

PAPER • OPEN ACCESS

Graph splicing rules with cycle graph and its complement on complete graphs

To cite this article: M N S Abdul Razak *et al* 2021 *J. Phys.: Conf. Ser.* **1988** 012067

View the [article online](#) for updates and enhancements.

A promotional banner for the 240th ECS Meeting. The banner features a colorful striped border at the top. On the left, the ECS logo is displayed in a green circle. To its right, the text reads "240th ECS Meeting" in large blue font, followed by "Oct 10-14, 2021, Orlando, Florida" in a smaller black font. Below this, it says "Register early and save up to 20% on registration costs" in bold black text, and "Early registration deadline Sep 13" in a smaller black font. At the bottom left, there is a red "REGISTER NOW" button. On the right side of the banner, there is a photograph of a diverse group of people in a professional setting, with a man in a white shirt and tie clapping and smiling in the foreground.

ECS **240th ECS Meeting**
Oct 10-14, 2021, Orlando, Florida
**Register early and save
up to 20% on registration costs**
Early registration deadline Sep 13
REGISTER NOW

Graph splicing rules with cycle graph and its complement on complete graphs

M N S Abdul Razak¹, W H Fong², and N H Sarmin³

^{1,2,3} Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

E-mail: ¹mnsyiham2@graduate.utm.my

Abstract. Graph splicing system is a notion originally used to illustrate the one-dimensional string of DNA splicing in the form of a graph. A graph splicing system is associated with a graph splicing scheme where graph splicing rules are defined. A graph splicing rule restricts the possible cuts to occur on the edges of the initial graph(s) in a graph splicing system. The subgraphs of the initial graph are used in the splicing rules to determine the edges that will be cut from the initial graph. The concept of graph splicing system can be applied on various types of graph, hence generates components of spliced graphs depending on the types of the graph splicing rules used. There is a graph splicing rule called as a cutting rule which can be applied on both linear graphs and circular graphs where the graphs are transformed into Pseudo-Linear Form. However, this cutting rule has limited various possible cuts that can occur on the complete graph. Therefore, in this research, the original concept of graph splicing system is applied on complete graphs as the initial graphs. Also, graph splicing rules involving subgraphs of the initial graphs which are also complete graphs, are considered and applied in the graph splicing system. Furthermore, the generated spliced graphs are obtained through the graph splicing system.

1. Introduction

Deoxyribonucleic acid, known as DNA, is an important molecule in a living organism. In biology, DNA is structured and described as a three-dimensional form of molecules [1]. The studies on DNA has been a great interest among researchers due to the important roles of DNA as the genetic information carrier, hence Head [2] introduced the theory of DNA splicing in 1987. In DNA splicing, the process is modeled in one-dimensional string; meanwhile the DNA structure is explained in three-dimensional form. Consequently, the intricacies process involving three-dimensional DNA structure seems to be inadequate to be described in one-dimensional string and has led to another notion called a graph splicing system [3]. In [3], Freund described the DNA splicing in the form of graphs where the DNA molecules are represented by the vertices of the graphs. A graph splicing scheme and a graph splicing rule are defined in a graph splicing system to restrict the cut at various edges of graphs and to reconnect them in another way.

There are various types of graph splicing systems such as regular graph splicing system, accepting graph splicing system [4] and n -cut splicing system [5]. Jeyabharathi et al. [5] introduced n -cut splicing to illustrate the cleavage patterns of the restriction enzymes in DNA splicing systems by representing the DNA molecules in the form of semigraphs where idea of semigraph is introduced by Sampathkumar [6]. Semigraph is said to be a direct generalization



of graphs in the form of various types of vertices and edges instead of a hypergraph which is in the form of sets. Some researchers have shown interest on the properties of semigraph such as Gaidhani et al. [7, 8] where they studied on the adjacency matrix of semigraph and energy of semigraph. Also, by the idea of semigraph and n -cut splicing, Jeyabharathi et al. [9] further their study on the relation of languages generated by the n -cut splicing and the norms of Parikh matrices. Besides that, Aisah et al. [10] also studied on the Parikh matrices for 2-cut and 4-cut splicing. Then, Thiagarajan et al. [11, 12] have studied the concept of folding technique on the n -cut spliced semigraph.

Besides that, Jeganathan and Rama [13] applied the concept of graph splicing systems by transforming the graphs in Pseudo-Linear Form (PLF). A new definition of graph splicing rule called as a cutting rule which is applicable to splice any types of graphs is introduced in [13]. The graphs are firstly transformed into PLF where the vertices are numbered in order and arranged in a line as per order i.e 1, 2, 3; which can be depicted as a path graph with edges going above or below the linear path. Since the cutting rule cut the graph vertically into two parts by only choosing an edge between two of the ordered vertices, hence this transformation is necessary to make sure that there is no more than one edge between the ordered vertices is cut. As an example, the authors transformed a complete graph into a PLF and applied a graph splicing system by using the cutting rule as defined.

However, the definition of cutting rule as in [13] and its application on a complete graph in PLF have limited the possible cut that can occur on a complete graph. Therefore in this paper, the definition of graph splicing rule and graph splicing scheme as in [3] are used and applied on complete graphs with n vertices.

This paper consists of four sections which are the introduction followed by the preliminaries where the basic ideas of graph splicing systems are discussed. Then, the main results on the splicing on complete graphs by using cycle graph and its complement in the graph splicing rules are discussed. The generalization of the splicing is then obtained and discussed. Finally, the results are summarized in the last section.

The next section discussed on some definitions and fundamental ideas of graph splicing system.

2. Preliminaries

A graph splicing system emphasizes on the behavior of the cuts at various edges on graphs and connects them in another way. The process of cutting and reconnecting of the graph is described by a splicing scheme, where at least a graph splicing rule is defined to restrict all the edges that will be cut. The definition of a graph splicing scheme is given in Definition 1.

Definition 1. [3] (Graph Splicing Scheme)

A graph splicing scheme is a pair $\sigma = (A, P)$ where A is a set of finite alphabets and P is a set of finite splicing rules.

A finite set P with k number of graph splicing rules can be written in the form $((h[1], E_c[1]), (h[2], E_c[2]), \dots, (h[k], E_c[k]); R)$ such that $k \geq 1$ where $k \in \mathbb{N}$ and for all i with $1 \leq i \leq k$, where

- $h[i] = (V'[i], E'[i])$ is a weakly connected graph, where $E'[i]$ is the edge of the i th graph splicing rule; simultaneously, $h[i]$ is any subgraph of the initial graph(s) used in the graph splicing system,
- $E_c[i] \subseteq E'[i]$ where $E_c[i]$ is the cutting pattern for the i th graph splicing rule,
- the vertices $V'[i]$ are mutually disjoint,

- R obeys the following rules:
 - (i) each edge $(n, m) \in E_c[i]$ is supposed to be divided into two parts; i.e the start part $(n, m]$ and the end part $[n, m)$,
 - (ii) the elements of R are of the form $((n, m], [n', m'))$, where (n, m) and (n', m') are edges from $\bigcup_{i \leq i \leq k} E_c[i]$,
 - (iii) every element from $\{(n, m], [n, m) \in \bigcup_{i \leq i \leq k} E_c[i]\}$ must appear exactly once in a pair of R .

Note that the set of alphabet A is empty throughout this paper unless otherwise stated. Next, the definition of a graph splicing system is given as follows.

Definition 2. [3] (Graph Splicing System)

A graph splicing system is a set $S = (\sigma, I)$ where σ is a graph splicing scheme and I is a set of the initial strings or graphs.

This research applies the concept of the above graph splicing system on complete graphs. Hence, the definition of a complete graph is given as Definition 3.

Definition 3. [14] (Complete Graph)

A simple graph is called a complete graph if and only if each pair of distinct vertices are adjacent. The complete graph with n vertices is denoted as K_n .

By the definition of a graph splicing scheme and a graph splicing rule, the edges that will be cut is restricted on any subgraph of the initial graph used in the graph splicing system, whereby the definition of a subgraph is stated in Definition 4.

Definition 4. [14] (Subgraph)

A graph $h = (V', E')$ is a subgraph of a graph $G = (V, E)$ if and only if $V' \subseteq V$ and $E' \subseteq E$. The symbols V' and E' denote the set of vertices and the set of edges in a subgraph h of G , respectively.

Since the complete graph is used as the initial graph and one of its subgraphs is the cycle graph, then the definition of a cycle graph and its complement is given in the following definition.

Definition 5. [14] (Cycle Graph)

The graph C_n ($n \geq 3$) is a cycle of length n on vertices $v_1, v_2, v_3, \dots, v_n$ with n edges $(v_1, v_2), (v_2, v_3), \dots, (v_n, v_1)$. The complement of C_n is denoted as $\overline{C_n}$.

The following shows a graph splicing rule denoted as P that can be written as

$$P = ((h, E_c); R),$$

where h is a subgraph of any initial graphs, E_c is the set of edges that will be cut and R is the recombination rule of the edges after splicing [3].

Next, example 2.1 is given to describe an example of a graph splicing system.

Example 2.1:

Let $h = (V', E')$ be a subgraph of a graph G such that $V' = \{1, 2, 3, 4, 5\}$ and $E' = \{(1, 2), (2, 3), (3, 4), (4, 5), (2, 5)\}$. Then, let the set of edges that will be cut in the graph splicing system be $E_c = \{(2, 3)\}$ and the recombination rule $R = \{(1', 1'), (2', 2')\}$. Hence, a graph splicing rule P can be written as

$$P = (((\{1, 2, 3, 4, 5\}, \{(1, 2), (2, 3), (3, 4), (4, 5), (2, 5)\}), \{(2, 3)\}); \{(1', 1'), (2', 2')\}),$$

which can also be depicted in the form of a graph as given in figure 1.

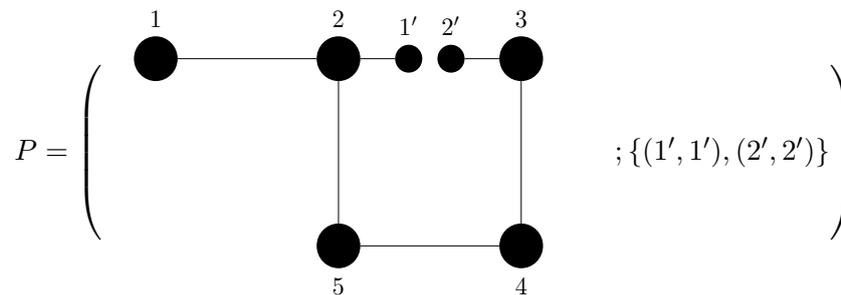


Figure 1. The graph splicing rule P in the form of a graph.

Consider an initial graph G represented in figure 2.

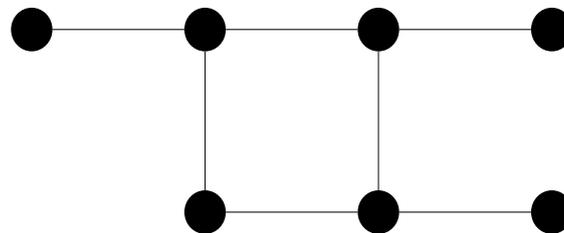


Figure 2. A graph G .

If the graph splicing rule P is applied on the graph G , then one of the possibilities to splice G by using P can be shown in the following figure.

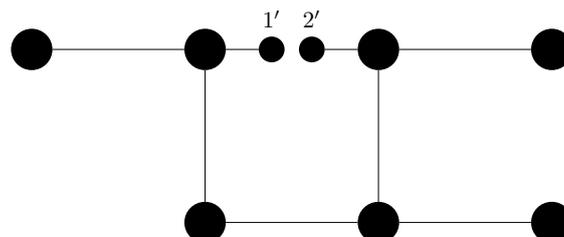


Figure 3. The graph G after it has been spliced by the graph splicing rule P .

Therefore, by following the recombination rule R as stated in the graph splicing rule P , it can be shown that the graph in figure 3 can recombine with a copy of itself as shown in figure 4 and form a new graph as shown in figure 5.

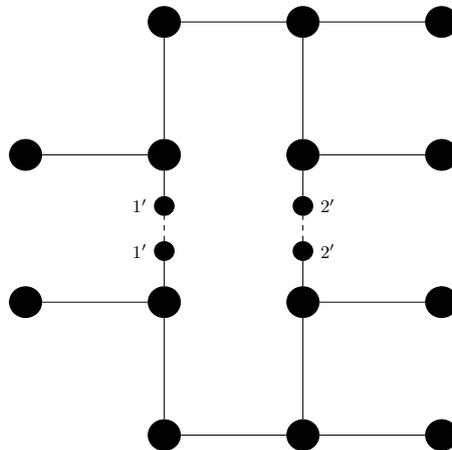


Figure 4. The recombination of two copies of the graph in figure 3.

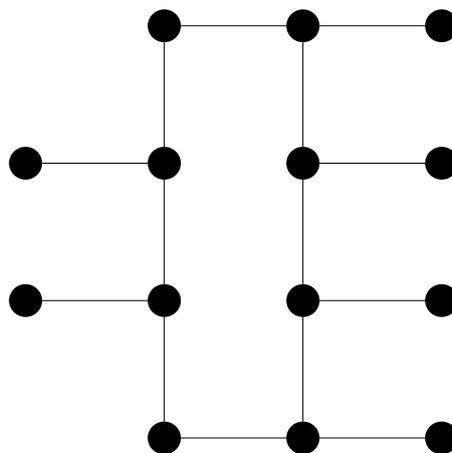


Figure 5. A new graph formed after the recombination.

The resulted graph can be recombined in another form based on the edge chosen to be cut. In the next section, two graph splicing rules with cycle graph and its complement are considered and applied on complete graphs.

3. Main results

In this section, graph splicing systems are applied on complete graphs. Two examples of graph splicing systems on complete graphs are shown to illustrate the splicing processes where two different graph splicing rules with cycle graph and its complement are applied on complete graph K_5 . Also, this section discusses on the generalization of splicing complete graphs with $n \geq 5$ number of vertices by considering cycle graph and its complement in the graph splicing rules.

3.1. Some examples of splicing on complete graph by using graph splicing rule with cycle graph and its complement

In this subsection, two graph splicing systems on complete graphs by using graph splicing rules with cycle graph C_5 and its complement $\overline{C_5}$ are discussed. Firstly, the graph splicing system is applied on complete graph K_5 as illustrated in example 3.1.

Example 3.1:

Let a complete graph K_5 be an initial graph and represented as follows:

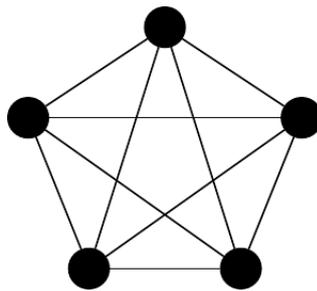


Figure 6. A complete graph K_5 .

Also, let a graph splicing rule P_1 with cycle graph C_5 be defined as follows:

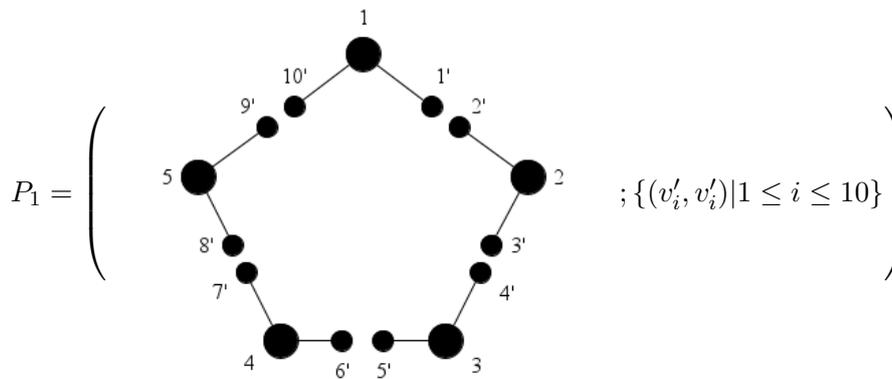


Figure 7. The graph splicing rule P_1 .

Then, the graph splicing rule P_1 is applied on the initial graph K_5 and generates a spliced graph as shown in figure 8.

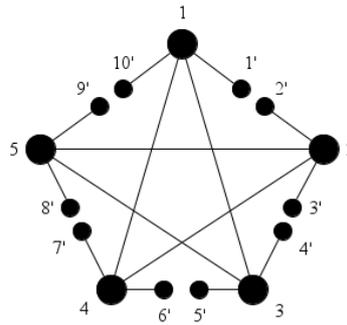


Figure 8. The generated spliced graphs after applying P_1 to K_5 .

From figure 8, it is shown that each vertex in the spliced graph is connected to two vertices and two overhangs. The symbols V_i and v_i' denote the vertices and overhangs of the spliced graph, respectively. Also, from the graph splicing rule P_1 , the recombination rule $E = \{(v_i', v_i') | 1 \leq i \leq 10\}$ where it is shown that if the overhang of a spliced graph is numbered as $1'$, hence it must also recombine with $1'$ overhang (in which from another copies of spliced graph). The recombination for two overhangs from two distinct copies of spliced graphs in figure 8 can be illustrated in figure 9 and the resulted graph is obtained and represented in figure 10 where the edges generated after the recombination are shown in grey colour.

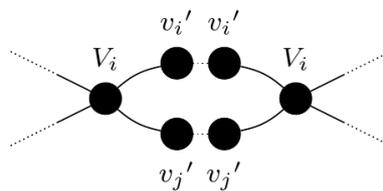


Figure 9. The recombination of two overhangs from two distinct copies of spliced graphs.

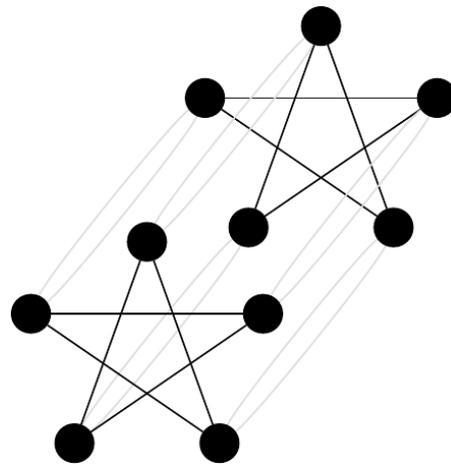


Figure 10. The resulted graphs after the recombination of two spliced graphs in 8.

From the above example, a graph splicing system is applied on complete graph K_5 by using a graph splicing rule with C_5 which generates a spliced graph in the form of a star pentagon and the resulted graph after the recombination is in the form of two connected star pentagons.

The next example shows a splicing on complete graph K_5 by using a graph splicing system with the complement of cycle graph C_5 , which is $\overline{C_5}$.

Example 3.2:

Recall the complete graph K_5 as in figure 6 and let it be the initial graph. Then, a graph splicing rule P_2 with $\overline{C_5}$ can be defined as follows.

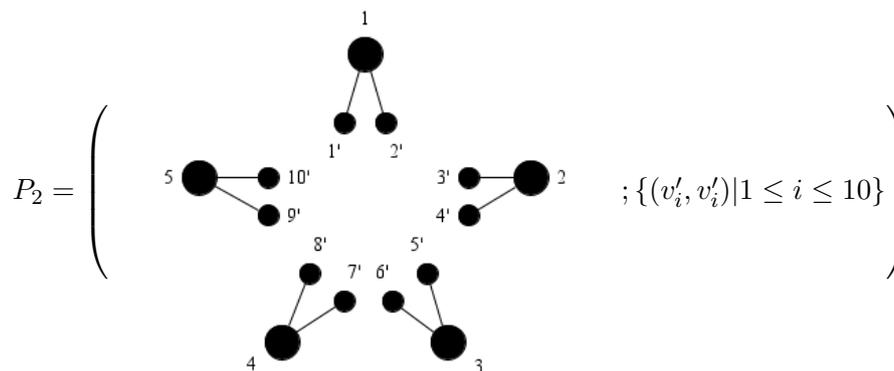


Figure 11. The graph splicing rule P_2

By splicing the initial graph K_5 by the graph splicing rule P_2 , then the generated spliced graph is represented in figure 12.

By observing figure 12, each vertex in the spliced graph is connected to two vertices and two overhangs. Again, as in figure 9, the overhang in the spliced graph will recombine with another overhang with the same number which is from another copy of the spliced graph. Hence,

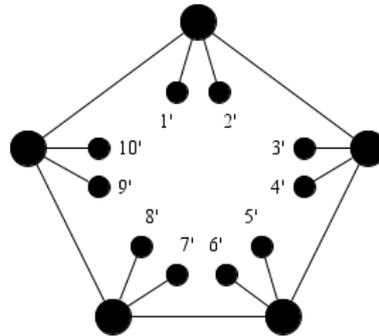


Figure 12. The spliced graph generated by applying P_2 on K_5 .

the resulted graph obtained after the recombination is represented as follows where the edges generated after recombination are shown in grey colour.

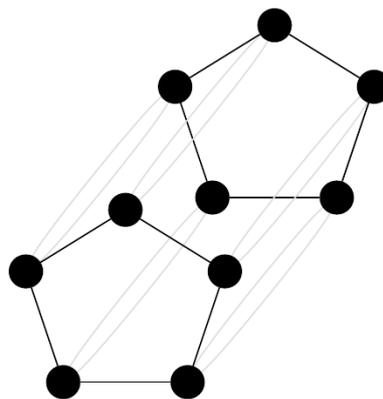


Figure 13. The resulted graph after the recombination of two spliced graphs in figure 12.

The above example shows a graph splicing system on a complete graph K_5 by using a graph splicing rule with $\overline{C_5}$. The graph splicing system generates a spliced graph which is in the form of a pentagon and after recombination, a resulted graph which consists of two connected pentagons are formed.

3.2. The generalization of splicing on complete graph by using graph splicing rule with cycle graph and its complement

By the graph splicing systems as shown in examples 3.1 and 3.2, it can be shown that splicing on complete graph by using graph splicing rules with cycle graph and its complement generates two spliced graphs which are in the form of a star pentagon and a pentagon, respectively. In general, the graph splicing systems in the above examples can be explained by considering complete graph K_n . To make sure that the resulted graphs after the recombination are either in the form of two connected star polygons or two connected polygons, the cycle graph and its complement that are considered are for $n \geq 5$. This is because by applying a graph splicing rule with C_3 on a complete graph K_3 will only result in three components of two connected vertices; meanwhile by applying a graph splicing rule with C_4 on a complete graph K_4 will only result in two intersecting planes.

The following two graph splicing rules P and P' with cycle graph and its complement can be written as

$$P = ((C_n, E'_1); \{(v'_i, v'_i) | 1 \leq i \leq 2n\})$$

and

$$P' = ((\overline{C_n}, E'_2); \{(v'_i, v'_i) | 1 \leq i \leq n(n-3)\}),$$

respectively, where E'_1 is the set of edges that will be cut by P and the edges must be in the set of edges of C_n ; on the other hand E'_2 is the set of edges that will be cut by P' in which the edges must be in the set of edges of $\overline{C_n}$.

It is known that each vertex in complete graph is connected to all vertices except itself i.e, each vertex is connected to $n - 1$ vertices. Therefore, by applying the graph splicing rule P on a complete graph K_n will generate a spliced graph where each vertex in the spliced graph is connected to $n - 3$ vertices and two overhangs; meanwhile, applying the graph splicing rule P' on a complete graph K_n will generate a spliced graph where each vertex in the spliced graph is connected to two vertices and $n - 3$ overhangs. Also, the spliced graph generated by applying P is in the form of a star polygon and the spliced graph generated by applying P' is in the form of a polygon.

The recombination of each overhang by following the restriction in the graph splicing rules P and P' can be described as in figure 9. Since the recombination rules restricted that if a overhang is numbered as v'_i , hence it will recombine with another overhang which is also numbered as v'_i . To achieve that, the overhang of the spliced graph needs to recombine with another overhang from another copy of the spliced graph which is numbered the same. The recombination of all overhangs resulted in new graphs which are in the form of two connected star polygons and two connected polygons by applying the graph splicing rules P and P' on complete graph K_n , respectively.

4. Conclusion

Complete graphs have different applications in real life problems in which one of them is to represent structures of molecules. Hence in this research, graph splicing on complete graphs by using graph splicing rules with a cycle graph and its complement has been discussed to study the behavior of the cuts at various edges and their recombination. Instead of using the modified graph splicing rule called as a cutting rule as defined in [13], this research applies the original concept of graph splicing as introduced in [3]. This is to ensure that there is no limitation on the possible edges that will be cut on the complete graphs. Also, two examples of splicings on complete graph with five vertices by using graph splicing rules with a cycle graph and its complement have been illustrated. Then, the generalization of the graph splicing rules have been shown. By applying a graph splicing rule with a cycle graph and its complement on a complete graph using the original graph splicing rule as introduced in [3], two spliced graphs in the form of a star polygon and a polygon, respectively, have been generated. Furthermore, two new graphs which are two connected star polygons and two connected polygons are obtained by the recombination of the generated spliced graphs. Moreover, the above results are different from the results in [13] where the graphs are always cut into two components due to the defined cutting rule. In this research it is shown that the edges can also be cut in a way in which the spliced graph does not result in two components. Also, the recombination of the spliced graphs results in new graphs instead of just another complete graph. Furthermore, the graph splicing rule used in the splicing system can be altered to achieve the desired results since there can be various subgraphs for every graph.

5. Acknowledgments

The authors would like to acknowledge Universiti Teknologi Malaysia (UTM) and Ministry of Higher Education Malaysia (MoHE) for the financial support through **Fundamental Research Grant Scheme (FRGS/1/2018/STG06/UTM/02/10)** and **(FRGS/1/2020/STG06/UTM/01/2)**. The authors would also like to thank UTM for the funding of this research through **UTM Fundamental Research Grant (UTMFR) Vote Number 20H70**.

REFERENCES

- [1] Jianwei M, Keith O H and David 2001 *An approach to three-dimensional structures of biomolecules by using single-molecule diffraction images* (USA: The National Academy of Sciences) pp 6641-6645
- [2] Head T 1987 *Bull. Math. Biol.* **49** 737-759
- [3] Freund R 1995 *1st Int. Symp. on Intelligence in Neural and Biological Systems (INBS'95)* (Washington, DC: IEEE) pp 189-194
- [4] Sankar M P, David N G and Thomas D G 2014 *Bio-Inspired Computing - Theories and Applications* ed Pan L, Păun *et al.* (Berlin, Heidelberg: Springer Berlin Heidelberg) pp 364-369
- [5] Jeyabharathi S, Padmashree J, Selvi S S and Thiagarajan K 2011 *6th Int. Conf. on Bio-Inspired Computing: Theories and Applications (BIC-TA)* (Washington, DC: IEEE Computer Society) pp 182-187
- [6] Sampathkumar E, Deshpande C M, Bam B Y, Pushpalatha L and Swaminathan V 2019 *Semigraphs and Their Applications* (India: Academy of Discrete Mathematics and Applications)
- [7] Gaidhani Y S, Deshpande C M and Athawale B P 2017 *Electron. Notes Discret. Math.* **63** 399-406
- [8] Gaidhani Y S, Deshpande C M and Pirzada S 2020 *AKCE Int. J. Graphs and Comb.* **16** 41-49
- [9] Jeyabharathi S 2016 *Int. J. Adv. Eng. Tech.* **VII** 81-84
- [10] Aisah I, Jayanti P R E and Supriatna A K 2019 *IOP Conf. Series: Materials Science and Engineering* (IOP Publishing) pp 1-8
- [11] Thiagarajan K, Padmashree J and Natarajan P 2016 *Indian J. Sci. Technol.* **9** 1-9
- [12] Thiagarajan K, Padmashree J and Jeyabharathi S 2013 *3rd Int. Conf. on Applied Mathematics and Pharmaceutical Sciences (ICAMPS'2013)* (Singapore) pp 29-30
- [13] Jeganathan L, Krithivasan K and Rama R 2008 Graph Splicing System 2013 *International Conference on Theoretical and Mathematical Foundations of Computer Science (TMFCS)* pp 72-79
- [14] Wilson R J 1996 *Introduction to Graph Theory* 4th ed (England: Addison Wesley Longman Limited)