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A Variant of Commutativity Degree and Its Generalized Conjugacy Class Graph

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The probability that two random elements in a group commute is called the commutativity degree. Recently, the concept of commutativity degree and its relation with graphs have been studied by many authors. In this research, a variant of the commutativity degree, namely the probability that an element of a group fixes a set is done for some metabelian groups. The set considered in this research is the set of pairs of commuting elements of order three. Using its definition, the probability that an element of the metabelian groups fixes the set is computed under conjugation action. Furthermore, its generalized conjugacy class graph is determined along with its properties.

Keywords: commutativity degree; probability that an element fixes a set; orbits; metabelian group, generalized conjugacy class graph.

1. INTRODUCTION

In mathematics, mainly in the area of algebra, group theory has been studied in various researches and its application has been done in many fields. One of the interesting concepts in group theory is the concept of commutativity degree. This concept is used to determine the abelianness of a group. This concept has been applied in many researches for some finite groups[1]. Meanwhile, the study of graph theory have been applied in many areas for example computer science, chemistry, physics as well as group theory. Since graph theory is an interesting way to show relations between objects, the research on the topic of commutativity degree can be presented in the form of a graph. The research in this topic is very crucial and from the graph obtained the graph properties can also be studied.

In this paper, the research focuses on the commutativity degree and its generalizations, namely the probability that an element of a group fixes a set. The objectives of this research are to find the probability that an element of a group fixes a set, to apply the result in graph theory and to find some properties of the graph. Since the probability that a group element fixes a set has not been done on the nonabelian metabelian groups of order 12, this research will cover on several nonabelian metabelian groups of order 12, namely the alternating group A_4, the dihedral group D_6 and the semidirect product T. The set Ω under this study is the set of all pairs of commuting elements in the form of (a,b), where a and b commute, $a \neq b$ and $\text{lcm}(|a|,|b|) = 3$. In addition, this study precisely focuses on generalized conjugacy class graph, where its properties such as chromatic number, clique number, independent number and dominating number are determined.

2. PRELIMINARIES

This section presents some definitions and properties related to this research.

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Definition 1: Metabelian Group[2]
A group G is metabelian if it has a normal subgroup A such that A and the factor group G/A are abelian.

Definition 2: Orbit[3]
Suppose G is a finite group that acts on a set Ω and $\omega \in \Omega$. The orbit of ω, denoted by $O(\omega)$, is the subset $O(\omega) = \{g\omega \mid g \in G, \omega \in \Omega\}$. In this research, the group action is conjugation action, hence the orbit is written as $O(\omega) = \{g\omega g^{-1} \mid g \in G, \omega \in \Omega\}$.

The probability that a pair of elements x and y selected randomly from a group G commute is called the commutativity degree and it was first introduced by Miller[4] in 1944. The definition of commutativity degree is stated in the following:

Definition 3: Commutativity Degree[4]
The probability that two random elements (x,y) in G commute is defined as follows:

$$P(G) = \frac{|\{(x,y) \in G \times G \mid xy = yx\}|}{|G|^2}.$$

This concept has been applied in many researches for some finite groups[1]. In 1968, Erdos and Turan explored this concept for symmetric group[5], S_n. Later in 1973, Gustafson introduced another method to compute the commutativity degree in which the conjugacy classes of a group are used[7]. This concept is defined in the following theorem.

Theorem 1
Let G be a finite group and $K(G)$ be the number of conjugacy classes in G. Then the commutativity degree $P(G)$, is given as:

$$P(G) = \frac{K(G)}{|G|}.$$

In our case, the number of conjugacy classes is the same as the number of orbits. Another study on the commutativity degree was done by Castalez in 2010 on some finite groups[6].

In this research, we used one of the extensions of the commutativity degree which is the probability that an element of a group fixes a set. The definition is given as follow:

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Definition 4: Probability That an Element of a Group Fixes a Set[8]

Let G be a group and Ω be the set of all pairs of commuting elements of G of size two in the form of (a,b) and lcm$(|a|,|b|) = 2$ where G acts on Ω. Then the probability that an element of a group fixes a set is

$$P_G(\Omega) = \frac{|\{(g,\omega)\in G\times\Omega \mid g\omega=\omega \text{ for } g\in G \text{ and } \omega\in\Omega\}|}{|G||\Omega|}$$

In this paper, we apply the concept of commutativity degree into graph theory. Before that, we provide some previous research in relating groups with graphs. There are various researches involving both group theory and graph theory which were done by previous researches. In 1990, a graph related to conjugacy classes was firstly introduced by Bertram[9]. The vertices of this graph are non-central conjugacy classes, where two vertices are linked if the cardinalities are not coprime. As a consequence, numerous works have been done on this graph and many results have been achieved. In this research, group theory will be particularly applied to the generalized conjugacy class graph. Recently, Omer *et al*. generalized the conjugacy class graph by defining the generalized conjugacy class graph whose vertices are non-central orbits under group action on a set[10]. Two vertices of this graph are adjacent if their cardinalities are not coprime. The definition of this graph is given as follows:

Definition 5: Generalized Conjugacy Class Graph[10]

Let G be a finite group and Ω is a set of G. Let A be the set of commuting elements in Ω, i.e. $A = \{\omega \in \Omega \mid \omega g = g\omega, g \in G\}$. Then the generalized conjugacy class graph, $\Gamma_G^{\Omega_c}$ is defined as a graph whose vertices are non-central orbits under group action on a set, that is $|V(\Gamma_G^{\Omega_c})| = K(\Omega) - |A|$. Two vertice ω_1 and ω_2 in $\Gamma_G^{\Omega_c}$ are adjacent if their cardinalities are not coprime, i.e. $\gcd(|\omega_1|, |\omega_2|) \neq 1$.

Definition 6: Chromatic Number[11], χ

The chromatic number, $\chi(\Gamma)$ is the maximum number of colors in a proper coloring of Γ.

Definition 7: Independent Number[11], α

A non-empty set S of vertices, $V(\Gamma)$ is called an independent set of Γ if there is an adjacent between two elements of S in Γ. The independent number is the number of vertices in maximum independent set and it is denoted by $\alpha(\Gamma)$.

Definition 8: Clique Number[11], ω

A clique is a complete subgraph in Γ. The size of the largest clique in Γ is called the clique number and is denoted by $\omega(\Gamma)$.

Definition 9: Dominating Number[11], γ

The dominating set $X \subseteq V(\Gamma)$ is a set where for each v outside $X, \exists\, x \in X$ such that v is adjacent to x. The minimum size of X is called the dominating number and it is denoted by $\gamma(\Gamma)$.

3. MAIN RESULTS

In this section, the probability that a group element fixes a set Ω is computed for all nonabelian metabelian groups of order 12, namely the alternating group A_4, the dihedral group D_6 and the semidirect product T. Then, based on the results, the generalized conjugacy class graph and its properties are found. We first define our set Ω in the following definition:

Definition 10: The Set Ω

The set Ω is the set of all pairs of commuting elements of $G \times G$ in the form of (a,b), $a \neq b$ and lcm$(|a|,|b|) = 3$. In symbols, we can write Ω as:

$$\Omega = \{(a,b) \in G \times G \mid ab = ba, a \neq b, lcm\,(|a|,|b|) = 3\}.$$

We give our results on the probability that an element of metabelian groups of order 12 fixes a set Ω in the following theorems:

Theorem 2

Let G be the dihedral group of order 12, D_6 and G acts on Ω by conjugation. Then the number of orbits of Ω is $K(\Omega) = 3$ and the probability that an element of D_6 fixes the set Ω is $P_{D_6}(\Omega) = \frac{1}{2}$.

Proof:

The orbits of the set $\Omega = \{(1, R_{120}), (1, R_{240}), (R_{120}, 1), (R_{240}, 1), (R_{120}, R_{240}), (R_{240}, R_{120})\}$ are listed as follows:

$$\omega_1 = O\big((1, R_{120})\big) = \{(1, R_{120}), (1, R_{240})\}$$
$$\omega_2 = O\big((R_{120}, R_{240})\big) = \{(R_{120}, R_{240}), (R_{240}, R_{120})\}$$
$$\omega_3 = O\big((R_{120}, 1)\big) = \{(R_{120}, 1), (R_{240}, 1)\}$$

Therefore, it is proven that the number of orbits of Ω is $K(\Omega) = 3$. By Theorem 1, the probability that an element of D_6 fixes the set Ω, $P_{D_6}(\Omega) = \frac{1}{2}$. ■

Theorem 3

Let G be the alternating group of order 12, A_4 and A_4 acts on the set Ω by conjugation action. Then the number of orbits of Ω is $K(\Omega) = 6$ and the probability that an element of A_4 fixes the set Ω is $P_{A_4}(\Omega) = \frac{1}{4}$.

Proof:

The orbits of the set $\Omega = \{\big((1), (123)\big), \big((1), (124)\big), ((1), (123)), \big((1,)(134)\big), \big((1), (142)\big), \big((1), (143)\big), \big((1), (234)\big), \big((1), (243)\big), \big((123), (1)\big), \big((123), (132)\big), \big((124), (1)\big), ((124), (142)), \big((132), (1)\big), \big((132), (123)\big), \big((134), (1)\big), ((134), (143)), \big((142), (1)\big), \big((142), (124)\big), ((143), (1)), ((143), (134)), \big((234), (1)\big), \big((234), (243)\big), \big((243), (1)\big), ((243), (234))\}$. are listed below:

$$\omega_1 = O\big((1), (123)\big) = \{\big((1), (123)\big), \big((1), (134)\big), \big((1), (142)\big), ((1), (243))\}.$$
$$\omega_2 = O\big((1), (124)\big) = \{\big((1), (124)\big), \big((1), (132)\big), \big((1), (143)\big), ((1), (234))\}.$$
$$\omega_3 = O\big((123), (1)\big) = \{\big((123), (1)\big), \big((134), (1)\big), \big((142), (1)\big), ((243), (1))\}.$$
$$\omega_4 = O\big((124), (1)\big) = \{\big((124), (1)\big), \big((132), (1)\big), \big((143), (1)\big), ((234), (1))\}.$$
$$\omega_5 = O\big((123), (132)\big) = \{\big((123), (132)\big), ((134), (143)), ((142), (124)), ((243), (234))\}.$$
$$\omega_6 = O\big((132), (123)\big) = \{\big((132), (123)\big), ((143), (134)), ((124), (142)), ((234), (243))\}.$$

Therefore, it is proven that the number of orbits of Ω is $K(\Omega) = 6$. Thus, by Theorem 1, the probability that an element of A_4 fixes the set Ω, $P_{A_4}(\Omega) = \frac{6}{24} = \frac{1}{4}$. ■

Theorem 4

Let G be the semidirect product of order 12, $T = \mathbb{Z}_3 \rtimes \mathbb{Z}_4$, where the elements $are\ T = \{1, a, a^2, a^3, b, b^2, ab, a^2b, a^3b, ab^2, a^2b^2, a^3b^2\}$ and the group acts on Ω by conjugation. Then the number of orbits of Ω is $K(\Omega) = 2$ and the probability that an element of T fixes the set Ω is $P_T(\Omega) = \frac{1}{2}$..

Proof:

The set Ω consists of four elements which are $\Omega = \{(1, b), (1, b^2), (b, 1), (b^2, 1)\}$. The orbits for the set Ω are:

$$\omega_1 = \{(1, b), (1, b^2)\}$$

$\omega_2 = \{(b, 1), (b^2, 1)\}$.

Thus, the number of orbits, $K(\Omega) = 2$. Hence, by Theorem 1, the probability that an element of T fixes the set Ω is $P_T(\Omega) = \frac{2}{4} = \frac{1}{2}$. ■

Now, based on the results on the probability, the generalized conjugacy class graph of the metabelian groups of order 12 are determined. The results are given in the following theorems:

Theorem 5

Let G be a dihedral group of order 12, D_6, and let Ω be a set of all pairs of commuting elements of $G \times G$ such that $\Omega = \{(a,b) \in G \times G \mid ab = ba, a \neq b, lcm\,(|a|,|b|) = 3\}$. If G acts on Ω by conjugation action, then $\Gamma_{D_6}^{\Omega}$ is a complete graph with 3 vertices, K_3.

Proof:

By using Definition 5, the vertices of the generalized conjugacy class graph are the non-central orbits under conjugation action on Ω such that $\left|V\left(\Gamma_{D_6}^{\Omega_c}\right)\right| = K(\Omega) - |Z(\Omega)|$in which the vertices are adjacent if their cardinalities are not coprime. Therefore, $\left|V\left(\Gamma_{D_6}^{\Omega_c}\right)\right| = 3 - 0 = 3$. Since the pair elements (ω_1, ω_2) are of order 1 and 3, and 3 and 3, thus the cardinalities are not coprime. Therefore, the generalized conjugacy class graph, $\Gamma_{D_6}^{\Omega_c}$ is K_3. ■

Proposition 1

The chromatic number for the generalized conjugacy class graph of the dihedral group of order 12, $\chi\left(\Gamma_{D_6}^{\Omega_c}\right) = 3$.

Proof:

There are three colours that can be applied on the vertices of $\Gamma_{D_6}^{\Omega_c}$because all of the vertices are connected to each other. Hence, the chromatic number of $\Gamma_{D_6}^{\Omega_c}$, $\chi\left(\Gamma_{D_6}^{\Omega_c}\right) = 3$. ■

Proposition 2

The independent number for the generalized conjugacy class graph of the dihedral group of order 12, $\alpha\left(\Gamma_{D_6}^{\Omega_c}\right) = 1$.

Proof:

All vertices of $\Gamma_{D_6}^{\Omega_c}$ are connected to each other. Hence, the independent set has an element which is 0. Therefore, the independent number of $\Gamma_{D_6}^{\Omega_c}$, $\alpha\left(\Gamma_{D_6}^{\Omega_c}\right) = 1$. ■

Proposition 3

The clique number for the generalized conjugacy class graph of the dihedral group of order 12, $\omega\left(\Gamma_{D_6}^{\Omega_c}\right) = 2$..

Proof:

K_2 is the largest complete subgraph of $\Gamma_{D_6}^{\Omega_c}$. These K_2 are $V\left(\Gamma_{sub1}\right) = \{\omega_1, \omega_2\}$, $V\left(\Gamma_{sub2}\right) = \{\omega_1, \omega_3\}$ and $V\left(\Gamma_{sub3}\right) = \{\omega_1, \omega_2\}$. Hence, the clique number of $\Gamma_{D_6}^{\Omega_c}$, $\omega\left(\Gamma_{D_6}^{\Omega_c}\right) = 2$.. ■

Proposition 4

The dominating number for the generalized conjugacy class graph of the dihedral group of order 12, $\gamma\left(\Gamma_{D_6}^{\Omega_c}\right) = 1$..

Proof:

The dominating number of $\Gamma_{D_6}^{\Omega_c}$ is 1 since 1 vertex is the minimum number that is needed so that the vertex dominates itself and others. Therefore, the dominating number of $\Gamma_{D_6}^{\Omega_c}$, $\gamma\left(\Gamma_{D_6}^{\Omega_c}\right) = 1$.. ■

Theorem 6

Let G be an alternating group of order 12, A_4, and let Ω be a set of all pairs of commuting elements of $G \times G$ such that $\Omega = \{(a,b) \in G \times G \mid ab = ba, a\ \ b, lcm\,(|a|,|b|) = 3\}$. If G acts on Ω by conjugaction action, then, $\Gamma_{A_4}^{\Omega_c}$ is a complete graph with six vertices.

Proof:

The cardinality of the center of the set Ω, $|(Z(\Omega)| = 0$. By using Definition 5, the vertices of the generalized conjugacy class graph of A_4 is $\left|V\left(\Gamma_{A_4}^{\Omega_c}\right)\right| = 6 - 0 = 6$.. From the orbits found in Theorem 3, the cardinality of all the orbits are 4. Hence, the cardinalities of all vertices are not coprime, resulting all of the vertices are adjacent. Therefore, $\Gamma_{A_4}^{\Omega_c}$ is a complete graph with six vertices. ■

Figure 2 shows the complete graph of six vertices, K_6 of $\Gamma_{A_4}^{\Omega_c}$.

Proposition 5

The chromatic number for the generalized conjugacy class graph of the alternating group of order 12, $\chi\left(\Gamma_{A_4}^{\Omega_c}\right) = 6$..

Proof:

There are six colours that can be applied on the vertices of $\Gamma_{A_4}^{\Omega_c}$ because all of the vertices are connected to each other. Therefore, the chromatic number of $\Gamma_{A_4}^{\Omega_c}$, $\chi\left(\Gamma_{A_4}^{\Omega_c}\right) = 6$.. ■

Proposition 6

The independent number for the generalized conjugacy class graph of the alternating group of order 12, $\alpha\left(\Gamma_{A_4}^{\Omega_c}\right) = 1$.

Proof:

All vertices of $\Gamma_{A_4}^{\Omega_c}$ are connected to each other. Hence, the only element in the independent set is 0. Therefore, the independent number of $\Gamma_{A_4}^{\Omega_c}$, $\alpha\left(\Gamma_{A_4}^{\Omega_c}\right) = 1$. ■

Proposition 7

The clique number for the generalized conjugacy class graph of the alternating group of order 12, $\omega\left(\Gamma_{A_4}^{\Omega_c}\right) = 5$..

Proof:

K_5 is the largest complete subgraph of $\Gamma_{A_4}^{\Omega_c}$. These K_5 are $V\left(\Gamma_{sub1}\right) = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, $V\left(\Gamma_{sub2}\right) = \{\omega_1, \omega_3, \omega_4, \omega_5, \omega_6\}$, $V\left(\Gamma_{sub3}\right) = \{\omega_1, \omega_2, \omega_4, \omega_5, \omega_6\}$, $V\left(\Gamma_{sub4}\right) = \{\omega_1, \omega_2, \omega_3, \omega_5, \omega_6\}$, $V\left(\Gamma_{sub5}\right) = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_6\}$ and $V\left(\Gamma_{sub6}\right) = \{\omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$. Hence, the clique number of $\Gamma_{A_4}^{\Omega_c}$, $\omega\left(\Gamma_{A_4}^{\Omega_c}\right) = 5$.. ■

Proposition 8

The dominating number for the generalized conjugacy class graph of the alternating group of order 12, $\gamma\left(\Gamma_{A_4}^{\Omega_c}\right) = 1$..

Proof:

The dominating number of $\Gamma_{A_4}^{\Omega_c}$ is 1 since 1 vertex is the minimum number that is needed so that the vertex dominates itself and others. Therefore, the dominating number of $\Gamma_{A_4}^{\Omega_c}$, $\gamma\left(\Gamma_{A_4}^{\Omega_c}\right) = 1$.. ■

Theorem 7

Let G be a semidirect product of order 12, $T = \mathbb{Z}_3 \rtimes \mathbb{Z}_4$ and let Ω be a set of all pairs of commuting elements of $G \times G$ such that $\Omega = \{(a,b) \in G \times G \mid ab = ba, a\ \ b, lcm\,(|a|,|b|) = 3\}$. If G acts on Ω by conjugaction action, then, $\Gamma_T^{\Omega_c}$ is a complete graph with two vertices K_2.

Proof:

By using Definition 5, the vertices of the generalized conjugacy class graph of T is $\left|V\left(\Gamma_T^{\Omega_c}\right)\right| = 2 - 0 = 2$. From the orbits found in Theorem 3, the cardinality of all the orbits are 2. Hence, the cardinalities of the vertices are not coprime, resulting both of the vertices are adjacent. Therefore, $\Gamma_T^{\Omega_c}$ is a complete graph with two vertices, K_2. ■

Figure 3 shows a complete graph of two vertices, K_2 of $\Gamma_T^{\Omega_c}$.

Proposition 9

The chromatic number for the generalized conjugacy class graph of the semidirect product of order 12, $\chi\left(\Gamma_T^{\Omega_c}\right) = 2$.

Proof:

There are two colors that can be applied on the vertices of $\Gamma_T^{\Omega_c}$ because both of the vertices are connected to each other. Therefore, the chromatic number of $\Gamma_T^{\Omega_c}$, $\chi\left(\Gamma_T^{\Omega_c}\right) = 2$.. ■

Proposition 10

The independent number for the generalized conjugacy class graph of the semidirect product of order 12, $\alpha\left(\Gamma_T^{\Omega_c}\right) = 1$.

Proof:

The vertices of $\Gamma_T^{\Omega_c}$ are connected to each other. Hence, the only element in the independent set is 0. Therefore, the independent number of $\Gamma_T^{\Omega_c}$, $\alpha\left(\Gamma_T^{\Omega_c}\right) = 1$. ■

Proposition 11

The clique number for the generalized conjugacy class graph of the semidirect product of order 12, $\omega\left(\Gamma_T^{\Omega_c}\right) = 0$..

Proof:

Since $\Gamma_T^{\Omega_c}$has only two vertices, there is no complete subgraph in $\Gamma_T^{\Omega_c}$. Hence, the clique number of $\Gamma_T^{\Omega_c}$, $\omega\left(\Gamma_T^{\Omega_c}\right) = 0$.. ■

Proposition 12

The dominating number for the generalized conjugacy class graph of semidirect product of order 12, $\gamma\left(\Gamma_T^{\Omega_c}\right) = 1$.

Proof:

The dominating number of $\Gamma_T^{\Omega_c}$ is 1 since 1 vertex is the minimum number that is needed so that the vertex dominates itself and others. Therefore, the dominating number of $\Gamma_T^{\Omega_c}$, $\gamma\left(\Gamma_T^{\Omega_c}\right) = 1$.. ■

4. CONCLUSION

In this research, the orbits of three nonabelian metabelian groups of order 12, namely the dihedral group D_6, the alternating group A_4 and the semidirect product, $T = \mathbb{Z}_3 \rtimes \mathbb{Z}_4$are determined. These orbits are used to find the probability that an element of the groups fixes the set Ω. It is proven that the probability that an element of the groups D_6, A_4 and T fixes the set Ω is $P_{D_6}(\Omega) = \frac{1}{2}$, $P_{A_4}(\Omega) = \frac{1}{4}$and $P_T(\Omega) = \frac{1}{2}$ respectively.

Next, the generalized conjugacy class graphs are constructed based on the orbits obtained. It is proven that the generalized conjugacy class graph of D_6 is a complete graph of three vertices, K_3 while the generalized conjugacy class graph for A_4 is a complete graph of six vertices, K_6 and the generalized conjugacy class graph of $T = \mathbb{Z}_3 \rtimes \mathbb{Z}_4$ is a complete graph of two vertices, K_2.

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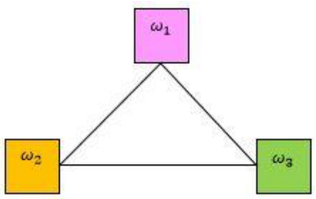
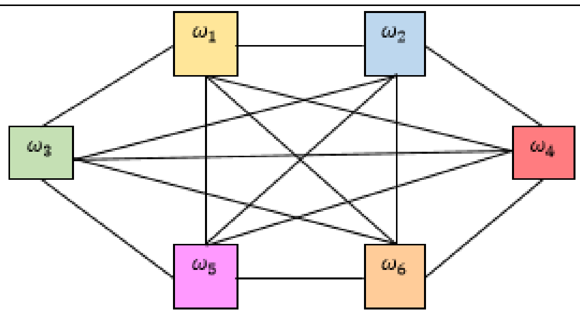
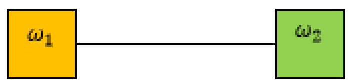


A complete graph of $\Gamma_{D_6}^{\Omega_c}$, K_3.



Complete graph of $\Gamma_{A_4}^{\Omega_c}$, K_6.



Complete Graph of $\Gamma_T^{\Omega_c}$, K_2.

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