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ENERGY OF CAYLEY GRAPHS FOR DIHEDRAL GROUPS FOR SPECIFIC SUBSET

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ABSTRACT

Let G be a finite group and S be a subset of G where S does not include the identity of G and is inverse closed. A Cayley graph of a group G with respect to the subset S is a graph where its vertices are the elements of G and two vertices a and b are connected if ab^{-1} is in the subset S. The energy of a Cayley graph is the sum of all absolute values of the eigenvalues of its adjacency matrix. In this paper, we consider a specific subset $S = \{b, ab, \ldots, a^{n-1}b\}$ for dihedral group of order $2n$, where $n \geq 3$ and find the Cayley graph with respect to the set. We also calculate the eigenvalues and compute the energy of the respected Cayley graphs. Finally, the generalization of the energy of the respected Cayley graphs is found.

Key words: Energy of graph, eigenvalues, adjacency matrix, Cayley graph, dihedral group

INTRODUCTION

There have been a grown of interests for many researchers to study on the energy of graph. According to Woods [1], the study on the energy of general simple graphs was first defined by Gutman in 1978 which are inspired from the Huckel Molecular Orbital (HMO) Theory proposed in 1930s by Huckel. The Huckel Molecular Orbital Theory has been used by chemists in approximating the energies related with π-electron orbitals in conjugated hydrocarbon molecules.

In 2009, Li et al. [2] in their book stated that in the early days, when computers were not widely available, the calculation of the HMO total π-electron energy was a serious problem. In order to overcome the difficulty, a variety of approaches have been offered to calculate the approximate calculation of the π-electron energy. Within the HMO approximation, the total energy of the π-electrons, denoted by ε is then obtained which is by summing individual electron energies. In conjugated hydrocarbons, the total number of π-electrons is equal to the number of vertices of the associated molecular graph. After some considerations, they have arrived at the definition of the energy ε which is the sum of the absolute values of the eigenvalues of the molecular graph.

In 2004, Bapat and Pati [3] have proved that the energy of a graph is never an odd integer in their research. Meanwhile, the properties that the energy of a graph is never the square root of an odd integer has been proven by Pirzada and Gutman [4] in 2008. There are also a few researchers who studied specifically the energy of unitary Cayley graphs (see [5], [6]).

The target of this study is to present the energy of the Cayley graphs associated to dihedral groups for subset $S = \{b, ab, \dots, a^{n-1}b\}$. The procedure consists of finding the elements, vertices and edges for the Cayley graphs of the dihedral groups, finding their isomorphisms, building the adjacency matrix for the Cayley graph, finding the spectrum of the adjacency matrix of the graphs and lastly calculating the energy of the graphs. Some properties and general formula for the energy will also be presented at the end of the study.

PRELIMINARIES

The followings are some definitions that are used in this work.

Definition 1 [7] Dihedral Group
If π_n is a regular polygon with n vertices, $v_1, v_2, \dots, v_n$ and center O, then the symmetry group $\Sigma(\pi n)$ is called the dihedral group with $2n$ elements, and it is denoted by D_{2n}.

Definition 2 [8] Cayley Graph of a Group
Let G be a finite group with identity 1. Let S be a subset of G satisfying $1 \notin S$ and $S = S^{-1}$; that is, $s \in S$ if and only if $s^{-1} \in S$. The Cayley graph $Cay(G; S)$ on G with *connection* set S is defined as follows:

- the vertices are the elements of G
- there is an edge joining g and h if and only if $h = sg$ for some $s \in S$.

The set of all Cayley graphs on G is denoted by $Cay(G)$.

Definition 3 [9] Adjacency Matrix
Let G be a graph with $V(G) = \{1, \dots, n\}$ and $E(G) = \{e_1, \dots, e_m\}$. The adjacency matrix of G denoted by $A(G)$ is the $n \times n$ matrix defined as follows. The rows and the columns of $A(G)$ are indexed by $V(G)$. If $i \neq j$ then the (i, j) −entry of $A(G)$ is 0 for vertices i and j nonadjacent, and the (i, j) −entry is 1 for i and j adjacent. The (i, i) −entry of $A(G)$ is 0 for $i = 1, \dots, n$ We often denote $A(G)$ simply by A.

Definition 4 [9] Energy of Graph
For any graph Γ, the energy of the graph is defined as $\varepsilon(\Gamma) = \sum_{i=1}^{n} |\lambda_i|$ where $\lambda_1, \cdots, \lambda_n$ are the eigenvalues of the adjacency matrix of Γ.

MAIN RESULTS

In this section, our main results are presented. Lemma 1 is used in proving Theorem 1.

Lemma 1
Let D_{2n} be dihedral group of order $2n$ where $n \geq 3$ and $S = \{b, ab, \dots, a^{n-1}b\}$ is a subset of D_{2n}. The Cayley graphs of D_{2n} with respect to the set S, $Cay(D_{2n}, S)$ are $K_{n,n}$. The eigenvalues of $Cay(D_{2n}, S)$ are 0 with multiplicity to $n - 2$ and $\pm n$ with multiplicity 1.

Theorem 1
Let D_{2n} be dihedral group of order $2n$ where $n \geq 3$ and $S = \{b, ab, \dots, a^{n-1}b\}$ is a subset of

D_{2n}. The energy of the Cayley graph of D_{2n} with respect to the set S, $\mathrm{E}\big(Cay(D_{2n}, S)\big)$ are $2n$.

CONCLUSION

For conclusion, it has been found that the energy of Cayley graph of the dihedral group of order $2n$ where $n \geq 3$ related to the subset $S = \{b, ab, \dots, a^{n-1}b\}$ is $2n$.

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