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The Nonabelian Tensor Square of a Bieberbach Group with Point Group $C_2 \times C_2$ of Dimension Three

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https://doi.org/10.26782/jmcms.2019.03.00024

Abstract

A Bieberbach group is a crystallographic group. This group is an extension of a finite point group and a free abelian group of finite rank. In this paper, a Bieberbach group with point group $C_2 \times C_2$ *of dimension three is chosen where its polycyclic presentation is shown to be consistent. The nonabelian tensor square of group is a specialization of more general of the nonabelian tensor product of group. The nonabelian tensor square of group is one of the homological functors which can reveal the properties of the groups. Also, the nonabelian tensor squares are one of the important elements on computing homological functors of groups. The main objective of this paper is to compute the nonabelian tensor square of a Bieberbach group with point group* $C_2 \times C_2$ *of dimension three by using the computational method for polycyclic groups. The finding showed that the nonabelian tensor square of the group is abelian and be presented in terms of its generators. The findings of this research can be used to compute the other homological functors of this group.*

Keywords : Bieberbach Group, Polycyclic Groups, Nonabelian Tensor Square

I. Introduction

A Bieberbach group is also known as a crystallographic group. This group is an extension of a free abelian group L of finite rank by a finite point group P which satisfies the short exact sequence

$$1 \longrightarrow L \xrightarrow{\varphi} G \xrightarrow{\phi} P \longrightarrow 1$$

such that $G/\varphi(L) \cong P$. New properties of crystallographic groups can be revealed by calculating the nonabelian tensor squares of the groups.

The nonabelian tensor square, $G \otimes G$ of a group G is generated by the symbols $g \otimes h,$ for all $g, h \in G,$ subject to relations

$$gg' \otimes h = ({}^{g}g' \otimes {}^{g}h)(g \otimes h) \text{ and } g \otimes hh' = (g \otimes h)({}^{h}g \otimes {}^{h}h')$$

for all $g, g', h, h' \in G,$ where ${}^{g}g' = gg'g^{-1}$. Brown and Loday (1987) have introduced the nonabelian tensor square as a specialization of more general nonabelian tensor products. Following that, many studies on computing the nonabelian tensor squares for various groups have been conducted. These include the 2-generator nilpotent of class two groups (Sarmin *et al.*, 1999; Kappe & Bacon, 2003), the free nilpotent groups (Blyth, Morse & Moravec, 2008) and the polycyclic groups (Blyth & Morse, 2009).

Masri (2009) has started the study of the nonabelian tensor squares of Bieberbach groups with certain point group and focused on Bieberbach groups with cyclic point group of order two. Next, the other studies which are related to the computation of the nonabelian tensor squares of Bieberbach groups with other point groups have also been done by other researchers such as the dihedral group (Mohd Idrus (2011); Wan Mohd Fauzi *et al. (*2014)), the cyclic group of order three and five (Mat Hassim, 2014), the symmetric point group (Tan *et al.*, 2016a, 2016b) and the elementary abelian 2-group point group (Abdul Ladi *et al.*, 2017).

In this paper, the nonabelian tensor square of $S_2(3)$, denoted as $S_2(3) \otimes S_2(3)$ is determined. The group of $S_2(3)$ is shown to be polycyclic group and the consistent polycyclic presentation of this group is given as the following (Abdul Ladi et al., 2017):

$$S_2(3) = \left\langle a_0, a_1, l_1, l_2, l_3 \;\middle|\; \begin{array}{l} a_0^2 = l_2^{-1}l_3^{-1}, a_1^2 = l_1^{-1}, {}^{a_0}a_1 = a_1 l_1^{-1} l_2^{-1}, \\ {}^{a_0}l_1 = l_1^{-1}, {}^{a_0}l_2 = l_2, {}^{a_0}l_3 = l_3, \\ {}^{a_1}l_1 = l_1, {}^{a_1}l_2 = l_2^{-1}, {}^{a_1}l_3 = l_3, \\ {}^{l_1}l_2 = l_2, {}^{l_1}l_3 = l_3, {}^{l_2}l_3 = l_3 \end{array} \right\rangle. \quad (1)$$

Preliminaries

The computation of the nonabelian tensor square in this study involves a group $\nu(G)$ which was introduced by Rocco (1991) as follows:

Definition 1 (Rocco, 1991)

Let G be a group with presentation $\langle G \mid R \rangle$ and let G^{φ} be an isomorphic copy of G via the mapping $\varphi : g \to g^{\varphi}$ for all $g \in G$. The group $\nu(G)$ is defined to be

$$\nu(G) = \left\langle G, G^{\varphi} \mid R, R^{\varphi}, {}^{x}[g, h^{\varphi}] = [{}^{x}g, ({}^{x}h)^{\varphi}] = {}^{x^{\varphi}}[g, h^{\varphi}], \forall x, g, h \in G \right\rangle.$$

Theorem 1 shows that the group $\nu(G)$ is related to the nonabelian tensor square of group G.

Theorem 1 (Ellis & Leonard, 1995)
Let G be a group. The mapping $\sigma : G \otimes G \to [G, G^{\varphi}] \lhd \nu(G)$ defined by $\sigma(g \otimes h) = [g, h^{\varphi}]$ for all g, h in G is an isomorphism.

Therefore, all the tensor computations can be done through the commutator computation within the subgroup fo $\nu(G)$, $[G, G^{\varphi}]$. Blyth and Morse (2009) showed that if G is polycyclic, then $\nu(G)$ is also polycyclic as given in the following proposition.

Proposition 1 (Blyth & Morse, 2009)
If G is polycyclic, then $\nu(G)$ is polycyclic.

In this study, the nonabelian tensor square of $S_2(3)$ is obtained by using the computational method for polycyclic group developed by Blyth and Morse (2009). Next, list of commutator identities in $\nu(G)$ with left conjugation are given as in the following. Let x, y and z be elements of a group G. Then

$$[xy, z] = {}^{x}[y, z] \cdot [x, z] \tag{2}$$

$$[x, yz] = [x, y] \cdot {}^{y}[x, z] \tag{3}$$

$$[x^{-1}, y] = [x^{-1}, [x, y]^{-1}] \cdot [x, y]^{-1} \tag{4}$$

$$[x, y^{-1}] = [y^{-1}, [x, y]^{-1}] \cdot [x, y]^{-1} \tag{5}$$

$$[x^{-1}, y^{-1}] = [x^{-1}, [y^{-1}, [x, y]]] \cdot [y^{-1}, [x, y]] \cdot$$
$$[x^{-1}, [x, y]] \cdot [x, y] \tag{6}$$

$${}^{z}[x, y] = [{}^{z}x, {}^{z}y] \tag{7}$$

Definition 2

The abelianization of a group G , $G^{ab} = G/G'$ is the quotient of group G by its derived subgroup, G'.

The next proposition shows the close relationship between the structure of the central subgroup of the nonabelian tensor square of group G, $\nabla(G)$ and G^{ab}.

Proposition 2 (Blyth *et al.*, 2010)
Let G be a group such that G^{ab} is finitely generated. Assume that G^{ab} is the direct product of the cyclic groups $\langle x_i G' \rangle$, for $i = 1, \ldots, s$ and set $E(G)$ to be

$\left\langle [x_i, x_j^{\varphi}] \mid i < j \right\rangle [G, G'^{\varphi}]$. Then the following hold:

(i) $\nabla(G)$ is generated by the elements of the set $\{[x_i, x_i^{\varphi}], [x_i, x_j^{\varphi}][x_j, x_i^{\varphi}] \mid 1 \le i < j \le s\}$;

(ii) $[G, G^{\varphi}] = \nabla(G)E(G)$.

The following propositions and theorem are some another commutator identities used in this paper.

Proposition 3 (Blyth & Morse, 2009; Rocco , 1991)

Let G be a group. Then the following relations hold in $\nu(G)$:

(i) $[g, g^{\varphi}]$ is central in $\nu(G)$ for all g in G;

(ii) $[g, g^{\varphi}] = 1$ for all g in G'.

Proposition 4 (Blyth & Morse, 2009)

Let g_1, g_2, g_3 and g_4 be elements of group G. Then in $\nu(G)$, $[[g_1, g_2], [g_3, g_4]^{\varphi}] = [[g_1, g_2^{\varphi}], [g_3, g_4^{\varphi}]]$.

Proposition 5 (Blyth *et al*., 2010)

Let G be any group. Then the following hold:

(i) If $g_1 \in G'$ or $g_2 \in G'$, then $[g_1, g_2^{\varphi}]^{-1} = [g_2, g_1^{\varphi}]$.

(ii) $[Z(G), (G')^{\varphi}] = 1$.

(iii) If A and B are two subgroups of G with $B \le G'$, then $[A, B^{\varphi}] = [B, A^{\varphi}]$. In particular, $[G, G'^{\varphi}] = [G', G^{\varphi}]$.

Proposition 6 (Blyth & Morse, 2009)

Let g and h be elements of G such that $[g, h] = 1$. Then, in $\nu(G)$,

(i) $[g^n, h^{\varphi}] = [g, h^{\varphi}]^n = [g, (h^{\varphi})^n]$ for all integers n;

(ii) $[g^n, (h^m)^{\varphi}][h^m, (g^n)^{\varphi}] = ([g, h^{\varphi}][h, g^{\varphi}])^{nm}$ for all integers n, m;

(iii) $[g, h^{\varphi}]$ is in the centre of $\nu(G)$.

Proposition 7 (Zomorodian, 2005)

Let A, B and C be abelian groups. The properties of the ordinary tensor product of two abelian groups are given as in the following.

(i) $C_0 \otimes A \cong A$,

(ii) $C_0 \otimes C_0 \cong C_0$,

(iii) $C_n \otimes C_m \cong C_{\gcd(n,m)}$, for $n, m \in \mathbb{N}$, and

(iv) $A \otimes (B \times C) = (A \otimes B) \times (A \otimes C)$.

where C_0 is a cyclic group of infinite order.

Theorem 2 (Brown *et al*., 1987)
Let G be a group. Then there exists a commutator mapping $\kappa : G \otimes G \to G'$ which is defined by $\kappa(g \otimes h) = [g, h]$. The kernel of κ is in the centre of $G \otimes G$.

The determination of the derived subgroup, the abelianization and the central subgroup of the nonabelian tensor square of $S_2(3)$ are given in the following proposition.

Proposition 8 (Abdul Ladi *et al*., 2017)
For group $S_2(3)$,

(i) The derived subgroup $S_2(3)' = \left\langle l_1^{-2}, l_1^{-1}l_2 \right\rangle$

(ii) The abelianization, $S_2(3)^{ab} = \left\langle a_0 S_2(3)', a_1 S_2(3)' \right\rangle \cong C_0 \times C_4$

Proposition 9 (Abdul Ladi *et al*., 2017)
The central subgroup of the nonabelian tensor square of $S_2(3)$ is given as the following:

$\nabla(S_2(3)) = \left\langle [a_0, a_0^{\varphi}], [a_1, a_1^{\varphi}], [a_0, a_1^{\varphi}][a_1, a_0^{\varphi}] \right\rangle \cong C_4 \times C_8 \times C_0.$

II. Main Results

In this section, the nonabelian tensor square of $S_2(3)$, denoted as $S_2(3) \otimes S_2(3)$ is computed.

Theorem 3

The nonabelian tensor square of $S_2(3) \otimes S_2(3)$ is abelian and isomorphic to $C_2 \times C_4 \times C_8 \times C_0^{\,3}$.

Proof. By Proposition 8, $S_2(3)' = \left\langle l_1^{-2}, l_1^{-1}l_2 \right\rangle$ and by Proposition 2, $E(S_2(3)) = \left\langle [a_0, a_1^{\varphi}] \right\rangle$ $[S_2(3), S_2(3)'^{\varphi}]$ where $[S_2(3), S_2(3)'^{\varphi}]$ is generated by generators $[a_0, l_1^{-2\varphi}]$, $[a_1, l_1^{-2\varphi}]$, $[l_1, l_1^{-2\varphi}]$, $[l_2, l_1^{-2\varphi}]$, $[l_3, l_1^{-2\varphi}]$, $[a_0, l_1^{-1}(^{\varphi}\ l$ $[a_1, l_1^{-1}(^{\varphi}\ l$ $[l_1, l_1^{-1}(^{\varphi}\ l$ $[l_2, l_1^{-1}(^{\varphi}\ l$ and $[l_3, (l_1^{-1}l_2)^{\varphi}]$.

However, some of these generators are identities and are shown as follows. Since $l_3 \in Z(S_2(3))$ and $l_1^{-2}, l_1^{-1}l_2 \in S_2(3)'$, then by Proposition 5(ii), $[l_3, l_1^{-2\varphi}] = 1$ and $[l_3, (l_1^{-1}l_2)^{\varphi}] = 1.$ Next, it is can be shown that,

$[l_1, l_1^{-2\varphi}] = [l_1, l_1^{-\varphi}]^{l_1^{-1}} [l_1, l_1^{-\varphi}]$ by (3)

$= [l_1, l_1^{-\varphi}]\,[l_1, l_1^{-\varphi}]$

$= [l_1^{-1}, l_1^{\varphi}]\,[l_1, l_1^{-\varphi}]$ by Proposition 6(i)

$= [a_1^{2}, l_1^{\varphi}][l_1, a_1^{2\varphi}]$ since $a_1^2 = l_1^{-1}$

$= [a_1, l_1^{2\varphi}][l_1^{2}, a_1^{\varphi}]$ by Proposition 6(i)

$= [a_1, l_1^{2\varphi}][a_1, l_1^{2\varphi}]^{-1}$ by Proposition 5(i)

$= 1$

$[l_2, l_1^{-2\varphi}] = [l_2, l_1^{-\varphi}]^{l_1^{-1}}[l_2, l_1^{-\varphi}]$ by (3)

$= [l_2, l_1^{-\varphi}]\,[l_2, l_1^{-\varphi}]$

$= {}^{a_0}[l_2, l_1^{-\varphi}][l_2, l_1^{-\varphi}]$ by Proposition 6(iii)

$= [l_2, l_1^{\varphi}][l_2, l_1^{-\varphi}]$ since ${}^{a_0}l_1^{-1} = l_1$

$= [l_2, l_1^{\varphi}][l_2, l_1^{\varphi}]^{-1}$ by Proposition 6(i)

$= 1$

Furthermore, some of the generators can be written as a product of other generators.

$[a_0, l_1^{-1\varphi}] = [l_1^{-1}, [a_0, l_1]^{-\varphi}] \cdot [a_0, l_1^{\varphi}]^{-1}$ by (5)

$= [l_1^{-1}, l_1^{2\varphi}][a_0, l_1^{\varphi}]^{-1}$ since $[a_0, l_1^{\varphi}] = l_1^{-2}$

$= [l_1, l_1^{-2\varphi}][a_0, l_1^{\varphi}]^{-1}$ by Proposition 6(i)

$= [a_0, l_1^{\varphi}]^{-1}$ since $[l_1, l_1^{-2\varphi}] = 1$

$[a_0, l_1^{-2\varphi}] = [a_0, l_1^{-\varphi}]^{l_1^{-1}}[a_0, l_1^{-\varphi}]$ by (3)

$= [a_0, l_1^{-\varphi}][a_0 l_1^{2}, l_1^{-\varphi}]$ since ${}^{l_1}a_0 = a_0 l_1^{-2}$

$= [a_0, l_1^{-\varphi}]^{a_0}[l_1^{2}, l_1^{-\varphi}][a_0, l_1^{-\varphi}]$ by (2)

$= [a_0, l_1^{-\varphi}]^{2}[l_1^{-2}, l_1^{\varphi}]$

$= [a_0, l_1^{-\varphi}]^{2}[l_1, l_1^{-2\varphi}]$ by Proposition 6(i)

$= [a_0, l_1^{-\varphi}]^{2}$ since $[l_1, l_1^{-2\varphi}] = 1$

$= [[a_0, l_1^{\varphi}]^{-1}]^{2}$ since $[a_0, l_1^{-1\varphi}] = [a_0, l_1^{\varphi}]^{-1}$

$= [a_0, l_1^{\varphi}]^{-2}$

By using similar arguments, $[a_1, l_1^{-2\varphi}] = [a_1, a_1^{\varphi}]^4$, $[a_0, (l_1^{-1}l_2)^{\varphi}] = [a_0, l_1^{\varphi}]^{-1}[a_0, l_2^{\varphi}]$, $[l_2, (l_1^{-1}l_2)^{\varphi}] = [l_1, (l_1^{-1}l_2)^{\varphi}]$, and $[a_1, (l_1^{-1}l_2)^{\varphi}] = [l_2^{2}, (l_1^{-1}l_2)^{\varphi}][a_1, (l_1^{-1}l_2)^{\varphi}]$. However, $[l_1, (l_1^{-1}l_2)^{\varphi}] = 1$.

Therefore, $[S_2(3), S_2(3)^{\varphi}] = \left\langle [a_0, l_1^{\varphi}], [a_0, l_2^{\varphi}], [a_1, a_1^{\varphi}] \right\rangle$. However, $[a_1, a_1^{\varphi}]$ is the element of $\nabla(S_2(3))$. Thus,
$E(S_2(3)) = \left\langle [a_0, l_1^{\varphi}], [a_1, l_1^{\varphi}], [a_0, l_2^{\varphi}] \right\rangle$.

By Proposition 2(ii),

$$\begin{aligned}
[S_2(3), S_2(3)^{\varphi}] &= \nabla(S_2(3))E(S_2(3)) \\
&= \left\langle [a_0, a_0^{\varphi}], [a_1, a_1^{\varphi}], [a_0, a_1^{\varphi}][a_1, a_0^{\varphi}], \right. \\
&\qquad \left. [a_0, a_1^{\varphi}], [a_0, l_1^{\varphi}], [a_0, l_2^{\varphi}] \right\rangle .
\end{aligned}$$

Next, the order of all the nontrivial generators of $[S_2(3), S_2(3)^{\varphi}]$ will be determined. By Proposition 9, $[a_0, a_0^{\varphi}]$ has infinite order, $[a_1, a_1^{\varphi}]$ has order 8 and $[a_0, a_1^{\varphi}][a_1, a_0^{\varphi}]$ has order 4. Besides,

$$\begin{aligned}
[a_0, l_2^{\varphi}] &= {}^{a_1}[a_0, l_2^{\varphi}] \\
&= [a_0 l_1^{-1} l_2, l_2^{-\varphi}] \\
&= {}^{a_0}[l_1^{-1} l_2, l_2^{-\varphi}][a_0, l_2^{-\varphi}] \\
&= [l_1 l_2, l_2^{-\varphi}][a_0, l_2^{\varphi}]^{-1} \\
&= {}^{a_0}[l_1 l_2, l_2^{-\varphi}][a_0, l_2^{\varphi}]^{-1} \\
&= [l_1^{-1} l_2, l_2^{-\varphi}][a_0, l_2^{\varphi}]^{-1} \\
&= [l_2^{-1}, (l_1^{-1} l_2)^{\varphi}]^{-1}[a_0, l_2^{\varphi}]^{-1} \\
&= [l_2, (l_1^{-1} l_2)^{\varphi}][a_0, l_2^{\varphi}]^{-1}
\end{aligned}$$

$$\begin{aligned}
[a_0, l_2^{\varphi}]^2 &= [l_2, (l_1^{-1} l_2)^{\varphi}] \\
&= [l_1, (l_1^{-1} l_2)^{\varphi}] \\
&= 1
\end{aligned}$$

It means that the order of $[a_0, l_2^{\varphi}]$ divides 2. So, the order of $[a_0, l_2^{\varphi}]$ is 2.

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By Theorem 2, $\kappa([a_0, a_1^{\varphi}]) = [a_0, a_1] = l_1^{-1}l_2$, and $\kappa([a_0, l_1^{\varphi}]) = [a_0, l_1] = l_1^{-2}$. Generators $l_1^{-1}l_2$ and l_1^{-2} are the elements of $S_2(3)'$ and have infinite order, hence $[a_0, a_1]$ and $[a_0, l_1]$ have infinite order.

Next, the six generators of $[S_2(3), S_2(3)^{\varphi}]$ will be shown to be independent. By Theorem 2, the generators of $[a_0, a_1^{\varphi}]$ and $[a_0, l_1^{\varphi}]$ are not in the kernel of κ. In the other words, $[a_0, a_1^{\varphi}]$ and $[a_0, l_1^{\varphi}]$ cannot be a product of others or it is a contradiction that it would be in the kernel of κ. By order restrictions, $[a_0, a_0^{\varphi}]$, $[a_1, a_1^{\varphi}]$ and $[a_0, a_1^{\varphi}][a_1, a_0^{\varphi}]$ are independent generators of $[S_2(3), S_2(3)^{\varphi}]$.

By Proposition 2, $[S_2(3), S_2(3)^{\varphi}] = \nabla(S_2(3))E(S_2(3))$. Since $\nabla(S_2(3))$ is normal then all the generator commute to each other. Hence, $\nabla(S_2(3))$ is abelian. In order to show $E(S_2(3))$ is abelian, we need to show that all elements commute in $E(S_2(3))$.

$$\begin{aligned}[][[a_0, a_1^{\varphi}], [a_0, l_1^{\varphi}]] &= [[a_0, a_1], [a_0, l_1]^{\varphi}] \\ &= [(l_1^{-1}l_2), l_1^{-2\varphi}] \\ &= [l_1^{-2}, (l_1^{-1}l_2)^{\varphi}]^{-1} \\ &= [l_1, (l_1^{-1}l_2)^{\varphi}]^{2} \\ &= 1\end{aligned}$$

$$\begin{aligned}[][[a_0, a_1^{\varphi}], [a_0, l_2^{\varphi}]] &= [[a_0, a_1], [a_0, l_2]^{\varphi}] \\ &= [(l_1^{-1}l_2), (1)^{\varphi}] \\ &= 1\end{aligned}$$

$$\begin{aligned}[][[a_0, l_1^{\varphi}], [a_0, l_2^{\varphi}]] &= [[a_0, l_1], [a_0, l_2]^{\varphi}] \\ &= [(1), l_1^{-2\varphi}] \\ &= 1\end{aligned}$$

By similar arguments,

$[[a_0, l_1^{\varphi}], [a_0, a_1^{\varphi}]] = 1,$

$[[a_0, l_2^{\varphi}], [a_0, a_1^{\varphi}]] = 1,$

$[[a_0, l_2^{\varphi}], [a_0, l_1^{\varphi}]] = 1.$

Since $[[x_1, y_1^{\varphi}],[x_2, y_2^{\varphi}]]=1$ for all $[x_1, y_1^{\varphi}],[x_2, y_2^{\varphi}]$ in $E(S_2(3)),$ then we can conclude that $E(S_2(3))$ is abelian. Therefore, we can conclude that,

$$S_2(3)\otimes S_2(3) = \nabla(S_2(3))E(S_2(3)) \cong C_2 \times C_4 \times C_8 \times C_0^3.$$

is abelian. □

III. Conclusion

In this paper, the nonabelian tensor square of a Bieberbach group with elementary abelian 2-group point group, $S_2(3)\otimes S_2(3)$ is computed and is shown to be abelian. The findings of this research can be used for further research in computing the other homological functors of this group.

IV. Acknowledgment

The first author would like to thank Universiti Pendidikan Sultan Idris (UPSI) for the funding of this research under Fundamental Research Grant Scheme (FRGS) under vote no. 2016-0084-102-02. The second author would like to express her appreciation to the Ministry of Education Malaysia (MOE) for her MyMaster Scholarship.

List of Symbols

G'	Derived Subgroup of G
G^{ab}	Abelianization of G
G/H	Quotient group of G by H
$G\otimes G$	Nonabelian Tensor Square of G
$G\times H$	Direct product of G and H
$G\cong H$	G is isomorphic to H
$[g,h]$	Commutator of g and h
${}^{g}h$	Left conjugation of h by g
C_n	Cyclic group of order n
C_0	Cyclic group of infinite order
$\langle X \mid R\rangle$	Group presentation by generators X and relators R
$\nabla(G)$	Central Subgroup of the nonabelian tensor square of group G
$S_i(j)$	i^{th} Bieberbach group with elementary abelian 2-group point group with dimension j
F_n^{ab}	Free abelian group of rank n

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