sara Litimein	1 143 143
Development of early warning system of dengue fever disease endemic: a computer simulation model Paian Sianturi	- 1 53 153
Controllability results for fractional integro-differential inclusions with infinite state dependent delay Sara Litimein and Atika Matallah	- 1 65 165
Numerical study of coupled natural convection with surface radiation in a cylindrical annular enclosure Belkacem Ould Said, Mohamed Amine Medebber, Noureddine Retiel, Aissa Abderrahmane and Mohammed El Ganaoui	l 1 75 175
Vibrational analysis and electronic proprieties of zwitterionic D-phenylalanine	185 185
	1 95 195

On the Analytical Solution for Stochastic Differential Equations with Lie Symme	try Anal-
ysis	201
Tugcem Partal, Zuhal Kucukarslan Yuzbasi and Ebru Cavlak Aslan	201
The local and global dynamics of a cancer tumor growth with multiphase struc	ture and
treatment model	205
Veli Shakhmurov, A. Maharramov and Bunyad Shahmurzada	205

The mathematical modelling of DNA splicing system with Palindromic and Non-Palindromic restriction enzymes

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Abstract: The mathematical modelling of DNA splicing system is introduced by Head where restriction enzymes and a ligase cleave and recombine DNA molecules in particular ways based on the cleavage pattern of restriction enzymes. The set of molecules resulting from the splicing system is called a splicing language, which can be analysed using formal language theory. The restriction enzymes are also known as restriction endonucleases which made up of three sites namely the crossing, left and right context. Palindrome is a sequence of string that reads the same forwards and backwards. The restriction enzymes also recognize palindromic and non-palindromic sequences. In this research, DNA splicing systems with one cutting site each of palindromic and non-palindromic restriction enzymes are modelled via Head's splicing system. The splicing languages from the splicing systems for same and different crossings are generalized and presented as theorems, which are proved using induction and direct methods respectively. Some examples of DNA splicing systems with palindromic and non-palindromic restriction enzymes are also provided to obtain the corresponding splicing languages using these theorems.

Keywords: DNA, splicing system, splicing language, palindromic, restriction enzyme.

1 Introduction

In 1987, DNA splicing system is introduced by Head [1] and mathematically modelled from a relation between formal language theory and molecular biology. The splicing system is also known as Head's Splicing System. In splicing systems, deoxyribonucleic acid (DNA) molecules are cut and recombined when react with a ligase and restriction enzymes which are biologically called as endodeoxyribonucleases [2].

The set of molecules resulting from a DNA splicing system is called as a splicing language which is simulated using formal language theory. A formal language consists of a set of strings of symbols from an alphabet [3]. Some notations in formal language theory, namely λ , +, \hat{u} and * which denote the empty string, union, concatenation and star-closure respectively, are used in this research [3]. By using the concepts in formal language theory, the splicing language from a splicing system is associated with three sets. The first set is the set of double stranded DNA (dsDNA) symbols from nitrogenous base pairings: adenine (A) pairs with thymine (T), while cytosine (C) pairs with guanine (G) [4]. The second set consists of initial DNA molecules taken from the sub sequences or pattern in protein or nucleotide chains [5]. Lastly, the third set consists of the cleavage pattern of restriction enzymes. The rule for the cleavage pattern of restriction enzymes is made up of three sites namely the crossing, left and right context [6]. The symbols \downarrow and \uparrow indicate the upper and lower cutting sites of the restriction enzymes respectively.

Throughout the years, notations in Head's splicing system had been extended and variant of splicing models had been developed namely Paun [7], Pixton [8], Goode-Pixton [9] and Yusof-Goode [10] splicing systems. The variant of splicing systems resulted in many types of splicing languages.

The splicing languages from different model of splicing system can be obtained based on the specific sequences of restriction enzymes. This research focuses on palindromic and non-palindromic sequences of restriction enzyme. Palindrome is a sequence of strings that reads the same forwards and backwards [11]. Previously, Fong [12] had studied the modelling of DNA splicing system with palindromic restriction enzyme. Research on DNA splicing system with non-palindromic restriction enzymes had also been done in [13].

In this paper, the generalisations of splicing languages from DNA splicing system with one cutting site of each palindromic and non-palindromic restriction enzymes for same and different crossings are presented.

In the next section, some preliminaries related to this research are given.

2 Preliminaries

In this research, DNA splicing systems with palindromic and non-palindromic restriction enzymes are modelled using Head's splicing system. The definitions of Head's splicing system and splicing language are stated in the following.

Definition 2.1 (1). Splicing System and Splicing Language A splicing system S = (A, I, B, C) consists of a finite alphabet A, a finite set I of initial strings in A^* , and finite sets B and C of triples (c, x, d) with c, x and d in A^* . Each such triple in B or C is called a pattern. For each such triple the string cxd is called a site and the string x is called a crossing. Patterns in B are called left patterns and patterns in C are called right patterns. The language L = L(S) generated by S consists of the strings in C and all strings that can be obtained by adjoining to cxd = cx

Next, the definition of a palindromic string is stated.

Definition 2.2 (14). **Palindromic String** A string I of a dsDNA is said to be palindromic if the sequence from the left to the right side of the upper single strand is equal to the sequence from the right to the left side of the lower single strand.

In this paper, generalisations of splicing languages from DNA splicing system involving palindromic and non-palindromic sequences for restriction enzymes are done. For example, the enzyme BfaI = 5' - CTAG - 3' is a palindromic restriction enzyme since the upper single strand of enzyme BfaI matches

with the lower single strand when read from backwards; while the enzyme BbvCIis a non-palindromic restriction enzyme since the upper single strand of enzyme BbvCI does not match with the lower single strand when read from backwards.

3 Results and findings

In this research, the splicing languages from DNA splicing systems involving one cutting site each of palindromic and non-palindromic restriction enzymes are generalised and given as theorems. These theorems are mathematically proved. Some examples of DNA splicing systems with one cutting site each of palindromic and non-palindromic restriction enzymes are provided to determine the corresponding splicing languages by using these theorems. The generalisation of splicing languages from DNA splicing system with one cutting site each of palindromic and non-palindromic restriction enzymes and same crossing is presented in Theorem 1.

Theorem 3.1. Given S = (A, I, B, C) is a DNA splicing system in which $A = \begin{pmatrix} A & C & G & T \\ T & G & C \end{pmatrix}$

 $is \ the \ set \ of \ dsDNA \ symbols, \ I = \left\{ \begin{array}{c} N_1N_1 \ \dots N_1X_1 \ Y \ X_2M \ M \dots MW_1 \ YW_2N_2N_2 \dots N_2 \\ N_1'N_1' \dots N_1'X_1'Y' X_2'M'M' \dots M'W_1'Y'W_2'N_2'N_2' \dots N_2' \end{array} \right\} \ is \ the \ set \\ consisting \ of \ an \ initial \ string \ with \ one \ cutting \ site \ each \ of \ palindromic \ and \ non-palindromic \ restriction \\ enzymes \ \begin{array}{c} X_1YX_2 \\ X_1'Y'X_2' \end{array} \ and \ \begin{array}{c} W_1 \ YW_2 \\ W_1Y'W_2' \end{array} \ respectively, \ set \end{array}$

 $B = \left\{ \begin{pmatrix} X_1 & Y & X_2 \\ X_1' & Y' & X_2' \end{pmatrix}, \begin{pmatrix} W_1 & Y & W_2 \\ W_1' & Y' & W_2' \end{pmatrix} \right\} \text{ is the set of cleavage pattern for the restriction}$ enzymes and set C is the empty set, the resulting splicing language consists of strings of the form

$$\begin{pmatrix} N_1N_1 \dots N_1 \\ N_1'N_1' \dots N_1' \end{pmatrix} + \begin{pmatrix} N_2'N_2' \dots N_2'W_2'YW_1'M'M' \dots M' \\ N_2N_2 \dots N_2W_2'YW_1MM \dots M \end{pmatrix} \begin{pmatrix} X_1 \\ X_1' \end{pmatrix} \begin{pmatrix} YX_2 & MM \dots M & W_1 \\ Y'X_2' & M'M' \dots M' & W_1' \end{pmatrix}^{n-1} \begin{pmatrix} W_2 & N_2N_2 \dots N_2 \\ W_2' & N_2'N_2' \dots N_2' + X_2'N_1N_1 \dots N_1 \end{pmatrix}$$
(3.1)

where $n \in \mathbb{Z}^+$, N_1 , X_1 , Y, X_2 , M, W_1 , W_2 and N_2 , denote arbitrary dsDNA symbol(s), N_1' , X_1' , Y', X_2' , M', W_2' are complementarities for N_1 , X_1 , Y, X_2 , M, W_1 , W_2' and N_2' are complementarities for N_1 , N_2 , N_2 , N_3 , N_4 , N_4 , N_4 , N_5 , and N_2 respectively, N_2' is the crossing, and

$$\left\{ \begin{array}{l} X_{1}YX_{2} & W_{1}Y\ W_{2} & W_{2}'Y'W_{1}' \\ X_{1}'Y'X_{2}' & W_{1}'Y'W_{2}' & W_{2}\ Y\ W_{1} \end{array} \right\} \notin \left\{ \begin{array}{l} N_{1}N_{1}\ \dots N_{1} & M\ M\ \dots\ M \\ N_{1}'N_{1}'\dots N_{1}' & M'M'\dots M' & N_{2}'N_{2}'\dots N_{2}' \end{array} \right\} \ .$$

Proof Suppose the restriction enzyme $X_1YX_2 \atop X_1'Y'X_2'$ is palindromic, so the base sequence of enzyme reads the same backwards and forwards:

$$rac{X_1YX_2}{X_1'Y'X_2'} = rac{X_2'Y'X_1'}{X_2YX_1}$$

Since the restriction enzyme $W_1 Y W_2 W_1' Y' W_2'$ is not palindromic, so the base sequence of enzyme is not the same backwards and forwards:

$$\frac{W_1Y\ W_2}{W_1'Y'W_2'} \neq \frac{W_2'Y'W_1'}{W_2Y\ W_1} \ .$$

Then
$$\frac{X_1}{X_1'} = \frac{X_2'}{X_2}$$
, $\frac{Y}{Y'} = \frac{Y'}{Y}$, $\frac{X_2}{X_2'} = \frac{X_1'}{X_1}$, $\frac{W_1}{W_1'} \neq \frac{W_2'}{W_2}$ and $\frac{W_2}{W_2'} \neq \frac{W_1'}{W_1}$. Then, the ini-

tial string $\frac{N_1N_1 \dots N_1X_1 \ Y \ X_2M \ M \dots MW_1 \ YW_2N_2N_2 \dots N_2}{N_1^{'}N_1^{'} \dots N_1^{'}X_1^{'}Y^{'}X_2^{'}M^{'}\dots M^{'}W_1^{'}Y^{'}W_2^{'}N_2^{'}\dots N_2^{'}} \text{ with the cutting site of the enzymes}$

$$\frac{X_1YX_2}{X_1'Y^{'}X_2^{'}}$$
 and $\frac{W_1\ YW_2}{W_1^{'}Y^{'}W_2^{'}}$ is shown respectively in the following:

for the first cutting site and

for the second cutting site.

The initial string can be written 180 degree wise as

$$N_{2}'N_{2}'\dots N_{2}'W_{2}'Y'W_{1}'M'M'\dots M'X_{2}'Y'X_{1}'N_{1}'N_{1}'\dots N_{1}' N_{2}N_{2}\dots N_{2}W_{2}Y W_{1}MM \dots MX_{2}Y X_{1}N_{1}N_{1}\dots N_{1}$$

$$(3.4)$$

Since $\frac{X_1}{X_1'} = \frac{X_2'}{X_2}$, $\frac{Y}{Y'} = \frac{Y'}{Y}$, $\frac{X_2}{X_2'} = \frac{X_1'}{X_1}$, string (3.4) can be written as

When the enzymes $X_1YX_2 \atop X_1'Y'X_2'$ and $W_1YW_2 \atop W_1'Y'W_2'$ are added to the initial string, (3.2) combines with (3.3) which gives

$$N_1 N_1 \dots N_1 X_1 Y W_2 N_2 N_2 \dots N_2 N_1' N_1' \dots N_1' X_1' Y' W_2' N_2' N_2' \dots N_2'$$

$$(3.6)$$

and

The results of the combination of strings (3.2) and (3.5) are

and

Next, when string (3.3) combines with string 3.5), the other new strings arise:

and

$$N_2'N_2'\dots N_2'W_2'Y\ W_1'M'M'\dots M'X_1 \qquad Y \qquad W_2N_2N_2\dots N_2 \\ N_2N_2\dots N_2W_2\ Y'\ W_1M\ M\ \dots MX_1' \qquad Y' \qquad W_2'N_2'N_2'\dots N_2' \quad .$$
 (3.9)

By using induction, this theorem can be proved. For n = 1, the string (3.1) is stated in (3.4), (3.6), (3.7) and (3.9). Next, let $n = k \in \mathbb{Z}^+$, string (3.1) becomes:

$$\begin{pmatrix} N_1 N_1 \dots N_1 \\ N_1' N_1' \dots N_1' \end{pmatrix} + \frac{N_2' N_2' \dots N_2' W_2' Y W_1' M' M' \dots M'}{N_2 N_2 \dots N_2 W_2 Y' W_1 M M \dots M} \end{pmatrix} \begin{pmatrix} X_1 \\ X_1' \end{pmatrix} \begin{pmatrix} Y X_2 & M M \dots M & W_1 \\ Y' X_2' & M' M' \dots M' & W_1 \end{pmatrix}^{k-1} \begin{pmatrix} W_2 & N_2 N_2 \dots N_2 \\ W_2' & N_2' N_2' \dots N_2' + X_2' N_1 N_1 \dots N_1 \end{pmatrix}$$
(3.10)

The following strings are among the strings in (3.10):

and

$$N_{2}'N_{2}'...N_{2}'W_{2}'YW_{1}'M'M'...M'X_{1} \left(\begin{array}{cccc} YX_{2} & M \ M \ ... \ M & W_{1} \\ Y'X_{2}' & M'M'...M' & W_{1}' \end{array} \right)^{k-1} \begin{array}{cccc} Y & X_{2}N_{1}'N_{1}'...N_{1}' \\ Y' & X_{2}'N_{1}N_{1}...N_{1} \end{array}$$
 (3.14)

By expanding strings (3.11), (3.12), (3.13) and (3.14), the string can be written respectively as:

and

$$\frac{N_{2}'N_{2}'...N_{2}'W_{2}'Y\ W_{1}'M'M'...M'X_{1}}{N_{2}N_{2}...N_{2}W_{2}\ Y'\ W_{1}MM\ ...MX_{1}'} + \frac{Y}{Y'} + \frac{X_{2}}{X_{2}'} \frac{M\ M\ ...\ M}{M'M'...M'} \frac{W_{1}}{W_{1}'} \left(\begin{array}{cccc} YX_{2} & M\ M\ ...\ M & W_{1} \\ Y'X_{2}' & M'M'...M' & W_{1}' \end{array} \right)^{k-2} Y X_{2}N_{1}'N_{1}'...N_{1}' \\ Y' X_{2}'N_{1}N_{1}...N_{1} & Y' X_{2}'N_{1}N_{1}...N_{1} \end{array}$$
(3.18)

Next, string (3.3) combines with (3.15) and (3.16) which produces new strings

and

Then, the other recombinations between string (3.8) with (3.17) and (3.18) can be shown as:

and

$$\frac{N_2'N_2'...N_2'W_2'YW_1'M'M'...M'X_1}{N_2N_2...N_2W_2} \frac{Y}{W_1M} \frac{X_2}{M} \frac{M}{M}...MX_1' \frac{Y}{Y} \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{Y}{Y} \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{Y}{Y} \frac{Y_2}{M} \frac{M}{M}...MW_1' \frac{Y}{Y_2} \frac{M}{M} \frac{M}{M}...MW_1' \frac{Y}{Y_2} \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{Y}{M} \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{Y}{M} \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{Y}{M} \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{Y}{M} \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{X_2}{M} \frac{X_2}{M} \frac{M}{M}...MW_1' \frac{X_2}{M} \frac{X_2}$$

By simplifying strings (3.19), (3.20), (3.21) and (3.22), the resulting strings are:

and

$$N_{2}'N_{2}'...N_{2}'W_{2}'YW_{1}'M'M'...M'X_{1} \left(\begin{array}{cccc} YX_{2} & M & M & ... & M & W_{1} \\ N_{2}N_{2}...N_{2}W_{2} & Y' & W_{1}M & M & ... & MX_{1}' \end{array}\right)^{(k+1)-1} Y X_{2}N_{1}'N_{1}'...N_{1}'$$

$$Y'X_{2}' & M'M'...M' & W_{1}'$$

$$Y'X_{2}' & M'M'...M' & W_{1}'$$

$$Y'X_{2}'N_{1}N_{1}...N_{1} ...$$
(3.26)

Therefore, from (3.23), (3.24), (3.25) and (3.26), the resulting splicing language can be summarized as

which depicts string (3.1) when n = k + 1. Hence, Theorem 1 is proved.

The splicing language from a DNA splicing system with one cutting site of each palindromic and non-palindromic restriction enzymes with same crossing is illustrated through Example 1.

 $\begin{aligned} \textbf{Example 3.1. } \textit{Given a DNA splicing system S} &= (A,\,I,\,B,\,C) \textit{ where } I = \left\{ \begin{array}{c} \text{ATCCGGGTCCGCGA} \\ \text{TAGGCCCAGGCGCT} \end{array} \right\} \textit{ is} \\ \textit{the set of initial string, set B} &= \left(\begin{array}{c} \mathbf{C} & \mathbf{,}\mathbf{CG}, & \mathbf{G} \\ \mathbf{G} & \mathbf{GC} & \mathbf{C} \end{array} \right), \left(\begin{array}{c} \mathbf{C} & \mathbf{,}\mathbf{CG}, & \mathbf{C} \\ \mathbf{G} & \mathbf{GC} & \mathbf{G} \end{array} \right) \textit{ is the set of cleavage pattern} \\ \textit{for the enzymes HpaII and AciI and set C is the empty set.} \end{aligned}$

Solution 3.1. The enzyme HpaII, $\begin{array}{c} 5'-CCGG-3'\\ 3'-GGCC-5' \end{array}$ is palindromic since the base sequence of enzyme

HpaII reads the same forwards and backwards; while AciI, 5'-CCGC-3' is not palindromic since the base sequence of enzyme AciI does not read the same forwards and backwards. The enzymes HpaII and AciI also have the same crossing, 3'-GC-5'

The initial string 5'-ATC CG GGTC CGA-3' has one cutting site each of the enzymes HpaII and AciI. Thus, by using Theorem 1, the splicing language is

where $n \in \mathbb{Z}^+$.

Next, the generalization of splicing languages from DNA splicing system with one cutting site each of two non-palindromic restriction enzymes and different crossings is presented in Theorem 3.2.

 $\begin{array}{l} \textbf{Theorem 3.2. Given } S = (A,\,I,\,B,\,C) \text{ is a DNA splicing system in which } A = \left\{ \begin{array}{l} A & C & G & T \\ T & G & C & A \end{array} \right\} \\ \text{is the set of dsDNA symbols, } I = \left\{ \begin{array}{l} N_1N_1 & \ldots & N_1X_1 & Y & X_2M & M & \ldots & MW_1 & ZW_2N_2N_2 & \ldots & N_2 \\ N_1'N_1' & \ldots & N_1'X_1'Y' & X_2'M'M' & \ldots & M'W_1'Z'W_2'N_2'N_2' & \ldots & N_2' \\ \end{array} \right\} \text{ is the set consisting of an initial string with two non-overlapping cutting sites of one palindromic and non-overlapping cutting sites of the set of the parameters of the set of th$

palindromic restriction enzymes $X_1YX_2 \atop X_1'Y'X_2'$ and $W_1ZW_2 \atop W_1'Z'W_2'$ respectively, set

$$B = \left\{ \left(\begin{array}{ccc} X_1 & ,Y, & X_2 \\ X_1^{'} & Y^{'} & X_2^{'} \end{array} \right), \left(\begin{array}{ccc} W_1 & ,Z, & W_2 \\ W_1^{'} & Z^{'} & W_2^{'} \end{array} \right) \right\}$$

is the set of cleavage pattern for the restriction enzymes and set C is the empty set, the resulting splicing language consists of strings of the form

$$\begin{pmatrix} N_1 N_1 \dots N_1 \\ N_1' N_1' \dots N_1' \end{pmatrix} + \begin{pmatrix} N_2' N_2' \dots N_2' W_2' Z' W_1' M' M' \dots M' \\ N_2' N_2 \dots N_2 W_2 \ Z \ W_1 M \ M \ \dots M \end{pmatrix} \begin{pmatrix} X_1 Y X_2 \\ X_1' Y' X_2' \end{pmatrix} \begin{pmatrix} M \ M \dots M \ W_1 \\ M' M' \dots M' W_1' \end{pmatrix} \begin{pmatrix} W_2 N_2 N_2 \dots N_2 \\ Y_2' N_2' N_2 \dots N_2' \end{pmatrix} + \begin{pmatrix} N_1' N_1' \dots N_1' \\ N_1 N_1 \dots N_1 \end{pmatrix}$$

where N_1 , N_1 , N_2 , N_3 , N_4 , N_4 , N_5 , N

$$\left\{ \begin{array}{l} X_1 Y X_2 \\ X_1' Y^{'} X_2^{'} \end{array}, \begin{array}{l} W_1 \ Z W_2 \\ W_1' Z^{'} W_2^{'} \end{array}, \begin{array}{l} W_2^{'} Z^{'} W_1^{'} \\ W_2 \ Z W_1 \end{array} \right\} \notin \left\{ \begin{array}{l} N_1 N_1 \ \dots \ N_1 \\ N_1^{'} N_1^{'} \dots \ N_1^{'} \end{array}, \begin{array}{l} M \ M \dots \ M \\ M^{'} M^{'} \dots \ M^{'} \end{array}, \begin{array}{l} N_2 N_2 \dots \ N_2 \\ N_2^{'} N_2^{'} \dots \ N_2^{'} \end{array} \right\} \ .$$

Proof Suppose the restriction enzyme $X_1YX_2 \atop X_1'Y'X_2'$ is palindromic, so the base sequence of enzyme reads the same backwards and forwards:

$$\frac{X_1YX_2}{X_1'Y'X_2'} = \frac{X_2'Y'X_1'}{X_2YX_1} .$$

Since the restriction enzyme $W_1 Z W_2 W_1' Z' W_2'$ is not palindromic, so the base sequence of enzyme is not the same backwards and forwards:

$$\frac{W_1 \ ZW_2}{W_1' Z' W_2'} \neq \frac{W_2' Z' W_1'}{W_2 \ ZW_1}$$

Then
$$X_1 = X_2'$$
, $Y_1 = Y'$, $X_2 = X_1'$, $W_1 \neq W_2'$, $Z \neq Z'$ and $W_2 \neq W_1'$

The initial string, $\frac{N_1N_1 \ldots N_1X_1YX_2M \ M \ldots MW_1 \ ZW_2N_2N_2 \ldots N_2}{N_1^{'}N_1^{'} \ldots N_1^{'}X_1^{'}Y^{'}X_2^{'}M^{'} \ldots M^{'}W_1^{'}Z^{'}W_2^{'}N_2^{'} \ldots N_2^{'}} \text{ with the cutting site}$

of the enzymes $X_1YX_2 \atop X_1'Y'X_2'$ and $X_1ZW_2 \atop W_1'Z'W_2'$ is shown respectively in the following:

for the first cutting site and

for the second cutting site.

The initial string can be written 180 degree wise as

$$N'_{2}N'_{2}...N'_{2}W'_{2}Z'W'_{1}M'M'...M'X'_{2}Y'X'_{1}N'_{1}N'_{1}...N'_{1} N_{2}N_{2}...N_{2}W_{2}ZW_{1}MM...MX_{2}YX_{1}N_{1}N_{1}...N_{1}$$
(3.29)

Since $\frac{X_1}{X_1'} = \frac{X_2'}{X_2}$, $\frac{Y}{Y'} = \frac{Y'}{Y}$, $\frac{X_2}{X_2'} = \frac{X_1'}{X_1}$, string (3.29) can be written as

Since $\frac{W_1}{W_1'} \neq \frac{W_2'}{W_2}$, $\frac{Z}{Z'} \neq \frac{Z'}{Z}$, $\frac{W_2}{W_2'} \neq \frac{W_1'}{W_1}$ and the crossings $\frac{Y}{Y'} \neq \frac{Z}{Z'}$ are different, then there is no new resulting molecule to combine with string (3.28).

When the enzymes $X_1YX_2 \atop X_1'Y'X_2'$ is added to the initial string, string (3.27) combines with (3.30) which gives

$$\begin{array}{cccc} N_1 N_1 & \dots & N_1 X_1 Y X_2 & N_1' N_1' \dots & N_1' \\ N_1' N_1' & \dots & N_1' X_1' Y' X_2' & N_1 N_1 \dots & N_1 \end{array} \tag{3.31}$$

and

$$N_{2}'N_{2}'...N_{2}'W_{2}'Z'W_{1}'M'M'...M'X_{1} \quad Y \quad X_{2} M M ...M W_{1} \quad Z \quad W_{2}N_{2}N_{2}...N_{2} N_{2}N_{2}...N_{2}W_{2} Z W_{1}M M ...MX_{1}' \quad Y' \quad X_{2}'M'M'...M'W_{1}' \quad Z' \quad W_{2}'N_{2}'N_{2}'...N_{2}'$$
 (3.32)

From (3.27), (3.30), (3.31) and (3.32), the resulting splicing language is simplified as

$$\left(\begin{array}{c} N_1 N_1 \dots N_1 \\ N_1' N_1' \dots N_1' \end{array} + \begin{array}{c} N_2' N_2' \dots N_2' W_2' Z' W_1' M' M' \dots M' \\ N_2 N_2 \dots N_2 W_2 \ Z \ W_1 M \ M \ \dots M \end{array} \right) \begin{array}{c} X_1 Y X_2 \\ X_1' Y' X_2' \end{array} \left(\begin{array}{c} M \ M \dots M \ W_1 \ Z \ W_2 N_2 N_2 \dots N_2 \\ M' M' \dots M' W_1' \ Z' \ W_2' N_2 N_2 \dots N_2' \end{array} + \begin{array}{c} N_1' N_1' \dots N_1' \\ N_1 N_1 \dots N_1 \end{array} \right)$$

Thus, Theorem 3.2 is proved.

Example 3.2 shows the splicing language from a DNA splicing system with one cutting site of each palindromic and non-palindromic restriction enzymes for different crossings.

Example 3.2. Given a DNA splicing system S = (A, I, B, C) where $I = \begin{bmatrix} TGTACGGACCGCGC \\ ACATGCCTGGCGCG \end{bmatrix}$ is

the set of initial string, set $B = \begin{pmatrix} G & , TA, & C \\ C & AT & G \end{pmatrix}, \begin{pmatrix} C & , CG, & C \\ G & GC & G \end{pmatrix}$ is the set of cleavage pattern for the enzymes CviAII and AciI and set C is the empty set.

Solution 3.2. The enzyme CviAII, 5'-GTAC-3' is palindromic since the base sequence of enzyme

CviAII reads the same forwards and backwards; while AciI, 3'-CCGC-3' is not palindromic since the base sequence of enzyme AciI does not read the same forwards and backwards. The enzymes CviAII and AciI have different crossings where crossing sites CviAII and AciI are 5'-TA-3' and 5'-CC-3' respectively.

The initial string 5'-TG TA CGGAC CG CGC -3' has one cutting site each of the enzymes CviAII and AciI. Thus, by using Theorem 2, the splicing language is

$$\begin{array}{c} 5'-\\ 3'-\\ \end{array} \left(\begin{array}{c} TG\\ AC \end{array} \right. + \left. \begin{array}{c} GCGCGGTCC\\ CGCGCCAGG \end{array} \right) \left. \begin{array}{c} GTAC\\ CATG\\ \end{array} \left(\begin{array}{c} GGACCGCGC\\ CCTGGCGCG\\ \end{array} \right. + \left. \begin{array}{c} CGC\\ GCG\\ -5' \end{array} \right) \right.$$

4 Conclusion

In this research, the generalizations of splicing languages from DNA splicing system with one cutting site of each palindromic and non-palindromic restriction enzymes for same and different crossings are presented as Theorem 1 and 2 respectively. The respective theorems are proved using induction and direct methods. By using these theorem, the splicing languages from different DNA splicing system are obtained when any initial string, palindromic and non-palindromic restriction enzymes are used.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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